

# Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/7.3.2-d-x^m-a+b-arctanh-c-x^n^p

Nasser M. Abbasi

May 24, 2020

Compiled on May 24, 2020 at 6:21pm

## Contents

<b>1</b>	<b>Introduction</b>	<b>11</b>
1.1	Listing of CAS systems tested . . . . .	11
1.2	Results . . . . .	12
1.3	Performance . . . . .	15
1.4	list of integrals that has no closed form antiderivative . . . . .	16
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	16
1.6	list of integrals solved by CAS but failed verification . . . . .	16
1.7	Timing . . . . .	17
1.8	Verification . . . . .	17
1.9	Important notes about some of the results . . . . .	17
1.10	Design of the test system . . . . .	19
<b>2</b>	<b>detailed summary tables of results</b>	<b>21</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	21
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	24
2.3	Detailed conclusion table specific for Rubi results . . . . .	73
<b>3</b>	<b>Listing of integrals</b>	<b>83</b>
3.1	$\int x^5 (a + b \tanh^{-1}(cx)) dx$ . . . . .	83
3.2	$\int x^4 (a + b \tanh^{-1}(cx)) dx$ . . . . .	87
3.3	$\int x^3 (a + b \tanh^{-1}(cx)) dx$ . . . . .	91

3.4	$\int x^2 (a + b \tanh^{-1}(cx)) dx$	95
3.5	$\int x (a + b \tanh^{-1}(cx)) dx$	99
3.6	$\int (a + b \tanh^{-1}(cx)) dx$	103
3.7	$\int \frac{a+b \tanh^{-1}(cx)}{x} dx$	106
3.8	$\int \frac{a+b \tanh^{-1}(cx)}{x^2} dx$	109
3.9	$\int \frac{a+b \tanh^{-1}(cx)}{x^3} dx$	113
3.10	$\int \frac{a+b \tanh^{-1}(cx)}{x^4} dx$	117
3.11	$\int \frac{a+b \tanh^{-1}(cx)}{x^5} dx$	121
3.12	$\int \frac{a+b \tanh^{-1}(cx)}{x^6} dx$	125
3.13	$\int x^5 (a + b \tanh^{-1}(cx))^2 dx$	129
3.14	$\int x^4 (a + b \tanh^{-1}(cx))^2 dx$	134
3.15	$\int x^3 (a + b \tanh^{-1}(cx))^2 dx$	139
3.16	$\int x^2 (a + b \tanh^{-1}(cx))^2 dx$	144
3.17	$\int x (a + b \tanh^{-1}(cx))^2 dx$	149
3.18	$\int (a + b \tanh^{-1}(cx))^2 dx$	153
3.19	$\int \frac{(a+b \tanh^{-1}(cx))^2}{x} dx$	157
3.20	$\int \frac{(a+b \tanh^{-1}(cx))^2}{x^2} dx$	161
3.21	$\int \frac{(a+b \tanh^{-1}(cx))^2}{x^3} dx$	165
3.22	$\int \frac{(a+b \tanh^{-1}(cx))^2}{x^4} dx$	170
3.23	$\int \frac{(a+b \tanh^{-1}(cx))^2}{x^5} dx$	175
3.24	$\int x^5 (a + b \tanh^{-1}(cx))^3 dx$	180
3.25	$\int x^4 (a + b \tanh^{-1}(cx))^3 dx$	187
3.26	$\int x^3 (a + b \tanh^{-1}(cx))^3 dx$	194
3.27	$\int x^2 (a + b \tanh^{-1}(cx))^3 dx$	200
3.28	$\int x (a + b \tanh^{-1}(cx))^3 dx$	206
3.29	$\int (a + b \tanh^{-1}(cx))^3 dx$	211
3.30	$\int \frac{(a+b \tanh^{-1}(cx))^3}{x} dx$	215
3.31	$\int \frac{(a+b \tanh^{-1}(cx))^3}{x^2} dx$	220
3.32	$\int \frac{(a+b \tanh^{-1}(cx))^3}{x^3} dx$	226

3.33	$\int \frac{(a+b \tanh^{-1}(cx))^3}{x^4} dx$	231
3.34	$\int \frac{(a+b \tanh^{-1}(cx))^3}{x^5} dx$	237
3.35	$\int (dx)^{5/2} (a + b \tanh^{-1}(cx)) dx$	243
3.36	$\int (dx)^{3/2} (a + b \tanh^{-1}(cx)) dx$	248
3.37	$\int \sqrt{dx} (a + b \tanh^{-1}(cx)) dx$	252
3.38	$\int \frac{a+b \tanh^{-1}(cx)}{\sqrt{dx}} dx$	257
3.39	$\int \frac{a+b \tanh^{-1}(cx)}{(dx)^{3/2}} dx$	261
3.40	$\int \frac{a+b \tanh^{-1}(cx)}{(dx)^{5/2}} dx$	265
3.41	$\int \frac{a+b \tanh^{-1}(cx)}{(dx)^{7/2}} dx$	270
3.42	$\int \frac{a+b \tanh^{-1}(cx)}{(dx)^{9/2}} dx$	275
3.43	$\int (dx)^m (a + b \tanh^{-1}(cx))^3 dx$	280
3.44	$\int (dx)^m (a + b \tanh^{-1}(cx))^2 dx$	283
3.45	$\int (dx)^m (a + b \tanh^{-1}(cx)) dx$	286
3.46	$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx$	289
3.47	$\int \frac{(dx)^m}{(a+b \tanh^{-1}(cx))^2} dx$	292
3.48	$\int (a + b \tanh^{-1}(cx))^p dx$	295
3.49	$\int (dx)^m (a + b \tanh^{-1}(cx))^p dx$	298
3.50	$\int x^7 (a + b \tanh^{-1}(cx^2)) dx$	301
3.51	$\int x^5 (a + b \tanh^{-1}(cx^2)) dx$	305
3.52	$\int x^3 (a + b \tanh^{-1}(cx^2)) dx$	309
3.53	$\int x (a + b \tanh^{-1}(cx^2)) dx$	313
3.54	$\int \frac{a+b \tanh^{-1}(cx^2)}{x} dx$	316
3.55	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^3} dx$	319
3.56	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^5} dx$	323
3.57	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^7} dx$	327
3.58	$\int x^4 (a + b \tanh^{-1}(cx^2)) dx$	331
3.59	$\int x^2 (a + b \tanh^{-1}(cx^2)) dx$	335
3.60	$\int (a + b \tanh^{-1}(cx^2)) dx$	339
3.61	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^2} dx$	343
3.62	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^4} dx$	347

3.63	$\int \frac{a+b \tanh^{-1}(cx^2)}{x^6} dx$	352
3.64	$\int x^7 (a + b \tanh^{-1}(cx^2))^2 dx$	357
3.65	$\int x^5 (a + b \tanh^{-1}(cx^2))^2 dx$	364
3.66	$\int x^3 (a + b \tanh^{-1}(cx^2))^2 dx$	371
3.67	$\int x (a + b \tanh^{-1}(cx^2))^2 dx$	378
3.68	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x} dx$	384
3.69	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^3} dx$	389
3.70	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^5} dx$	395
3.71	$\int x^4 (a + b \tanh^{-1}(cx^2))^2 dx$	403
3.72	$\int x^2 (a + b \tanh^{-1}(cx^2))^2 dx$	412
3.73	$\int (a + b \tanh^{-1}(cx^2))^2 dx$	421
3.74	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^2} dx$	429
3.75	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^4} dx$	438
3.76	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^6} dx$	447
3.77	$\int x^3 (a + b \tanh^{-1}(cx^2))^3 dx$	456
3.78	$\int x (a + b \tanh^{-1}(cx^2))^3 dx$	466
3.79	$\int \frac{(a+b \tanh^{-1}(cx^2))^3}{x} dx$	474
3.80	$\int \frac{(a+b \tanh^{-1}(cx^2))^3}{x^3} dx$	479
3.81	$\int \frac{(a+b \tanh^{-1}(cx^2))^3}{x^5} dx$	483
3.82	$\int (dx)^{5/2} (a + b \tanh^{-1}(cx^2)) dx$	487
3.83	$\int (dx)^{3/2} (a + b \tanh^{-1}(cx^2)) dx$	493
3.84	$\int \sqrt{dx} (a + b \tanh^{-1}(cx^2)) dx$	499
3.85	$\int \frac{a+b \tanh^{-1}(cx^2)}{\sqrt{dx}} dx$	505
3.86	$\int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{3/2}} dx$	512
3.87	$\int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{5/2}} dx$	519
3.88	$\int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{7/2}} dx$	526
3.89	$\int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{9/2}} dx$	533
3.90	$\int \sqrt{dx} (a + b \tanh^{-1}(cx^2))^2 dx$	540

3.91	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{\sqrt{dx}} dx$	546
3.92	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{(dx)^{3/2}} dx$	552
3.93	$\int \frac{(a+b \tanh^{-1}(cx^2))^2}{(dx)^{5/2}} dx$	558
3.94	$\int (dx)^m (a+b \tanh^{-1}(cx^2))^3 dx$	564
3.95	$\int (dx)^m (a+b \tanh^{-1}(cx^2))^2 dx$	567
3.96	$\int (dx)^m (a+b \tanh^{-1}(cx^2)) dx$	570
3.97	$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$	574
3.98	$\int \frac{(dx)^m}{(a+b \tanh^{-1}(cx^2))^2} dx$	577
3.99	$\int x^{11} (a+b \tanh^{-1}(cx^3)) dx$	580
3.100	$\int x^8 (a+b \tanh^{-1}(cx^3)) dx$	584
3.101	$\int x^5 (a+b \tanh^{-1}(cx^3)) dx$	588
3.102	$\int x^2 (a+b \tanh^{-1}(cx^3)) dx$	592
3.103	$\int \frac{a+b \tanh^{-1}(cx^3)}{x} dx$	595
3.104	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^4} dx$	598
3.105	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^7} dx$	602
3.106	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^{10}} dx$	606
3.107	$\int x^3 (a+b \tanh^{-1}(cx^3)) dx$	610
3.108	$\int (a+b \tanh^{-1}(cx^3)) dx$	616
3.109	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^3} dx$	621
3.110	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^6} dx$	626
3.111	$\int x^7 (a+b \tanh^{-1}(cx^3)) dx$	631
3.112	$\int x^4 (a+b \tanh^{-1}(cx^3)) dx$	636
3.113	$\int x (a+b \tanh^{-1}(cx^3)) dx$	641
3.114	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^2} dx$	646
3.115	$\int \frac{a+b \tanh^{-1}(cx^3)}{x^5} dx$	651
3.116	$\int x^{11} (a+b \tanh^{-1}(cx^3))^2 dx$	656
3.117	$\int x^8 (a+b \tanh^{-1}(cx^3))^2 dx$	663
3.118	$\int x^5 (a+b \tanh^{-1}(cx^3))^2 dx$	670
3.119	$\int x^2 (a+b \tanh^{-1}(cx^3))^2 dx$	677

3.120	$\int \frac{(a+b \tanh^{-1}(cx^3))^2}{x} dx$	683
3.121	$\int \frac{(a+b \tanh^{-1}(cx^3))^2}{x^4} dx$	688
3.122	$\int \frac{(a+b \tanh^{-1}(cx^3))^2}{x^7} dx$	694
3.123	$\int \frac{(a+b \tanh^{-1}(cx^3))^2}{x^{10}} dx$	702
3.124	$\int x^8 (a+b \tanh^{-1}(cx^3))^3 dx$	710
3.125	$\int x^5 (a+b \tanh^{-1}(cx^3))^3 dx$	721
3.126	$\int x^2 (a+b \tanh^{-1}(cx^3))^3 dx$	731
3.127	$\int \frac{(a+b \tanh^{-1}(cx^3))^3}{x} dx$	739
3.128	$\int \frac{(a+b \tanh^{-1}(cx^3))^3}{x^4} dx$	744
3.129	$\int \frac{(a+b \tanh^{-1}(cx^3))^3}{x^7} dx$	748
3.130	$\int (dx)^m (a+b \tanh^{-1}(cx^3))^3 dx$	752
3.131	$\int (dx)^m (a+b \tanh^{-1}(cx^3))^2 dx$	755
3.132	$\int (dx)^m (a+b \tanh^{-1}(cx^3)) dx$	758
3.133	$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^3)} dx$	762
3.134	$\int \frac{(dx)^m}{(a+b \tanh^{-1}(cx^3))^2} dx$	765
3.135	$\int x^3 (a+b \tanh^{-1}(\frac{c}{x})) dx$	768
3.136	$\int x^2 (a+b \tanh^{-1}(\frac{c}{x})) dx$	772
3.137	$\int x (a+b \tanh^{-1}(\frac{c}{x})) dx$	776
3.138	$\int (a+b \tanh^{-1}(\frac{c}{x})) dx$	780
3.139	$\int \frac{a+b \tanh^{-1}(\frac{c}{x})}{x} dx$	783
3.140	$\int \frac{a+b \tanh^{-1}(\frac{c}{x})}{x^2} dx$	786
3.141	$\int \frac{a+b \tanh^{-1}(\frac{c}{x})}{x^3} dx$	790
3.142	$\int \frac{a+b \tanh^{-1}(\frac{c}{x})}{x^4} dx$	794
3.143	$\int x^3 (a+b \tanh^{-1}(\frac{c}{x}))^2 dx$	798
3.144	$\int x^2 (a+b \tanh^{-1}(\frac{c}{x}))^2 dx$	808
3.145	$\int x (a+b \tanh^{-1}(\frac{c}{x}))^2 dx$	817
3.146	$\int (a+b \tanh^{-1}(\frac{c}{x}))^2 dx$	826
3.147	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^2}{x} dx$	832

3.148	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^2}{x^2} dx$	837
3.149	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^2}{x^3} dx$	843
3.150	$\int x^3 (a+b \tanh^{-1}(\frac{c}{x}))^3 dx$	852
3.151	$\int x^2 (a+b \tanh^{-1}(\frac{c}{x}))^3 dx$	858
3.152	$\int x (a+b \tanh^{-1}(\frac{c}{x}))^3 dx$	864
3.153	$\int (a+b \tanh^{-1}(\frac{c}{x}))^3 dx$	869
3.154	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^3}{x} dx$	874
3.155	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^3}{x^2} dx$	880
3.156	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x}))^3}{x^3} dx$	888
3.157	$\int x^7 (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	894
3.158	$\int x^5 (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	898
3.159	$\int x^3 (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	902
3.160	$\int x (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	906
3.161	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x} dx$	910
3.162	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^3} dx$	913
3.163	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^5} dx$	917
3.164	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^7} dx$	921
3.165	$\int x^4 (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	925
3.166	$\int x^2 (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	930
3.167	$\int (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	935
3.168	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^2} dx$	939
3.169	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^4} dx$	944
3.170	$\int \frac{a+b \tanh^{-1}(\frac{c}{x^2})}{x^6} dx$	949
3.171	$\int x^3 (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	954
3.172	$\int x (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	963
3.173	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x} dx$	970
3.174	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x^3} dx$	975

3.175	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x^5} dx$	981
3.176	$\int x^4 (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	990
3.177	$\int x^2 (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	1000
3.178	$\int (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	1010
3.179	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x^2} dx$	1020
3.180	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x^4} dx$	1031
3.181	$\int \frac{(a+b \tanh^{-1}(\frac{c}{x^2}))^2}{x^6} dx$	1041
3.182	$\int (dx)^m (a+b \tanh^{-1}(\frac{c}{x^2}))^3 dx$	1051
3.183	$\int (dx)^m (a+b \tanh^{-1}(\frac{c}{x^2}))^2 dx$	1054
3.184	$\int (dx)^m (a+b \tanh^{-1}(\frac{c}{x^2})) dx$	1057
3.185	$\int \frac{(dx)^m}{a+b \tanh^{-1}(\frac{c}{x^2})} dx$	1061
3.186	$\int \frac{(dx)^m}{(a+b \tanh^{-1}(\frac{c}{x^2}))^2} dx$	1064
3.187	$\int x^3 (a+b \tanh^{-1}(c\sqrt{x})) dx$	1067
3.188	$\int x^2 (a+b \tanh^{-1}(c\sqrt{x})) dx$	1071
3.189	$\int x (a+b \tanh^{-1}(c\sqrt{x})) dx$	1075
3.190	$\int (a+b \tanh^{-1}(c\sqrt{x})) dx$	1079
3.191	$\int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x} dx$	1083
3.192	$\int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^2} dx$	1086
3.193	$\int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^3} dx$	1090
3.194	$\int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^4} dx$	1094
3.195	$\int x^3 (a+b \tanh^{-1}(c\sqrt{x}))^2 dx$	1098
3.196	$\int x^2 (a+b \tanh^{-1}(c\sqrt{x}))^2 dx$	1102
3.197	$\int x (a+b \tanh^{-1}(c\sqrt{x}))^2 dx$	1106
3.198	$\int (a+b \tanh^{-1}(c\sqrt{x}))^2 dx$	1109
3.199	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x} dx$	1112
3.200	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^2} dx$	1117
3.201	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^3} dx$	1121



3.202	$\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$	.1125
3.203	$\int x^2 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$	.1130
3.204	$\int x (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$	.1135
3.205	$\int (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$	.1139
3.206	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x} dx$	.1142
3.207	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^2} dx$	.1148
3.208	$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^3} dx$	.1151
3.209	$\int x^{3/2} \tanh^{-1}(\sqrt{x}) dx$	.1155
3.210	$\int \sqrt{x} \tanh^{-1}(\sqrt{x}) dx$	.1159
3.211	$\int \frac{\tanh^{-1}(\sqrt{x})}{\sqrt{x}} dx$	.1162
3.212	$\int \frac{\tanh^{-1}(\sqrt{x})}{x^{3/2}} dx$	.1165
3.213	$\int x^3 (a + b \tanh^{-1}(cx^{3/2})) dx$	.1169
3.214	$\int x^2 (a + b \tanh^{-1}(cx^{3/2})) dx$	.1175
3.215	$\int x (a + b \tanh^{-1}(cx^{3/2})) dx$	.1179
3.216	$\int (a + b \tanh^{-1}(cx^{3/2})) dx$	.1185
3.217	$\int \frac{a+b \tanh^{-1}(cx^{3/2})}{x} dx$	.1191
3.218	$\int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^2} dx$	.1194
3.219	$\int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^3} dx$	.1199
3.220	$\int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^4} dx$	.1204
3.221	$\int x^2 (a + b \tanh^{-1}(cx^{3/2}))^2 dx$	.1208
3.222	$\int \frac{(a+b \tanh^{-1}(cx^{3/2}))^2}{x} dx$	.1211
3.223	$\int \frac{(a+b \tanh^{-1}(cx^{3/2}))^2}{x^4} dx$	.1216
3.224	$\int x^2 (a + b \tanh^{-1}(cx^n)) dx$	.1219
3.225	$\int x (a + b \tanh^{-1}(cx^n)) dx$	.1222
3.226	$\int (a + b \tanh^{-1}(cx^n)) dx$	.1225
3.227	$\int \frac{a+b \tanh^{-1}(cx^n)}{x} dx$	.1228
3.228	$\int \frac{a+b \tanh^{-1}(cx^n)}{x^2} dx$	.1231
3.229	$\int \frac{a+b \tanh^{-1}(cx^n)}{x^3} dx$	.1234
3.230	$\int \frac{a+b \tanh^{-1}(cx^n)}{x^4} dx$	.1237
3.231	$\int x (a + b \tanh^{-1}(cx^n))^2 dx$	.1240

3.232	$\int (a + b \tanh^{-1}(cx^n))^2 dx$	.1243
3.233	$\int \frac{(a+b \tanh^{-1}(cx^n))^2}{x} dx$	.1246
3.234	$\int \frac{(a+b \tanh^{-1}(cx^n))^2}{x^2} dx$	.1251
3.235	$\int \frac{(a+b \tanh^{-1}(cx^n))^2}{x^3} dx$	.1254
3.236	$\int \frac{\tanh^{-1}(ax^n)}{x} dx$	.1257
3.237	$\int \frac{\tanh^{-1}(ax^5)}{x} dx$	.1260
3.238	$\int \tanh^{-1}\left(\frac{1}{x}\right) dx$	.1263
3.239	$\int (dx)^m (a + b \tanh^{-1}(cx^n))^3 dx$	.1266
3.240	$\int (dx)^m (a + b \tanh^{-1}(cx^n))^2 dx$	.1269
3.241	$\int (dx)^m (a + b \tanh^{-1}(cx^n)) dx$	.1272
3.242	$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx$	.1276
3.243	$\int \frac{(dx)^m}{(a+b \tanh^{-1}(cx^n))^2} dx$	.1279

#### 4 Listing of Grading functions

1283

# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 243 ]. This is test number [ 192 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 88.89 ( 216 )	% 11.11 ( 27 )
Mathematica	% 95.06 ( 231 )	% 4.94 ( 12 )
Maple	% 80.66 ( 196 )	% 19.34 ( 47 )
Maxima	% 43.62 ( 106 )	% 56.38 ( 137 )
Fricas	% 60.08 ( 146 )	% 39.92 ( 97 )
Sympy	% 30.45 ( 74 )	% 69.55 ( 169 )
Giac	% 51.03 ( 124 )	% 48.97 ( 119 )

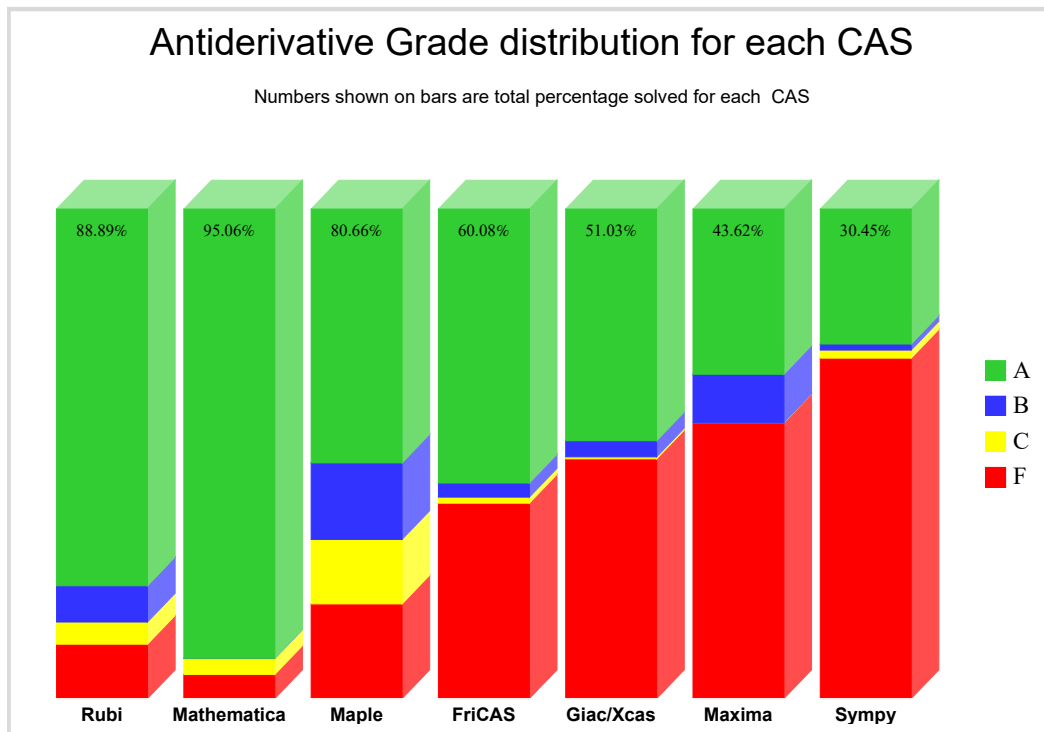
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

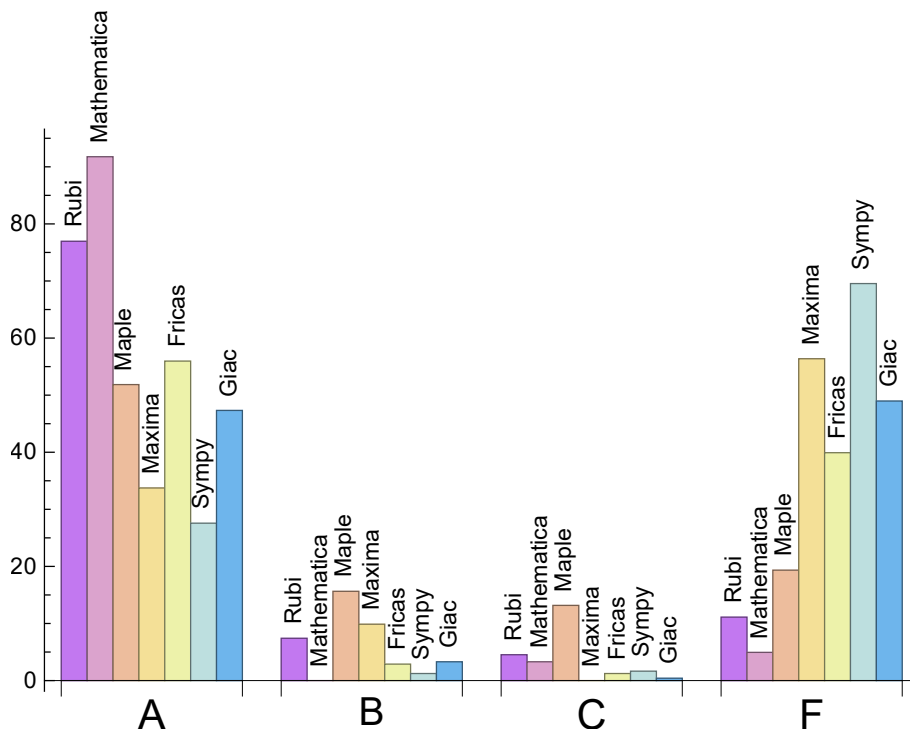
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	76.95	7.41	4.53	11.11
Mathematica	91.77	0.	3.29	4.94
Maple	51.85	15.64	13.17	19.34
Maxima	33.74	9.88	0.	56.38
Fricas	55.97	2.88	1.23	39.92
Sympy	27.57	1.23	1.65	69.55
Giac	47.33	3.29	0.41	48.97

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	199.6	1.33	74.	1.
Mathematica	0.47	121.66	1.07	99.	1.11
Maple	0.56	3909.02	39.16	81.	1.22
Maxima	1.04	177.82	1.77	76.5	1.51
Fricas	1.79	332.4	3.56	141.	2.92
Sympy	17.01	188.93	3.09	80.	1.47
Giac	1.27	119.34	1.41	90.	1.6

## 1.4 list of integrals that has no closed form antiderivative

{43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {64, 65, 66, 67, 69, 70, 77, 78, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 143, 144, 145, 146, 148, 149, 155, 171, 172, 174, 175}

**Mathematica** {14, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 65, 67, 69, 73, 74, 77, 78, 80, 81, 117, 119, 121, 123, 124, 125, 126, 128, 129, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 172, 173, 174, 178, 179, 202, 203, 204, 205, 207, 208}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.



**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

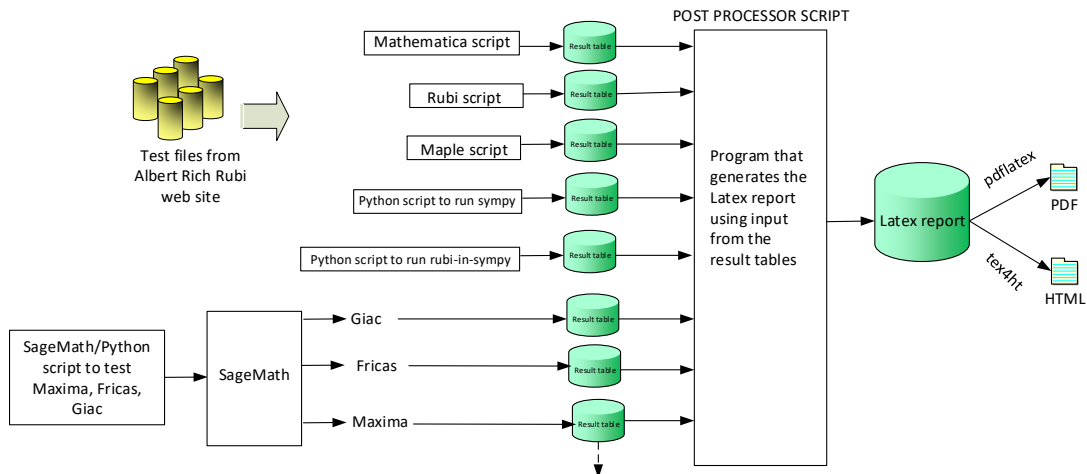
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 71, 72, 73, 74, 75, 76, 79, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 120, 127, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 154, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 206, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243 }

B grade: { 65, 67, 69, 77, 78, 117, 119, 121, 123, 124, 125, 126, 144, 146, 148, 155, 172, 174 }

C grade: { 64, 66, 70, 116, 118, 122, 143, 145, 149, 171, 175 }

F grade: { 80, 81, 90, 91, 92, 93, 128, 129, 150, 151, 152, 153, 156, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 207, 208, 221, 223 }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 154, 155, 156, 157, 158, 159, 160, 161,

162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 178, 179, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243 }

B grade: { }

C grade: { 31, 33, 80, 128, 151, 153, 227, 236 }

F grade: { 71, 72, 75, 76, 90, 91, 92, 93, 176, 177, 180, 181 }

## 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 18, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 67, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 148, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 7, 13, 14, 15, 16, 17, 20, 21, 22, 23, 29, 54, 66, 70, 78, 118, 122, 126, 139, 143, 144, 145, 146, 149, 155, 161, 175, 191, 195, 196, 197, 198, 200, 201, 217, 221, 227, 236 }

C grade: { 19, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 77, 103, 125, 147, 150, 151, 152, 153, 154, 156, 199, 202, 203, 204, 205, 206, 207, 208, 222, 233, 237 }

F grade: { 45, 64, 65, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 90, 91, 92, 93, 96, 116, 117, 120, 121, 123, 124, 127, 128, 129, 132, 171, 172, 173, 176, 177, 178, 179, 180, 181, 184, 223, 224, 225, 226, 228, 229, 230, 241 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 64, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 110, 112, 114, 116, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 171, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 209, 210, 211, 212, 214, 220, 231, 232, 234, 235, 238, 242, 243 }

B grade: { 17, 21, 23, 66, 70, 118, 122, 149, 175, 191, 197, 198, 200, 201, 202, 203, 204, 207, 208, 217, 221, 223, 236, 237 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 54, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 107, 109, 111, 113, 115, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 199, 205, 206, 213, 215, 216, 218, 219, 222, 224, 225, 226, 227, 228, 229, 230, 233, 239, 240, 241 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 70, 82, 83, 84, 85, 86, 87, 88, 89, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 201, 209, 210, 211, 212, 214, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 198, 200, 221, 223, 227, 236 }

C grade: { 213, 215, 216 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 49, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 217, 222, 224, 225, 226, 228, 229, 230, 233, 237, 241 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 43, 44, 46, 47, 48, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 159, 162, 163, 164, 165, 166, 167, 168, 169, 170, 175, 192, 193, 200, 201, 210, 232, 234, 238 }

B grade: { 209, 211, 212 }

C grade: { 37, 158, 160, 171 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 45, 49, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 236, 237, 239, 240, 241, 242, 243 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 63, 64, 66, 94, 95, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 182, 183, 185, 186, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 214, 216, 218, 219, 220, 231, 232, 234, 235, 238, 239, 240, 242, 243 }

B grade: { 60, 61, 62, 85, 86, 87, 88, 89 }

C grade: { 213 }

F grade: { 7, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 54, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 215, 217, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	67	95	153	63	104
normalized size	1	1.	1.37	1.14	1.61	2.59	1.07	1.76
time (sec)	N/A	0.033	0.01	0.007	0.969	1.954	2.132	1.173

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	60	74	153	68	84
normalized size	1	1.	1.09	1.05	1.3	2.68	1.19	1.47
time (sec)	N/A	0.043	0.009	0.007	0.966	2.056	1.616	1.156



Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	82	127	53	92
normalized size	1	1.	1.46	1.21	1.71	2.65	1.1	1.92
time (sec)	N/A	0.029	0.009	0.007	0.96	2.025	1.167	1.235

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	51	51	59	127	58	72
normalized size	1	1.	1.11	1.11	1.28	2.76	1.26	1.57
time (sec)	N/A	0.035	0.008	0.007	0.968	1.969	0.945	1.232

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	68	104	42	80
normalized size	1	1.	1.59	1.32	1.84	2.81	1.14	2.16
time (sec)	N/A	0.017	0.007	0.007	0.985	1.908	0.592	1.209

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	41	97	27	54
normalized size	1	1.	1.	0.97	1.37	3.23	0.9	1.8
time (sec)	N/A	0.013	0.003	0.002	0.976	1.947	0.387	1.172

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	47	0	0	0	0
normalized size	1	1.	0.92	1.81	0.	0.	0.	0.
time (sec)	N/A	0.015	0.009	0.013	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	45	53	116	41	63
normalized size	1	1.	1.08	1.25	1.47	3.22	1.14	1.75
time (sec)	N/A	0.026	0.008	0.01	0.976	1.87	0.946	1.152

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	59	49	61	95	36	77
normalized size	1	1.	1.59	1.32	1.65	2.57	0.97	2.08
time (sec)	N/A	0.022	0.008	0.01	0.979	1.933	0.74	1.202

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	59	66	140	70	80
normalized size	1	1.	1.09	1.09	1.22	2.59	1.3	1.48
time (sec)	N/A	0.036	0.008	0.01	0.967	1.928	1.561	1.206

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	70	58	81	117	46	92
normalized size	1	1.	1.46	1.21	1.69	2.44	0.96	1.92
time (sec)	N/A	0.027	0.008	0.01	0.987	1.926	1.166	1.214

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	70	68	82	166	80	92
normalized size	1	1.	1.08	1.05	1.26	2.55	1.23	1.42
time (sec)	N/A	0.041	0.009	0.01	0.973	1.979	2.782	1.222

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	164	314	290	437	211	259
normalized size	1	1.	1.13	2.17	2.	3.01	1.46	1.79
time (sec)	N/A	0.327	0.068	0.023	1.013	2.188	4.115	1.302

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	161	306	0	0	0	0
normalized size	1	1.	0.99	1.89	0.	0.	0.	0.
time (sec)	N/A	0.302	0.452	0.016	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	132	278	255	354	168	215
normalized size	1	1.	1.17	2.46	2.26	3.13	1.49	1.9
time (sec)	N/A	0.223	0.059	0.015	0.977	2.097	2.244	1.257

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	122	270	0	0	0	0
normalized size	1	1.	0.94	2.08	0.	0.	0.	0.
time (sec)	N/A	0.201	0.257	0.013	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	90	239	213	269	114	163
normalized size	1	1.	1.2	3.19	2.84	3.59	1.52	2.17
time (sec)	N/A	0.114	0.062	0.016	0.994	2.233	1.089	1.213

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	82	123	0	0	0	0
normalized size	1	1.	1.11	1.66	0.	0.	0.	0.
time (sec)	N/A	0.101	0.143	0.049	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	120	701	0	0	0	0
normalized size	1	1.	1.03	5.99	0.	0.	0.	0.
time (sec)	N/A	0.264	0.072	0.214	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	71	71	94	248	0	0	0	0
normalized size	1	1.	1.32	3.49	0.	0.	0.	0.
time (sec)	N/A	0.145	0.144	0.021	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	101	253	204	300	126	0
normalized size	1	1.	1.26	3.16	2.55	3.75	1.58	0.
time (sec)	N/A	0.133	0.068	0.019	1.004	2.367	1.455	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	145	339	0	0	0	0
normalized size	1	1.	1.12	2.61	0.	0.	0.	0.
time (sec)	N/A	0.231	0.344	0.02	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	164	290	302	386	184	0
normalized size	1	1.	1.4	2.48	2.58	3.3	1.57	0.
time (sec)	N/A	0.228	0.069	0.026	0.998	2.38	2.601	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	247	247	305	1330	0	0	0	0
normalized size	1	1.	1.23	5.38	0.	0.	0.	0.
time (sec)	N/A	0.964	0.704	0.754	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	383	1275	0	0	0	0
normalized size	1	1.	1.46	4.87	0.	0.	0.	0.
time (sec)	N/A	0.774	0.735	0.823	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	245	1245	0	0	0	0
normalized size	1	1.	1.32	6.73	0.	0.	0.	0.
time (sec)	N/A	0.569	0.492	0.602	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	250	1177	0	0	0	0
normalized size	1	1.	1.27	5.97	0.	0.	0.	0.
time (sec)	N/A	0.449	0.52	0.477	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	161	6097	0	0	0	0
normalized size	1	1.	1.31	49.57	0.	0.	0.	0.
time (sec)	N/A	0.25	0.284	0.342	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	161	261	0	0	0	0
normalized size	1	1.	1.49	2.42	0.	0.	0.	0.
time (sec)	N/A	0.217	0.26	0.082	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	178	1470	0	0	0	0
normalized size	1	1.	0.97	7.99	0.	0.	0.	0.
time (sec)	N/A	0.449	0.128	0.113	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	196	1583	0	0	0	0
normalized size	1	1.	1.92	15.52	0.	0.	0.	0.
time (sec)	N/A	0.268	0.321	0.183	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	123	123	192	5098	0	0	0	0
normalized size	1	1.	1.56	41.45	0.	0.	0.	0.
time (sec)	N/A	0.294	0.272	0.316	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	323	1838	0	0	0	0
normalized size	1	1.	1.62	9.19	0.	0.	0.	0.
time (sec)	N/A	0.498	0.861	0.849	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	295	1281	0	0	0	0
normalized size	1	1.	1.58	6.85	0.	0.	0.	0.
time (sec)	N/A	0.626	0.639	0.873	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	128	107	0	660	0	0
normalized size	1	1.	1.03	0.86	0.	5.32	0.	0.
time (sec)	N/A	0.086	0.073	0.036	0.	2.339	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	115	93	0	586	0	0
normalized size	1	1.	1.08	0.88	0.	5.53	0.	0.
time (sec)	N/A	0.067	0.055	0.011	0.	2.253	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	114	89	0	509	586	0
normalized size	1	1.	1.08	0.84	0.	4.8	5.53	0.
time (sec)	N/A	0.061	0.05	0.012	0.	2.229	20.006	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	98	70	0	487	0	119
normalized size	1	1.	1.15	0.82	0.	5.73	0.	1.4
time (sec)	N/A	0.052	0.032	0.013	0.	2.324	0.	1.162

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	99	78	0	494	0	127
normalized size	1	1.	1.16	0.92	0.	5.81	0.	1.49
time (sec)	N/A	0.055	0.043	0.013	0.	2.16	0.	1.27

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	94	0	558	0	163
normalized size	1	1.	1.	0.88	0.	5.21	0.	1.52
time (sec)	N/A	0.063	0.06	0.014	0.	2.212	0.	1.343

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	108	94	0	583	0	159
normalized size	1	1.	1.01	0.88	0.	5.45	0.	1.49
time (sec)	N/A	0.066	0.048	0.014	0.	2.28	0.	1.265

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	122	108	0	628	0	188
normalized size	1	1.	0.98	0.86	0.	5.02	0.	1.5
time (sec)	N/A	0.078	0.066	0.015	0.	2.452	0.	1.423



Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	3.519	1.024	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	2.321	0.914	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	59	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.067	0.925	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.253	0.654	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.5	0.658	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.005	0.441	0.201	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.326	0.378	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	93	135	58	105
normalized size	1	1.	1.44	1.22	1.72	2.5	1.07	1.94
time (sec)	N/A	0.039	0.016	0.022	0.953	1.999	29.879	1.206

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	62	134	85	77
normalized size	1	1.	1.1	0.94	1.29	2.79	1.77	1.6
time (sec)	N/A	0.035	0.014	0.009	0.982	2.059	21.035	1.17

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	78	112	48	93
normalized size	1	1.	1.56	1.33	1.81	2.6	1.12	2.16
time (sec)	N/A	0.03	0.013	0.012	0.955	2.02	14.08	1.184

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	108	71	66
normalized size	1	1.	1.14	1.	1.35	2.92	1.92	1.78
time (sec)	N/A	0.015	0.007	0.003	0.956	2.065	10.068	1.188

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	124	0	0	0	0
normalized size	1	1.	0.93	4.13	0.	0.	0.	0.
time (sec)	N/A	0.033	0.013	0.029	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	55	130	80	69
normalized size	1	1.	1.12	1.22	1.38	3.25	2.	1.72
time (sec)	N/A	0.026	0.011	0.013	0.981	2.002	17.27	1.172

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	69	103	41	90
normalized size	1	1.	1.59	1.34	1.68	2.51	1.	2.2
time (sec)	N/A	0.027	0.011	0.013	0.959	1.939	12.464	1.195

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	69	150	97	88
normalized size	1	1.	1.09	1.12	1.23	2.68	1.73	1.57
time (sec)	N/A	0.034	0.011	0.015	0.975	2.11	34.431	1.333

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	93	53	0	473	185	103
normalized size	1	1.	1.43	0.82	0.	7.28	2.85	1.58
time (sec)	N/A	0.035	0.024	0.011	0.	2.009	19.363	1.274

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	448	581	101
normalized size	1	1.	1.44	0.81	0.	7.11	9.22	1.6
time (sec)	N/A	0.033	0.019	0.01	0.	2.229	12.964	1.277

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	57	37	0	394	139	112
normalized size	1	1.	1.3	0.84	0.	8.95	3.16	2.55
time (sec)	N/A	0.024	0.02	0.007	0.	2.074	8.394	1.195

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	75	42	0	385	699	107
normalized size	1	1.	1.63	0.91	0.	8.37	15.2	2.33
time (sec)	N/A	0.026	0.017	0.013	0.	2.125	15.145	1.225

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	440	813	132
normalized size	1	1.	1.44	0.81	0.	6.98	12.9	2.1
time (sec)	N/A	0.033	0.027	0.014	0.	2.134	21.291	1.352

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	51	0	458	833	123
normalized size	1	1.	1.44	0.81	0.	7.27	13.22	1.95
time (sec)	N/A	0.032	0.027	0.013	0.	2.141	30.884	1.533

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	293	375	206	236
normalized size	1	5.09	1.17	0.	2.34	3.	1.65	1.89
time (sec)	N/A	1.547	0.073	180.	1.007	2.043	39.731	1.503

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0
normalized size	1	3.67	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.296	0.278	180.	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	251	292	163	186
normalized size	1	5.76	1.16	2.71	2.76	3.21	1.79	2.04
time (sec)	N/A	0.965	0.06	0.163	0.976	2.113	19.12	1.382

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	94	207	99	144	0	0	0	0
normalized size	1	2.2	1.05	1.53	0.	0.	0.	0.
time (sec)	N/A	0.514	0.066	0.035	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	141	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.336	0.101	0.133	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	87	237	119	0	0	0	0	0
normalized size	1	2.72	1.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.632	0.157	180.	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	236	324	175	0
normalized size	1	4.09	1.26	2.92	2.68	3.68	1.99	0.
time (sec)	N/A	1.057	0.083	0.188	1.003	2.164	26.066	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1173	1173	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.334	9.308	0.216	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1129	1129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.061	9.058	0.184	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	958	958	566	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	1.468	2.562	0.125	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	942	942	566	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.335	3.369	0.175	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1102	1102	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.842	2.554	0.175	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1176	1176	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.99	3.012	0.167	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	141	479	185	751	0	0	0	0
normalized size	1	3.4	1.31	5.33	0.	0.	0.	0.
time (sec)	N/A	4.216	0.464	0.269	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	134	390	213	298	0	0	0	0
normalized size	1	2.91	1.59	2.22	0.	0.	0.	0.
time (sec)	N/A	2.323	0.222	0.007	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	211	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.56	0.191	0.138	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	125	0	222	0	0	0	0	0
normalized size	1	0.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.78	0.414	0.194	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	139	0	218	0	0	0	0	0
normalized size	1	0.	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	1.558	0.29	0.231	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	241	302	0	124	0	0
normalized size	1	1.	0.76	0.95	0.	0.39	0.	0.
time (sec)	N/A	0.334	0.125	0.022	0.	2.293	0.	0.



Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	240	292	0	109	0	0
normalized size	1	1.	0.76	0.92	0.	0.34	0.	0.
time (sec)	N/A	0.3	0.093	0.016	0.	2.272	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	227	280	0	80	0	0
normalized size	1	1.	0.75	0.93	0.	0.27	0.	0.
time (sec)	N/A	0.251	0.072	0.011	0.	2.16	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	227	273	0	72	0	666
normalized size	1	1.	0.8	0.96	0.	0.25	0.	2.34
time (sec)	N/A	0.242	0.059	0.013	0.	2.229	0.	1.243

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	268	272	0	81	0	682
normalized size	1	1.	0.94	0.95	0.	0.28	0.	2.39
time (sec)	N/A	0.248	0.103	0.013	0.	2.266	0.	1.906

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	268	280	0	89	0	698
normalized size	1	1.	0.89	0.93	0.	0.3	0.	2.32
time (sec)	N/A	0.25	0.106	0.01	0.	2.268	0.	3.186

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	275	292	0	105	0	717
normalized size	1	1.	0.87	0.92	0.	0.33	0.	2.26
time (sec)	N/A	0.278	0.086	0.016	0.	2.237	0.	9.78

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	281	302	0	109	0	720
normalized size	1	1.	0.89	0.95	0.	0.34	0.	2.27
time (sec)	N/A	0.287	0.098	0.014	0.	2.236	0.	25.291

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6327	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	61.946	0.313	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6177	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	59.351	0.213	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6334	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	94.015	0.272	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F(-2)	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6520	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	50.738	0.275	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.751	0.122	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	1.167	0.119	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.066	0.117	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.348	0.063	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.414	0.068	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	78	66	93	138	0	105
normalized size	1	1.	1.44	1.22	1.72	2.56	0.	1.94
time (sec)	N/A	0.038	0.02	0.009	0.953	2.042	0.	1.202

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	45	62	134	0	77
normalized size	1	1.	1.1	0.94	1.29	2.79	0.	1.6
time (sec)	N/A	0.036	0.016	0.008	1.021	1.916	0.	1.196

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	57	78	113	0	93
normalized size	1	1.	1.56	1.33	1.81	2.63	0.	2.16
time (sec)	N/A	0.03	0.016	0.012	0.98	1.799	0.	1.149

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	108	0	66
normalized size	1	1.	1.14	1.	1.35	2.92	0.	1.78
time (sec)	N/A	0.02	0.008	0.003	1.011	1.961	0.	1.169

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	92	0	0	0	0
normalized size	1	1.	0.93	3.07	0.	0.	0.	0.
time (sec)	N/A	0.034	0.014	0.029	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	45	49	55	130	0	69
normalized size	1	1.	1.12	1.22	1.38	3.25	0.	1.72
time (sec)	N/A	0.027	0.012	0.013	1.021	1.98	0.	1.134

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	65	55	69	104	0	90
normalized size	1	1.	1.59	1.34	1.68	2.54	0.	2.2
time (sec)	N/A	0.027	0.018	0.013	1.013	1.936	0.	1.162

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	63	69	150	0	88
normalized size	1	1.	1.09	1.12	1.23	2.68	0.	1.57
time (sec)	N/A	0.036	0.012	0.014	0.965	2.157	0.	1.171

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	196	184	0	2611	0	279
normalized size	1	1.	1.13	1.06	0.	15.01	0.	1.6
time (sec)	N/A	0.22	0.042	0.015	0.	2.077	0.	1.317

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	136	99	143	674	0	147
normalized size	1	1.	1.35	0.98	1.42	6.67	0.	1.46
time (sec)	N/A	0.097	0.043	0.006	1.448	1.869	0.	1.184

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	187	159	0	598	0	223
normalized size	1	1.	1.13	0.96	0.	3.62	0.	1.35
time (sec)	N/A	0.197	0.052	0.01	0.	1.927	0.	1.298

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	196	172	144	367	0	169
normalized size	1	1.	1.7	1.5	1.25	3.19	0.	1.47
time (sec)	N/A	0.095	0.05	0.014	1.449	1.767	0.	1.218

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	198	186	0	659	0	281
normalized size	1	1.	1.12	1.06	0.	3.74	0.	1.6
time (sec)	N/A	0.284	0.03	0.013	0.	1.884	0.	1.493

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	198	114	161	414	0	170
normalized size	1	1.	1.69	0.97	1.38	3.54	0.	1.45
time (sec)	N/A	0.097	0.028	0.007	1.501	1.791	0.	1.186

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	187	177	0	622	0	269
normalized size	1	1.	1.13	1.07	0.	3.77	0.	1.63
time (sec)	N/A	0.249	0.026	0.01	0.	1.858	0.	1.31

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	183	105	149	312	0	143
normalized size	1	1.	1.76	1.01	1.43	3.	0.	1.38
time (sec)	N/A	0.083	0.029	0.005	1.528	1.829	0.	1.183

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	196	172	0	547	0	261
normalized size	1	1.	1.13	0.99	0.	3.14	0.	1.5
time (sec)	N/A	0.267	0.052	0.013	0.	1.673	0.	2.087

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	125	636	146	0	293	381	0	236
normalized size	1	5.09	1.17	0.	2.34	3.05	0.	1.89
time (sec)	N/A	1.546	0.076	180.	0.986	1.855	0.	1.289

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	146	536	132	0	0	0	0	0
normalized size	1	3.67	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	1.309	0.272	180.	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	F(-2)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	91	524	106	247	251	292	0	186
normalized size	1	5.76	1.16	2.71	2.76	3.21	0.	2.04
time (sec)	N/A	0.985	0.056	0.18	1.123	1.689	0.	1.198

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	96	207	99	145	0	0	0	0
normalized size	1	2.16	1.03	1.51	0.	0.	0.	0.
time (sec)	N/A	0.592	0.152	0.004	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	143	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	0.111	0.144	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	90	237	117	0	0	0	0	0
normalized size	1	2.63	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.619	0.158	180.	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	F(-1)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	88	360	111	257	236	324	0	0
normalized size	1	4.09	1.26	2.92	2.68	3.68	0.	0.
time (sec)	N/A	1.06	0.083	0.201	0.997	1.771	0.	0.



Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	144	420	159	0	0	0	0	0
normalized size	1	2.92	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	1.311	0.363	0.181	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	231	1421	334	0	0	0	0	0
normalized size	1	6.15	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	6.35	0.47	0.217	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	139	479	185	750	0	0	0	0
normalized size	1	3.45	1.33	5.4	0.	0.	0.	0.
time (sec)	N/A	4.204	0.299	0.316	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	130	390	191	295	0	0	0	0
normalized size	1	3.	1.47	2.27	0.	0.	0.	0.
time (sec)	N/A	2.467	0.282	0.006	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	214	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.565	0.18	0.151	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	120	0	223	0	0	0	0	0
normalized size	1	0.	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.779	0.395	0.208	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	136	0	218	0	0	0	0	0
normalized size	1	0.	1.6	0.	0.	0.	0.	0.
time (sec)	N/A	1.575	0.279	0.23	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.81	0.125	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.174	0.125	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.062	0.119	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.349	0.07	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.361	0.069	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	67	62	77	113	46	84
normalized size	1	1.	1.34	1.24	1.54	2.26	0.92	1.68
time (sec)	N/A	0.032	0.01	0.013	0.979	1.668	1.759	1.168

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	67	57	117	49	66
normalized size	1	1.	1.11	1.49	1.27	2.6	1.09	1.47
time (sec)	N/A	0.033	0.008	0.013	0.961	1.769	1.109	1.168

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	56	53	59	90	36	72
normalized size	1	1.	1.44	1.36	1.51	2.31	0.92	1.85
time (sec)	N/A	0.02	0.008	0.01	0.97	1.687	0.487	1.141

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	48	39	85	24	57
normalized size	1	1.	1.	1.66	1.34	2.93	0.83	1.97
time (sec)	N/A	0.013	0.003	0.01	0.95	1.679	0.317	1.131

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	63	0	0	0	0
normalized size	1	1.	0.93	2.1	0.	0.	0.	0.
time (sec)	N/A	0.032	0.012	0.014	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	37	50	115	39	66
normalized size	1	1.	1.09	1.06	1.43	3.29	1.11	1.89
time (sec)	N/A	0.021	0.008	0.004	0.957	1.694	1.852	1.133

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	57	70	103	44	77
normalized size	1	1.	1.4	1.33	1.63	2.4	1.02	1.79
time (sec)	N/A	0.027	0.009	0.007	0.955	1.761	2.114	1.164

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	57	74	144	68	85
normalized size	1	1.	1.09	1.	1.3	2.53	1.19	1.49
time (sec)	N/A	0.041	0.009	0.007	0.962	1.672	3.641	1.129

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	123	812	131	328	255	350	158	209
normalized size	1	6.6	1.07	2.67	2.07	2.85	1.28	1.7
time (sec)	N/A	1.703	0.061	0.007	0.982	1.677	3.443	1.222

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	142	695	145	391	0	0	0	0
normalized size	1	4.89	1.02	2.75	0.	0.	0.	0.
time (sec)	N/A	1.389	0.303	0.032	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	A	A	A	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	83	574	92	287	184	254	104	159
normalized size	1	6.92	1.11	3.46	2.22	3.06	1.25	1.92
time (sec)	N/A	1.041	0.047	0.025	0.976	1.762	1.05	1.211

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	74	370	97	282	0	0	0	0
normalized size	1	5.	1.31	3.81	0.	0.	0.	0.
time (sec)	N/A	0.402	0.122	0.012	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	114	780	0	0	0	0
normalized size	1	1.	0.86	5.86	0.	0.	0.	0.
time (sec)	N/A	0.314	0.092	0.162	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	87	205	101	144	0	0	0	0
normalized size	1	2.36	1.16	1.66	0.	0.	0.	0.
time (sec)	N/A	0.507	0.097	0.004	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	87	707	119	284	223	288	124	0
normalized size	1	8.13	1.37	3.26	2.56	3.31	1.43	0.
time (sec)	N/A	1.247	0.067	0.016	0.989	1.867	1.962	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	203	0	286	1410	0	0	0	0
normalized size	1	0.	1.41	6.95	0.	0.	0.	0.
time (sec)	N/A	4.39	0.603	0.332	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	217	0	316	2033	0	0	0	0
normalized size	1	0.	1.46	9.37	0.	0.	0.	0.
time (sec)	N/A	3.302	0.744	0.02	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	135	0	193	5536	0	0	0	0
normalized size	1	0.	1.43	41.01	0.	0.	0.	0.
time (sec)	N/A	2.245	0.318	0.278	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	108	0	198	1756	0	0	0	0
normalized size	1	0.	1.83	16.26	0.	0.	0.	0.
time (sec)	N/A	0.701	0.263	0.121	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	171	1631	0	0	0	0
normalized size	1	1.	0.82	7.84	0.	0.	0.	0.
time (sec)	N/A	0.509	0.191	0.07	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	126	387	205	298	0	0	0	0
normalized size	1	3.07	1.63	2.37	0.	0.	0.	0.
time (sec)	N/A	2.237	0.122	0.004	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	139	0	195	6645	0	0	0	0
normalized size	1	0.	1.4	47.81	0.	0.	0.	0.
time (sec)	N/A	2.102	0.334	0.266	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	73	64	84	122	51	96
normalized size	1	1.	1.35	1.19	1.56	2.26	0.94	1.78
time (sec)	N/A	0.042	0.014	0.016	0.99	1.75	18.497	1.332

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	65	57	124	75	70
normalized size	1	1.	1.11	1.44	1.27	2.76	1.67	1.56
time (sec)	N/A	0.033	0.012	0.019	0.968	1.711	13.194	1.245

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	55	66	97	41	84
normalized size	1	1.	1.44	1.28	1.53	2.26	0.95	1.95
time (sec)	N/A	0.03	0.011	0.006	0.96	1.585	15.366	1.254

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	52	46	99	61	63
normalized size	1	1.	1.15	1.53	1.35	2.91	1.79	1.85
time (sec)	N/A	0.018	0.007	0.016	0.982	1.647	10.123	1.386

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	154	0	0	0	0
normalized size	1	1.	0.93	5.13	0.	0.	0.	0.
time (sec)	N/A	0.033	0.014	0.03	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	50	126	76	70
normalized size	1	1.	1.14	1.	1.35	3.41	2.05	1.89
time (sec)	N/A	0.02	0.009	0.004	0.969	1.775	18.503	1.275



Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	57	76	109	49	89
normalized size	1	1.	1.42	1.27	1.69	2.42	1.09	1.98
time (sec)	N/A	0.032	0.011	0.009	0.967	1.546	45.385	1.214

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	45	74	151	94	88
normalized size	1	1.	1.09	0.79	1.3	2.65	1.65	1.54
time (sec)	N/A	0.04	0.013	0.006	0.98	1.678	47.101	1.16

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	88	53	0	440	588	90
normalized size	1	1.	1.4	0.84	0.	6.98	9.33	1.43
time (sec)	N/A	0.035	0.021	0.014	0.	1.836	42.206	1.303

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	86	51	0	421	624	93
normalized size	1	1.	1.41	0.84	0.	6.9	10.23	1.52
time (sec)	N/A	0.033	0.018	0.008	0.	1.774	22.228	1.258

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	54	39	0	351	473	77
normalized size	1	1.	1.23	0.89	0.	7.98	10.75	1.75
time (sec)	N/A	0.024	0.016	0.01	0.	1.764	16.727	1.215

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	72	44	0	385	593	84
normalized size	1	1.	1.57	0.96	0.	8.37	12.89	1.83
time (sec)	N/A	0.031	0.018	0.01	0.	1.774	13.88	1.232

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	0	451	706	97
normalized size	1	1.	1.38	0.85	0.	6.94	10.86	1.49
time (sec)	N/A	0.037	0.031	0.012	0.	1.848	25.699	1.301

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	90	55	0	468	668	100
normalized size	1	1.	1.38	0.85	0.	7.2	10.28	1.54
time (sec)	N/A	0.038	0.024	0.011	0.	1.785	52.415	1.314

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	A	A	C	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	94	599	104	0	212	278	151	180
normalized size	1	6.37	1.11	0.	2.26	2.96	1.61	1.91
time (sec)	N/A	1.296	0.06	180.	0.978	1.817	14.765	1.275

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	94	404	107	0	0	0	0	0
normalized size	1	4.3	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.696	0.134	180.	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	177	0	0	0	0	0
normalized size	1	1.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	0.066	0.483	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	99	207	114	144	0	0	0	0
normalized size	1	2.09	1.15	1.45	0.	0.	0.	0.
time (sec)	N/A	0.529	0.09	0.004	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	B	B	A	A	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	97	770	131	682002	247	308	172	0
normalized size	1	7.94	1.35	7030.95	2.55	3.18	1.77	0.
time (sec)	N/A	1.53	0.081	88.027	1.057	1.799	41.77	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1214	1214	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.712	5.646	0.685	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1172	1172	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.208	2.978	0.641	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1549	1549	565	0	0	0	0	0
normalized size	1	1.	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	2.249	3.406	0.549	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1117	1117	568	0	0	0	0	0
normalized size	1	1.	0.51	0.	0.	0.	0.	0.
time (sec)	N/A	2.15	3.037	0.656	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1263	1263	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.585	2.662	0.603	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1337	1337	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.79	2.61	0.665	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	2.437	0.664	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	1.625	0.38	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	68	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.067	0.294	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.403	0.185	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.39	0.146	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	114	84	116	213	0	142
normalized size	1	1.	1.3	0.95	1.32	2.42	0.	1.61
time (sec)	N/A	0.042	0.031	0.026	1.023	1.787	0.	1.204

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	101	75	105	185	0	131
normalized size	1	1.	1.35	1.	1.4	2.47	0.	1.75
time (sec)	N/A	0.035	0.024	0.026	0.958	1.779	0.	1.17

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	88	66	93	159	0	119
normalized size	1	1.	1.42	1.06	1.5	2.56	0.	1.92
time (sec)	N/A	0.024	0.021	0.027	1.002	1.761	0.	1.194

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	42	50	72	131	0	90
normalized size	1	1.	1.08	1.28	1.85	3.36	0.	2.31
time (sec)	N/A	0.02	0.024	0.024	0.971	1.719	0.	1.22

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	63	82	0	0	0
normalized size	1	1.	1.	2.17	2.83	0.	0.	0.
time (sec)	N/A	0.032	0.011	0.036	1.493	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	67	55	69	122	231	90
normalized size	1	1.	1.68	1.38	1.72	3.05	5.78	2.25
time (sec)	N/A	0.023	0.024	0.033	0.961	1.746	40.854	1.332

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	86	64	86	149	342	105
normalized size	1	1.	1.43	1.07	1.43	2.48	5.7	1.75
time (sec)	N/A	0.026	0.026	0.033	0.956	1.652	179.21	1.327

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	99	73	97	174	0	119
normalized size	1	1.	1.36	1.	1.33	2.38	0.	1.63
time (sec)	N/A	0.034	0.03	0.037	0.957	1.627	0.	1.399

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	A	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	211	0	224	396	358	662	0	0
normalized size	1	0.	1.06	1.88	1.7	3.14	0.	0.
time (sec)	N/A	0.025	0.112	0.056	0.988	2.126	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	A	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	173	0	194	358	325	567	0	0
normalized size	1	0.	1.12	2.07	1.88	3.28	0.	0.
time (sec)	N/A	0.024	0.104	0.049	1.005	2.187	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	A	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	129	0	160	317	290	479	0	0
normalized size	1	0.	1.24	2.46	2.25	3.71	0.	0.
time (sec)	N/A	0.014	0.086	0.051	1.027	2.204	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	85	0	115	272	236	382	0	0
normalized size	1	0.	1.35	3.2	2.78	4.49	0.	0.
time (sec)	N/A	0.006	0.061	0.05	1.014	1.991	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	164	742	0	0	0	0
normalized size	1	1.	1.13	5.12	0.	0.	0.	0.
time (sec)	N/A	0.317	0.083	0.316	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	85	0	129	292	235	375	680	0
normalized size	1	0.	1.52	3.44	2.76	4.41	8.	0.
time (sec)	N/A	0.024	0.11	0.054	1.02	1.834	42.93	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	133	0	178	332	316	471	972	0
normalized size	1	0.	1.34	2.5	2.38	3.54	7.31	0.
time (sec)	N/A	0.024	0.128	0.062	0.985	1.924	179.052	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	374	0	418	1518	2662	0	0	0
normalized size	1	0.	1.12	4.06	7.12	0.	0.	0.
time (sec)	N/A	0.023	1.194	1.179	4.152	0.	0.	0.



Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	304	0	351	1423	2132	0	0	0
normalized size	1	0.	1.15	4.68	7.01	0.	0.	0.
time (sec)	N/A	0.024	0.789	0.326	3.732	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	234	0	285	1339	1598	0	0	0
normalized size	1	0.	1.22	5.72	6.83	0.	0.	0.
time (sec)	N/A	0.014	0.527	0.286	3.391	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	201	6235	0	0	0	0
normalized size	1	0.	1.42	43.91	0.	0.	0.	0.
time (sec)	N/A	0.006	0.277	0.388	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	248	1542	0	0	0	0
normalized size	1	1.	1.11	6.88	0.	0.	0.	0.
time (sec)	N/A	0.512	0.199	0.173	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	230	5199	713	0	0	0
normalized size	1	0.	1.62	36.61	5.02	0.	0.	0.
time (sec)	N/A	0.022	0.307	0.412	5.865	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	B	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	234	0	333	1365	949	0	0	0
normalized size	1	0.	1.42	5.83	4.06	0.	0.	0.
time (sec)	N/A	0.023	0.692	0.31	6.649	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	31	35	32	112	128	49
normalized size	1	1.	0.82	0.92	0.84	2.95	3.37	1.29
time (sec)	N/A	0.016	0.016	0.024	0.971	1.653	18.109	1.179

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	25	30	26	97	39	42
normalized size	1	1.	0.81	0.97	0.84	3.13	1.26	1.35
time (sec)	N/A	0.013	0.011	0.024	0.977	1.706	1.717	1.168

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	76	87	34
normalized size	1	1.	1.	0.85	1.1	3.8	4.35	1.7
time (sec)	N/A	0.009	0.008	0.025	0.954	1.708	0.892	1.182

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	24	100	126	41
normalized size	1	1.	1.	1.21	1.	4.17	5.25	1.71
time (sec)	N/A	0.01	0.018	0.03	0.958	1.808	2.417	1.164

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	0	4629	0	296
normalized size	1	1.	1.17	1.02	0.	24.36	0.	1.56
time (sec)	N/A	0.301	0.053	0.036	0.	10.99	0.	1.492

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	57	78	142	0	97
normalized size	1	1.	1.53	1.16	1.59	2.9	0.	1.98
time (sec)	N/A	0.034	0.026	0.029	0.995	1.856	0.	1.228

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	222	194	0	4730	0	0
normalized size	1	1.	1.17	1.02	0.	24.89	0.	0.
time (sec)	N/A	0.238	0.034	0.036	0.	10.529	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	141	179	0	4289	0	251
normalized size	1	1.	0.83	1.05	0.	25.23	0.	1.48
time (sec)	N/A	0.287	0.101	0.032	0.	10.769	0.	1.199

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	32	63	84	0	0	0
normalized size	1	1.	0.94	1.85	2.47	0.	0.	0.
time (sec)	N/A	0.033	0.02	0.036	1.594	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	205	167	0	647	0	232
normalized size	1	1.	1.19	0.97	0.	3.76	0.	1.35
time (sec)	N/A	0.221	0.034	0.034	0.	1.933	0.	1.47

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	220	180	0	618	0	270
normalized size	1	1.	1.17	0.96	0.	3.29	0.	1.44
time (sec)	N/A	0.287	0.058	0.037	0.	1.996	0.	1.514

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	73	55	69	132	0	90
normalized size	1	1.	1.55	1.17	1.47	2.81	0.	1.91
time (sec)	N/A	0.032	0.029	0.036	0.985	1.878	0.	1.212

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	B	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	101	0	122	284	251	402	0	0
normalized size	1	0.	1.21	2.81	2.49	3.98	0.	0.
time (sec)	N/A	0.024	0.093	0.049	1.009	1.998	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	167	785	0	0	0	0
normalized size	1	1.	1.07	5.03	0.	0.	0.	0.
time (sec)	N/A	0.32	0.132	0.334	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	B	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	96	0	123	0	236	402	0	0
normalized size	1	0.	1.28	0.	2.46	4.19	0.	0.
time (sec)	N/A	0.024	0.139	0.28	0.998	1.891	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	73	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.052	0.12	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	73	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	0.045	0.21	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.032	0.191	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	76	0	456	0	0
normalized size	1	1.	1.08	2.11	0.	12.67	0.	0.
time (sec)	N/A	0.035	0.075	0.036	0.	1.819	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	66	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.061	0.102	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	73	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.041	0.109	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.042	0.112	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	8.618	0.162	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	1.897	0.187	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	146	880	0	0	0	0
normalized size	1	1.	0.99	5.95	0.	0.	0.	0.
time (sec)	N/A	0.313	0.128	0.33	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	12.352	0.109	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	12.317	0.109	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	61	198	423	0	0
normalized size	1	1.	1.1	2.03	6.6	14.1	0.	0.
time (sec)	N/A	0.021	0.039	0.034	1.266	1.731	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	85	140	0	0	0
normalized size	1	1.	0.92	3.54	5.83	0.	0.	0.
time (sec)	N/A	0.021	0.011	0.095	0.989	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	30	20	63	15	38
normalized size	1	1.	0.89	1.58	1.05	3.32	0.79	2.
time (sec)	N/A	0.006	0.002	0.043	0.955	1.607	0.258	1.18

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	7.971	0.232	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	5.791	0.214	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	88	77	0	0	0	0	0
normalized size	1	1.05	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.087	0.279	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.373	0.058	0.	0.	0.	0.



Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	1.753	0.275	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [178] had the largest ratio of [ 2.417 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	12	0.25
2	A	4	3	1.	12	0.25
3	A	4	3	1.	12	0.25
4	A	4	3	1.	12	0.25
5	A	3	3	1.	10	0.3
6	A	3	2	1.	8	0.25
7	A	1	1	1.	12	0.083
8	A	5	5	1.	12	0.417
9	A	3	3	1.	12	0.25
10	A	4	3	1.	12	0.25
11	A	4	3	1.	12	0.25
12	A	4	3	1.	12	0.25
13	A	16	7	1.	14	0.5
14	A	14	9	1.	14	0.643
15	A	11	7	1.	14	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	9	8	1.	14	0.571
17	A	6	5	1.	12	0.417
18	A	5	5	1.	10	0.5
19	A	6	5	1.	14	0.357
20	A	4	4	1.	14	0.286
21	A	8	7	1.	14	0.5
22	A	8	7	1.	14	0.5
23	A	13	8	1.	14	0.571
24	A	33	11	1.	14	0.786
25	A	24	11	1.	14	0.786
26	A	18	10	1.	14	0.714
27	A	12	9	1.	14	0.643
28	A	8	8	1.	12	0.667
29	A	5	6	1.	10	0.6
30	A	8	6	1.	14	0.429
31	A	5	6	1.	14	0.429
32	A	7	6	1.	14	0.429
33	A	14	11	1.	14	0.786
34	A	16	8	1.	14	0.571
35	A	7	6	1.	16	0.375
36	A	6	6	1.	16	0.375
37	A	6	6	1.	16	0.375
38	A	5	5	1.	16	0.312
39	A	5	5	1.	16	0.312
40	A	6	6	1.	16	0.375
41	A	6	6	1.	16	0.375
42	A	7	6	1.	16	0.375
43	A	0	0	0.	0	0.
44	A	0	0	0.	0	0.
45	A	2	2	1.	14	0.143
46	A	0	0	0.	0	0.
47	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	0	0	0.	0	0.
49	A	0	0	0.	0	0.
50	A	5	4	1.	14	0.286
51	A	4	3	1.	14	0.214
52	A	4	4	1.	14	0.286
53	A	2	2	1.	12	0.167
54	A	2	2	1.	14	0.143
55	A	5	5	1.	14	0.357
56	A	4	4	1.	14	0.286
57	A	4	3	1.	14	0.214
58	A	5	5	1.	14	0.357
59	A	5	5	1.	14	0.357
60	A	5	4	1.	10	0.4
61	A	4	4	1.	14	0.286
62	A	5	5	1.	14	0.357
63	A	5	5	1.	14	0.357
64	C	62	19	5.09	16	1.187
65	B	53	19	3.67	16	1.187
66	C	44	16	5.76	16	1.
67	B	28	12	2.2	14	0.857
68	A	7	6	1.	16	0.375
69	B	24	13	2.72	16	0.812
70	C	46	23	4.09	16	1.438
71	A	102	26	1.	16	1.625
72	A	86	26	1.	16	1.625
73	A	69	21	1.	12	1.75
74	A	47	21	1.	16	1.313
75	A	64	24	1.	16	1.5
76	A	77	24	1.	16	1.5
77	B	155	30	3.4	16	1.875
78	B	82	23	2.91	14	1.643
79	A	9	7	1.	16	0.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	F	0	0	N/A	0	N/A
81	F	0	0	N/A	0	N/A
82	A	17	14	1.	18	0.778
83	A	17	14	1.	18	0.778
84	A	16	13	1.	18	0.722
85	A	16	13	1.	18	0.722
86	A	16	13	1.	18	0.722
87	A	16	13	1.	18	0.722
88	A	17	14	1.	18	0.778
89	A	17	14	1.	18	0.778
90	F	0	0	N/A	0	N/A
91	F	0	0	N/A	0	N/A
92	F	0	0	N/A	0	N/A
93	F	0	0	N/A	0	N/A
94	A	0	0	0.	0	0.
95	A	0	0	0.	0	0.
96	A	3	3	1.	16	0.188
97	A	0	0	0.	0	0.
98	A	0	0	0.	0	0.
99	A	5	4	1.	14	0.286
100	A	4	3	1.	14	0.214
101	A	4	4	1.	14	0.286
102	A	2	2	1.	14	0.143
103	A	2	2	1.	14	0.143
104	A	5	5	1.	14	0.357
105	A	4	4	1.	14	0.286
106	A	4	3	1.	14	0.214
107	A	12	8	1.	14	0.571
108	A	9	8	1.	10	0.8
109	A	11	7	1.	14	0.5
110	A	9	9	1.	14	0.643
111	A	12	8	1.	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	9	9	1.	14	0.643
113	A	11	7	1.	12	0.583
114	A	8	8	1.	14	0.571
115	A	12	8	1.	14	0.571
116	C	62	19	5.09	16	1.187
117	B	53	19	3.67	16	1.187
118	C	44	16	5.76	16	1.
119	B	28	12	2.16	16	0.75
120	A	7	6	1.	16	0.375
121	B	24	13	2.63	16	0.812
122	C	46	23	4.09	16	1.438
123	B	59	24	2.92	16	1.5
124	B	239	32	6.15	16	2.
125	B	155	30	3.45	16	1.875
126	B	82	23	3.	16	1.438
127	A	9	7	1.	16	0.438
128	F	0	0	N/A	0	N/A
129	F	0	0	N/A	0	N/A
130	A	0	0	0.	0	0.
131	A	0	0	0.	0	0.
132	A	3	3	1.	16	0.188
133	A	0	0	0.	0	0.
134	A	0	0	0.	0	0.
135	A	5	4	1.	14	0.286
136	A	5	4	1.	14	0.286
137	A	4	4	1.	12	0.333
138	A	4	3	1.	10	0.3
139	A	2	2	1.	14	0.143
140	A	2	2	1.	14	0.143
141	A	4	4	1.	14	0.286
142	A	5	4	1.	14	0.286
143	C	88	34	6.6	16	2.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	B	73	34	4.89	16	2.125
145	C	58	32	6.92	14	2.286
146	B	31	14	5.	12	1.167
147	A	7	6	1.	16	0.375
148	B	28	12	2.36	16	0.75
149	C	66	23	8.13	16	1.438
150	F	0	0	N/A	0	N/A
151	F	0	0	N/A	0	N/A
152	F	0	0	N/A	0	N/A
153	F	0	0	N/A	0	N/A
154	A	9	7	1.	16	0.438
155	B	82	23	3.07	16	1.438
156	F	0	0	N/A	0	N/A
157	A	6	5	1.	14	0.357
158	A	5	4	1.	14	0.286
159	A	5	5	1.	14	0.357
160	A	3	3	1.	12	0.25
161	A	2	2	1.	14	0.143
162	A	2	2	1.	14	0.143
163	A	5	5	1.	14	0.357
164	A	5	4	1.	14	0.286
165	A	6	6	1.	14	0.429
166	A	6	6	1.	14	0.429
167	A	6	5	1.	10	0.5
168	A	5	5	1.	14	0.357
169	A	6	6	1.	14	0.429
170	A	6	6	1.	14	0.429
171	C	59	33	6.37	16	2.063
172	B	34	19	4.3	14	1.357
173	A	7	6	1.	16	0.375
174	B	28	12	2.09	16	0.75
175	C	67	23	7.94	16	1.438

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	97	33	1.	16	2.063
177	A	79	33	1.	16	2.063
178	A	99	29	1.	12	2.417
179	A	71	29	1.	16	1.812
180	A	104	30	1.	16	1.875
181	A	129	30	1.	16	1.875
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	4	4	1.	16	0.25
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	7	4	1.	16	0.25
188	A	6	4	1.	16	0.25
189	A	5	4	1.	14	0.286
190	A	5	4	1.	12	0.333
191	A	2	2	1.	16	0.125
192	A	4	4	1.	16	0.25
193	A	5	4	1.	16	0.25
194	A	6	4	1.	16	0.25
195	F	0	0	N/A	0	N/A
196	F	0	0	N/A	0	N/A
197	F	0	0	N/A	0	N/A
198	F	0	0	N/A	0	N/A
199	A	7	6	1.	18	0.333
200	F	0	0	N/A	0	N/A
201	F	0	0	N/A	0	N/A
202	F	0	0	N/A	0	N/A
203	F	0	0	N/A	0	N/A
204	F	0	0	N/A	0	N/A
205	F	0	0	N/A	0	N/A
206	A	9	7	1.	18	0.389
207	F	0	0	N/A	0	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	F	0	0	N/A	0	N/A
209	A	3	2	1.	12	0.167
210	A	3	2	1.	12	0.167
211	A	2	2	1.	12	0.167
212	A	4	4	1.	12	0.333
213	A	13	9	1.	16	0.562
214	A	5	5	1.	16	0.312
215	A	13	9	1.	14	0.643
216	A	13	8	1.	12	0.667
217	A	2	2	1.	16	0.125
218	A	12	8	1.	16	0.5
219	A	13	9	1.	16	0.562
220	A	5	5	1.	16	0.312
221	F	0	0	N/A	0	N/A
222	A	7	6	1.	18	0.333
223	F	0	0	N/A	0	N/A
224	A	2	2	1.	14	0.143
225	A	2	2	1.	12	0.167
226	A	3	2	1.	10	0.2
227	A	2	2	1.	14	0.143
228	A	2	2	1.	14	0.143
229	A	2	2	1.	14	0.143
230	A	2	2	1.	14	0.143
231	A	0	0	0.	0	0.
232	A	0	0	0.	0	0.
233	A	7	6	1.	16	0.375
234	A	0	0	0.	0	0.
235	A	0	0	0.	0	0.
236	A	2	2	1.	10	0.2
237	A	2	2	1.	10	0.2
238	A	3	3	1.	4	0.75
239	A	0	0	0.	0	0.

Continued on next page



Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	0	0	0.	0	0.
241	A	3	3	1.05	16	0.188
242	A	0	0	0.	0	0.
243	A	0	0	0.	0	0.



# Chapter 3

## Listing of integrals

### 3.1 $\int x^5 (a + b \tanh^{-1}(cx)) dx$

Optimal. Leaf size=59

$$\frac{1}{6}x^6 (a + b \tanh^{-1}(cx)) + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \tanh^{-1}(cx)}{6c^6} + \frac{bx^5}{30c}$$

[Out] (b\*x)/(6\*c^5) + (b\*x^3)/(18\*c^3) + (b\*x^5)/(30\*c) - (b\*ArcTanh[c\*x])/(6\*c^6) + (x^6\*(a + b\*ArcTanh[c\*x]))/6

---

**Rubi [A]** time = 0.0329148, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 302, 206}

$$\frac{1}{6}x^6 (a + b \tanh^{-1}(cx)) + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \tanh^{-1}(cx)}{6c^6} + \frac{bx^5}{30c}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x)/(6\*c^5) + (b\*x^3)/(18\*c^3) + (b\*x^5)/(30\*c) - (b\*ArcTanh[c\*x])/(6\*c^6) + (x^6\*(a + b\*ArcTanh[c\*x]))/6

Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int x^5 (a + b \tanh^{-1}(cx)) dx &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx)) - \frac{1}{6} (bc) \int \frac{x^6}{1 - c^2 x^2} dx \\ &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx)) - \frac{1}{6} (bc) \int \left( -\frac{1}{c^6} - \frac{x^2}{c^4} - \frac{x^4}{c^2} + \frac{1}{c^6 (1 - c^2 x^2)} \right) dx \\ &= \frac{bx}{6c^5} + \frac{bx^3}{18c^3} + \frac{bx^5}{30c} + \frac{1}{6} x^6 (a + b \tanh^{-1}(cx)) - \frac{b \int \frac{1}{1 - c^2 x^2} dx}{6c^5} \\ &= \frac{bx}{6c^5} + \frac{bx^3}{18c^3} + \frac{bx^5}{30c} - \frac{b \tanh^{-1}(cx)}{6c^6} + \frac{1}{6} x^6 (a + b \tanh^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.0095314, size = 81, normalized size = 1.37

$$\frac{ax^6}{6} + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} + \frac{b \log(1 - cx)}{12c^6} - \frac{b \log(cx + 1)}{12c^6} + \frac{bx^5}{30c} + \frac{1}{6} bx^6 \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c*x]),x]
```

```
[Out] (b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) + (a*x^6)/6 + (b*x^6*ArcT
anh[c*x])/6 + (b*Log[1 - c*x])/(12*c^6) - (b*Log[1 + c*x])/(12*c^6)
```

---

**Maple [A]** time = 0.007, size = 67, normalized size = 1.1

$$\frac{x^6 a}{6} + \frac{bx^6 \operatorname{Arctanh}(cx)}{6} + \frac{bx^5}{30c} + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} + \frac{b \ln(cx-1)}{12c^6} - \frac{b \ln(cx+1)}{12c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arctanh(c*x)),x)`

[Out]  $\frac{1}{6}x^6a + \frac{1}{6}bx^6\operatorname{arctanh}(cx) + \frac{1}{30}bx^5/c + \frac{1}{18}bx^3/c^3 + \frac{1}{6}bx/c^5 + \frac{1}{12/c^6}b\ln(cx-1) - \frac{1}{12/c^6}b\ln(cx+1)$

---

**Maxima [A]** time = 0.969474, size = 95, normalized size = 1.61

$$\frac{1}{6}ax^6 + \frac{1}{180} \left( 30x^6 \operatorname{artanh}(cx) + c \left( \frac{2(3c^4x^5 + 5c^2x^3 + 15x)}{c^6} - \frac{15 \log(cx+1)}{c^7} + \frac{15 \log(cx-1)}{c^7} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{6}ax^6 + \frac{1}{180}(30x^6\operatorname{arctanh}(cx) + c(2(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx+1)/c^7 + 15\log(cx-1)/c^7))*b$

---

**Fricas [A]** time = 1.95422, size = 153, normalized size = 2.59

$$\frac{30ac^6x^6 + 6bc^5x^5 + 10bc^3x^3 + 30bcx + 15(bc^6x^6 - b) \log\left(-\frac{cx+1}{cx-1}\right)}{180c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out]  $\frac{1}{180}(30a*c^6*x^6 + 6*b*c^5*x^5 + 10*b*c^3*x^3 + 30*b*c*x + 15*(b*c^6*x^6 - b)*\log(-(c*x + 1)/(c*x - 1)))/c^6$

---

**Sympy [A]** time = 2.13222, size = 63, normalized size = 1.07

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atanh}(cx)}{6} + \frac{bx^5}{30c} + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \operatorname{atanh}(cx)}{6c^6} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*atanh(c\*x)),x)

[Out] Piecewise((a\*x\*\*6/6 + b\*x\*\*6\*atanh(c\*x)/6 + b\*x\*\*5/(30\*c) + b\*x\*\*3/(18\*c\*\*3) + b\*x/(6\*c\*\*5) - b\*atanh(c\*x)/(6\*c\*\*6), Ne(c, 0)), (a\*x\*\*6/6, True))

---

**Giac [A]** time = 1.173, size = 104, normalized size = 1.76

$$\frac{1}{12} bx^6 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{6} ax^6 + \frac{bx^5}{30c} + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b \log(cx+1)}{12c^6} + \frac{b \log(cx-1)}{12c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] 1/12\*b\*x^6\*log(-(c\*x + 1)/(c\*x - 1)) + 1/6\*a\*x^6 + 1/30\*b\*x^5/c + 1/18\*b\*x^3/c^3 + 1/6\*b\*x/c^5 - 1/12\*b\*log(c\*x + 1)/c^6 + 1/12\*b\*log(c\*x - 1)/c^6

### 3.2 $\int x^4 (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=57

$$\frac{1}{5}x^5 (a + b \tanh^{-1}(cx)) + \frac{bx^2}{10c^3} + \frac{b \log(1 - c^2x^2)}{10c^5} + \frac{bx^4}{20c}$$

[Out] (b\*x^2)/(10\*c^3) + (b\*x^4)/(20\*c) + (x^5\*(a + b\*ArcTanh[c\*x]))/5 + (b\*Log[1 - c^2\*x^2])/(10\*c^5)

**Rubi [A]** time = 0.0426034, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 266, 43}

$$\frac{1}{5}x^5 (a + b \tanh^{-1}(cx)) + \frac{bx^2}{10c^3} + \frac{b \log(1 - c^2x^2)}{10c^5} + \frac{bx^4}{20c}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x^2)/(10\*c^3) + (b\*x^4)/(20\*c) + (x^5\*(a + b\*ArcTanh[c\*x]))/5 + (b\*Log[1 - c^2\*x^2])/(10\*c^5)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

`x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

### Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \tanh^{-1}(cx)) dx &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx)) - \frac{1}{5} (bc) \int \frac{x^5}{1 - c^2 x^2} dx \\
 &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx)) - \frac{1}{10} (bc) \operatorname{Subst} \left( \int \frac{x^2}{1 - c^2 x} dx, x, x^2 \right) \\
 &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx)) - \frac{1}{10} (bc) \operatorname{Subst} \left( \int \left( -\frac{1}{c^4} - \frac{x}{c^2} - \frac{1}{c^4 (-1 + c^2 x)} \right) dx, x, x^2 \right) \\
 &= \frac{bx^2}{10c^3} + \frac{bx^4}{20c} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx)) + \frac{b \log(1 - c^2 x^2)}{10c^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.0085123, size = 62, normalized size = 1.09

$$\frac{ax^5}{5} + \frac{bx^2}{10c^3} + \frac{b \log(1 - c^2 x^2)}{10c^5} + \frac{bx^4}{20c} + \frac{1}{5} bx^5 \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcTanh[c\*x]), x]

[Out] (b\*x^2)/(10\*c^3) + (b\*x^4)/(20\*c) + (a\*x^5)/5 + (b\*x^5\*ArcTanh[c\*x])/5 + (b\*Log[1 - c^2\*x^2])/(10\*c^5)

**Maple [A]** time = 0.007, size = 60, normalized size = 1.1

$$\frac{ax^5}{5} + \frac{bx^5 \operatorname{Artanh}(cx)}{5} + \frac{bx^4}{20c} + \frac{bx^2}{10c^3} + \frac{b \ln(cx - 1)}{10c^5} + \frac{b \ln(cx + 1)}{10c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arctanh(c\*x)), x)

[Out] 1/5\*a\*x^5+1/5\*b\*x^5\*arctanh(c\*x)+1/20\*b\*x^4/c+1/10\*b\*x^2/c^3+1/10/c^5\*b\*ln(c\*x-1)+1/10/c^5\*b\*ln(c\*x+1)



---

**Maxima [A]** time = 0.966233, size = 74, normalized size = 1.3

$$\frac{1}{5}ax^5 + \frac{1}{20}\left(4x^5 \operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2x^2}{c^4} + \frac{2\log(c^2x^2 - 1)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x)),x, algorithm="maxima")

[Out] 1/5\*a\*x^5 + 1/20\*(4\*x^5\*arctanh(c\*x) + c\*((c^2\*x^4 + 2\*x^2)/c^4 + 2\*log(c^2\*x^2 - 1)/c^6))\*b

---

**Fricas [A]** time = 2.05552, size = 153, normalized size = 2.68

$$\frac{2bc^5x^5 \log\left(-\frac{cx+1}{cx-1}\right) + 4ac^5x^5 + bc^4x^4 + 2bc^2x^2 + 2b \log(c^2x^2 - 1)}{20c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] 1/20\*(2\*b\*c^5\*x^5\*log(-(c\*x + 1)/(c\*x - 1)) + 4\*a\*c^5\*x^5 + b\*c^4\*x^4 + 2\*b\*c^2\*x^2 + 2\*b\*log(c^2\*x^2 - 1))/c^5

---

**Sympy [A]** time = 1.61565, size = 68, normalized size = 1.19

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atanh}(cx)}{5} + \frac{bx^4}{20c} + \frac{bx^2}{10c^3} + \frac{b \log\left(x - \frac{1}{c}\right)}{5c^5} + \frac{b \operatorname{atanh}(cx)}{5c^5} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*atanh(c\*x)),x)

[Out] Piecewise((a\*x\*\*5/5 + b\*x\*\*5\*atanh(c\*x)/5 + b\*x\*\*4/(20\*c) + b\*x\*\*2/(10\*c\*\*3) + b\*log(x - 1/c)/(5\*c\*\*5) + b\*atanh(c\*x)/(5\*c\*\*5), Ne(c, 0)), (a\*x\*\*5/5,

True))

---

**Giac [A]** time = 1.15635, size = 84, normalized size = 1.47

$$\frac{1}{10}bx^5 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{5}ax^5 + \frac{bx^4}{20c} + \frac{bx^2}{10c^3} + \frac{b \log(c^2x^2-1)}{10c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] 1/10\*b\*x^5\*log(-(c\*x + 1)/(c\*x - 1)) + 1/5\*a\*x^5 + 1/20\*b\*x^4/c + 1/10\*b\*x^2/c^3 + 1/10\*b\*log(c^2\*x^2 - 1)/c^5

### 3.3 $\int x^3 (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=48

$$\frac{1}{4}x^4(a + b \tanh^{-1}(cx)) + \frac{bx}{4c^3} - \frac{b \tanh^{-1}(cx)}{4c^4} + \frac{bx^3}{12c}$$

[Out] (b\*x)/(4\*c^3) + (b\*x^3)/(12\*c) - (b\*ArcTanh[c\*x])/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x]))/4

**Rubi [A]** time = 0.0289794, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 302, 206}

$$\frac{1}{4}x^4(a + b \tanh^{-1}(cx)) + \frac{bx}{4c^3} - \frac{b \tanh^{-1}(cx)}{4c^4} + \frac{bx^3}{12c}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x)/(4\*c^3) + (b\*x^3)/(12\*c) - (b\*ArcTanh[c\*x])/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x]))/4

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p_.*((d_.)*(x_))^(m_.), x_Symbol]
  >: Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] >: Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] >: Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \tanh^{-1}(cx)) dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx)) - \frac{1}{4}(bc) \int \frac{x^4}{1 - c^2x^2} dx \\
 &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx)) - \frac{1}{4}(bc) \int \left( -\frac{1}{c^4} - \frac{x^2}{c^2} + \frac{1}{c^4(1 - c^2x^2)} \right) dx \\
 &= \frac{bx}{4c^3} + \frac{bx^3}{12c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx)) - \frac{b \int \frac{1}{1 - c^2x^2} dx}{4c^3} \\
 &= \frac{bx}{4c^3} + \frac{bx^3}{12c} - \frac{b \tanh^{-1}(cx)}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))
 \end{aligned}$$

**Mathematica [A]** time = 0.0086165, size = 70, normalized size = 1.46

$$\frac{ax^4}{4} + \frac{bx}{4c^3} + \frac{b \log(1 - cx)}{8c^4} - \frac{b \log(cx + 1)}{8c^4} + \frac{bx^3}{12c} + \frac{1}{4}bx^4 \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x]), x]

[Out] (b\*x)/(4\*c^3) + (b\*x^3)/(12\*c) + (a\*x^4)/4 + (b\*x^4\*ArcTanh[c\*x])/4 + (b\*Log[1 - c\*x])/(8\*c^4) - (b\*Log[1 + c\*x])/(8\*c^4)

**Maple [A]** time = 0.007, size = 58, normalized size = 1.2

$$\frac{x^4a}{4} + \frac{bx^4 \operatorname{Arctanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} + \frac{b \ln(cx - 1)}{8c^4} - \frac{b \ln(cx + 1)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x)), x)

[Out] 1/4\*x^4\*a+1/4\*b\*x^4\*arctanh(c\*x)+1/12\*b\*x^3/c+1/4\*b\*x/c^3+1/8/c^4\*b\*ln(c\*x-1)-1/8/c^4\*b\*ln(c\*x+1)

**Maxima [A]** time = 0.959693, size = 82, normalized size = 1.71

$$\frac{1}{4}ax^4 + \frac{1}{24}\left(6x^4 \operatorname{artanh}(cx) + c\left(\frac{2(c^2x^3 + 3x)}{c^4} - \frac{3\log(cx+1)}{c^5} + \frac{3\log(cx-1)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out] `1/4*a*x^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b`

**Fricas [A]** time = 2.02546, size = 127, normalized size = 2.65

$$\frac{6ac^4x^4 + 2bc^3x^3 + 6bcx + 3(bc^4x^4 - b)\log\left(-\frac{cx+1}{cx-1}\right)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out] `1/24*(6*a*c^4*x^4 + 2*b*c^3*x^3 + 6*b*c*x + 3*(b*c^4*x^4 - b)*log(-(c*x + 1)/(c*x - 1)))/c^4`

**Sympy [A]** time = 1.16716, size = 53, normalized size = 1.1

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \operatorname{atanh}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*atanh(c*x)),x)`

[Out] `Piecewise((a*x**4/4 + b*x**4*atanh(c*x)/4 + b*x**3/(12*c) + b*x/(4*c**3) - b*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))`

**Giac [A]** time = 1.2352, size = 92, normalized size = 1.92

$$\frac{1}{8}bx^4 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{4}ax^4 + \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b \log(cx+1)}{8c^4} + \frac{b \log(cx-1)}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] 1/8\*b\*x^4\*log(-(c\*x + 1)/(c\*x - 1)) + 1/4\*a\*x^4 + 1/12\*b\*x^3/c + 1/4\*b\*x/c^3 - 1/8\*b\*log(c\*x + 1)/c^4 + 1/8\*b\*log(c\*x - 1)/c^4

### 3.4 $\int x^2 (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=46

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) + \frac{b \log(1 - c^2x^2)}{6c^3} + \frac{bx^2}{6c}$$

[Out] (b\*x^2)/(6\*c) + (x^3\*(a + b\*ArcTanh[c\*x]))/3 + (b\*Log[1 - c^2\*x^2])/(6\*c^3)

**Rubi [A]** time = 0.03505, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 266, 43}

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) + \frac{b \log(1 - c^2x^2)}{6c^3} + \frac{bx^2}{6c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x^2)/(6\*c) + (x^3\*(a + b\*ArcTanh[c\*x]))/3 + (b\*Log[1 - c^2\*x^2])/(6\*c^3)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
  > Simp[(((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[(((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] > Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] > Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + b \tanh^{-1}(cx)) dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) - \frac{1}{3}(bc) \int \frac{x^3}{1 - c^2x^2} dx \\
 &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) - \frac{1}{6}(bc) \text{Subst} \left( \int \frac{x}{1 - c^2x} dx, x, x^2 \right) \\
 &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) - \frac{1}{6}(bc) \text{Subst} \left( \int \left( -\frac{1}{c^2} - \frac{1}{c^2(-1 + c^2x)} \right) dx, x, x^2 \right) \\
 &= \frac{bx^2}{6c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx)) + \frac{b \log(1 - c^2x^2)}{6c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0079837, size = 51, normalized size = 1.11

$$\frac{ax^3}{3} + \frac{b \log(1 - c^2x^2)}{6c^3} + \frac{bx^2}{6c} + \frac{1}{3}bx^3 \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x^2)/(6\*c) + (a\*x^3)/3 + (b\*x^3\*ArcTanh[c\*x])/3 + (b\*Log[1 - c^2\*x^2])/(6\*c^3)

**Maple [A]** time = 0.007, size = 51, normalized size = 1.1

$$\frac{x^3 a}{3} + \frac{bx^3 \text{Artanh}(cx)}{3} + \frac{bx^2}{6c} + \frac{b \ln(cx - 1)}{6c^3} + \frac{b \ln(cx + 1)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x)),x)

[Out] 1/3\*x^3\*a+1/3\*b\*x^3\*arctanh(c\*x)+1/6\*b\*x^2/c+1/6/c^3\*b\*ln(c\*x-1)+1/6/c^3\*b\*ln(c\*x+1)



**Maxima [A]** time = 0.968135, size = 59, normalized size = 1.28

$$\frac{1}{3} ax^3 + \frac{1}{6} \left( 2x^3 \operatorname{artanh}(cx) + c \left( \frac{x^2}{c^2} + \frac{\log(c^2x^2 - 1)}{c^4} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/6\*(2\*x^3\*arctanh(c\*x) + c\*(x^2/c^2 + log(c^2\*x^2 - 1)/c^4))\*b

**Fricas [A]** time = 1.96861, size = 127, normalized size = 2.76

$$\frac{bc^3x^3 \log\left(-\frac{cx+1}{cx-1}\right) + 2ac^3x^3 + bc^2x^2 + b \log(c^2x^2 - 1)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] 1/6\*(b\*c^3\*x^3\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a\*c^3\*x^3 + b\*c^2\*x^2 + b\*log(c^2\*x^2 - 1))/c^3

**Sympy [A]** time = 0.944725, size = 58, normalized size = 1.26

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atanh}(cx)}{3} + \frac{bx^2}{6c} + \frac{b \log\left(x - \frac{1}{c}\right)}{3c^3} + \frac{b \operatorname{atanh}(cx)}{3c^3} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x)),x)

[Out] Piecewise((a\*x\*\*3/3 + b\*x\*\*3\*atanh(c\*x)/3 + b\*x\*\*2/(6\*c) + b\*log(x - 1/c)/(3\*c\*\*3) + b\*atanh(c\*x)/(3\*c\*\*3), Ne(c, 0)), (a\*x\*\*3/3, True))

**Giac [A]** time = 1.23154, size = 72, normalized size = 1.57

$$\frac{1}{6}bx^3 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{3}ax^3 + \frac{bx^2}{6c} + \frac{b \log(c^2x^2-1)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] 1/6*b*x^3*log(-(c*x + 1)/(c*x - 1)) + 1/3*a*x^3 + 1/6*b*x^2/c + 1/6*b*log(c^2*x^2 - 1)/c^3
```

### 3.5 $\int x (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=37

$$\frac{1}{2}x^2(a + b \tanh^{-1}(cx)) - \frac{b \tanh^{-1}(cx)}{2c^2} + \frac{bx}{2c}$$

[Out] (b\*x)/(2\*c) - (b\*ArcTanh[c\*x])/(2\*c^2) + (x^2\*(a + b\*ArcTanh[c\*x]))/2

**Rubi [A]** time = 0.0172118, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {5916, 321, 206}

$$\frac{1}{2}x^2(a + b \tanh^{-1}(cx)) - \frac{b \tanh^{-1}(cx)}{2c^2} + \frac{bx}{2c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcTanh[c\*x]), x]

[Out] (b\*x)/(2\*c) - (b\*ArcTanh[c\*x])/(2\*c^2) + (x^2\*(a + b\*ArcTanh[c\*x]))/2

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

$Q[a, 0] \parallel \text{Lt}Q[b, 0]$

### Rubi steps

$$\begin{aligned} \int x(a + b \tanh^{-1}(cx)) dx &= \frac{1}{2}x^2(a + b \tanh^{-1}(cx)) - \frac{1}{2}(bc) \int \frac{x^2}{1 - c^2x^2} dx \\ &= \frac{bx}{2c} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx)) - \frac{b \int \frac{1}{1 - c^2x^2} dx}{2c} \\ &= \frac{bx}{2c} - \frac{b \tanh^{-1}(cx)}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.0072913, size = 59, normalized size = 1.59

$$\frac{ax^2}{2} + \frac{b \log(1 - cx)}{4c^2} - \frac{b \log(cx + 1)}{4c^2} + \frac{1}{2}bx^2 \tanh^{-1}(cx) + \frac{bx}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x]),x]

[Out] (b\*x)/(2\*c) + (a\*x^2)/2 + (b\*x^2\*ArcTanh[c\*x])/2 + (b\*Log[1 - c\*x])/(4\*c^2) - (b\*Log[1 + c\*x])/(4\*c^2)

**Maple [A]** time = 0.007, size = 49, normalized size = 1.3

$$\frac{ax^2}{2} + \frac{bx^2 \text{Artanh}(cx)}{2} + \frac{bx}{2c} + \frac{b \ln(cx - 1)}{4c^2} - \frac{b \ln(cx + 1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c\*x)),x)

[Out] 1/2\*a\*x^2+1/2\*b\*x^2\*arctanh(c\*x)+1/2\*b\*x/c+1/4/c^2\*b\*ln(c\*x-1)-1/4/c^2\*b\*ln(c\*x+1)

**Maxima [A]** time = 0.98538, size = 68, normalized size = 1.84

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \operatorname{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/4\*(2\*x^2\*arctanh(c\*x) + c\*(2\*x/c^2 - log(c\*x + 1)/c^3 + log(c\*x - 1)/c^3))\*b

**Fricas [A]** time = 1.90822, size = 104, normalized size = 2.81

$$\frac{2ac^2x^2 + 2bcx + (bc^2x^2 - b)\log\left(-\frac{cx+1}{cx-1}\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] 1/4\*(2\*a\*c^2\*x^2 + 2\*b\*c\*x + (b\*c^2\*x^2 - b)\*log(-(c\*x + 1)/(c\*x - 1)))/c^2

**Sympy [A]** time = 0.592059, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}(cx)}{2} + \frac{bx}{2c} - \frac{b \operatorname{atanh}(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x)),x)

[Out] Piecewise((a\*x\*\*2/2 + b\*x\*\*2\*atanh(c\*x)/2 + b\*x/(2\*c) - b\*atanh(c\*x)/(2\*c\*\*2), Ne(c, 0)), (a\*x\*\*2/2, True))

**Giac [A]** time = 1.20851, size = 80, normalized size = 2.16

$$\frac{1}{4}bx^2 \log\left(-\frac{cx+1}{cx-1}\right) + \frac{1}{2}ax^2 + \frac{bx}{2c} - \frac{b \log(cx+1)}{4c^2} + \frac{b \log(cx-1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] 1/4\*b\*x^2\*log(-(c\*x + 1)/(c\*x - 1)) + 1/2\*a\*x^2 + 1/2\*b\*x/c - 1/4\*b\*log(c\*x + 1)/c^2 + 1/4\*b\*log(c\*x - 1)/c^2

### 3.6 $\int (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=30

$$ax + \frac{b \log(1 - c^2 x^2)}{2c} + bx \tanh^{-1}(cx)$$

[Out] a\*x + b\*x\*ArcTanh[c\*x] + (b\*Log[1 - c^2\*x^2])/(2\*c)

**Rubi [A]** time = 0.0125908, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5910, 260}

$$ax + \frac{b \log(1 - c^2 x^2)}{2c} + bx \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*x], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x] + (b\*Log[1 - c^2\*x^2])/(2\*c)

#### Rule 5910

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTanh[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned} \int (a + b \tanh^{-1}(cx)) dx &= ax + b \int \tanh^{-1}(cx) dx \\ &= ax + bx \tanh^{-1}(cx) - (bc) \int \frac{x}{1 - c^2 x^2} dx \\ &= ax + bx \tanh^{-1}(cx) + \frac{b \log(1 - c^2 x^2)}{2c} \end{aligned}$$

**Mathematica [A]** time = 0.0028612, size = 30, normalized size = 1.

$$ax + \frac{b \log(1 - c^2 x^2)}{2c} + bx \tanh^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*x], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x] + (b\*Log[1 - c^2\*x^2])/(2\*c)

**Maple [A]** time = 0.002, size = 29, normalized size = 1.

$$ax + bx \operatorname{Artanh}(cx) + \frac{b \ln(-c^2 x^2 + 1)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctanh(c\*x), x)

[Out] a\*x+b\*x\*arctanh(c\*x)+1/2\*b\*ln(-c^2\*x^2+1)/c

**Maxima [A]** time = 0.975951, size = 41, normalized size = 1.37

$$ax + \frac{(2cx \operatorname{artanh}(cx) + \log(-c^2 x^2 + 1))b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x), x, algorithm="maxima")

[Out] a\*x + 1/2\*(2\*c\*x\*arctanh(c\*x) + log(-c^2\*x^2 + 1))\*b/c

**Fricas [A]** time = 1.94706, size = 97, normalized size = 3.23

$$\frac{bcx \log\left(-\frac{cx+1}{cx-1}\right) + 2acx + b \log(c^2 x^2 - 1)}{2c}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x),x, algorithm="fricas")

[Out] 1/2\*(b\*c\*x\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a\*c\*x + b\*log(c^2\*x^2 - 1))/c

**Sympy [A]** time = 0.386652, size = 27, normalized size = 0.9

$$ax + b \begin{cases} x \operatorname{atanh}(cx) + \frac{\log(cx+1)}{c} - \frac{\operatorname{atanh}(cx)}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c\*x),x)

[Out] a\*x + b\*Piecewise((x\*atanh(c\*x) + log(c\*x + 1)/c - atanh(c\*x)/c, Ne(c, 0)), (0, True))

**Giac [A]** time = 1.17245, size = 54, normalized size = 1.8

$$\frac{1}{2} \left( x \log \left( -\frac{cx+1}{cx-1} \right) + \frac{\log(|c^2x^2-1|)}{c} \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x),x, algorithm="giac")

[Out] 1/2\*(x\*log(-(c\*x + 1)/(c\*x - 1)) + log(abs(c^2\*x^2 - 1))/c)\*b + a\*x

$$3.7 \quad \int \frac{a+b \tanh^{-1}(cx)}{x} dx$$

**Optimal.** Leaf size=26

$$-\frac{1}{2}b\text{PolyLog}(2, -cx) + \frac{1}{2}b\text{PolyLog}(2, cx) + a \log(x)$$

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x)])/2 + (b\*PolyLog[2, c\*x])/2

**Rubi [A]** time = 0.0148648, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {5912}

$$-\frac{1}{2}b\text{PolyLog}(2, -cx) + \frac{1}{2}b\text{PolyLog}(2, cx) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/x, x]

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x)])/2 + (b\*PolyLog[2, c\*x])/2

**Rule 5912**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) / ; FreeQ[{a, b, c}, x]

**Rubi steps**

$$\int \frac{a + b \tanh^{-1}(cx)}{x} dx = a \log(x) - \frac{1}{2}b\text{Li}_2(-cx) + \frac{1}{2}b\text{Li}_2(cx)$$

**Mathematica [A]** time = 0.0093242, size = 24, normalized size = 0.92

$$\frac{1}{2}b(\text{PolyLog}(2, cx) - \text{PolyLog}(2, -cx)) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/x,x]

[Out] a\*Log[x] + (b\*(-PolyLog[2, -(c\*x)] + PolyLog[2, c\*x]))/2

**Maple [B]** time = 0.013, size = 47, normalized size = 1.8

$$a \ln(cx) + b \ln(cx) \operatorname{Arctanh}(cx) - \frac{b \operatorname{dilog}(cx)}{2} - \frac{b \operatorname{dilog}(cx+1)}{2} - \frac{b \ln(cx) \ln(cx+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/x,x)

[Out] a\*ln(c\*x)+b\*ln(c\*x)\*arctanh(c\*x)-1/2\*b\*dilog(c\*x)-1/2\*b\*dilog(c\*x+1)-1/2\*b\*ln(c\*x)\*ln(c\*x+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} b \int \frac{\log(cx+1) - \log(-cx+1)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x,x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(c\*x + 1) - log(-c\*x + 1))/x, x) + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{artanh}(cx) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x) + a)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/x,x)

[Out] Integral((a + b\*atanh(c\*x))/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)/x, x)

$$3.8 \quad \int \frac{a+b \tanh^{-1}(cx)}{x^2} dx$$

Optimal. Leaf size=36

$$-\frac{a+b \tanh^{-1}(cx)}{x} - \frac{1}{2}bc \log(1-c^2x^2) + bc \log(x)$$

[Out]  $-\left(\frac{a + b \operatorname{ArcTanh}[c*x]}{x}\right) + b*c*\operatorname{Log}[x] - (b*c*\operatorname{Log}[1 - c^2*x^2])/2$

**Rubi [A]** time = 0.0262703, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5916, 266, 36, 29, 31}

$$-\frac{a+b \tanh^{-1}(cx)}{x} - \frac{1}{2}bc \log(1-c^2x^2) + bc \log(x)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*x])/x^2, x]$

[Out]  $-\left(\frac{a + b*\operatorname{ArcTanh}[c*x]}{x}\right) + b*c*\operatorname{Log}[x] - (b*c*\operatorname{Log}[1 - c^2*x^2])/2$

#### Rule 5916

$\operatorname{Int}[(a_.) + \operatorname{ArcTanh}[(c_.)*(x_.)]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \operatorname{Simp}[\frac{(d*x)^{(m+1)}*(a + b*\operatorname{ArcTanh}[c*x])^p}{d*(m+1)}, x] - \operatorname{Dist}[\frac{(b*c*p)}{d*(m+1)}, \operatorname{Int}[\frac{(d*x)^{(m+1)}*(a + b*\operatorname{ArcTanh}[c*x])^{(p-1)}}{(1 - c^2*x^2)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x$  &&  $\operatorname{IGtQ}[p, 0]$  &&  $(\operatorname{EqQ}[p, 1] \mid \mid \operatorname{IntegerQ}[m])$  &&  $\operatorname{NeQ}[m, -1]$

#### Rule 266

$\operatorname{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol]$   $\rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /;$   $\operatorname{FreeQ}\{a, b, m, n, p\}, x$  &&  $\operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

#### Rule 36

$\operatorname{Int}[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol]$   $\rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*x), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[1/(c + d*x), x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d\}, x$  &&  $\operatorname{NeQ}[b*c - a*d, 0]$

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{x^2} dx &= -\frac{a + b \tanh^{-1}(cx)}{x} + (bc) \int \frac{1}{x(1 - c^2x^2)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx)}{x} + \frac{1}{2}(bc) \operatorname{Subst}\left(\int \frac{1}{x(1 - c^2x)} dx, x, x^2\right) \\
 &= -\frac{a + b \tanh^{-1}(cx)}{x} + \frac{1}{2}(bc) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^2\right) + \frac{1}{2}(bc^3) \operatorname{Subst}\left(\int \frac{1}{1 - c^2x} dx, x, x^2\right) \\
 &= -\frac{a + b \tanh^{-1}(cx)}{x} + bc \log(x) - \frac{1}{2}bc \log(1 - c^2x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.0079244, size = 39, normalized size = 1.08

$$-\frac{a}{x} - \frac{1}{2}bc \log(1 - c^2x^2) + bc \log(x) - \frac{b \tanh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^2, x]
```

```
[Out] -(a/x) - (b*ArcTanh[c*x])/x + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

**Maple [A]** time = 0.01, size = 45, normalized size = 1.3

$$-\frac{a}{x} - \frac{b \operatorname{Arctanh}(cx)}{x} - \frac{cb \ln(cx - 1)}{2} + cb \ln(cx) - \frac{cb \ln(cx + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))/x^2,x)`

[Out]  $-a/x - b/x * \operatorname{arctanh}(c*x) - 1/2 * c * b * \ln(c*x - 1) + c * b * \ln(c*x) - 1/2 * c * b * \ln(c*x + 1)$

**Maxima [A]** time = 0.97607, size = 53, normalized size = 1.47

$$-\frac{1}{2} \left( c(\log(c^2x^2 - 1) - \log(x^2)) + \frac{2 \operatorname{artanh}(cx)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/x^2,x, algorithm="maxima")`

[Out]  $-1/2 * (c * (\log(c^2 * x^2 - 1) - \log(x^2))) + 2 * \operatorname{arctanh}(c*x) / x * b - a/x$

**Fricas [A]** time = 1.87, size = 116, normalized size = 3.22

$$\frac{bcx \log(c^2x^2 - 1) - 2bcx \log(x) + b \log\left(-\frac{cx+1}{cx-1}\right) + 2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/x^2,x, algorithm="fricas")`

[Out]  $-1/2 * (b * c * x * \log(c^2 * x^2 - 1) - 2 * b * c * x * \log(x) + b * \log(-(c*x + 1) / (c*x - 1)) + 2 * a) / x$

**Sympy [A]** time = 0.946261, size = 41, normalized size = 1.14

$$\begin{cases} -\frac{a}{x} + bc \log(x) - bc \log\left(x - \frac{1}{c}\right) - bc \operatorname{atanh}(cx) - \frac{b \operatorname{atanh}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))/x**2,x)`

[Out] Piecewise((-a/x + b\*c\*log(x) - b\*c\*log(x - 1/c) - b\*c\*atanh(c\*x) - b\*atanh(c\*x)/x, Ne(c, 0)), (-a/x, True))

**Giac [A]** time = 1.15233, size = 63, normalized size = 1.75

$$-\frac{1}{2}bc \log(c^2x^2 - 1) + bc \log(x) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{2x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^2,x, algorithm="giac")

[Out] -1/2\*b\*c\*log(c^2\*x^2 - 1) + b\*c\*log(x) - 1/2\*b\*log(-(c\*x + 1)/(c\*x - 1))/x - a/x



$$3.9 \quad \int \frac{a+b \tanh^{-1}(cx)}{x^3} dx$$

**Optimal.** Leaf size=37

$$-\frac{a+b \tanh^{-1}(cx)}{2x^2} + \frac{1}{2}bc^2 \tanh^{-1}(cx) - \frac{bc}{2x}$$

[Out]  $-(b*c)/(2*x) + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)$

**Rubi [A]** time = 0.0216618, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 325, 206}

$$-\frac{a+b \tanh^{-1}(cx)}{2x^2} + \frac{1}{2}bc^2 \tanh^{-1}(cx) - \frac{bc}{2x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/x^3, x]

[Out]  $-(b*c)/(2*x) + (b*c^2*ArcTanh[c*x])/2 - (a + b*ArcTanh[c*x])/(2*x^2)$

#### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx)}{x^3} dx &= -\frac{a + b \tanh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc) \int \frac{1}{x^2(1 - c^2x^2)} dx \\ &= -\frac{bc}{2x} - \frac{a + b \tanh^{-1}(cx)}{2x^2} + \frac{1}{2}(bc^3) \int \frac{1}{1 - c^2x^2} dx \\ &= -\frac{bc}{2x} + \frac{1}{2}bc^2 \tanh^{-1}(cx) - \frac{a + b \tanh^{-1}(cx)}{2x^2} \end{aligned}$$

**Mathematica [A]** time = 0.0083632, size = 59, normalized size = 1.59

$$-\frac{a}{2x^2} - \frac{1}{4}bc^2 \log(1 - cx) + \frac{1}{4}bc^2 \log(cx + 1) - \frac{b \tanh^{-1}(cx)}{2x^2} - \frac{bc}{2x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/x^3,x]

[Out] -a/(2\*x^2) - (b\*c)/(2\*x) - (b\*ArcTanh[c\*x])/(2\*x^2) - (b\*c^2\*Log[1 - c\*x])/4 + (b\*c^2\*Log[1 + c\*x])/4

**Maple [A]** time = 0.01, size = 49, normalized size = 1.3

$$-\frac{a}{2x^2} - \frac{b \operatorname{Artanh}(cx)}{2x^2} - \frac{bc}{2x} - \frac{c^2b \ln(cx - 1)}{4} + \frac{c^2b \ln(cx + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/x^3,x)

[Out] -1/2\*a/x^2-1/2\*b/x^2\*arctanh(c\*x)-1/2\*b\*c/x-1/4\*c^2\*b\*ln(c\*x-1)+1/4\*c^2\*b\*ln(c\*x+1)

**Maxima [A]** time = 0.978649, size = 61, normalized size = 1.65

$$\frac{1}{4} \left( \left( c \log(cx+1) - c \log(cx-1) - \frac{2}{x} \right) c - \frac{2 \operatorname{artanh}(cx)}{x^2} \right) b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^3,x, algorithm="maxima")

[Out] 1/4\*((c\*log(c\*x + 1) - c\*log(c\*x - 1) - 2/x)\*c - 2\*arctanh(c\*x)/x^2)\*b - 1/2\*a/x^2

**Fricas [A]** time = 1.93332, size = 95, normalized size = 2.57

$$\frac{2bcx - (bc^2x^2 - b) \log\left(-\frac{cx+1}{cx-1}\right) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^3,x, algorithm="fricas")

[Out] -1/4\*(2\*b\*c\*x - (b\*c^2\*x^2 - b)\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a)/x^2

**Sympy [A]** time = 0.739748, size = 36, normalized size = 0.97

$$-\frac{a}{2x^2} + \frac{bc^2 \operatorname{atanh}(cx)}{2} - \frac{bc}{2x} - \frac{b \operatorname{atanh}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/x\*\*3,x)

[Out] -a/(2\*x\*\*2) + b\*c\*\*2\*atanh(c\*x)/2 - b\*c/(2\*x) - b\*atanh(c\*x)/(2\*x\*\*2)

**Giac [A]** time = 1.2016, size = 77, normalized size = 2.08

$$\frac{1}{4} bc^2 \log(cx+1) - \frac{1}{4} bc^2 \log(cx-1) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{4x^2} - \frac{bcx+a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*c^2*log(c*x + 1) - 1/4*b*c^2*log(c*x - 1) - 1/4*b*log(-(c*x + 1)/(c*x  
- 1))/x^2 - 1/2*(b*c*x + a)/x^2
```

### 3.10 $\int \frac{a+b \tanh^{-1}(cx)}{x^4} dx$

**Optimal.** Leaf size=54

$$-\frac{a+b \tanh^{-1}(cx)}{3x^3} - \frac{1}{6}bc^3 \log(1-c^2x^2) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{6x^2}$$

[Out]  $-(b*c)/(6*x^2) - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^2])/6$

**Rubi [A]** time = 0.0360336, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 266, 44}

$$-\frac{a+b \tanh^{-1}(cx)}{3x^3} - \frac{1}{6}bc^3 \log(1-c^2x^2) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{6x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/x^4, x]

[Out]  $-(b*c)/(6*x^2) - (a + b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^2])/6$

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{x^4} dx &= -\frac{a + b \tanh^{-1}(cx)}{3x^3} + \frac{1}{3}(bc) \int \frac{1}{x^3(1 - c^2x^2)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx)}{3x^3} + \frac{1}{6}(bc) \text{Subst} \left( \int \frac{1}{x^2(1 - c^2x)} dx, x, x^2 \right) \\
 &= -\frac{a + b \tanh^{-1}(cx)}{3x^3} + \frac{1}{6}(bc) \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{c^2}{x} - \frac{c^4}{-1 + c^2x} \right) dx, x, x^2 \right) \\
 &= -\frac{bc}{6x^2} - \frac{a + b \tanh^{-1}(cx)}{3x^3} + \frac{1}{3}bc^3 \log(x) - \frac{1}{6}bc^3 \log(1 - c^2x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.008483, size = 59, normalized size = 1.09

$$-\frac{a}{3x^3} - \frac{1}{6}bc^3 \log(1 - c^2x^2) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{6x^2} - \frac{b \tanh^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/x^4, x]

[Out] -a/(3\*x^3) - (b\*c)/(6\*x^2) - (b\*ArcTanh[c\*x])/(3\*x^3) + (b\*c^3\*Log[x])/3 - (b\*c^3\*Log[1 - c^2\*x^2])/6

**Maple [A]** time = 0.01, size = 59, normalized size = 1.1

$$-\frac{a}{3x^3} - \frac{b \text{Artanh}(cx)}{3x^3} - \frac{c^3 b \ln(cx - 1)}{6} - \frac{bc}{6x^2} + \frac{c^3 b \ln(cx)}{3} - \frac{c^3 b \ln(cx + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/x^4, x)

[Out] -1/3\*a/x^3-1/3\*b/x^3\*arctanh(c\*x)-1/6\*c^3\*b\*ln(c\*x-1)-1/6\*b\*c/x^2+1/3\*c^3\*b\*ln(c\*x)-1/6\*c^3\*b\*ln(c\*x+1)

---

**Maxima [A]** time = 0.967296, size = 66, normalized size = 1.22

$$-\frac{1}{6} \left( \left( c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + \frac{1}{x^2} \right) c + \frac{2 \operatorname{artanh}(cx)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^4,x, algorithm="maxima")

[Out] -1/6\*((c^2\*log(c^2\*x^2 - 1) - c^2\*log(x^2) + 1/x^2)\*c + 2\*arctanh(c\*x)/x^3)  
\*b - 1/3\*a/x^3

---

**Fricas [A]** time = 1.92786, size = 140, normalized size = 2.59

$$\frac{bc^3 x^3 \log(c^2 x^2 - 1) - 2bc^3 x^3 \log(x) + bcx + b \log\left(-\frac{cx+1}{cx-1}\right) + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^4,x, algorithm="fricas")

[Out] -1/6\*(b\*c^3\*x^3\*log(c^2\*x^2 - 1) - 2\*b\*c^3\*x^3\*log(x) + b\*c\*x + b\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a)/x^3

---

**Sympy [A]** time = 1.5614, size = 70, normalized size = 1.3

$$\begin{cases} -\frac{a}{3x^3} + \frac{bc^3 \log(x)}{3} - \frac{bc^3 \log\left(x - \frac{1}{c}\right)}{3} - \frac{bc^3 \operatorname{atanh}(cx)}{3} - \frac{bc}{6x^2} - \frac{b \operatorname{atanh}(cx)}{3x^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/x\*\*4,x)

[Out] Piecewise((-a/(3\*x\*\*3) + b\*c\*\*3\*log(x)/3 - b\*c\*\*3\*log(x - 1/c)/3 - b\*c\*\*3\*a\*tanh(c\*x)/3 - b\*c/(6\*x\*\*2) - b\*atanh(c\*x)/(3\*x\*\*3), Ne(c, 0)), (-a/(3\*x\*\*3))

, True))

---

**Giac [A]** time = 1.20603, size = 80, normalized size = 1.48

$$-\frac{1}{6}bc^3 \log(c^2x^2 - 1) + \frac{1}{3}bc^3 \log(x) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{6x^3} - \frac{bcx + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^4,x, algorithm="giac")

[Out] -1/6\*b\*c^3\*log(c^2\*x^2 - 1) + 1/3\*b\*c^3\*log(x) - 1/6\*b\*log(-(c\*x + 1)/(c\*x - 1))/x^3 - 1/6\*(b\*c\*x + 2\*a)/x^3



$$3.11 \quad \int \frac{a+b \tanh^{-1}(cx)}{x^5} dx$$

**Optimal.** Leaf size=48

$$-\frac{a+b \tanh^{-1}(cx)}{4x^4} - \frac{bc^3}{4x} + \frac{1}{4}bc^4 \tanh^{-1}(cx) - \frac{bc}{12x^3}$$

[Out]  $-(b*c)/(12*x^3) - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c*x])/(4*x^4)$

**Rubi [A]** time = 0.0266312, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 325, 206}

$$-\frac{a+b \tanh^{-1}(cx)}{4x^4} - \frac{bc^3}{4x} + \frac{1}{4}bc^4 \tanh^{-1}(cx) - \frac{bc}{12x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/x^5, x]

[Out]  $-(b*c)/(12*x^3) - (b*c^3)/(4*x) + (b*c^4*ArcTanh[c*x])/4 - (a + b*ArcTanh[c*x])/(4*x^4)$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx)}{x^5} dx &= -\frac{a + b \tanh^{-1}(cx)}{4x^4} + \frac{1}{4}(bc) \int \frac{1}{x^4(1 - c^2x^2)} dx \\ &= -\frac{bc}{12x^3} - \frac{a + b \tanh^{-1}(cx)}{4x^4} + \frac{1}{4}(bc^3) \int \frac{1}{x^2(1 - c^2x^2)} dx \\ &= -\frac{bc}{12x^3} - \frac{bc^3}{4x} - \frac{a + b \tanh^{-1}(cx)}{4x^4} + \frac{1}{4}(bc^5) \int \frac{1}{1 - c^2x^2} dx \\ &= -\frac{bc}{12x^3} - \frac{bc^3}{4x} + \frac{1}{4}bc^4 \tanh^{-1}(cx) - \frac{a + b \tanh^{-1}(cx)}{4x^4} \end{aligned}$$

**Mathematica [A]** time = 0.0082598, size = 70, normalized size = 1.46

$$-\frac{a}{4x^4} - \frac{bc^3}{4x} - \frac{1}{8}bc^4 \log(1 - cx) + \frac{1}{8}bc^4 \log(cx + 1) - \frac{bc}{12x^3} - \frac{b \tanh^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])/x^5, x]
```

```
[Out] -a/(4*x^4) - (b*c)/(12*x^3) - (b*c^3)/(4*x) - (b*ArcTanh[c*x])/(4*x^4) - (b
*c^4*Log[1 - c*x])/8 + (b*c^4*Log[1 + c*x])/8
```

**Maple [A]** time = 0.01, size = 58, normalized size = 1.2

$$-\frac{a}{4x^4} - \frac{b \operatorname{Arctanh}(cx)}{4x^4} - \frac{c^4 b \ln(cx - 1)}{8} - \frac{bc}{12x^3} - \frac{bc^3}{4x} + \frac{c^4 b \ln(cx + 1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/x^5, x)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctanh(c*x)-1/8*c^4*b*ln(c*x-1)-1/12*b*c/x^3-1/4*b*c^
3/x+1/8*c^4*b*ln(c*x+1)
```

---

**Maxima [A]** time = 0.987132, size = 81, normalized size = 1.69

$$\frac{1}{24} \left( \left( 3c^3 \log(cx+1) - 3c^3 \log(cx-1) - \frac{2(3c^2x^2+1)}{x^3} \right) c - \frac{6 \operatorname{artanh}(cx)}{x^4} \right) b - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^5,x, algorithm="maxima")

[Out] 1/24\*((3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) - 2\*(3\*c^2\*x^2 + 1)/x^3)\*c - 6\*arctanh(c\*x)/x^4)\*b - 1/4\*a/x^4

---

**Fricas [A]** time = 1.92559, size = 117, normalized size = 2.44

$$\frac{6bc^3x^3 + 2bcx - 3(bc^4x^4 - b) \log\left(-\frac{cx+1}{cx-1}\right) + 6a}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^5,x, algorithm="fricas")

[Out] -1/24\*(6\*b\*c^3\*x^3 + 2\*b\*c\*x - 3\*(b\*c^4\*x^4 - b)\*log(-(c\*x + 1)/(c\*x - 1)) + 6\*a)/x^4

---

**Sympy [A]** time = 1.16585, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4} + \frac{bc^4 \operatorname{atanh}(cx)}{4} - \frac{bc^3}{4x} - \frac{bc}{12x^3} - \frac{b \operatorname{atanh}(cx)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/x\*\*5,x)

[Out] -a/(4\*x\*\*4) + b\*c\*\*4\*atanh(c\*x)/4 - b\*c\*\*3/(4\*x) - b\*c/(12\*x\*\*3) - b\*atanh(c\*x)/(4\*x\*\*4)

---

**Giac [A]** time = 1.21371, size = 92, normalized size = 1.92

$$\frac{1}{8} bc^4 \log(cx + 1) - \frac{1}{8} bc^4 \log(cx - 1) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{8x^4} - \frac{3bc^3x^3 + bcx + 3a}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^5,x, algorithm="giac")

[Out] 1/8\*b\*c^4\*log(c\*x + 1) - 1/8\*b\*c^4\*log(c\*x - 1) - 1/8\*b\*log(-(c\*x + 1)/(c\*x - 1))/x^4 - 1/12\*(3\*b\*c^3\*x^3 + b\*c\*x + 3\*a)/x^4

### 3.12 $\int \frac{a+b \tanh^{-1}(cx)}{x^6} dx$

**Optimal.** Leaf size=65

$$-\frac{a+b \tanh^{-1}(cx)}{5x^5} - \frac{bc^3}{10x^2} - \frac{1}{10}bc^5 \log(1-c^2x^2) + \frac{1}{5}bc^5 \log(x) - \frac{bc}{20x^4}$$

[Out]  $-(b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10$

**Rubi [A]** time = 0.0413205, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5916, 266, 44}

$$-\frac{a+b \tanh^{-1}(cx)}{5x^5} - \frac{bc^3}{10x^2} - \frac{1}{10}bc^5 \log(1-c^2x^2) + \frac{1}{5}bc^5 \log(x) - \frac{bc}{20x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/x^6, x]

[Out]  $-(b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (a + b*ArcTanh[c*x])/(5*x^5) + (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10$

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx)}{x^6} dx &= -\frac{a + b \tanh^{-1}(cx)}{5x^5} + \frac{1}{5}(bc) \int \frac{1}{x^5(1 - c^2x^2)} dx \\ &= -\frac{a + b \tanh^{-1}(cx)}{5x^5} + \frac{1}{10}(bc) \text{Subst} \left( \int \frac{1}{x^3(1 - c^2x)} dx, x, x^2 \right) \\ &= -\frac{a + b \tanh^{-1}(cx)}{5x^5} + \frac{1}{10}(bc) \text{Subst} \left( \int \left( \frac{1}{x^3} + \frac{c^2}{x^2} + \frac{c^4}{x} - \frac{c^6}{-1 + c^2x} \right) dx, x, x^2 \right) \\ &= -\frac{bc}{20x^4} - \frac{bc^3}{10x^2} - \frac{a + b \tanh^{-1}(cx)}{5x^5} + \frac{1}{5}bc^5 \log(x) - \frac{1}{10}bc^5 \log(1 - c^2x^2) \end{aligned}$$

**Mathematica [A]** time = 0.0089695, size = 70, normalized size = 1.08

$$-\frac{a}{5x^5} - \frac{bc^3}{10x^2} - \frac{1}{10}bc^5 \log(1 - c^2x^2) + \frac{1}{5}bc^5 \log(x) - \frac{bc}{20x^4} - \frac{b \tanh^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/x^6,x]

[Out] -a/(5\*x^5) - (b\*c)/(20\*x^4) - (b\*c^3)/(10\*x^2) - (b\*ArcTanh[c\*x])/(5\*x^5) + (b\*c^5\*Log[x])/5 - (b\*c^5\*Log[1 - c^2\*x^2])/10

**Maple [A]** time = 0.01, size = 68, normalized size = 1.1

$$-\frac{a}{5x^5} - \frac{b \text{Artanh}(cx)}{5x^5} - \frac{c^5 b \ln(cx - 1)}{10} - \frac{bc}{20x^4} - \frac{bc^3}{10x^2} + \frac{c^5 b \ln(cx)}{5} - \frac{c^5 b \ln(cx + 1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/x^6,x)

[Out] -1/5\*a/x^5-1/5\*b/x^5\*arctanh(c\*x)-1/10\*c^5\*b\*ln(c\*x-1)-1/20\*b\*c/x^4-1/10\*b\*c^3/x^2+1/5\*c^5\*b\*ln(c\*x)-1/10\*c^5\*b\*ln(c\*x+1)

---

**Maxima [A]** time = 0.972758, size = 82, normalized size = 1.26

$$-\frac{1}{20} \left( \left( 2c^4 \log(c^2x^2 - 1) - 2c^4 \log(x^2) + \frac{2c^2x^2 + 1}{x^4} \right) c + \frac{4 \operatorname{artanh}(cx)}{x^5} \right) b - \frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^6,x, algorithm="maxima")

[Out] -1/20\*((2\*c^4\*log(c^2\*x^2 - 1) - 2\*c^4\*log(x^2) + (2\*c^2\*x^2 + 1)/x^4)\*c + 4\*arctanh(c\*x)/x^5)\*b - 1/5\*a/x^5

---

**Fricas [A]** time = 1.97896, size = 166, normalized size = 2.55

$$\frac{2bc^5x^5 \log(c^2x^2 - 1) - 4bc^5x^5 \log(x) + 2bc^3x^3 + bcx + 2b \log\left(-\frac{cx+1}{cx-1}\right) + 4a}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/20\*(2\*b\*c^5\*x^5\*log(c^2\*x^2 - 1) - 4\*b\*c^5\*x^5\*log(x) + 2\*b\*c^3\*x^3 + b\*c\*x + 2\*b\*log(-(c\*x + 1)/(c\*x - 1)) + 4\*a)/x^5

---

**Sympy [A]** time = 2.78227, size = 80, normalized size = 1.23

$$\begin{cases} -\frac{a}{5x^5} + \frac{bc^5 \log(x)}{5} - \frac{bc^5 \log\left(x - \frac{1}{c}\right)}{5} - \frac{bc^5 \operatorname{atanh}(cx)}{5} - \frac{bc^3}{10x^2} - \frac{bc}{20x^4} - \frac{b \operatorname{atanh}(cx)}{5x^5} & \text{for } c \neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/x\*\*6,x)

[Out] Piecewise((-a/(5\*x\*\*5) + b\*c\*\*5\*log(x)/5 - b\*c\*\*5\*log(x - 1/c)/5 - b\*c\*\*5\*a\*tanh(c\*x)/5 - b\*c\*\*3/(10\*x\*\*2) - b\*c/(20\*x\*\*4) - b\*atanh(c\*x)/(5\*x\*\*5), Ne(

`c, 0)), (-a/(5*x**5), True))`

**Giac [A]** time = 1.22231, size = 92, normalized size = 1.42

$$-\frac{1}{10}bc^5 \log(c^2x^2 - 1) + \frac{1}{5}bc^5 \log(x) - \frac{b \log\left(-\frac{cx+1}{cx-1}\right)}{10x^5} - \frac{2bc^3x^3 + bcx + 4a}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/x^6,x, algorithm="giac")`

[Out] `-1/10*b*c^5*log(c^2*x^2 - 1) + 1/5*b*c^5*log(x) - 1/10*b*log(-(c*x + 1)/(c*x - 1))/x^5 - 1/20*(2*b*c^3*x^3 + b*c*x + 4*a)/x^5`



### 3.13 $\int x^5 (a + b \tanh^{-1}(cx))^2 dx$

**Optimal.** Leaf size=145

$$\frac{bx^3(a + b \tanh^{-1}(cx))}{9c^3} + \frac{abx}{3c^5} - \frac{(a + b \tanh^{-1}(cx))^2}{6c^6} + \frac{1}{6}x^6(a + b \tanh^{-1}(cx))^2 + \frac{bx^5(a + b \tanh^{-1}(cx))}{15c} + \frac{b^2x^4}{60c^2} + \frac{4b^3x^3}{45c^3}$$

[Out] (a\*b\*x)/(3\*c^5) + (4\*b^2\*x^2)/(45\*c^4) + (b^2\*x^4)/(60\*c^2) + (b^2\*x\*ArcTanh[c\*x])/(3\*c^5) + (b\*x^3\*(a + b\*ArcTanh[c\*x]))/(9\*c^3) + (b\*x^5\*(a + b\*ArcTanh[c\*x]))/(15\*c) - (a + b\*ArcTanh[c\*x])^2/(6\*c^6) + (x^6\*(a + b\*ArcTanh[c\*x])^2)/6 + (23\*b^2\*Log[1 - c^2\*x^2])/(90\*c^6)

**Rubi [A]** time = 0.326833, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{bx^3(a + b \tanh^{-1}(cx))}{9c^3} + \frac{abx}{3c^5} - \frac{(a + b \tanh^{-1}(cx))^2}{6c^6} + \frac{1}{6}x^6(a + b \tanh^{-1}(cx))^2 + \frac{bx^5(a + b \tanh^{-1}(cx))}{15c} + \frac{b^2x^4}{60c^2} + \frac{4b^3x^3}{45c^3}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (a\*b\*x)/(3\*c^5) + (4\*b^2\*x^2)/(45\*c^4) + (b^2\*x^4)/(60\*c^2) + (b^2\*x\*ArcTanh[c\*x])/(3\*c^5) + (b\*x^3\*(a + b\*ArcTanh[c\*x]))/(9\*c^3) + (b\*x^5\*(a + b\*ArcTanh[c\*x]))/(15\*c) - (a + b\*ArcTanh[c\*x])^2/(6\*c^6) + (x^6\*(a + b\*ArcTanh[c\*x])^2)/6 + (23\*b^2\*Log[1 - c^2\*x^2])/(90\*c^6)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/((d + e*x^2)^(m - 1)), x]
```

```
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \tanh^{-1}(cx))^2 dx &= \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^2 - \frac{1}{3}(bc) \int \frac{x^6 (a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx \\
&= \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^2 + \frac{b \int x^4 (a + b \tanh^{-1}(cx)) dx}{3c} - \frac{b \int \frac{x^4 (a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx}{3c} \\
&= \frac{bx^5 (a + b \tanh^{-1}(cx))}{15c} + \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^2 - \frac{1}{15}b^2 \int \frac{x^5}{1 - c^2x^2} dx + \frac{b \int x^2 (a + b \tanh^{-1}(cx)) dx}{3c} \\
&= \frac{bx^3 (a + b \tanh^{-1}(cx))}{9c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))}{15c} + \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^2 - \frac{1}{30}b^2 \int \frac{x^5}{1 - c^2x^2} dx \\
&= \frac{abx}{3c^5} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{9c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))}{15c} - \frac{(a + b \tanh^{-1}(cx))^2}{6c^6} + \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^2 \\
&= \frac{abx}{3c^5} + \frac{b^2x^2}{30c^4} + \frac{b^2x^4}{60c^2} + \frac{b^2x \tanh^{-1}(cx)}{3c^5} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{9c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))}{15c} \\
&= \frac{abx}{3c^5} + \frac{4b^2x^2}{45c^4} + \frac{b^2x^4}{60c^2} + \frac{b^2x \tanh^{-1}(cx)}{3c^5} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{9c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))}{15c}
\end{aligned}$$

**Mathematica [A]** time = 0.0678579, size = 164, normalized size = 1.13

$$\frac{30a^2c^6x^6 + 12abc^5x^5 + 20abc^3x^3 + 4bcx \tanh^{-1}(cx) (15ac^5x^5 + b(3c^4x^4 + 5c^2x^2 + 15)) + 60abcx + 2b(15a + 23b) \log(1 - cx)}{180c^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (60\*a\*b\*c\*x + 16\*b^2\*c^2\*x^2 + 20\*a\*b\*c^3\*x^3 + 3\*b^2\*c^4\*x^4 + 12\*a\*b\*c^5\*x^5 + 30\*a^2\*c^6\*x^6 + 4\*b\*c\*x\*(15\*a\*c^5\*x^5 + b\*(15 + 5\*c^2\*x^2 + 3\*c^4\*x^4))\*ArcTanh[c\*x] + 30\*b^2\*(-1 + c^6\*x^6)\*ArcTanh[c\*x]^2 + 2\*b\*(15\*a + 23\*b)\*Log[1 - c\*x] - 30\*a\*b\*Log[1 + c\*x] + 46\*b^2\*Log[1 + c\*x])/(180\*c^6)

**Maple [B]** time = 0.023, size = 314, normalized size = 2.2

$$\frac{x^6 a^2}{6} + \frac{b^2 x^6 (\operatorname{Artanh}(cx))^2}{6} + \frac{b^2 \operatorname{Artanh}(cx) x^5}{15c} + \frac{b^2 \operatorname{Artanh}(cx) x^3}{9c^3} + \frac{b^2 x \operatorname{Artanh}(cx)}{3c^5} + \frac{b^2 \operatorname{Artanh}(cx) \ln(cx - 1)}{6c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arctanh(c*x))^2,x)`

[Out]  $\frac{1}{6}x^6a^2 + \frac{1}{6}b^2x^6\operatorname{arctanh}(cx)^2 + \frac{1}{15}cb^2\operatorname{arctanh}(cx)x^5 + \frac{1}{9}c^3b^2\operatorname{arctanh}(cx)x^3 + \frac{1}{3}b^2x\operatorname{arctanh}(cx)/c^5 + \frac{1}{6}c^6b^2\operatorname{arctanh}(cx)\ln(cx-1) - \frac{1}{6}c^6b^2\operatorname{arctanh}(cx)\ln(cx+1) + \frac{1}{24}c^6b^2\ln(cx-1)^2 - \frac{1}{12}c^6b^2\ln(cx-1)\ln(1/2+1/2cx) - \frac{1}{12}c^6b^2\ln(-1/2cx+1/2)\ln(cx+1) + \frac{1}{12}c^6b^2\ln(-1/2cx+1/2)\ln(1/2+1/2cx) + \frac{1}{24}c^6b^2\ln(cx+1)^2 + \frac{1}{60}b^2x^4/c^2 + \frac{4}{45}b^2x^2/c^4 + \frac{23}{90}c^6b^2\ln(cx-1) + \frac{23}{90}c^6b^2\ln(cx+1) + \frac{1}{3}abx^6\operatorname{arctanh}(cx) + \frac{1}{15}cx^5ab + \frac{1}{9}abx^3/c^3 + \frac{1}{3}abx/c^5 + \frac{1}{6}c^6ab\ln(cx-1) - \frac{1}{6}c^6ab\ln(cx+1)$

**Maxima [A]** time = 1.01312, size = 290, normalized size = 2.

$$\frac{1}{6}b^2x^6\operatorname{artanh}(cx)^2 + \frac{1}{6}a^2x^6 + \frac{1}{90}\left(30x^6\operatorname{artanh}(cx) + c\left(\frac{2(3c^4x^5 + 5c^2x^3 + 15x)}{c^6} - \frac{15\log(cx+1)}{c^7} + \frac{15\log(cx-1)}{c^7}\right)\right)ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{6}b^2x^6\operatorname{arctanh}(cx)^2 + \frac{1}{6}a^2x^6 + \frac{1}{90}(30x^6\operatorname{arctanh}(cx) + c(2(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx+1)/c^7 + 15\log(cx-1)/c^7))ab + \frac{1}{360}(4c(2(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx+1)/c^7 + 15\log(cx-1)/c^7)\operatorname{arctanh}(cx) + (6c^4x^4 + 32c^2x^2 - 2(15\log(cx-1) - 46)\log(cx+1) + 15\log(cx+1)^2 + 15\log(cx-1)^2 + 92\log(cx-1))/c^6)b^2$

**Fricas [A]** time = 2.18829, size = 437, normalized size = 3.01

$$\frac{60a^2c^6x^6 + 24abc^5x^5 + 6b^2c^4x^4 + 40abc^3x^3 + 32b^2c^2x^2 + 120abcx + 15(b^2c^6x^6 - b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4(15ab - 23b^2)\log(cx-1)}{360c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="fricas")`

```
[Out] 1/360*(60*a^2*c^6*x^6 + 24*a*b*c^5*x^5 + 6*b^2*c^4*x^4 + 40*a*b*c^3*x^3 + 3
2*b^2*c^2*x^2 + 120*a*b*c*x + 15*(b^2*c^6*x^6 - b^2)*log(-(c*x + 1)/(c*x -
1))^2 - 4*(15*a*b - 23*b^2)*log(c*x + 1) + 4*(15*a*b + 23*b^2)*log(c*x - 1)
+ 4*(15*a*b*c^6*x^6 + 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 + 15*b^2*c*x)*log(-(c*
x + 1)/(c*x - 1)))/c^6
```

**Sympy [A]** time = 4.11455, size = 211, normalized size = 1.46

$$\left\{ \frac{a^2x^6}{6} + \frac{abx^6 \operatorname{atanh}(cx)}{3} + \frac{abx^5}{15c} + \frac{abx^3}{9c^3} + \frac{abx}{3c^5} - \frac{ab \operatorname{atanh}(cx)}{3c^6} + \frac{b^2x^6 \operatorname{atanh}^2(cx)}{6} + \frac{b^2x^5 \operatorname{atanh}(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3 \operatorname{atanh}(cx)}{9c^3} + \frac{4b^2x^2}{45c^4} + \frac{b^2x}{45c^5} + \frac{b^2}{45c^6} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atanh(c*x)/3 + a*b*x**5/(15*c) + a*b*x**3
/(9*c**3) + a*b*x/(3*c**5) - a*b*atanh(c*x)/(3*c**6) + b**2*x**6*atanh(c*x)
**2/6 + b**2*x**5*atanh(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atanh
(c*x)/(9*c**3) + 4*b**2*x**2/(45*c**4) + b**2*x*atanh(c*x)/(3*c**5) + 23*b*
**2*log(x - 1/c)/(45*c**6) - b**2*atanh(c*x)**2/(6*c**6) + 23*b**2*atanh(c*x
)/(45*c**6), Ne(c, 0)), (a**2*x**6/6, True))
```

**Giac [A]** time = 1.30206, size = 259, normalized size = 1.79

$$\frac{1}{6}a^2x^6 + \frac{abx^5}{15c} + \frac{b^2x^4}{60c^2} + \frac{1}{24}\left(b^2x^6 - \frac{b^2}{c^6}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2 + \frac{abx^3}{9c^3} + \frac{1}{90}\left(15abx^6 + \frac{3b^2x^5}{c} + \frac{5b^2x^3}{c^3} + \frac{15b^2x}{c^5}\right)\log\left(-\frac{cx+1}{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/6*a^2*x^6 + 1/15*a*b*x^5/c + 1/60*b^2*x^4/c^2 + 1/24*(b^2*x^6 - b^2/c^6)*
log(-(c*x + 1)/(c*x - 1))^2 + 1/9*a*b*x^3/c^3 + 1/90*(15*a*b*x^6 + 3*b^2*x^
5/c + 5*b^2*x^3/c^3 + 15*b^2*x/c^5)*log(-(c*x + 1)/(c*x - 1)) + 4/45*b^2*x^
2/c^4 + 1/3*a*b*x/c^5 - 1/90*(15*a*b - 23*b^2)*log(c*x + 1)/c^6 + 1/90*(15*
a*b + 23*b^2)*log(c*x - 1)/c^6
```

### 3.14 $\int x^4 \left(a + b \tanh^{-1}(cx)\right)^2 dx$

**Optimal.** Leaf size=162

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{5c^5} + \frac{bx^2(a + b \tanh^{-1}(cx))}{5c^3} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{5c^5} + \frac{1}{5}x^5(a$$

[Out]  $(3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) + (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^5)$

**Rubi [A]** time = 0.302228, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{5c^5} + \frac{bx^2(a + b \tanh^{-1}(cx))}{5c^3} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{5c^5} + \frac{1}{5}x^5(a$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c\*x])^2,x]

[Out]  $(3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) - (3*b^2*ArcTanh[c*x])/(10*c^5) + (b*x^2*(a + b*ArcTanh[c*x]))/(5*c^3) + (b*x^4*(a + b*ArcTanh[c*x]))/(10*c) + (a + b*ArcTanh[c*x])^2/(5*c^5) + (x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/(5*c^5) - (b^2*PolyLog[2, 1 - 2/(1 - c*x)])/(5*c^5)$

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

$c, d, e, f, g, x$  && EqQ[ $c, 2*d$ ] && EqQ[ $e^2*f + d^2*g, 0$ ]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \tanh^{-1}(cx))^2 dx &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^2 - \frac{1}{5} (2bc) \int \frac{x^5 (a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx \\
 &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^2 + \frac{(2b) \int x^3 (a + b \tanh^{-1}(cx)) dx}{5c} - \frac{(2b) \int \frac{x^3 (a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx}{5c} \\
 &= \frac{bx^4 (a + b \tanh^{-1}(cx))}{10c} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^2 - \frac{1}{10} b^2 \int \frac{x^4}{1 - c^2 x^2} dx + \frac{(2b) \int x (a + b \tanh^{-1}(cx)) dx}{5c} \\
 &= \frac{bx^2 (a + b \tanh^{-1}(cx))}{5c^3} + \frac{bx^4 (a + b \tanh^{-1}(cx))}{10c} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^2 \\
 &= \frac{3b^2 x}{10c^4} + \frac{b^2 x^3}{30c^2} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{5c^3} + \frac{bx^4 (a + b \tanh^{-1}(cx))}{10c} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5} \\
 &= \frac{3b^2 x}{10c^4} + \frac{b^2 x^3}{30c^2} - \frac{3b^2 \tanh^{-1}(cx)}{10c^5} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{5c^3} + \frac{bx^4 (a + b \tanh^{-1}(cx))}{10c} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5} \\
 &= \frac{3b^2 x}{10c^4} + \frac{b^2 x^3}{30c^2} - \frac{3b^2 \tanh^{-1}(cx)}{10c^5} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{5c^3} + \frac{bx^4 (a + b \tanh^{-1}(cx))}{10c} + \frac{(a + b \tanh^{-1}(cx))^2}{5c^5}
 \end{aligned}$$

**Mathematica [A]** time = 0.451515, size = 161, normalized size = 0.99

$$\frac{6b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + 6a^2 c^5 x^5 + 3abc^4 x^4 + 6abc^2 x^2 + 6ab \log(c^2 x^2 - 1) + 3b \tanh^{-1}(cx) \left(4ac^5 x^5 + b(c^4 x^4 + 30c^5)\right)}{30c^5}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (-9\*a\*b + 9\*b^2\*c\*x + 6\*a\*b\*c^2\*x^2 + b^2\*c^3\*x^3 + 3\*a\*b\*c^4\*x^4 + 6\*a^2\*c^5\*x^5 + 6\*b^2\*(-1 + c^5\*x^5)\*ArcTanh[c\*x]^2 + 3\*b\*ArcTanh[c\*x]\*(4\*a\*c^5\*x^5 + b\*(-3 + 2\*c^2\*x^2 + c^4\*x^4) - 4\*b\*Log[1 + E^(-2\*ArcTanh[c\*x])]) + 6\*a\*



$b \cdot \text{Log}[-1 + c^2 x^2] + 6 b^2 \text{PolyLog}[2, -E^{-2 \text{ArcTanh}[c x]}] / (30 c^5)$

**Maple [B]** time = 0.016, size = 306, normalized size = 1.9

$$\frac{x^5 a^2}{5} + \frac{x^5 b^2 (\text{Artanh}(cx))^2}{5} + \frac{b^2 \text{Artanh}(cx) x^4}{10c} + \frac{b^2 \text{Artanh}(cx) x^2}{5c^3} + \frac{b^2 \text{Artanh}(cx) \ln(cx-1)}{5c^5} + \frac{b^2 \text{Artanh}(cx) \ln(cx+1)}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*arctanh(c*x))^2,x)`

[Out]  $1/5 x^5 a^2 + 1/5 x^5 b^2 \text{arctanh}(cx)^2 + 1/10 c b^2 \text{arctanh}(cx) x^4 + 1/5 c^3 b^2 \text{arctanh}(cx) x^2 + 1/5 c^5 b^2 \text{arctanh}(cx) \ln(cx-1) + 1/5 c^5 b^2 \text{arctanh}(cx) \ln(cx+1) + 1/30 b^2 x^3 / c^2 + 3/10 b^2 x / c^4 + 3/20 c^5 b^2 \ln(cx-1) - 3/20 c^5 b^2 \ln(cx+1) + 1/20 c^5 b^2 \ln(cx-1)^2 - 1/5 c^5 b^2 \text{dilog}(1/2 + 1/2 cx) - 1/10 c^5 b^2 \ln(cx-1) \ln(1/2 + 1/2 cx) + 1/10 c^5 b^2 \ln(-1/2 cx + 1/2) \ln(cx+1) - 1/10 c^5 b^2 \ln(-1/2 cx + 1/2) \ln(1/2 + 1/2 cx) - 1/20 c^5 b^2 \ln(cx+1)^2 + 2/5 x^5 a b \text{arctanh}(cx) + 1/10 c x^4 a b + 1/5 a b x^2 / c^3 + 1/5 c^5 a b \ln(cx-1) + 1/5 c^5 a b \ln(cx+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out]  $1/5 a^2 x^5 + 1/10 (4 x^5 \text{arctanh}(cx) + c((c^2 x^4 + 2 x^2)/c^4 + 2 \log(c^2 x^2 - 1)/c^6)) a b - 1/36000 (24 c^6 (2 (3 c^4 x^5 + 5 c^2 x^3 + 15 x)/c^{10} - 15 \log(cx+1)/c^{11} + 15 \log(cx-1)/c^{11}) - 45 c^5 ((c^2 x^4 + 2 x^2)/c^8 + 2 \log(c^2 x^2 - 1)/c^{10}) - 108000 c^5 \text{integrate}(1/150 x^5 \log(cx+1)/(c^6 x^2 - c^4), x) + 50 c^4 (2 (c^2 x^3 + 3 x)/c^8 - 3 \log(cx+1)/c^9 + 3 \log(cx-1)/c^9) - 300 c^3 (x^2/c^6 + \log(c^2 x^2 - 1)/c^8) + 900 c^2 (2 x/c^6 - \log(cx+1)/c^7 + \log(cx-1)/c^7) - 540000 c \text{integrate}(1/150 x \log(cx+1)/(c^6 x^2 - c^4), x) - 60 (30 c^5 x^5 \log(cx+1)^2 + (12 c^5 x^5 - 15 c^4 x^4 + 20 c^3 x^3 - 30 c^2 x^2 + 60 c x - 60 (c^5 x^5 + 1) \log(cx+1)) \log(-cx+1))/c^5 - (72 (cx-1)^5 (25 \log(-cx+1)^2 - 10 \log(-cx+1) + 2) + 1125 (cx-1)^4 (8 \log(-cx+1)^2 - 4 \log(-cx+1) + 2) + 1125 (cx-1)^3 (16 \log(-cx+1) - 4 \log(-cx+1) + 2) + 1125 (cx-1)^2 (8 \log(-cx+1) - 4 \log(-cx+1) + 2) + 1125 (cx-1) (4 \log(-cx+1) - 2 \log(-cx+1) + 1) + 1125) \log(-cx+1) + 1125) \log(-cx+1) + 1125$

1) + 1) + 2000\*(c\*x - 1)^3\*(9\*log(-c\*x + 1)^2 - 6\*log(-c\*x + 1) + 2) + 9000\*(c\*x - 1)^2\*(2\*log(-c\*x + 1)^2 - 2\*log(-c\*x + 1) + 1) + 9000\*(c\*x - 1)\*(log(-c\*x + 1)^2 - 2\*log(-c\*x + 1) + 2))/c^5 + 1800\*log(150\*c^6\*x^2 - 150\*c^4)/c^5 - 540000\*integrate(1/150\*log(c\*x + 1)/(c^6\*x^2 - c^4), x))\*b^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^2x^4 \operatorname{artanh}(cx)^2 + 2abx^4 \operatorname{artanh}(cx) + a^2x^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^4\*arctanh(c\*x)^2 + 2\*a\*b\*x^4\*arctanh(c\*x) + a^2\*x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{atanh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*atanh(c\*x))\*\*2,x)

[Out] Integral(x\*\*4\*(a + b\*atanh(c\*x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^2\*x^4, x)

### 3.15 $\int x^3 (a + b \tanh^{-1}(cx))^2 dx$

**Optimal.** Leaf size=113

$$\frac{abx}{2c^3} - \frac{(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 + \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2 \log(1 - c^2x^2)}{3c^4} + \frac{b^2x \tanh^{-1}(cx)}{2c^3}$$

[Out] (a\*b\*x)/(2\*c^3) + (b^2\*x^2)/(12\*c^2) + (b^2\*x\*ArcTanh[c\*x])/(2\*c^3) + (b\*x^3\*(a + b\*ArcTanh[c\*x]))/(6\*c) - (a + b\*ArcTanh[c\*x])^2/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x])^2)/4 + (b^2\*Log[1 - c^2\*x^2])/(3\*c^4)

**Rubi [A]** time = 0.222507, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5916, 5980, 266, 43, 5910, 260, 5948}

$$\frac{abx}{2c^3} - \frac{(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 + \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2 \log(1 - c^2x^2)}{3c^4} + \frac{b^2x \tanh^{-1}(cx)}{2c^3}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (a\*b\*x)/(2\*c^3) + (b^2\*x^2)/(12\*c^2) + (b^2\*x\*ArcTanh[c\*x])/(2\*c^3) + (b\*x^3\*(a + b\*ArcTanh[c\*x]))/(6\*c) - (a + b\*ArcTanh[c\*x])^2/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x])^2)/4 + (b^2\*Log[1 - c^2\*x^2])/(3\*c^4)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

]

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \tanh^{-1}(cx))^2 dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 - \frac{1}{2}(bc) \int \frac{x^4 (a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx \\
&= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 + \frac{b \int x^2 (a + b \tanh^{-1}(cx)) dx}{2c} - \frac{b \int \frac{x^{2(a+b \tanh^{-1}(cx))}}{1-c^2x^2} dx}{2c} \\
&= \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 - \frac{1}{6}b^2 \int \frac{x^3}{1 - c^2x^2} dx + \frac{b \int (a + b \tanh^{-1}(cx))}{2c} \\
&= \frac{abx}{2c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} - \frac{(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 - \frac{1}{12}b^2 \int \frac{x^3}{1 - c^2x^2} dx \\
&= \frac{abx}{2c^3} + \frac{b^2x \tanh^{-1}(cx)}{2c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} - \frac{(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2 \\
&= \frac{abx}{2c^3} + \frac{b^2x^2}{12c^2} + \frac{b^2x \tanh^{-1}(cx)}{2c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))}{6c} - \frac{(a + b \tanh^{-1}(cx))^2}{4c^4} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.0588042, size = 132, normalized size = 1.17

$$\frac{3a^2c^4x^4 + 2abc^3x^3 + 2bcx \tanh^{-1}(cx) (3ac^3x^3 + b(c^2x^2 + 3)) + 6abcx + b(3a + 4b) \log(1 - cx) - 3ab \log(cx + 1) + b^2c^2}{12c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (6\*a\*b\*c\*x + b^2\*c^2\*x^2 + 2\*a\*b\*c^3\*x^3 + 3\*a^2\*c^4\*x^4 + 2\*b\*c\*x\*(3\*a\*c^3\*x^3 + b\*(3 + c^2\*x^2))\*ArcTanh[c\*x] + 3\*b^2\*(-1 + c^4\*x^4)\*ArcTanh[c\*x]^2 + b\*(3\*a + 4\*b)\*Log[1 - c\*x] - 3\*a\*b\*Log[1 + c\*x] + 4\*b^2\*Log[1 + c\*x])/ (12\*c^4)

**Maple [B]** time = 0.015, size = 278, normalized size = 2.5

$$\frac{a^2x^4}{4} + \frac{b^2x^4 (\operatorname{Artanh}(cx))^2}{4} + \frac{b^2 \operatorname{Artanh}(cx) x^3}{6c} + \frac{b^2x \operatorname{Artanh}(cx)}{2c^3} + \frac{b^2 \operatorname{Artanh}(cx) \ln(cx - 1)}{4c^4} - \frac{b^2 \operatorname{Artanh}(cx) \ln(cx + 1)}{4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x))^2,x)

```
[Out] 1/4*a^2*x^4+1/4*b^2*x^4*arctanh(c*x)^2+1/6/c*b^2*arctanh(c*x)*x^3+1/2*b^2*x
*arctanh(c*x)/c^3+1/4/c^4*b^2*arctanh(c*x)*ln(c*x-1)-1/4/c^4*b^2*arctanh(c*
x)*ln(c*x+1)+1/16/c^4*b^2*ln(c*x-1)^2-1/8/c^4*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)
-1/8/c^4*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/8/c^4*b^2*ln(-1/2*c*x+1/2)*ln(1/2
+1/2*c*x)+1/16/c^4*b^2*ln(c*x+1)^2+1/12*b^2*x^2/c^2+1/3/c^4*b^2*ln(c*x-1)+1
/3/c^4*b^2*ln(c*x+1)+1/2*x^4*a*b*arctanh(c*x)+1/6*a*b*x^3/c+1/2*a*b*x/c^3+1
/4/c^4*a*b*ln(c*x-1)-1/4/c^4*a*b*ln(c*x+1)
```

**Maxima [A]** time = 0.976638, size = 255, normalized size = 2.26

$$\frac{1}{4}b^2x^4 \operatorname{artanh}(cx)^2 + \frac{1}{4}a^2x^4 + \frac{1}{12} \left( 6x^4 \operatorname{artanh}(cx) + c \left( \frac{2(c^2x^3 + 3x)}{c^4} - \frac{3 \log(cx + 1)}{c^5} + \frac{3 \log(cx - 1)}{c^5} \right) \right) ab + \frac{1}{48} \left( 4c \left( \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctanh(c*x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c*x) + c*(2*
(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b + 1/48*
(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*arct
anh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x + 1
)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2
```

**Fricas [A]** time = 2.09657, size = 354, normalized size = 3.13

$$\frac{12a^2c^4x^4 + 8abc^3x^3 + 4b^2c^2x^2 + 24abcx + 3(b^2c^4x^4 - b^2) \log\left(-\frac{cx+1}{cx-1}\right)^2 - 4(3ab - 4b^2) \log(cx + 1) + 4(3ab + 4b^2) \log(cx - 1)}{48c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*a^2*c^4*x^4 + 8*a*b*c^3*x^3 + 4*b^2*c^2*x^2 + 24*a*b*c*x + 3*(b^2*
c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*x + 1)
+ 4*(3*a*b + 4*b^2)*log(c*x - 1) + 4*(3*a*b*c^4*x^4 + b^2*c^3*x^3 + 3*b^2*
c*x)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

**Sympy [A]** time = 2.24423, size = 168, normalized size = 1.49

$$\left\{ \begin{array}{l} \frac{a^2x^4}{4} + \frac{abx^4 \operatorname{atanh}(cx)}{2} + \frac{abx^3}{6c} + \frac{abx}{2c^3} - \frac{ab \operatorname{atanh}(cx)}{2c^4} + \frac{b^2x^4 \operatorname{atanh}^2(cx)}{4} + \frac{b^2x^3 \operatorname{atanh}(cx)}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2x \operatorname{atanh}(cx)}{2c^3} + \frac{2b^2 \log\left(x - \frac{1}{c}\right)}{3c^4} - \frac{b^2 \operatorname{atanh}(cx)}{3c^4} \\ \frac{a^2x^4}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x))\*\*2,x)

[Out] Piecewise((a\*\*2\*x\*\*4/4 + a\*b\*x\*\*4\*atanh(c\*x)/2 + a\*b\*x\*\*3/(6\*c) + a\*b\*x/(2\*c\*\*3) - a\*b\*atanh(c\*x)/(2\*c\*\*4) + b\*\*2\*x\*\*4\*atanh(c\*x)\*\*2/4 + b\*\*2\*x\*\*3\*atanh(c\*x)/(6\*c) + b\*\*2\*x\*\*2/(12\*c\*\*2) + b\*\*2\*x\*atanh(c\*x)/(2\*c\*\*3) + 2\*b\*\*2\*log(x - 1/c)/(3\*c\*\*4) - b\*\*2\*atanh(c\*x)\*\*2/(4\*c\*\*4) + 2\*b\*\*2\*atanh(c\*x)/(3\*c\*\*4), Ne(c, 0)), (a\*\*2\*x\*\*4/4, True))

**Giac [A]** time = 1.25719, size = 215, normalized size = 1.9

$$\frac{1}{4} a^2 x^4 + \frac{abx^3}{6c} + \frac{1}{16} \left( b^2 x^4 - \frac{b^2}{c^4} \right) \log \left( -\frac{cx+1}{cx-1} \right)^2 + \frac{b^2 x^2}{12c^2} + \frac{1}{12} \left( 3abx^4 + \frac{b^2 x^3}{c} + \frac{3b^2 x}{c^3} \right) \log \left( -\frac{cx+1}{cx-1} \right) + \frac{abx}{2c^3} - \frac{(3ab - b^2)}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x))^2,x, algorithm="giac")

[Out] 1/4\*a^2\*x^4 + 1/6\*a\*b\*x^3/c + 1/16\*(b^2\*x^4 - b^2/c^4)\*log(-(c\*x + 1)/(c\*x - 1))^2 + 1/12\*b^2\*x^2/c^2 + 1/12\*(3\*a\*b\*x^4 + b^2\*x^3/c + 3\*b^2\*x/c^3)\*log(-(c\*x + 1)/(c\*x - 1)) + 1/2\*a\*b\*x/c^3 - 1/12\*(3\*a\*b - 4\*b^2)\*log(c\*x + 1)/c^4 + 1/12\*(3\*a\*b + 4\*b^2)\*log(c\*x - 1)/c^4

### 3.16 $\int x^2 \left( a + b \tanh^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=130

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{3c^3} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{3c^3} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx))^2 + \frac{bx^2(a}{3c^3}$$

[Out] (b^2\*x)/(3\*c^2) - (b^2\*ArcTanh[c\*x])/(3\*c^3) + (b\*x^2\*(a + b\*ArcTanh[c\*x]))/(3\*c) + (a + b\*ArcTanh[c\*x])^2/(3\*c^3) + (x^3\*(a + b\*ArcTanh[c\*x])^2)/3 - (2\*b\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/(3\*c^3) - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x)])/(3\*c^3)

**Rubi [A]** time = 0.200816, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{3c^3} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{3c^3} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx))^2 + \frac{bx^2(a}{3c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (b^2\*x)/(3\*c^2) - (b^2\*ArcTanh[c\*x])/(3\*c^3) + (b\*x^2\*(a + b\*ArcTanh[c\*x]))/(3\*c) + (a + b\*ArcTanh[c\*x])^2/(3\*c^3) + (x^3\*(a + b\*ArcTanh[c\*x])^2)/3 - (2\*b\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/(3\*c^3) - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x)])/(3\*c^3)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^(m_.)/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```



```
]^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 5984

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx))^2 dx &= \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2 - \frac{1}{3} (2bc) \int \frac{x^3 (a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx \\
&= \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2 + \frac{(2b) \int x (a + b \tanh^{-1}(cx)) dx}{3c} - \frac{(2b) \int \frac{x(a+b \tanh^{-1}(cx))}{1-c^2 x^2} dx}{3c} \\
&= \frac{bx^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2 - \frac{1}{3} b^2 \int \frac{x^2}{1 - c^2} \\
&= \frac{b^2 x}{3c^2} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2 - \frac{2b(a - \\
&= \frac{b^2 x}{3c^2} - \frac{b^2 \tanh^{-1}(cx)}{3c^3} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2 \\
&= \frac{b^2 x}{3c^2} - \frac{b^2 \tanh^{-1}(cx)}{3c^3} + \frac{bx^2 (a + b \tanh^{-1}(cx))}{3c} + \frac{(a + b \tanh^{-1}(cx))^2}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.257234, size = 122, normalized size = 0.94

$$\frac{b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + a^2 c^3 x^3 + abc^2 x^2 + ab \log(c^2 x^2 - 1) + b \tanh^{-1}(cx) \left(2ac^3 x^3 + bc^2 x^2 - 2b \log\left(e^{-2 \tanh^{-1}(cx)}\right)\right)}{3c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (b^2\*c\*x + a\*b\*c^2\*x^2 + a^2\*c^3\*x^3 + b^2\*(-1 + c^3\*x^3)\*ArcTanh[c\*x]^2 + b\*ArcTanh[c\*x]\*(-b + b\*c^2\*x^2 + 2\*a\*c^3\*x^3 - 2\*b\*Log[1 + E^(-2\*ArcTanh[c\*x])])) + a\*b\*Log[-1 + c^2\*x^2] + b^2\*PolyLog[2, -E^(-2\*ArcTanh[c\*x])])/(3\*c^3)

**Maple [B]** time = 0.013, size = 270, normalized size = 2.1

$$\frac{x^3 a^2}{3} + \frac{b^2 x^3 (\text{Artanh}(cx))^2}{3} + \frac{b^2 \text{Artanh}(cx) x^2}{3c} + \frac{b^2 \text{Artanh}(cx) \ln(cx - 1)}{3c^3} + \frac{b^2 \text{Artanh}(cx) \ln(cx + 1)}{3c^3} + \frac{b^2 x}{3c^2} + \frac{b^2 \ln}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x))^2,x)`

[Out]  $\frac{1}{3}x^3a^2 + \frac{1}{3}b^2x^3\operatorname{arctanh}(cx)^2 + \frac{1}{3}cb^2\operatorname{arctanh}(cx)x^2 + \frac{1}{3}c^3b^2\operatorname{arctanh}(cx)\ln(cx-1) + \frac{1}{3}c^3b^2\operatorname{arctanh}(cx)\ln(cx+1) + \frac{1}{3}b^2x/c^2 + \frac{1}{6}c^3b^2\ln(cx-1) - \frac{1}{6}c^3b^2\ln(cx+1) + \frac{1}{12}c^3b^2\ln(cx-1)^2 - \frac{1}{3}c^3b^2\operatorname{dilog}\left(\frac{1}{2} + \frac{1}{2}cx\right) - \frac{1}{6}c^3b^2\ln(cx-1)\ln\left(\frac{1}{2} + \frac{1}{2}cx\right) - \frac{1}{6}c^3b^2\ln\left(-\frac{1}{2}cx + \frac{1}{2}\right)\ln\left(\frac{1}{2} + \frac{1}{2}cx\right) + \frac{1}{6}c^3b^2\ln\left(-\frac{1}{2}cx + \frac{1}{2}\right)\ln(cx+1) - \frac{1}{12}c^3b^2\ln(cx+1)^2 + \frac{2}{3}abx^3\operatorname{arctanh}(cx) + \frac{1}{3}abx^2/c + \frac{1}{3}c^3ab\ln(cx-1) + \frac{1}{3}c^3ab\ln(cx+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3 + \frac{1}{3}\left(2x^3\operatorname{artanh}(cx) + c\left(\frac{x^2}{c^2} + \frac{\log(c^2x^2 - 1)}{c^4}\right)\right)ab - \frac{1}{216}\left(2c^4\left(\frac{2(c^2x^3 + 3x)}{c^6} - \frac{3\log(cx+1)}{c^7} + \frac{3\log(cx-1)}{c^7}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{3}a^2x^3 + \frac{1}{3}(2x^3\operatorname{arctanh}(cx) + c(x^2/c^2 + \log(c^2x^2 - 1)/c^4))ab - \frac{1}{216}(2c^4(2(c^2x^3 + 3x)/c^6 - 3\log(cx+1)/c^7 + 3\log(cx-1)/c^7) - 3c^3(x^2/c^4 + \log(c^2x^2 - 1)/c^6) - 648c^3\operatorname{integrate}(1/9x^3\log(cx+1)/(c^4x^2 - c^2), x) + 9c^2(2x/c^4 - \log(cx+1)/c^5 + \log(cx-1)/c^5) - 324c\operatorname{integrate}(1/9x\log(cx+1)/(c^4x^2 - c^2), x) - 6(3c^3x^3\log(cx+1)^2 + (2c^3x^3 - 3c^2x^2 + 6cx - 6(c^3x^3 + 1)\log(cx+1))\log(-cx+1))/c^3 - (2(cx-1)^3(9\log(-cx+1)^2 - 6\log(-cx+1) + 2) + 27(cx-1)^2(2\log(-cx+1)^2 - 2\log(-cx+1) + 1) + 54(cx-1)(\log(-cx+1)^2 - 2\log(-cx+1) + 2))/c^3 + 18\log(9c^4x^2 - 9c^2)/c^3 - 324\operatorname{integrate}(1/9\log(cx+1)/(c^4x^2 - c^2), x))b^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^2x^2\operatorname{artanh}(cx)^2 + 2abx^2\operatorname{artanh}(cx) + a^2x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2, x)`

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c*x))**2,x)`

[Out] `Integral(x**2*(a + b*atanh(c*x))**2, x)`

---

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x) + a)^2*x^2, x)`

### 3.17 $\int x \left( a + b \tanh^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=75

$$-\frac{(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^2 + \frac{abx}{c} + \frac{b^2 \log(1 - c^2x^2)}{2c^2} + \frac{b^2x \tanh^{-1}(cx)}{c}$$

[Out] (a\*b\*x)/c + (b^2\*x\*ArcTanh[c\*x])/c - (a + b\*ArcTanh[c\*x])^2/(2\*c^2) + (x^2\*(a + b\*ArcTanh[c\*x])^2)/2 + (b^2\*Log[1 - c^2\*x^2])/(2\*c^2)

**Rubi [A]** time = 0.114009, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5916, 5980, 5910, 260, 5948}

$$-\frac{(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^2 + \frac{abx}{c} + \frac{b^2 \log(1 - c^2x^2)}{2c^2} + \frac{b^2x \tanh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] (a\*b\*x)/c + (b^2\*x\*ArcTanh[c\*x])/c - (a + b\*ArcTanh[c\*x])^2/(2\*c^2) + (x^2\*(a + b\*ArcTanh[c\*x])^2)/2 + (b^2\*Log[1 - c^2\*x^2])/(2\*c^2)

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((f_.)*(x_))^(m_.)/((d_.) + (
e_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p]/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x (a + b \tanh^{-1}(cx))^2 dx &= \frac{1}{2}x^2 (a + b \tanh^{-1}(cx))^2 - (bc) \int \frac{x^2 (a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx \\
&= \frac{1}{2}x^2 (a + b \tanh^{-1}(cx))^2 + \frac{b \int (a + b \tanh^{-1}(cx)) dx}{c} - \frac{b \int \frac{a + b \tanh^{-1}(cx)}{1 - c^2x^2} dx}{c} \\
&= \frac{abx}{c} - \frac{(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx))^2 + \frac{b^2 \int \tanh^{-1}(cx) dx}{c} \\
&= \frac{abx}{c} + \frac{b^2x \tanh^{-1}(cx)}{c} - \frac{(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx))^2 - b^2 \int \frac{x}{1 - c^2x^2} dx \\
&= \frac{abx}{c} + \frac{b^2x \tanh^{-1}(cx)}{c} - \frac{(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx))^2 + \frac{b^2 \log(1 - c^2x^2)}{2c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0623569, size = 90, normalized size = 1.2

$$\frac{a^2c^2x^2 + 2abcx + b(a + b) \log(1 - cx) - ab \log(cx + 1) + 2bcx \tanh^{-1}(cx)(acx + b) + b^2(c^2x^2 - 1) \tanh^{-1}(cx)^2 + b^2 \log(c^2x^2 - 1)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*x])^2, x]
```

[Out]  $(2*a*b*c*x + a^2*c^2*x^2 + 2*b*c*x*(b + a*c*x)*\text{ArcTanh}[c*x] + b^2*(-1 + c^2*x^2)*\text{ArcTanh}[c*x]^2 + b*(a + b)*\text{Log}[1 - c*x] - a*b*\text{Log}[1 + c*x] + b^2*\text{Log}[1 + c*x])/(2*c^2)$

**Maple [B]** time = 0.016, size = 239, normalized size = 3.2

$$\frac{a^2x^2}{2} + \frac{x^2b^2(\text{Artanh}(cx))^2}{2} + \frac{b^2x\text{Artanh}(cx)}{c} + \frac{b^2\text{Artanh}(cx)\ln(cx-1)}{2c^2} - \frac{b^2\text{Artanh}(cx)\ln(cx+1)}{2c^2} + \frac{b^2(\ln(cx-1) - \ln(cx+1))}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x))^2,x)`

[Out]  $1/2*a^2*x^2+1/2*x^2*b^2*\text{arctanh}(c*x)^2+b^2*x*\text{arctanh}(c*x)/c+1/2/c^2*b^2*\text{arctanh}(c*x)*\ln(c*x-1)-1/2/c^2*b^2*\text{arctanh}(c*x)*\ln(c*x+1)+1/8/c^2*b^2*\ln(c*x-1)^2-1/4/c^2*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x)+1/2/c^2*b^2*\ln(c*x-1)+1/2/c^2*b^2*\ln(c*x+1)-1/4/c^2*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1)+1/4/c^2*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x)+1/8/c^2*b^2*\ln(c*x+1)^2+b*x^2*a*\text{arctanh}(c*x)+a*b*x/c+1/2/c^2*a*b*\ln(c*x-1)-1/2/c^2*a*b*\ln(c*x+1)$

**Maxima [B]** time = 0.99353, size = 213, normalized size = 2.84

$$\frac{1}{2}b^2x^2\text{artanh}(cx)^2 + \frac{1}{2}a^2x^2 + \frac{1}{2}\left(2x^2\text{artanh}(cx) + c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right)\right)ab + \frac{1}{8}\left(4c\left(\frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3}\right) + \frac{2x}{c} - \frac{\log(cx+1)}{c^2} + \frac{\log(cx-1)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out]  $1/2*b^2*x^2*\text{arctanh}(c*x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*\text{arctanh}(c*x) + c*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3))*a*b + 1/8*(4*c*(2*x/c^2 - \log(c*x + 1)/c^3 + \log(c*x - 1)/c^3)*\text{arctanh}(c*x) - (2*(\log(c*x - 1) - 2)*\log(c*x + 1) - \log(c*x + 1)^2 - \log(c*x - 1)^2 - 4*\log(c*x - 1))/c^2)*b^2$

**Fricas [A]** time = 2.23298, size = 269, normalized size = 3.59

$$\frac{4a^2c^2x^2 + 8abcx + (b^2c^2x^2 - b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2 - 4(ab - b^2)\log(cx+1) + 4(ab + b^2)\log(cx-1) + 4(abc^2x^2 + b^2cx)\log\left(\frac{cx+1}{cx-1}\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*c^2*x^2 + 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(a*b - b^2)*log(c*x + 1) + 4*(a*b + b^2)*log(c*x - 1) + 4*(a*b*c^2*x^2 + b^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

**Sympy [A]** time = 1.08891, size = 114, normalized size = 1.52

$$\left\{ \begin{array}{l} \frac{a^2x^2}{2} + abx^2 \operatorname{atanh}(cx) + \frac{abx}{c} - \frac{ab \operatorname{atanh}(cx)}{c^2} + \frac{b^2x^2 \operatorname{atanh}^2(cx)}{2} + \frac{b^2x \operatorname{atanh}(cx)}{c} + \frac{b^2 \log\left(x - \frac{1}{c}\right)}{c^2} - \frac{b^2 \operatorname{atanh}^2(cx)}{2c^2} + \frac{b^2 \operatorname{atanh}(cx)}{c^2} \\ \frac{a^2x^2}{2} \end{array} \right. \text{ for other}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atanh(c*x) + a*b*x/c - a*b*atanh(c*x)/c**2 + b**2*x**2*atanh(c*x)**2/2 + b**2*x*atanh(c*x)/c + b**2*log(x - 1/c)/c**2 - b**2*atanh(c*x)**2/(2*c**2) + b**2*atanh(c*x)/c**2, Ne(c, 0)), (a**2*x**2/2, True))
```

**Giac [A]** time = 1.21261, size = 163, normalized size = 2.17

$$\frac{1}{2}a^2x^2 + \frac{1}{8}\left(b^2x^2 - \frac{b^2}{c^2}\right)\log\left(\frac{-cx+1}{cx-1}\right)^2 + \frac{abx}{c} + \frac{1}{2}\left(abx^2 + \frac{b^2x}{c}\right)\log\left(\frac{-cx+1}{cx-1}\right) - \frac{(ab-b^2)\log(cx+1)}{2c^2} + \frac{(ab+b^2)\log(cx-1)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*x^2 + 1/8*(b^2*x^2 - b^2/c^2)*log(-(c*x + 1)/(c*x - 1))^2 + a*b*x/c + 1/2*(a*b*x^2 + b^2*x/c)*log(-(c*x + 1)/(c*x - 1)) - 1/2*(a*b - b^2)*log(c*x + 1)/c^2 + 1/2*(a*b + b^2)*log(c*x - 1)/c^2
```



### 3.18 $\int (a + b \tanh^{-1}(cx))^2 dx$

**Optimal.** Leaf size=74

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c} + x(a + b \tanh^{-1}(cx))^2 + \frac{(a + b \tanh^{-1}(cx))^2}{c} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{c}$$

[Out] (a + b\*ArcTanh[c\*x])^2/c + x\*(a + b\*ArcTanh[c\*x])^2 - (2\*b\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/c - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x)])/c

**Rubi [A]** time = 0.101124, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5910, 5984, 5918, 2402, 2315}

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c} + x(a + b \tanh^{-1}(cx))^2 + \frac{(a + b \tanh^{-1}(cx))^2}{c} - \frac{2b \log\left(\frac{2}{1-cx}\right)(a + b \tanh^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^2, x]

[Out] (a + b\*ArcTanh[c\*x])^2/c + x\*(a + b\*ArcTanh[c\*x])^2 - (2\*b\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/c - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x)])/c

#### Rule 5910

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTanh[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

#### Rule 5918

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[(a + b\*ArcTanh[c\*x])^p\*Log[2/(1 + (e\*x)/d)]/e, x] + Dist[(b\*c\*

p)/e, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int (a + b \tanh^{-1}(cx))^2 dx &= x(a + b \tanh^{-1}(cx))^2 - (2bc) \int \frac{x(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx \\
 &= \frac{(a + b \tanh^{-1}(cx))^2}{c} + x(a + b \tanh^{-1}(cx))^2 - (2b) \int \frac{a + b \tanh^{-1}(cx)}{1 - cx} dx \\
 &= \frac{(a + b \tanh^{-1}(cx))^2}{c} + x(a + b \tanh^{-1}(cx))^2 - \frac{2b(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{c} + (2b^2) \int \frac{1}{1-cx} dx \\
 &= \frac{(a + b \tanh^{-1}(cx))^2}{c} + x(a + b \tanh^{-1}(cx))^2 - \frac{2b(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{c} - \frac{(2b^2) \operatorname{Li}_2\left(\frac{2}{1-cx}\right)}{c} \\
 &= \frac{(a + b \tanh^{-1}(cx))^2}{c} + x(a + b \tanh^{-1}(cx))^2 - \frac{2b(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{c} - \frac{b^2 \operatorname{Li}_2\left(\frac{2}{1-cx}\right)}{c}
 \end{aligned}$$

**Mathematica [A]** time = 0.142661, size = 82, normalized size = 1.11

$$\frac{b^2 \operatorname{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + a(acx + b \log(1 - c^2x^2)) + 2b \tanh^{-1}(cx) \left(acx - b \log\left(e^{-2 \tanh^{-1}(cx)} + 1\right)\right) + b^2(cx - 1) \operatorname{Li}_2\left(\frac{2}{1-cx}\right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^2, x]

[Out] (b^2\*(-1 + c\*x)\*ArcTanh[c\*x]^2 + 2\*b\*ArcTanh[c\*x]\*(a\*c\*x - b\*Log[1 + E^(-2\*ArcTanh[c\*x])]) + a\*(a\*c\*x + b\*Log[1 - c^2\*x^2]) + b^2\*PolyLog[2, -E^(-2\*Ar

$c \operatorname{Tanh}[c*x])]/c$

**Maple [A]** time = 0.049, size = 123, normalized size = 1.7

$$x b^2 (\operatorname{Artanh}(cx))^2 + 2xab \operatorname{Artanh}(cx) + \frac{b^2 (\operatorname{Artanh}(cx))^2}{c} - 2 \frac{\operatorname{Artanh}(cx) b^2}{c} \ln \left( \frac{(cx+1)^2}{-c^2 x^2 + 1} + 1 \right) + a^2 x - \frac{b^2}{c} \operatorname{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^2,x)`

[Out] `x*b^2*arctanh(c*x)^2+2*x*a*b*arctanh(c*x)+1/c*b^2*arctanh(c*x)^2-2/c*arctanh(c*x)*ln((c*x+1)^2/(-c^2*x^2+1)+1)*b^2+a^2*x-1/c*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))*b^2+1/c*a*b*ln(-c^2*x^2+1)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4} \left( c^2 \left( \frac{2x}{c^2} - \frac{\log(cx+1)}{c^3} + \frac{\log(cx-1)}{c^3} \right) - 6c \int \frac{x \log(cx+1)}{c^2 x^2 - 1} dx - \frac{(cx-1)(\log(-cx+1)^2 - 2 \log(-cx+1) + 2)}{c} \right) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2,x, algorithm="maxima")`

[Out] `-1/4*(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - 6*c*integrate(x*log(c*x + 1)/(c^2*x^2 - 1), x) - (c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2)/c - (c*x*log(c*x + 1)^2 + 2*(c*x - (c*x + 1)*log(c*x + 1))*log(-c*x + 1))/c + log(c^2*x^2 - 1)/c - 2*integrate(log(c*x + 1)/(c^2*x^2 - 1), x))*b^2 + a^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2,x, algorithm="fricas")`

[Out] `integral(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))**2,x)`

[Out] `Integral((a + b*atanh(c*x))**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x) + a)^2, x)`

$$3.19 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{x} dx$$

**Optimal.** Leaf size=117

$$-b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) + b \operatorname{PolyLog}\left(2, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx)) + \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)$$

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x))]/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x))]/2
```

**Rubi [A]** time = 0.263671, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5914, 6052, 5948, 6058, 6610}

$$-b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) + b \operatorname{PolyLog}\left(2, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx)) + \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^2/x, x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - b*(a + b*ArcTanh[c*x])*
PolyLog[2, 1 - 2/(1 - c*x)] + b*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 -
c*x)] + (b^2*PolyLog[3, 1 - 2/(1 - c*x))]/2 - (b^2*PolyLog[3, -1 + 2/(1 -
c*x))]/2
```

#### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^p_/(x_), x_Symbol] := Simp[2*(a +
b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c^p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x))]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

#### Rule 6052

```
Int[(ArcTanh[u_] * ((a_.) + ArcTanh[(c_.)*(x_)]) * (b_.)^p_)/((d_.) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u])*(a + b*ArcTanh[c*x])^p]/(d +
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u])*(a + b*ArcTanh[c*x])^p]/(d + e*
```

```
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0]
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x]
&& EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_] * ((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.))/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= -Simp[((a + b*ArcTanh[c*x])^p * PolyLog[2, 1 - u]) / (2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1) * PolyLog[2, 1 - u]) / (d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_) * PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w * PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx &= 2(a + b \tanh^{-1}(cx))^2 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - (4bc) \int \frac{(a + b \tanh^{-1}(cx)) \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right)}{1 - c^2x^2} dx \\ &= 2(a + b \tanh^{-1}(cx))^2 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) + (2bc) \int \frac{(a + b \tanh^{-1}(cx)) \log\left(\frac{2}{1 - cx}\right)}{1 - c^2x^2} dx - (2bc) \int \frac{1}{1 - c^2x^2} dx \\ &= 2(a + b \tanh^{-1}(cx))^2 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - b(a + b \tanh^{-1}(cx)) \operatorname{Li}_2\left(1 - \frac{2}{1 - cx}\right) + b(a + b \tanh^{-1}(cx)) \operatorname{Li}_2\left(\frac{2}{1 - cx}\right) - (2bc) \int \frac{1}{1 - c^2x^2} dx \\ &= 2(a + b \tanh^{-1}(cx))^2 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - b(a + b \tanh^{-1}(cx)) \operatorname{Li}_2\left(1 - \frac{2}{1 - cx}\right) + b(a + b \tanh^{-1}(cx)) \operatorname{Li}_2\left(\frac{2}{1 - cx}\right) - (2bc) \int \frac{1}{1 - c^2x^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.0715302, size = 120, normalized size = 1.03

$$\frac{1}{2}b \left( 2 \operatorname{PolyLog}\left(2, \frac{cx + 1}{1 - cx}\right) (a + b \tanh^{-1}(cx)) - 2 \operatorname{PolyLog}\left(2, \frac{cx + 1}{cx - 1}\right) (a + b \tanh^{-1}(cx)) + b \left( \operatorname{PolyLog}\left(3, \frac{cx + 1}{cx - 1}\right) - \operatorname{PolyLog}\left(3, \frac{cx + 1}{1 - cx}\right) \right) \right) - (2bc) \int \frac{1}{1 - c^2x^2} dx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^2*ArcTanh[(1 + c*x)/(-1 + c*x)] + (b*(2*(a + b*ArcTanh[c*x])*PolyLog[2, (1 + c*x)/(1 - c*x)] - 2*(a + b*ArcTanh[c*x])*PolyLog[2, (1 + c*x)/(-1 + c*x)] + b*(-PolyLog[3, (1 + c*x)/(1 - c*x)] + PolyLog[3, (1 + c*x)/(-1 + c*x)])))/2
```

**Maple [C]** time = 0.214, size = 701, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+b^2*ln(c*x)*arctanh(c*x)^2-b^2*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+1/2*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-b^2*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^2*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*b^2*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+2*a*b*ln(c*x)*arctanh(c*x)-a*b*ln(c*x)*ln(c*x+1)-a*b*dilog(c*x)-a*b*dilog(c*x+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2(\log(cx+1) - \log(-cx+1))^2}{4x} + \frac{ab(\log(cx+1) - \log(-cx+1))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

[Out]  $a^2 \log(x) + \text{integrate}(1/4*b^2*(\log(c*x + 1) - \log(-c*x + 1))^2/x + a*b*(\log(c*x + 1) - \log(-c*x + 1))/x, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))**2/x,x)`

[Out] `Integral((a + b*atanh(c*x))**2/x, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x) + a)^2/x, x)`



$$3.20 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{x^2} dx$$

**Optimal.** Leaf size=71

$$b^2(-c)\text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + c(a+b \tanh^{-1}(cx))^2 - \frac{(a+b \tanh^{-1}(cx))^2}{x} + 2bc \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx))$$

[Out] c\*(a + b\*ArcTanh[c\*x])^2 - (a + b\*ArcTanh[c\*x])^2/x + 2\*b\*c\*(a + b\*ArcTanh[c\*x])\*Log[2 - 2/(1 + c\*x)] - b^2\*c\*PolyLog[2, -1 + 2/(1 + c\*x)]

**Rubi [A]** time = 0.145297, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {5916, 5988, 5932, 2447}

$$b^2(-c)\text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + c(a+b \tanh^{-1}(cx))^2 - \frac{(a+b \tanh^{-1}(cx))^2}{x} + 2bc \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx))$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^2/x^2,x]

[Out] c\*(a + b\*ArcTanh[c\*x])^2 - (a + b\*ArcTanh[c\*x])^2/x + 2\*b\*c\*(a + b\*ArcTanh[c\*x])\*Log[2 - 2/(1 + c\*x)] - b^2\*c\*PolyLog[2, -1 + 2/(1 + c\*x)]

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^ (p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] :> Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^p_/((x_.)*((d_) + (e_.)*(x_.))), x
_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] -
Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \tanh^{-1}(cx))^2}{x^2} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{x} + (2bc) \int \frac{a + b \tanh^{-1}(cx)}{x(1 - c^2x^2)} dx \\ &= c(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{x} + (2bc) \int \frac{a + b \tanh^{-1}(cx)}{x(1 + cx)} dx \\ &= c(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{x} + 2bc(a + b \tanh^{-1}(cx)) \log\left(2 - \frac{2}{1 + cx}\right) - (2bc) \int \frac{a + b \tanh^{-1}(cx)}{x(1 + cx)} dx \\ &= c(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{x} + 2bc(a + b \tanh^{-1}(cx)) \log\left(2 - \frac{2}{1 + cx}\right) - b^2 \int \frac{a + b \tanh^{-1}(cx)}{x(1 + cx)} dx \end{aligned}$$

**Mathematica [A]** time = 0.143522, size = 94, normalized size = 1.32

$$\frac{-b^2cx \operatorname{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx)}\right) - a(a + bcx \log(1 - c^2x^2) - 2bcx \log(cx)) + 2b \tanh^{-1}(cx) \left(bcx \log\left(1 - e^{-2 \tanh^{-1}(cx)}\right) - \frac{2}{1 + cx}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*x])^2/x^2, x]
```

```
[Out] (b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(-a + b*c*x*Log[1 - E^(-2
*ArcTanh[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 - c^2*x^2]) - b^2*
c*x*PolyLog[2, E^(-2*ArcTanh[c*x])])/x
```

---

**Maple [B]** time = 0.021, size = 248, normalized size = 3.5

$$-\frac{a^2}{x} - \frac{b^2 (\operatorname{Artanh}(cx))^2}{x} - cb^2 \operatorname{Artanh}(cx) \ln(cx-1) + 2cb^2 \ln(cx) \operatorname{Artanh}(cx) - cb^2 \operatorname{Artanh}(cx) \ln(cx+1) - \frac{cb^2}{x} \ln(cx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^2/x^2,x)`

[Out]  $-a^2/x - b^2/x * \operatorname{arctanh}(c*x)^2 - c*b^2 * \operatorname{arctanh}(c*x) * \ln(c*x-1) + 2*c*b^2 * \ln(c*x) * \operatorname{arctanh}(c*x) - c*b^2 * \operatorname{arctanh}(c*x) * \ln(c*x+1) - 1/4*c*b^2 * \ln(c*x-1)^2 + c*b^2 * \operatorname{dilog}(1/2+1/2*c*x) + 1/2*c*b^2 * \ln(c*x-1) * \ln(1/2+1/2*c*x) + 1/2*c*b^2 * \ln(-1/2*c*x+1/2) * \ln(1/2+1/2*c*x) - 1/2*c*b^2 * \ln(-1/2*c*x+1/2) * \ln(c*x+1) + 1/4*c*b^2 * \ln(c*x+1)^2 - c*b^2 * \operatorname{dilog}(c*x) - c*b^2 * \operatorname{dilog}(c*x+1) - c*b^2 * \ln(c*x) * \ln(c*x+1) - 2*a*b/x * \operatorname{arctanh}(c*x) - c*a*b * \ln(c*x-1) + 2*c*a*b * \ln(c*x) - c*a*b * \ln(c*x+1)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(c(\log(c^2x^2-1) - \log(x^2)) + \frac{2 \operatorname{artanh}(cx)}{x}\right)ab - \frac{1}{4}b^2\left(\frac{\log(-cx+1)^2}{x} + \int -\frac{(cx-1)\log(cx+1)^2 + 2(cx-(cx-1))}{cx^3-x^2} dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")`

[Out]  $-(c*(\log(c^2*x^2-1) - \log(x^2)) + 2*\operatorname{arctanh}(c*x)/x)*a*b - 1/4*b^2*(\log(-c*x+1)^2/x + \operatorname{integrate}(-((c*x-1)*\log(c*x+1)^2 + 2*(c*x-(c*x-1))*\log(c*x+1))*\log(-c*x+1))/(c*x^3-x^2), x)) - a^2/x$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x))**2/x**2,x)`

[Out] `Integral((a + b*atanh(c*x))**2/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x) + a)^2/x^2, x)`

$$3.21 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{x^3} dx$$

**Optimal.** Leaf size=80

$$\frac{1}{2}c^2(a+b \tanh^{-1}(cx))^2 - \frac{(a+b \tanh^{-1}(cx))^2}{2x^2} - \frac{bc(a+b \tanh^{-1}(cx))}{x} - \frac{1}{2}b^2c^2 \log(1-c^2x^2) + b^2c^2 \log(x)$$

[Out] -((b\*c\*(a + b\*ArcTanh[c\*x]))/x) + (c^2\*(a + b\*ArcTanh[c\*x])^2)/2 - (a + b\*ArcTanh[c\*x])^2/(2\*x^2) + b^2\*c^2\*Log[x] - (b^2\*c^2\*Log[1 - c^2\*x^2])/2

**Rubi [A]** time = 0.13302, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5916, 5982, 266, 36, 29, 31, 5948}

$$\frac{1}{2}c^2(a+b \tanh^{-1}(cx))^2 - \frac{(a+b \tanh^{-1}(cx))^2}{2x^2} - \frac{bc(a+b \tanh^{-1}(cx))}{x} - \frac{1}{2}b^2c^2 \log(1-c^2x^2) + b^2c^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^2/x^3,x]

[Out] -((b\*c\*(a + b\*ArcTanh[c\*x]))/x) + (c^2\*(a + b\*ArcTanh[c\*x])^2)/2 - (a + b\*ArcTanh[c\*x])^2/(2\*x^2) + b^2\*c^2\*Log[x] - (b^2\*c^2\*Log[1 - c^2\*x^2])/2

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5982

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.)/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
, x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^
2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(p_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^2}{x^3} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + (bc) \int \frac{a + b \tanh^{-1}(cx)}{x^2(1 - c^2x^2)} dx \\
&= -\frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + (bc) \int \frac{a + b \tanh^{-1}(cx)}{x^2} dx + (bc^3) \int \frac{a + b \tanh^{-1}(cx)}{1 - c^2x^2} dx \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + (b^2c^2) \int \frac{1}{x(1 - cx)} dx \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{2}(b^2c^2) \text{Subst} \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{2}(b^2c^2) \text{Subst} \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{2x^2} + b^2c^2 \log(x) - \frac{1}{2}b^2c^2 \log(1 - cx)
\end{aligned}$$

**Mathematica [A]** time = 0.0676631, size = 101, normalized size = 1.26

$$\frac{a^2 + bc^2x^2(a + b) \log(1 - cx) - bc^2x^2(a - b) \log(cx + 1) + 2abcx + 2b \tanh^{-1}(cx)(a + bcx) - 2b^2c^2x^2 \log(x) - b^2(c^2x^2 - 1) \log(x)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])^2/x^3,x]

[Out]  $-\frac{(a^2 + 2abcx + 2b^2c^2x^2 \log(1 - cx) - 2b^2c^2x^2 \log(cx + 1) + 2abcx + 2b \operatorname{Arctanh}(cx)(a + bcx) - 2b^2c^2x^2 \log(x) - b^2(c^2x^2 - 1) \log(x))}{2x^2}$

**Maple [B]** time = 0.019, size = 253, normalized size = 3.2

$$-\frac{a^2}{2x^2} - \frac{b^2(\operatorname{Arctanh}(cx))^2}{2x^2} - \frac{cb^2 \operatorname{Arctanh}(cx)}{x} - \frac{c^2b^2 \operatorname{Arctanh}(cx) \ln(cx - 1)}{2} + \frac{c^2b^2 \operatorname{Arctanh}(cx) \ln(cx + 1)}{2} - \frac{c^2b^2(\ln(cx - 1) + \ln(cx + 1))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^2/x^3,x)

[Out]  $-1/2*a^2/x^2-1/2*b^2/x^2*\operatorname{arctanh}(c*x)^2-c*b^2*\operatorname{arctanh}(c*x)/x-1/2*c^2*b^2*\operatorname{arctanh}(c*x)*\ln(c*x-1)+1/2*c^2*b^2*\operatorname{arctanh}(c*x)*\ln(c*x+1)-1/8*c^2*b^2*\ln(c*x-1)^2+1/4*c^2*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x)-1/2*c^2*b^2*\ln(c*x-1)+c^2*b^2*\ln(c*x)-1/2*c^2*b^2*\ln(c*x+1)-1/4*c^2*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x)+1/4*c^2*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1)-1/8*c^2*b^2*\ln(c*x+1)^2-a*b/x^2*\operatorname{arctanh}(c*x)-a*b*c/x-1/2*c^2*a*b*\ln(c*x-1)+1/2*c^2*a*b*\ln(c*x+1)$

**Maxima [B]** time = 1.00399, size = 204, normalized size = 2.55

$$\frac{1}{2} \left( \left( c \log(cx+1) - c \log(cx-1) - \frac{2}{x} \right) c - \frac{2 \operatorname{artanh}(cx)}{x^2} \right) ab + \frac{1}{8} \left( (2(\log(cx-1) - 2)\log(cx+1) - \log(cx+1)^2 - \log(cx-1)^2 - 4\log(cx-1) + 8\log(x))c^2 + 4(c \log(cx+1) - c \log(cx-1) - \frac{2}{x})c^2 \operatorname{arctanh}(cx) \right) b^2 - \frac{1}{2} b^2 \operatorname{arctanh}(cx)^2 / x^2 - \frac{1}{2} a^2 / x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")`

[Out]  $1/2*((c*\log(c*x + 1) - c*\log(c*x - 1) - 2/x)*c - 2*\operatorname{arctanh}(c*x)/x^2)*a*b + 1/8*((2*(\log(c*x - 1) - 2)*\log(c*x + 1) - \log(c*x + 1)^2 - \log(c*x - 1)^2 - 4*\log(c*x - 1) + 8*\log(x))*c^2 + 4*(c*\log(c*x + 1) - c*\log(c*x - 1) - 2/x)*c^2*\operatorname{arctanh}(c*x))*b^2 - 1/2*b^2*\operatorname{arctanh}(c*x)^2/x^2 - 1/2*a^2/x^2$

**Fricas [A]** time = 2.36725, size = 300, normalized size = 3.75

$$\frac{8b^2c^2x^2 \log(x) + 4(ab - b^2)c^2x^2 \log(cx+1) - 4(ab + b^2)c^2x^2 \log(cx-1) - 8abcx + (b^2c^2x^2 - b^2) \log\left(-\frac{cx+1}{cx-1}\right)^2 - 4a^2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")`

[Out]  $1/8*(8*b^2*c^2*x^2*\log(x) + 4*(a*b - b^2)*c^2*x^2*\log(c*x + 1) - 4*(a*b + b^2)*c^2*x^2*\log(c*x - 1) - 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*\log(-(c*x + 1)/(c*x - 1)))^2 - 4*a^2 - 4*(b^2*c*x + a*b)*\log(-(c*x + 1)/(c*x - 1)))/x^2$

**Sympy [A]** time = 1.45457, size = 126, normalized size = 1.58

$$\left\{ \begin{array}{l} -\frac{a^2}{2x^2} + abc^2 \operatorname{atanh}(cx) - \frac{abc}{x} - \frac{ab \operatorname{atanh}(cx)}{x^2} + b^2c^2 \log(x) - b^2c^2 \log\left(x - \frac{1}{c}\right) + \frac{b^2c^2 \operatorname{atanh}^2(cx)}{2} - b^2c^2 \operatorname{atanh}(cx) - \frac{b^2c \operatorname{atanh}(cx)}{x} \\ -\frac{a^2}{2x^2} \end{array} \right.$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*2/x\*\*3,x)

[Out] Piecewise((-a\*\*2/(2\*x\*\*2) + a\*b\*c\*\*2\*atanh(c\*x) - a\*b\*c/x - a\*b\*atanh(c\*x)/x\*\*2 + b\*\*2\*c\*\*2\*log(x) - b\*\*2\*c\*\*2\*log(x - 1/c) + b\*\*2\*c\*\*2\*atanh(c\*x)\*\*2/2 - b\*\*2\*c\*\*2\*atanh(c\*x) - b\*\*2\*c\*atanh(c\*x)/x - b\*\*2\*atanh(c\*x)\*\*2/(2\*x\*\*2), Ne(c, 0)), (-a\*\*2/(2\*x\*\*2), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^2/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^2/x^3, x)

$$3.22 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{x^4} dx$$

**Optimal.** Leaf size=130

$$-\frac{1}{3}b^2c^3 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + \frac{1}{3}c^3(a+b \tanh^{-1}(cx))^2 + \frac{2}{3}bc^3 \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx)) - \frac{bc(a+b \tanh^{-1}(cx))}{3x^2}$$

[Out]  $-(b^2*c^2)/(3*x) + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/3$

**Rubi [A]** time = 0.230829, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447}

$$-\frac{1}{3}b^2c^3 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + \frac{1}{3}c^3(a+b \tanh^{-1}(cx))^2 + \frac{2}{3}bc^3 \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx)) - \frac{bc(a+b \tanh^{-1}(cx))}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^2/x^4, x]

[Out]  $-(b^2*c^2)/(3*x) + (b^2*c^3*ArcTanh[c*x])/3 - (b*c*(a + b*ArcTanh[c*x]))/(3*x^2) + (c^3*(a + b*ArcTanh[c*x])^2)/3 - (a + b*ArcTanh[c*x])^2/(3*x^3) + (2*b*c^3*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)])/3 - (b^2*c^3*PolyLog[2, -1 + 2/(1 + c*x)])/3$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5982

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_.)/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x]

, x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^2}{x^4} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{3x^3} + \frac{1}{3}(2bc) \int \frac{a + b \tanh^{-1}(cx)}{x^3(1 - c^2x^2)} dx \\
&= -\frac{(a + b \tanh^{-1}(cx))^2}{3x^3} + \frac{1}{3}(2bc) \int \frac{a + b \tanh^{-1}(cx)}{x^3} dx + \frac{1}{3}(2bc^3) \int \frac{a + b \tanh^{-1}(cx)}{x(1 - c^2x^2)} dx \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{3x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{3x^3} + \frac{1}{3}(b^2c^2) \int \frac{dx}{x^2(1 - c^2x^2)} \\
&= -\frac{b^2c^2}{3x} - \frac{bc(a + b \tanh^{-1}(cx))}{3x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{3x^3} + \frac{2}{3}bc^3(a + b \tanh^{-1}(cx)) \\
&= -\frac{b^2c^2}{3x} + \frac{1}{3}b^2c^3 \tanh^{-1}(cx) - \frac{bc(a + b \tanh^{-1}(cx))}{3x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.34386, size = 145, normalized size = 1.12

$$\frac{b^2c^3x^3 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx)}\right) + a^2 - 2abc^3x^3 \log(cx) + abc^3x^3 \log(1 - c^2x^2) + b \tanh^{-1}(cx) \left(2a - bc^3x^3 - 2bc^3x^3 \log\left(\frac{1 - c^2x^2}{1 + c^2x^2}\right)\right)}{3x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^2/x^4, x]

[Out]  $-(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - c^3*x^3)*\text{ArcTanh}[c*x]^2 + b*\text{ArcTanh}[c*x]*(2*a + b*c*x - b*c^3*x^3 - 2*b*c^3*x^3*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c*x])}])) - 2*a*b*c^3*x^3*\text{Log}[c*x] + a*b*c^3*x^3*\text{Log}[1 - c^2*x^2] + b^2*c^3*x^3*\text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c*x])}])]/(3*x^3)$

**Maple [B]** time = 0.02, size = 339, normalized size = 2.6

$$\frac{a^2}{3x^3} - \frac{b^2(\text{Artanh}(cx))^2}{3x^3} - \frac{c^3b^2\text{Artanh}(cx)\ln(cx-1)}{3} - \frac{cb^2\text{Artanh}(cx)}{3x^2} + \frac{2c^3b^2\ln(cx)\text{Artanh}(cx)}{3} - \frac{c^3b^2\text{Artanh}(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^2/x^4, x)

```
[Out] -1/3*a^2/x^3-1/3*b^2/x^3*arctanh(c*x)^2-1/3*c^3*b^2*arctanh(c*x)*ln(c*x-1)-
1/3*c*b^2*arctanh(c*x)/x^2+2/3*c^3*b^2*ln(c*x)*arctanh(c*x)-1/3*c^3*b^2*arc
tanh(c*x)*ln(c*x+1)-1/3*b^2*c^2/x-1/6*c^3*b^2*ln(c*x-1)+1/6*c^3*b^2*ln(c*x+
1)-1/12*c^3*b^2*ln(c*x-1)^2+1/3*c^3*b^2*dilog(1/2+1/2*c*x)+1/6*c^3*b^2*ln(c
*x-1)*ln(1/2+1/2*c*x)+1/6*c^3*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-1/6*c^3*
b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+1/12*c^3*b^2*ln(c*x+1)^2-1/3*c^3*b^2*dilog(c
*x)-1/3*c^3*b^2*dilog(c*x+1)-1/3*c^3*b^2*ln(c*x)*ln(c*x+1)-2/3*a*b/x^3*arct
anh(c*x)-1/3*c^3*a*b*ln(c*x-1)-1/3*c^3*a*b/x^2+2/3*c^3*a*b*ln(c*x)-1/3*c^3*a*
b*ln(c*x+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3} \left( c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + \frac{1}{x^2} c + \frac{2 \operatorname{artanh}(cx)}{x^3} \right) ab - \frac{1}{12} b^2 \left( \frac{\log(-cx + 1)^2}{x^3} + 3 \int -\frac{3(cx - 1) \log(cx + 1)^2}{x^3} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3)
*a*b - 1/12*b^2*(log(-c*x + 1)^2/x^3 + 3*integrate(-1/3*(3*(c*x - 1)*log(c*
x + 1)^2 + 2*(c*x - 3*(c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^4),
x)) - 1/3*a^2/x^3
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^4, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*2/x\*\*4,x)

[Out] Integral((a + b\*atanh(c\*x))\*\*2/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^2/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^2/x^4, x)

$$3.23 \quad \int \frac{(a+b \tanh^{-1}(cx))^2}{x^5} dx$$

**Optimal.** Leaf size=117

$$\frac{1}{4}c^4(a+b \tanh^{-1}(cx))^2 - \frac{bc^3(a+b \tanh^{-1}(cx))}{2x} - \frac{bc(a+b \tanh^{-1}(cx))}{6x^3} - \frac{(a+b \tanh^{-1}(cx))^2}{4x^4} - \frac{b^2c^2}{12x^2} - \frac{1}{3}b^2c^4 \log\left(\frac{1-c^2x^2}{1-c^2x^2}\right)$$

[Out]  $-(b^2c^2)/(12x^2) - (bc*(a + b*ArcTanh[c*x]))/(6x^3) - (bc^3*(a + b*ArcTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3$

**Rubi [A]** time = 0.228329, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5916, 5982, 266, 44, 36, 29, 31, 5948}

$$\frac{1}{4}c^4(a+b \tanh^{-1}(cx))^2 - \frac{bc^3(a+b \tanh^{-1}(cx))}{2x} - \frac{bc(a+b \tanh^{-1}(cx))}{6x^3} - \frac{(a+b \tanh^{-1}(cx))^2}{4x^4} - \frac{b^2c^2}{12x^2} - \frac{1}{3}b^2c^4 \log\left(\frac{1-c^2x^2}{1-c^2x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^2/x^5, x]

[Out]  $-(b^2c^2)/(12x^2) - (bc*(a + b*ArcTanh[c*x]))/(6x^3) - (bc^3*(a + b*ArcTanh[c*x]))/(2*x) + (c^4*(a + b*ArcTanh[c*x])^2)/4 - (a + b*ArcTanh[c*x])^2/(4*x^4) + (2*b^2*c^4*Log[x])/3 - (b^2*c^4*Log[1 - c^2*x^2])/3$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x])^(p-1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5982

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_.))/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] :> Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m+2)\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x]

2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[  
Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b,  
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[  
ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &  
& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m  
+ n + 2, 0])

### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[b/(b\*c  
- a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x],  
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 5948

Int[((a\_) + ArcTanh[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symb  
ol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b,  
, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^2}{x^5} dx &= -\frac{(a + b \tanh^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc) \int \frac{a + b \tanh^{-1}(cx)}{x^4(1 - c^2x^2)} dx \\
&= -\frac{(a + b \tanh^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc) \int \frac{a + b \tanh^{-1}(cx)}{x^4} dx + \frac{1}{2}(bc^3) \int \frac{a + b \tanh^{-1}(cx)}{x^2(1 - c^2x^2)} dx \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{6x^3} - \frac{(a + b \tanh^{-1}(cx))^2}{4x^4} + \frac{1}{6}(b^2c^2) \int \frac{1}{x^3(1 - c^2x^2)} dx + \frac{1}{2}(bc^3) \int \frac{1}{x^2(1 - c^2x^2)} dx \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{6x^3} - \frac{bc^3(a + b \tanh^{-1}(cx))}{2x} + \frac{1}{4}c^4(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{4x^4} \\
&= -\frac{bc(a + b \tanh^{-1}(cx))}{6x^3} - \frac{bc^3(a + b \tanh^{-1}(cx))}{2x} + \frac{1}{4}c^4(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{4x^4} \\
&= -\frac{b^2c^2}{12x^2} - \frac{bc(a + b \tanh^{-1}(cx))}{6x^3} - \frac{bc^3(a + b \tanh^{-1}(cx))}{2x} + \frac{1}{4}c^4(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{4x^4} \\
&= -\frac{b^2c^2}{12x^2} - \frac{bc(a + b \tanh^{-1}(cx))}{6x^3} - \frac{bc^3(a + b \tanh^{-1}(cx))}{2x} + \frac{1}{4}c^4(a + b \tanh^{-1}(cx))^2 - \frac{(a + b \tanh^{-1}(cx))^2}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0691125, size = 164, normalized size = 1.4

$$\frac{3a^2 + 6abc^3x^3 + 3abc^4x^4 \log(1 - cx) - 3abc^4x^4 \log(cx + 1) + 2b \tanh^{-1}(cx) (3a + 3bc^3x^3 + bcx) + 2abcx + b^2c^2x^2 - 8}{12x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])^2/x^5, x]

[Out]  $-(3a^2 + 2a*bcx + b^2c^2x^2 + 6a*bc^3x^3 + 2b*(3a + bcx + 3b*c^3x^3)*\text{ArcTanh}[c*x] - 3b^2*(-1 + c^4x^4)*\text{ArcTanh}[c*x]^2 - 8b^2c^4x^4*\text{Log}[x] + 3a*bc^4x^4*\text{Log}[1 - cx] + 4b^2c^4x^4*\text{Log}[1 - cx] - 3a*bc^4x^4*\text{Log}[1 + cx] + 4b^2c^4x^4*\text{Log}[1 + cx])/(12x^4)$

**Maple [B]** time = 0.026, size = 290, normalized size = 2.5

$$-\frac{a^2}{4x^4} - \frac{b^2(\text{Artanh}(cx))^2}{4x^4} - \frac{c^4b^2\text{Artanh}(cx)\ln(cx-1)}{4} - \frac{cb^2\text{Artanh}(cx)}{6x^3} - \frac{c^3b^2\text{Artanh}(cx)}{2x} + \frac{c^4b^2\text{Artanh}(cx)\ln}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^2/x^5,x)`

[Out] 
$$-1/4*a^2/x^4-1/4*b^2/x^4*arctanh(c*x)^2-1/4*c^4*b^2*arctanh(c*x)*\ln(c*x-1)-1/6*c*b^2*arctanh(c*x)/x^3-1/2*c^3*b^2*arctanh(c*x)/x+1/4*c^4*b^2*arctanh(c*x)*\ln(c*x+1)-1/16*c^4*b^2*\ln(c*x-1)^2+1/8*c^4*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x)-1/8*c^4*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x)+1/8*c^4*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1)-1/16*c^4*b^2*\ln(c*x+1)^2-1/3*c^4*b^2*\ln(c*x-1)-1/12*b^2*c^2/x^2+2/3*c^4*b^2*\ln(c*x)-1/3*c^4*b^2*\ln(c*x+1)-1/2*a*b/x^4*arctanh(c*x)-1/4*c^4*a*b*\ln(c*x-1)-1/6*a*b*c/x^3-1/2*c^3*a*b/x+1/4*c^4*a*b*\ln(c*x+1)$$

**Maxima [B]** time = 0.997564, size = 302, normalized size = 2.58

$$\frac{1}{12} \left( \left( 3c^3 \log(cx+1) - 3c^3 \log(cx-1) - \frac{2(3c^2x^2+1)}{x^3} \right) c - \frac{6 \operatorname{artanh}(cx)}{x^4} \right) ab + \frac{1}{48} \left( \left( 32c^2 \log(x) - \frac{3c^2x^2 \log(cx+1)^2}{x^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")`

[Out] 
$$1/12*((3*c^3*\log(c*x + 1) - 3*c^3*\log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c - 6*arctanh(c*x)/x^4)*a*b + 1/48*((32*c^2*\log(x) - (3*c^2*x^2*\log(c*x + 1)^2 + 3*c^2*x^2*\log(c*x - 1)^2 + 16*c^2*x^2*\log(c*x - 1) - 2*(3*c^2*x^2*\log(c*x - 1) - 8*c^2*x^2)*\log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*\log(c*x + 1) - 3*c^3*\log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2 - 1/4*b^2*arctanh(c*x)^2/x^4 - 1/4*a^2/x^4$$

**Fricas [A]** time = 2.37999, size = 386, normalized size = 3.3

$$\frac{32b^2c^4x^4 \log(x) + 4(3ab - 4b^2)c^4x^4 \log(cx+1) - 4(3ab + 4b^2)c^4x^4 \log(cx-1) - 24abc^3x^3 - 4b^2c^2x^2 - 8abcx + 3(b^2c^4x^4 \log(x) + 4(3ab - 4b^2)c^4x^4 \log(cx+1) - 4(3ab + 4b^2)c^4x^4 \log(cx-1) - 24abc^3x^3 - 4b^2c^2x^2 - 8abcx + 3b^2c^4x^4 \log(x))}{48x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")`

[Out] 
$$1/48*(32*b^2*c^4*x^4*\log(x) + 4*(3*a*b - 4*b^2)*c^4*x^4*\log(c*x + 1) - 4*(3*a*b + 4*b^2)*c^4*x^4*\log(c*x - 1) - 24*a*b*c^3*x^3 - 4*b^2*c^2*x^2 - 8*a*b$$

$*c*x + 3*(b^2*c^4*x^4 - b^2)*\log(-(c*x + 1)/(c*x - 1))^2 - 12*a^2 - 4*(3*b^2*c^3*x^3 + b^2*c*x + 3*a*b)*\log(-(c*x + 1)/(c*x - 1))/x^4$

**Sympy [A]** time = 2.60062, size = 184, normalized size = 1.57

$$\left\{ \begin{array}{l} -\frac{a^2}{4x^4} + \frac{abc^4 \operatorname{atanh}(cx)}{2} - \frac{abc^3}{2x} - \frac{abc}{6x^3} - \frac{ab \operatorname{atanh}(cx)}{2x^4} + \frac{2b^2c^4 \log(x)}{3} - \frac{2b^2c^4 \log\left(x - \frac{1}{c}\right)}{3} + \frac{b^2c^4 \operatorname{atanh}^2(cx)}{4} - \frac{2b^2c^4 \operatorname{atanh}(cx)}{3} - \frac{b^2c^3 \operatorname{atanh}(cx)}{2x} \\ -\frac{a^2}{4x^4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*2/x\*\*5,x)

[Out] Piecewise((-a\*\*2/(4\*x\*\*4) + a\*b\*c\*\*4\*atanh(c\*x)/2 - a\*b\*c\*\*3/(2\*x) - a\*b\*c/(6\*x\*\*3) - a\*b\*atanh(c\*x)/(2\*x\*\*4) + 2\*b\*\*2\*c\*\*4\*log(x)/3 - 2\*b\*\*2\*c\*\*4\*log(x - 1/c)/3 + b\*\*2\*c\*\*4\*atanh(c\*x)\*\*2/4 - 2\*b\*\*2\*c\*\*4\*atanh(c\*x)/3 - b\*\*2\*c\*\*3\*atanh(c\*x)/(2\*x) - b\*\*2\*c\*\*2/(12\*x\*\*2) - b\*\*2\*c\*atanh(c\*x)/(6\*x\*\*3) - b\*\*2\*atanh(c\*x)\*\*2/(4\*x\*\*4), Ne(c, 0)), (-a\*\*2/(4\*x\*\*4), True))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^2/x^5,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^2/x^5, x)

### 3.24 $\int x^5 (a + b \tanh^{-1}(cx))^3 dx$

**Optimal.** Leaf size=247

$$\frac{23b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{30c^6} + \frac{b^2 x^4 (a + b \tanh^{-1}(cx))}{20c^2} + \frac{4b^2 x^2 (a + b \tanh^{-1}(cx))}{15c^4} - \frac{23b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{15c^6}$$

[Out]  $(19b^3x)/(60c^5) + (b^3x^3)/(60c^3) - (19b^3\text{ArcTanh}[cx])/(60c^6) + (4b^2x^2(a + b\text{ArcTanh}[cx]))/(15c^4) + (b^2x^4(a + b\text{ArcTanh}[cx]))/(20c^2) + (23b(a + b\text{ArcTanh}[cx])^2)/(30c^6) + (bx(a + b\text{ArcTanh}[cx])^2)/(2c^5) + (bx^3(a + b\text{ArcTanh}[cx])^2)/(6c^3) + (bx^5(a + b\text{ArcTanh}[cx])^2)/(10c) - (a + b\text{ArcTanh}[cx])^3/(6c^6) + (x^6(a + b\text{ArcTanh}[cx])^3)/6 - (23b^2(a + b\text{ArcTanh}[cx])\text{Log}[2/(1 - cx)])/(15c^6) - (23b^3\text{PolyLog}[2, 1 - 2/(1 - cx)])/(30c^6)$

**Rubi [A]** time = 0.963608, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {5916, 5980, 302, 206, 321, 5984, 5918, 2402, 2315, 5910, 5948}

$$\frac{23b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{30c^6} + \frac{b^2 x^4 (a + b \tanh^{-1}(cx))}{20c^2} + \frac{4b^2 x^2 (a + b \tanh^{-1}(cx))}{15c^4} - \frac{23b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{15c^6}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x])^3,x]

[Out]  $(19b^3x)/(60c^5) + (b^3x^3)/(60c^3) - (19b^3\text{ArcTanh}[cx])/(60c^6) + (4b^2x^2(a + b\text{ArcTanh}[cx]))/(15c^4) + (b^2x^4(a + b\text{ArcTanh}[cx]))/(20c^2) + (23b(a + b\text{ArcTanh}[cx])^2)/(30c^6) + (bx(a + b\text{ArcTanh}[cx])^2)/(2c^5) + (bx^3(a + b\text{ArcTanh}[cx])^2)/(6c^3) + (bx^5(a + b\text{ArcTanh}[cx])^2)/(10c) - (a + b\text{ArcTanh}[cx])^3/(6c^6) + (x^6(a + b\text{ArcTanh}[cx])^3)/6 - (23b^2(a + b\text{ArcTanh}[cx])\text{Log}[2/(1 - cx)])/(15c^6) - (23b^3\text{PolyLog}[2, 1 - 2/(1 - cx)])/(30c^6)$

#### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol]  
 :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In

tegerQ[m]) && NeQ[m, -1]

### Rule 5980

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*((f\_.)\*(x\_.))^(m\_)) / ((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcTanh[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcTanh[c\*x])^p) / (d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 302

Int[(x\_)^(m\_) / ((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x] / Rt[a, 2]) / (Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 321

Int[((c\_.)\*(x\_.))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1)) / (b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1)) / (b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)\*(x\_.)) / ((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1) / (b\*e\*(p + 1)), x] + Dist[1 / (c\*d), Int[(a + b\*ArcTanh[c\*x])^p / (1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5918

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.) / ((d\_) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p \* Log[2 / (1 + (e\*x)/d)]) / e, x] + Dist[(b\*c\*p) / e, Int[((a + b\*ArcTanh[c\*x])^(p - 1) \* Log[2 / (1 + (e\*x)/d)]) / (1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p, x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^p/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x^5 (a + b \tanh^{-1}(cx))^3 dx &= \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^3 - \frac{1}{2}(bc) \int \frac{x^6 (a + b \tanh^{-1}(cx))^2}{1 - c^2x^2} dx \\
&= \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^3 + \frac{b \int x^4 (a + b \tanh^{-1}(cx))^2 dx}{2c} - \frac{b \int \frac{x^4 (a + b \tanh^{-1}(cx))^2}{1 - c^2x^2} dx}{2c} \\
&= \frac{bx^5 (a + b \tanh^{-1}(cx))^2}{10c} + \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^3 - \frac{1}{5}b^2 \int \frac{x^5 (a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx + \dots \\
&= \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{6c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))^2}{10c} + \frac{1}{6}x^6 (a + b \tanh^{-1}(cx))^3 + \frac{b \int (a + b \tanh^{-1}(cx))}{6c^3} \\
&= \frac{b^2x^4 (a + b \tanh^{-1}(cx))}{20c^2} + \frac{bx (a + b \tanh^{-1}(cx))^2}{2c^5} + \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{6c^3} + \frac{bx^5 (a + b \tanh^{-1}(cx))}{6c^3} \\
&= \frac{4b^2x^2 (a + b \tanh^{-1}(cx))}{15c^4} + \frac{b^2x^4 (a + b \tanh^{-1}(cx))}{20c^2} + \frac{23b (a + b \tanh^{-1}(cx))^2}{30c^6} + \frac{bx (a + b \tanh^{-1}(cx))}{30c^6} \\
&= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} + \frac{4b^2x^2 (a + b \tanh^{-1}(cx))}{15c^4} + \frac{b^2x^4 (a + b \tanh^{-1}(cx))}{20c^2} + \frac{23b (a + b \tanh^{-1}(cx))}{30c^6} \\
&= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} - \frac{19b^3 \tanh^{-1}(cx)}{60c^6} + \frac{4b^2x^2 (a + b \tanh^{-1}(cx))}{15c^4} + \frac{b^2x^4 (a + b \tanh^{-1}(cx))}{20c^2} \\
&= \frac{19b^3x}{60c^5} + \frac{b^3x^3}{60c^3} - \frac{19b^3 \tanh^{-1}(cx)}{60c^6} + \frac{4b^2x^2 (a + b \tanh^{-1}(cx))}{15c^4} + \frac{b^2x^4 (a + b \tanh^{-1}(cx))}{20c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.703991, size = 305, normalized size = 1.23

$$46b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + b \tanh^{-1}(cx) \left(30a^2c^6x^6 + 4abcx(3c^4x^4 + 5c^2x^2 + 15) + b^2(3c^4x^4 + 16c^2x^2 - 19) - 9\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (-19\*a\*b^2 + 30\*a^2\*b\*c\*x + 19\*b^3\*c\*x + 16\*a\*b^2\*c^2\*x^2 + 10\*a^2\*b\*c^3\*x^3 + b^3\*c^3\*x^3 + 3\*a\*b^2\*c^4\*x^4 + 6\*a^2\*b\*c^5\*x^5 + 10\*a^3\*c^6\*x^6 + 2\*b^2\*(b\*(-23 + 15\*c\*x + 5\*c^3\*x^3 + 3\*c^5\*x^5) + 15\*a\*(-1 + c^6\*x^6))\*ArcTanh[c\*x]^2 + 10\*b^3\*(-1 + c^6\*x^6)\*ArcTanh[c\*x]^3 + b\*ArcTanh[c\*x]\*(30\*a^2\*c^6\*x^6 + 4\*a\*b\*c\*x\*(15 + 5\*c^2\*x^2 + 3\*c^4\*x^4) + b^2\*(-19 + 16\*c^2\*x^2 + 3\*c^4\*x^4) - 9)

$$4x^4) - 92b^2 \text{Log}[1 + E^{(-2 \text{ArcTanh}[c*x])}] + 15a^2b \text{Log}[1 - c*x] - 15a^2b \text{Log}[1 + c*x] + 46ab^2 \text{Log}[1 - c^2x^2] + 46b^3 \text{PolyLog}[2, -E^{(-2 \text{ArcTanh}[c*x])}]] / (60c^6)$$

**Maple [C]** time = 0.754, size = 1330, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^5(a+b \cdot \text{arctanh}(c \cdot x))^3, x)$

[Out]  $\frac{1}{4} \frac{I}{c^6 b^3 \pi} \text{csgn}(I(c*x+1)/(-c^2*x^2+1)^{(1/2)}) * \text{csgn}(I(c*x+1)^2/(c^2*x^2-1))^{2*} \text{arctanh}(c*x)^2 + \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I((c*x+1)^2/(-c^2*x^2+1)+1)) * \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^{2*} \text{arctanh}(c*x)^2 - \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)) * \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^{2*} \text{arctanh}(c*x)^2 + \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I(c*x+1)/(-c^2*x^2+1)^{(1/2)})^{2*} \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)) * \text{arctanh}(c*x)^2 + \frac{1}{6} c^3 b^3 \text{arctanh}(c*x)^2 x^3 + \frac{1}{2} c^5 b^3 \text{arctanh}(c*x)^2 x + \frac{1}{2} a b^2 x^6 \text{arctanh}(c*x)^2 + \frac{1}{2} a^2 b x^6 \text{arctanh}(c*x) + \frac{23}{30} c^6 a b^2 \ln(c*x-1) + \frac{23}{30} c^6 a b^2 \ln(c*x+1) + \frac{1}{4} c^6 a^2 b \ln(c*x-1) - \frac{1}{4} c^6 a^2 b \ln(c*x+1) + \frac{1}{8} c^6 a b^2 \ln(c*x-1)^2 + \frac{19}{60} b^3 x/c^5 + \frac{1}{60} b^3 x^3/c^3 - \frac{19}{60} b^3 \text{arctanh}(c*x)/c^6 + \frac{1}{6} x^6 a^3 - \frac{1}{3} c^6 b^3 + \frac{1}{10} c x^5 a^2 b + \frac{4}{15} c^4 x^2 a b^2 + \frac{1}{2} c^5 x a^2 b + \frac{1}{6} c^3 a^2 b x^3 + \frac{1}{20} c^2 a b^2 x^4 + \frac{1}{8} c^6 a b^2 \ln(c*x+1)^2 + \frac{1}{4} c^6 b^3 \text{arctanh}(c*x)^2 \ln(c*x-1) - \frac{1}{4} c^6 b^3 \text{arctanh}(c*x)^2 \ln(c*x+1) - \frac{23}{15} c^6 b^3 \text{arctanh}(c*x) * \ln(1+I(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - \frac{23}{15} c^6 b^3 \text{arctanh}(c*x) * \ln(1-I(c*x+1)/(-c^2*x^2+1)^{(1/2)}) + \frac{1}{2} c^6 b^3 \text{arctanh}(c*x)^2 \ln((c*x+1)/(-c^2*x^2+1)^{(1/2)}) + \frac{4}{15} c^4 b^3 \text{arctanh}(c*x) x^2 + \frac{1}{20} c^2 b^3 \text{arctanh}(c*x) x^4 + \frac{1}{10} c b^3 \text{arctanh}(c*x)^2 x^5 - \frac{1}{6} c^6 b^3 \text{arctanh}(c*x)^3 + \frac{23}{30} c^6 b^3 \text{arctanh}(c*x)^2 - \frac{23}{15} c^6 b^3 \text{dilog}(1-I(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - \frac{23}{15} c^6 b^3 \text{dilog}(1+I(c*x+1)/(-c^2*x^2+1)^{(1/2)}) + \frac{1}{6} b^3 x^6 \text{arctanh}(c*x)^3 - \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I((c*x+1)^2/(-c^2*x^2+1)+1)) * \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)) * \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1)) * \text{arctanh}(c*x)^2 + \frac{1}{5} c a b^2 x^5 \text{arctanh}(c*x) + \frac{1}{3} c^3 a b^2 \text{arctanh}(c*x) x^3 + \frac{1}{c^5} a b^2 \text{arctanh}(c*x) x + \frac{1}{2} c^6 a b^2 \text{arctanh}(c*x) * \ln(c*x-1) - \frac{1}{2} c^6 a b^2 \text{arctanh}(c*x) * \ln(c*x+1) - \frac{1}{4} c^6 a b^2 \ln(c*x-1) * \ln(1/2 + 1/2 c*x) - \frac{1}{4} c^6 a b^2 \ln(-1/2 c*x + 1/2) * \ln(c*x+1) + \frac{1}{4} c^6 a b^2 \ln(-1/2 c*x + 1/2) * \ln(1/2 + 1/2 c*x) - \frac{1}{4} \frac{I}{c^6 b^3 \pi} \text{arctanh}(c*x)^2 + \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I(c*x+1)^2/(c^2*x^2-1))^{3*} \text{arctanh}(c*x)^2 - \frac{1}{4} \frac{I}{c^6 b^3 \pi} \text{csgn}(I((c*x+1)^2/(-c^2*x^2+1)+1))^{3*} \text{arctanh}(c*x)^2 + \frac{1}{8} \frac{I}{c^6 b^3 \pi} \text{csgn}(I(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^{3*} \text{arctanh}(c*x)^2 + \frac{1}{4} \frac{I}{c^6 b^3 \pi} \text{csgn}(I((c*x+1)^2/(-c^2*x^2+1)+1))^{2*} \text{arctanh}(c*x)^2$



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}ab^2x^6\operatorname{arctanh}(cx)^2 + \frac{1}{6}a^3x^6 + \frac{1}{60}(30x^6\operatorname{arctanh}(cx) + c*(2*(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx + 1)/c^7 + 15\log(cx - 1)/c^7))*a^2b + \frac{1}{120}(4c*(2*(3c^4x^5 + 5c^2x^3 + 15x)/c^6 - 15\log(cx + 1)/c^7 + 15\log(cx - 1)/c^7)*\operatorname{arctanh}(cx) + (6c^4x^4 + 32c^2x^2 - 2*(15\log(cx - 1) - 46)\log(cx + 1) + 15\log(cx + 1)^2 + 15\log(cx - 1)^2 + 92\log(cx - 1))/c^6)*a*b^2 - \frac{1}{1728000}(500c^7*((2c^4x^6 + 3c^2x^4 + 6x^2)/c^{11} + 6\log(c^2x^2 - 1)/c^{13}) + 728c^6*(2*(3c^4x^5 + 5c^2x^3 + 15x)/c^{11} - 15\log(cx + 1)/c^{12} + 15\log(cx - 1)/c^{12}) + 1485c^5*((c^2x^4 + 2x^2)/c^9 + 2\log(c^2x^2 - 1)/c^{11}) - 622080000c^5\operatorname{integrate}(1/3600x^5\log(cx + 1)/(c^7x^2 - c^5), x) + 9750c^4*(2*(c^2x^3 + 3x)/c^9 - 3\log(cx + 1)/c^{10} + 3\log(cx - 1)/c^{10}) - 2700c^3*(x^2/c^7 + \log(c^2x^2 - 1)/c^9) - 1036800000c^3\operatorname{integrate}(1/3600x^3\log(cx + 1)/(c^7x^2 - c^5), x) + 227700c^2*(2x/c^7 - \log(cx + 1)/c^8 + \log(cx - 1)/c^8) - 5495040000c*\operatorname{integrate}(1/3600x*\log(cx + 1)/(c^7x^2 - c^5), x) + (1000*(36\log(-cx + 1)^3 - 18\log(-cx + 1)^2 + 6\log(-cx + 1) - 1)*(cx - 1)^6 + 1728*(125\log(-cx + 1)^3 - 75\log(-cx + 1)^2 + 30\log(-cx + 1) - 6)*(cx - 1)^5 + 16875*(32\log(-cx + 1)^3 - 24\log(-cx + 1)^2 + 12\log(-cx + 1) - 3)*(cx - 1)^4 + 80000*(9\log(-cx + 1)^3 - 9\log(-cx + 1)^2 + 6\log(-cx + 1) - 2)*(cx - 1)^3 + 135000*(4\log(-cx + 1)^3 - 6\log(-cx + 1)^2 + 6\log(-cx + 1) - 3)*(cx - 1)^2 + 216000*(\log(-cx + 1)^3 - 3\log(-cx + 1)^2 + 6\log(-cx + 1) - 6)*(cx - 1))/c^6 - 60*(600*(c^6x^6 - 1)*\log(cx + 1)^3 + 240*(3c^5x^5 + 5c^3x^3 + 15cx)*\log(cx + 1)^2 - 30*(10c^6x^6 - 12c^5x^5 + 15c^4x^4 - 20c^3x^3 + 30c^2x^2 - 60cx - 60*(c^6x^6 - 1)*\log(cx + 1) + 37)*\log(-cx + 1)^2 + (100c^6x^6 + 264c^5x^5 - 165c^4x^4 + 1140c^3x^3 - 1230c^2x^2 - 1800*(c^6x^6 - 1)*\log(cx + 1)^2 + 8820cx - 480*(3c^5x^5 + 5c^3x^3 + 15cx + 23)*\log(cx + 1))*\log(-cx + 1))/c^6 + 264600*\log(3600c^7x^2 - 3600c^5)/c^6 - 2384640000*\operatorname{integrate}(1/3600*\log(cx + 1)/(c^7x^2 - c^5), x))*b^3$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral( $b^3x^5 \operatorname{artanh}(cx)^3 + 3ab^2x^5 \operatorname{artanh}(cx)^2 + 3a^2bx^5 \operatorname{artanh}(cx) + a^3x^5, x$ )

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^5\*arctanh(c\*x)^3 + 3\*a\*b^2\*x^5\*arctanh(c\*x)^2 + 3\*a^2\*b\*x^5\*arctanh(c\*x) + a^3\*x^5, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*atanh(c\*x))\*\*3,x)

[Out] Integral(x\*\*5\*(a + b\*atanh(c\*x))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3\*x^5, x)

### 3.25 $\int x^4 \left( a + b \tanh^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=262

$$-\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) \left(a + b \tanh^{-1}(cx)\right)}{5c^5} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{10c^5} + \frac{b^2 x^3 \left(a + b \tanh^{-1}(cx)\right)}{10c^2} + \frac{9ab^2 x}{10c^4} + \frac{3bx^2}{10c^4}$$

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
+ (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

**Rubi [A]** time = 0.773954, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {5916, 5980, 266, 43, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$-\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) \left(a + b \tanh^{-1}(cx)\right)}{5c^5} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{10c^5} + \frac{b^2 x^3 \left(a + b \tanh^{-1}(cx)\right)}{10c^2} + \frac{9ab^2 x}{10c^4} + \frac{3bx^2}{10c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTanh[c*x])^3, x]
```

```
[Out] (9*a*b^2*x)/(10*c^4) + (b^3*x^2)/(20*c^3) + (9*b^3*x*ArcTanh[c*x])/(10*c^4)
+ (b^2*x^3*(a + b*ArcTanh[c*x]))/(10*c^2) - (9*b*(a + b*ArcTanh[c*x])^2)/(
20*c^5) + (3*b*x^2*(a + b*ArcTanh[c*x])^2)/(10*c^3) + (3*b*x^4*(a + b*ArcTa
nh[c*x])^2)/(20*c) + (a + b*ArcTanh[c*x])^3/(5*c^5) + (x^5*(a + b*ArcTanh[c
*x])^3)/5 - (3*b*(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/(5*c^5) + (b^3*Lo
g[1 - c^2*x^2])/(2*c^5) - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 -
c*x)])/(5*c^5) + (3*b^3*PolyLog[3, 1 - 2/(1 - c*x)])/(10*c^5)
```

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
```

tegerQ[m]) && NeQ[m, -1]

### Rule 5980

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_)^(m\_))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[f^2/e, Int[(f\*x)^(m - 2)\*(a + b\*ArcTanh[c\*x])^p, x], x] - Dist[(d\*f^2)/e, Int[((f\*x)^(m - 2)\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 5910

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcTanh[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 5984

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e

}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
]
```

### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int x^4 (a + b \tanh^{-1}(cx))^3 dx &= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3 - \frac{1}{5} (3bc) \int \frac{x^5 (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx \\
&= \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3 + \frac{(3b) \int x^3 (a + b \tanh^{-1}(cx))^2 dx}{5c} - \frac{(3b) \int \frac{x^3 (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx}{5c} \\
&= \frac{3bx^4 (a + b \tanh^{-1}(cx))^2}{20c} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3 - \frac{1}{10} (3b^2) \int \frac{x^4 (a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx \\
&= \frac{3bx^2 (a + b \tanh^{-1}(cx))^2}{10c^3} + \frac{3bx^4 (a + b \tanh^{-1}(cx))^2}{20c} + \frac{(a + b \tanh^{-1}(cx))^3}{5c^5} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3 \\
&= \frac{b^2 x^3 (a + b \tanh^{-1}(cx))}{10c^2} + \frac{3bx^2 (a + b \tanh^{-1}(cx))^2}{10c^3} + \frac{3bx^4 (a + b \tanh^{-1}(cx))^2}{20c} + \frac{(a + b \tanh^{-1}(cx))^3}{5c^5} \\
&= \frac{9ab^2 x}{10c^4} + \frac{b^2 x^3 (a + b \tanh^{-1}(cx))}{10c^2} - \frac{9b (a + b \tanh^{-1}(cx))^2}{20c^5} + \frac{3bx^2 (a + b \tanh^{-1}(cx))^2}{10c^3} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3 \\
&= \frac{9ab^2 x}{10c^4} + \frac{9b^3 x \tanh^{-1}(cx)}{10c^4} + \frac{b^2 x^3 (a + b \tanh^{-1}(cx))}{10c^2} - \frac{9b (a + b \tanh^{-1}(cx))^2}{20c^5} + \frac{3bx^2 (a + b \tanh^{-1}(cx))^2}{10c^3} \\
&= \frac{9ab^2 x}{10c^4} + \frac{b^3 x^2}{20c^3} + \frac{9b^3 x \tanh^{-1}(cx)}{10c^4} + \frac{b^2 x^3 (a + b \tanh^{-1}(cx))}{10c^2} - \frac{9b (a + b \tanh^{-1}(cx))^2}{20c^5} + \frac{1}{5} x^5 (a + b \tanh^{-1}(cx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.735381, size = 383, normalized size = 1.46

$$12b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) (a + b \tanh^{-1}(cx)) + 6b^3 \text{PolyLog}\left(3, -e^{-2 \tanh^{-1}(cx)}\right) + 3a^2 bc^4 x^4 + 6a^2 bc^2 x^2 + 6a^2 b \log$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (-b^3 + 18\*a\*b^2\*c\*x + 6\*a^2\*b\*c^2\*x^2 + b^3\*c^2\*x^2 + 2\*a\*b^2\*c^3\*x^3 + 3\*a^2\*b\*c^4\*x^4 + 4\*a^3\*c^5\*x^5 - 18\*a\*b^2\*ArcTanh[c\*x] + 18\*b^3\*c\*x\*ArcTanh[c\*x] + 12\*a\*b^2\*c^2\*x^2\*ArcTanh[c\*x] + 2\*b^3\*c^3\*x^3\*ArcTanh[c\*x] + 6\*a\*b^2\*c^4\*x^4\*ArcTanh[c\*x] + 12\*a^2\*b\*c^5\*x^5\*ArcTanh[c\*x] - 12\*a\*b^2\*ArcTanh[c\*x]^2 - 9\*b^3\*ArcTanh[c\*x]^2 + 6\*b^3\*c^2\*x^2\*ArcTanh[c\*x]^2 + 3\*b^3\*c^4\*x^4\*ArcTanh[c\*x]^2 + 12\*a\*b^2\*c^5\*x^5\*ArcTanh[c\*x]^2 - 4\*b^3\*ArcTanh[c\*x]^3 + 4\*b^3\*c^5\*x^5\*ArcTanh[c\*x]^3 - 24\*a\*b^2\*ArcTanh[c\*x]\*Log[1 + E^(-2\*ArcTanh[c\*x])] - 12\*b^3\*ArcTanh[c\*x]^2\*Log[1 + E^(-2\*ArcTanh[c\*x])] + 6\*a^2\*b\*Log[1

$-c^2x^2] + 10b^3\text{Log}[1 - c^2x^2] + 12b^2(a + b\text{ArcTanh}[cx])\text{PolyLog}[2, -E^{(-2\text{ArcTanh}[cx])}] + 6b^3\text{PolyLog}[3, -E^{(-2\text{ArcTanh}[cx])}]]/(20c^5)$

**Maple [C]** time = 0.823, size = 1275, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4(a+b\text{arctanh}(cx))^3, x)$

[Out]  $3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I*(cx+1)^2/(c^2x^2-1)/((cx+1)^2/(-c^2x^2+1)+1))^2\text{csgn}(I*(cx+1)^2/(c^2x^2-1))*\text{Pi}-3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I*(cx+1)/(-c^2x^2+1)^{(1/2)})^2\text{csgn}(I*(cx+1)^2/(c^2x^2-1))*\text{Pi}-1/20/c^5b^3+1/5x^5a^3+1/20b^3x^2/c^3+3/10/c^5b^3\text{arctanh}(cx)^2\ln(cx+1)-3/5/c^5b^3\text{arctanh}(cx)^2\ln((cx+1)/(-c^2x^2+1)^{(1/2)})-3/5/c^5b^3\text{arctanh}(cx)*\text{polylog}(2, -(cx+1)^2/(-c^2x^2+1))+3/5x^5a^2b\text{arctanh}(cx)+3/5x^5a*b^2\text{arctanh}(cx)^2+1/10a*b^2x^3/c^2+9/10a*b^2x/c^4+9/10b^3x\text{arctanh}(cx)/c^4+3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I/((cx+1)^2/(-c^2x^2+1)+1))*\text{csgn}(I*(cx+1)^2/(c^2x^2-1)/((cx+1)^2/(-c^2x^2+1)+1))*\text{csgn}(I*(cx+1)^2/(c^2x^2-1))*\text{Pi}-3/10I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I*(cx+1)/(-c^2x^2+1)^{(1/2)})*\text{csgn}(I*(cx+1)^2/(c^2x^2-1))^2\text{Pi}-3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I/((cx+1)^2/(-c^2x^2+1)+1))*\text{csgn}(I*(cx+1)^2/(c^2x^2-1)/((cx+1)^2/(-c^2x^2+1)+1))^2\text{Pi}-3/10I/c^5b^3\text{arctanh}(cx)^2\text{Pi}+3/5/c^5a*b^2\text{arctanh}(cx)*\ln(cx-1)+3/5/c^5a*b^2\text{arctanh}(cx)*\ln(cx+1)-3/10/c^5a*b^2*\ln(cx-1)*\ln(1/2+1/2*cx)+3/10/c^5a*b^2*\ln(-1/2*cx+1/2)*\ln(cx+1)-3/10/c^5a*b^2*\ln(-1/2*cx+1/2)*\ln(1/2+1/2*cx)+3/10/c^5a*b^2*x^4\text{arctanh}(cx)+3/5/c^3a*b^2\text{arctanh}(cx)*x^2+3/20/c^4a^2*b+3/10/c^3a^2*b*x^2+3/20/c^5b^3\text{arctanh}(cx)^2*x^4+3/10/c^3b^3\text{arctanh}(cx)^2*x^2+1/10/c^2b^3\text{arctanh}(cx)*x^3+9/20/c^5a*b^2*\ln(cx-1)-9/20/c^5a*b^2*\ln(cx+1)+3/10/c^5a^2*b*\ln(cx-1)+3/10/c^5a^2*b*\ln(cx+1)+3/20/c^5a*b^2*\ln(cx-1)^2-3/20/c^5a*b^2*\ln(cx+1)^2-3/5/c^5b^3\text{arctanh}(cx)^2*\ln(2)-3/5/c^5a*b^2*\text{dilog}(1/2+1/2*cx)+3/10/c^5b^3\text{arctanh}(cx)^2*\ln(cx-1)+1/5x^5b^3\text{arctanh}(cx)^3+1/5/c^5b^3\text{arctanh}(cx)^3-9/20/c^5b^3\text{arctanh}(cx)^2+3/10/c^5b^3\text{polylog}(3, -(cx+1)^2/(-c^2x^2+1))-1/c^5b^3*\ln((cx+1)^2/(-c^2x^2+1)+1)+1/c^5b^3\text{arctanh}(cx)-3/10I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I/((cx+1)^2/(-c^2x^2+1)+1))^3\text{Pi}-3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I*(cx+1)^2/(c^2x^2-1)/((cx+1)^2/(-c^2x^2+1)+1))^3\text{Pi}-3/20I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I*(cx+1)^2/(c^2x^2-1))^3\text{Pi}+3/10I/c^5b^3\text{arctanh}(cx)^2\text{csgn}(I/((cx+1)^2/(-c^2x^2+1)+1))^2\text{Pi}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{5}a^3x^5 + \frac{3}{20}\left(4x^5 \operatorname{artanh}(cx) + c\left(\frac{c^2x^4 + 2x^2}{c^4} + \frac{2\log(c^2x^2 - 1)}{c^6}\right)\right)a^2b - \frac{2(b^3c^5x^5 - b^3)\log(-cx + 1)^3 - 3(4ab^2c^5x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x))^3,x, algorithm="maxima")

[Out] 1/5\*a^3\*x^5 + 3/20\*(4\*x^5\*arctanh(c\*x) + c\*((c^2\*x^4 + 2\*x^2)/c^4 + 2\*log(c^2\*x^2 - 1)/c^6))\*a^2\*b - 1/80\*(2\*(b^3\*c^5\*x^5 - b^3)\*log(-c\*x + 1)^3 - 3\*(4\*a\*b^2\*c^5\*x^5 + b^3\*c^4\*x^4 + 2\*b^3\*c^2\*x^2 + 2\*(b^3\*c^5\*x^5 + b^3)\*log(c\*x + 1))\*log(-c\*x + 1)^2)/c^5 - integrate(-1/40\*(5\*(b^3\*c^5\*x^5 - b^3\*c^4\*x^4)\*log(c\*x + 1)^3 + 30\*(a\*b^2\*c^5\*x^5 - a\*b^2\*c^4\*x^4)\*log(c\*x + 1)^2 - 3\*(4\*a\*b^2\*c^5\*x^5 + b^3\*c^4\*x^4 + 2\*b^3\*c^2\*x^2 + 5\*(b^3\*c^5\*x^5 - b^3\*c^4\*x^4)\*log(c\*x + 1)^2 - 2\*(10\*a\*b^2\*c^4\*x^4 - (10\*a\*b^2\*c^5 + b^3\*c^5)\*x^5 - b^3)\*log(c\*x + 1))\*log(-c\*x + 1))/(c^5\*x - c^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3x^4 \operatorname{artanh}(cx)^3 + 3ab^2x^4 \operatorname{artanh}(cx)^2 + 3a^2bx^4 \operatorname{artanh}(cx) + a^3x^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^4\*arctanh(c\*x)^3 + 3\*a\*b^2\*x^4\*arctanh(c\*x)^2 + 3\*a^2\*b\*x^4\*arctanh(c\*x) + a^3\*x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*atanh(c\*x))\*\*3,x)

[Out] Integral(x\*\*4\*(a + b\*atanh(c\*x))\*\*3, x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3\*x^4, x)

### 3.26 $\int x^3 \left(a + b \tanh^{-1}(cx)\right)^3 dx$

**Optimal.** Leaf size=185

$$-\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^4} + \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} - \frac{2b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} - \frac{(a + b \tanh^{-1}(cx))^3}{4c^4}$$

[Out] (b^3\*x)/(4\*c^3) - (b^3\*ArcTanh[c\*x])/(4\*c^4) + (b^2\*x^2\*(a + b\*ArcTanh[c\*x]))/(4\*c^2) + (b\*(a + b\*ArcTanh[c\*x])^2)/c^4 + (3\*b\*x\*(a + b\*ArcTanh[c\*x])^2)/(4\*c^3) + (b\*x^3\*(a + b\*ArcTanh[c\*x])^2)/(4\*c) - (a + b\*ArcTanh[c\*x])^3/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x])^3)/4 - (2\*b^2\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/c^4 - (b^3\*PolyLog[2, 1 - 2/(1 - c\*x)])/c^4

**Rubi [A]** time = 0.569391, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5916, 5980, 321, 206, 5984, 5918, 2402, 2315, 5910, 5948}

$$-\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c^4} + \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} - \frac{2b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} - \frac{(a + b \tanh^{-1}(cx))^3}{4c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (b^3\*x)/(4\*c^3) - (b^3\*ArcTanh[c\*x])/(4\*c^4) + (b^2\*x^2\*(a + b\*ArcTanh[c\*x]))/(4\*c^2) + (b\*(a + b\*ArcTanh[c\*x])^2)/c^4 + (3\*b\*x\*(a + b\*ArcTanh[c\*x])^2)/(4\*c^3) + (b\*x^3\*(a + b\*ArcTanh[c\*x])^2)/(4\*c) - (a + b\*ArcTanh[c\*x])^3/(4\*c^4) + (x^4\*(a + b\*ArcTanh[c\*x])^3)/4 - (2\*b^2\*(a + b\*ArcTanh[c\*x])\*Log[2/(1 - c\*x)])/c^4 - (b^3\*PolyLog[2, 1 - 2/(1 - c\*x)])/c^4

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 321

```
Int[(((c_.)*(x_.))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 5910

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.), x\_Symbol] := Simp[x\*(a + b\*ArcTanh[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^ (p\_.)/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \tanh^{-1}(cx))^3 dx &= \frac{1}{4} x^4 (a + b \tanh^{-1}(cx))^3 - \frac{1}{4} (3bc) \int \frac{x^4 (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx \\
 &= \frac{1}{4} x^4 (a + b \tanh^{-1}(cx))^3 + \frac{(3b) \int x^2 (a + b \tanh^{-1}(cx))^2 dx}{4c} - \frac{(3b) \int \frac{x^{2(a+b \tanh^{-1}(cx))^2}}{1 - c^2 x^2} dx}{4c} \\
 &= \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{4c} + \frac{1}{4} x^4 (a + b \tanh^{-1}(cx))^3 - \frac{1}{2} b^2 \int \frac{x^3 (a + b \tanh^{-1}(cx))}{1 - c^2 x^2} dx + \\
 &= \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{4c} - \frac{(a + b \tanh^{-1}(cx))^3}{4c^4} + \frac{1}{4} x^4 (a + b \tanh^{-1}(cx))^3 \\
 &= \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} + \frac{b (a + b \tanh^{-1}(cx))^2}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{4c^3} \\
 &= \frac{b^3 x}{4c^3} + \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} + \frac{b (a + b \tanh^{-1}(cx))^2}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} + \frac{bx^3 (a + b \tanh^{-1}(cx))^2}{4c^3} \\
 &= \frac{b^3 x}{4c^3} - \frac{b^3 \tanh^{-1}(cx)}{4c^4} + \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} + \frac{b (a + b \tanh^{-1}(cx))^2}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3} \\
 &= \frac{b^3 x}{4c^3} - \frac{b^3 \tanh^{-1}(cx)}{4c^4} + \frac{b^2 x^2 (a + b \tanh^{-1}(cx))}{4c^2} + \frac{b (a + b \tanh^{-1}(cx))^2}{c^4} + \frac{3bx (a + b \tanh^{-1}(cx))^2}{4c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.49171, size = 245, normalized size = 1.32

$$8b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + 2b \tanh^{-1}(cx) \left(3a^2 c^4 x^4 + 2abcx (c^2 x^2 + 3) + b^2 (c^2 x^2 - 1) - 8b^2 \log\left(e^{-2 \tanh^{-1}(cx)} + 1\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x])^3,x]

[Out]  $(-2*a*b^2 + 6*a^2*b*c*x + 2*b^3*c*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c^3*x^3 + 2*a^3*c^4*x^4 + 2*b^2*(b*(-4 + 3*c*x + c^3*x^3) + 3*a*(-1 + c^4*x^4))*\text{ArcTanh}[c*x]^2 + 2*b^3*(-1 + c^4*x^4)*\text{ArcTanh}[c*x]^3 + 2*b*\text{ArcTanh}[c*x]*(3*a^2*c^4*x^4 + b^2*(-1 + c^2*x^2) + 2*a*b*c*x*(3 + c^2*x^2) - 8*b^2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}]) + 3*a^2*b*\text{Log}[1 - c*x] - 3*a^2*b*\text{Log}[1 + c*x] + 8*a*b^2*\text{Log}[1 - c^2*x^2] + 8*b^3*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}])/(8*c^4)$

**Maple [C]** time = 0.602, size = 1245, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x))^3,x)

[Out]  $-1/4/c^4*b^3*\text{arctanh}(c*x)^3 + 1/c^4*b^3*\text{arctanh}(c*x)^2 - 2/c^4*b^3*\text{dilog}(1 - I*(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - 2/c^4*b^3*\text{dilog}(1 + I*(c*x+1)/(-c^2*x^2+1)^{(1/2)}) + 1/4*x^4*b^3*\text{arctanh}(c*x)^3 - 3/16*I/c^4*b^3*\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*\text{arctanh}(c*x)^2 + 1/4*b^3*x/c^3 - 1/4*b^3*\text{arctanh}(c*x)/c^4 + 1/4*x^4*a^3 - 3/16*I/c^4*b^3*\text{Pi}*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\text{arctanh}(c*x)^2 + 3/16*I/c^4*b^3*\text{Pi}*c\text{sgn}(I*(c*x+1)/(-c^2*x^2+1)^{(1/2)})^2*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*\text{arctanh}(c*x)^2 + 3/16*I/c^4*b^3*\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\text{arctanh}(c*x)^2 + 3/8*I/c^4*b^3*\text{Pi}*c\text{sgn}(I*(c*x+1)/(-c^2*x^2+1)^{(1/2)})*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^2*\text{arctanh}(c*x)^2 + 1/2/c*a*b^2*\text{arctanh}(c*x)*x^3 + 3/2/c^3*a*b^2*\text{arctanh}(c*x)*x + 3/4/c^4*a*b^2*\text{arctanh}(c*x)*\ln(c*x-1) - 3/4/c^4*a*b^2*\text{arctanh}(c*x)*\ln(c*x+1) - 3/8/c^4*a*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x) - 3/8/c^4*a*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1) + 3/8/c^4*a*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x) - 3/8*I/c^4*b^3*\text{Pi}*c\text{sgn}(I*(c*x+1)/(-c^2*x^2+1)^{(1/2)})*\text{arctanh}(c*x)^2 + 1/4/c^2*x^2*a*b^2 + 3/4/c^3*x*a^2*b + 1/4/c*a^2*b*x^3 + 3/4*a*b^2*x^4*\text{arctanh}(c*x)^2 + 3/4*x^4*a^2*b*\text{arctanh}(c*x) + 3/8/c^4*b^3*\text{arctanh}(c*x)^2*\ln(c*x-1) - 3/8/c^4*b^3*\text{arctanh}(c*x)^2*\ln(c*x+1) - 2/c^4*b^3*\text{arctanh}(c*x)*\ln(1+I*(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - 2/c^4*b^3*\text{arctanh}(c*x)*\ln(1-I*(c*x+1)/(-c^2*x^2+1)^{(1/2)}) + 3/4/c^4*b^3*\text{arctanh}(c*x)^2*\ln((c*x+1)/(-c^2*x^2+1)^{(1/2)}) + 1/c^4*a*b^2*\ln(c*x-1) + 1/c^4*a*b^2*\ln(c*x+1) + 3/8/c^4*a^2*b*\ln(c*x-1) - 3/8/c^4*a^2*b*\ln(c*x+1) + 3/16/c^4*a*b^2*\ln(c*x-1)^2 + 3/16/c^4*a*b^2*\ln(c*x+1)^2 + 1/4/c^2*b^3*\text{arctanh}(c*x)*x^2 + 1/4/c*b^3*\text{arctanh}(c*x)^2*x^3 + 3/4/c^3*b^3*\text{arctanh}(c*x)^2*x - 1/4/c^4*b^3 + 3/8*$

$$\begin{aligned} & I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\operatorname{arctanh}(c*x)^2+3/16*I/c^4 \\ & *b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*\operatorname{arctanh}( \\ & c*x)^2+3/16*I/c^4*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*\operatorname{arctanh}(c*x)^2-3/8 \\ & *I/c^4*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*\operatorname{arctanh}(c*x)^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{3}{4}a^2b^2x^4\operatorname{arctanh}(cx)^2 + \frac{1}{4}a^3x^4 + \frac{1}{8}(6x^4\operatorname{arctanh}(cx) + c(2(c^2x^3 + 3x)/c^4 - 3\log(cx + 1)/c^5 + 3\log(cx - 1)/c^5))a^2b + \frac{1}{16}(4c(2(c^2x^3 + 3x)/c^4 - 3\log(cx + 1)/c^5 + 3\log(cx - 1)/c^5) \operatorname{arctanh}(cx) + (4c^2x^2 - 2(3\log(cx - 1) - 8)\log(cx + 1) + 3\log(cx + 1)^2 + 3\log(cx - 1)^2 + 16\log(cx - 1)))/c^4)a^2b - \frac{1}{9216}(27c^5((c^2x^4 + 2x^2)/c^7 + 2\log(c^2x^2 - 1)/c^9) + 74c^4(2(c^2x^3 + 3x)/c^7 - 3\log(cx + 1)/c^8 + 3\log(cx - 1)/c^8) + 60c^3(x^2/c^5 + \log(c^2x^2 - 1)/c^7) - 221184c^3\operatorname{integrate}(1/96x^3\log(cx + 1)/(c^5x^2 - c^3), x) + 1692c^2(2x/c^5 - \log(cx + 1)/c^6 + \log(cx - 1)/c^6) - 1105920c^2\operatorname{integrate}(1/96x\log(cx + 1)/(c^5x^2 - c^3), x) + (9(32\log(-cx + 1))^3 - 24\log(-cx + 1)^2 + 12\log(-cx + 1) - 3)(cx - 1)^4 + 128(9\log(-cx + 1))^3 - 9\log(-cx + 1)^2 + 6\log(-cx + 1) - 2)(cx - 1)^3 + 432(4\log(-cx + 1))^3 - 6\log(-cx + 1)^2 + 6\log(-cx + 1) - 3)(cx - 1)^2 + 1152(\log(-cx + 1))^3 - 3\log(-cx + 1)^2 + 6\log(-cx + 1) - 6)(cx - 1)/c^4 - 12(24(c^4x^4 - 1)\log(cx + 1)^3 + 48(c^3x^3 + 3cx)\log(cx + 1)^2 - 6(3c^4x^4 - 4c^3x^3 + 6c^2x^2 - 12cx - 12)(c^4x^4 - 1)\log(cx + 1) + 7)\log(-cx + 1)^2 + (9c^4x^4 + 28c^3x^3 - 18c^2x^2 - 72)(c^4x^4 - 1)\log(cx + 1)^2 + 300cx - 96(c^3x^3 + 3cx + 4)\log(cx + 1))\log(-cx + 1)/c^4 + 1800\log(96c^5x^2 - 96c^3)/c^4 - 442368\operatorname{integrate}(1/96\log(cx + 1)/(c^5x^2 - c^3), x))b^3$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3x^3 \operatorname{artanh}(cx)^3 + 3ab^2x^3 \operatorname{artanh}(cx)^2 + 3a^2bx^3 \operatorname{artanh}(cx) + a^3x^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctanh(c*x)^3 + 3*a*b^2*x^3*arctanh(c*x)^2 + 3*a^2*b*x^3*
arctanh(c*x) + a^3*x^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*x))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*x^3, x)
```

### 3.27 $\int x^2 \left( a + b \tanh^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=197

$$\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^3} + \frac{b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{2c^3} + \frac{ab^2x}{c^2} - \frac{b(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3}$$

[Out] (a\*b^2\*x)/c^2 + (b^3\*x\*ArcTanh[c\*x])/c^2 - (b\*(a + b\*ArcTanh[c\*x])^2)/(2\*c^3) + (b\*x^2\*(a + b\*ArcTanh[c\*x])^2)/(2\*c) + (a + b\*ArcTanh[c\*x])^3/(3\*c^3) + (x^3\*(a + b\*ArcTanh[c\*x])^3)/3 - (b\*(a + b\*ArcTanh[c\*x])^2\*Log[2/(1 - c\*x)])/c^3 + (b^3\*Log[1 - c^2\*x^2])/(2\*c^3) - (b^2\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, 1 - 2/(1 - c\*x)])/c^3 + (b^3\*PolyLog[3, 1 - 2/(1 - c\*x)])/c^3

**Rubi [A]** time = 0.449067, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {5916, 5980, 5910, 260, 5948, 5984, 5918, 6058, 6610}

$$\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^3} + \frac{b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{2c^3} + \frac{ab^2x}{c^2} - \frac{b(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (a\*b^2\*x)/c^2 + (b^3\*x\*ArcTanh[c\*x])/c^2 - (b\*(a + b\*ArcTanh[c\*x])^2)/(2\*c^3) + (b\*x^2\*(a + b\*ArcTanh[c\*x])^2)/(2\*c) + (a + b\*ArcTanh[c\*x])^3/(3\*c^3) + (x^3\*(a + b\*ArcTanh[c\*x])^3)/3 - (b\*(a + b\*ArcTanh[c\*x])^2\*Log[2/(1 - c\*x)])/c^3 + (b^3\*Log[1 - c^2\*x^2])/(2\*c^3) - (b^2\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, 1 - 2/(1 - c\*x)])/c^3 + (b^3\*PolyLog[3, 1 - 2/(1 - c\*x)])/c^3

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980



```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_)]/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

### Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*ArcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 5984

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 6058

```
Int[(Log[u_]*)((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[(a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u]]/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u]]/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
```

e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + b \tanh^{-1}(cx))^3 dx &= \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^3 - (bc) \int \frac{x^3 (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx \\
 &= \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^3 + \frac{b \int x (a + b \tanh^{-1}(cx))^2 dx}{c} - \frac{b \int \frac{x (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx}{c} \\
 &= \frac{bx^2 (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^3 - b^2 \int \frac{x^2 (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx \\
 &= \frac{bx^2 (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^3 - \frac{b(a + b \tanh^{-1}(cx))^2}{c} \int \frac{x}{1 - c^2 x^2} dx \\
 &= \frac{ab^2 x}{c^2} - \frac{b(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{bx^2 (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx))^3 \\
 &= \frac{ab^2 x}{c^2} + \frac{b^3 x \tanh^{-1}(cx)}{c^2} - \frac{b(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{bx^2 (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3} \\
 &= \frac{ab^2 x}{c^2} + \frac{b^3 x \tanh^{-1}(cx)}{c^2} - \frac{b(a + b \tanh^{-1}(cx))^2}{2c^3} + \frac{bx^2 (a + b \tanh^{-1}(cx))^2}{2c} + \frac{(a + b \tanh^{-1}(cx))^3}{3c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.520302, size = 250, normalized size = 1.27

$$\frac{6ab^2 \left( \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + (c^3 x^3 - 1) \tanh^{-1}(cx)^2 + \tanh^{-1}(cx) \left( c^2 x^2 - 2 \log\left( e^{-2 \tanh^{-1}(cx)} + 1 \right) - 1 \right) + cx \right) + b^3}{c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (3\*a^2\*b\*c^2\*x^2 + 2\*a^3\*c^3\*x^3 + 6\*a^2\*b\*c^3\*x^3\*ArcTanh[c\*x] + 3\*a^2\*b\*L  
og[1 - c^2\*x^2] + 6\*a\*b^2\*(c\*x + (-1 + c^3\*x^3)\*ArcTanh[c\*x]^2 + ArcTanh[c\*

$$x]*(-1 + c^2*x^2 - 2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}]) + \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}] + b^3*(6*c*x*\text{ArcTanh}[c*x] - 3*\text{ArcTanh}[c*x]^2 + 3*c^2*x^2*\text{ArcTanh}[c*x]^2 - 2*\text{ArcTanh}[c*x]^3 + 2*c^3*x^3*\text{ArcTanh}[c*x]^3 - 6*\text{ArcTanh}[c*x]^2*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x])}] + 3*\text{Log}[1 - c^2*x^2] + 6*\text{ArcTanh}[c*x]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x])}] + 3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c*x])}]))/(6*c^3)$$

**Maple [C]** time = 0.477, size = 1177, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(a+b*\text{arctanh}(c*x))^3, x)$

[Out]  $1/c^3*b^3*\text{arctanh}(c*x)+1/3/c^3*b^3*\text{arctanh}(c*x)^3-1/2/c^3*b^3*\text{arctanh}(c*x)^2+1/2/c^3*b^3*\text{polylog}(3, -(c*x+1)^2/(-c^2*x^2+1))-1/c^3*b^3*\ln((c*x+1)^2/(-c^2*x^2+1)+1)+1/3*x^3*b^3*\text{arctanh}(c*x)^3+a*b^2*x/c^2+b^3*x*\text{arctanh}(c*x)/c^2+1/c*a*b^2*\text{arctanh}(c*x)*x^2+1/c^3*a*b^2*\text{arctanh}(c*x)*\ln(c*x-1)+1/c^3*a*b^2*\text{arctanh}(c*x)*\ln(c*x+1)-1/2/c^3*a*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x)+1/2/c^3*a*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1)-1/2/c^3*a*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x)-1/2*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{Pi}+1/3*x^3*a^3+1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1))*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*\text{csgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))*\text{Pi}+1/2/c*a^2*b*x^2+a^2*b*x^3*\text{arctanh}(c*x)+a*b^2*x^3*\text{arctanh}(c*x)^2+1/2/c^3*a*b^2*\ln(c*x-1)-1/2/c^3*a*b^2*\ln(c*x+1)+1/2/c^3*a^2*b*\ln(c*x-1)+1/2/c^3*a^2*b*\ln(c*x+1)+1/4/c^3*a*b^2*\ln(c*x-1)^2-1/4/c^3*a*b^2*\ln(c*x+1)^2-1/c^3*b^3*\text{arctanh}(c*x)^2*\ln(2)-1/c^3*a*b^2*\text{dilog}(1/2+1/2*c*x)+1/2/c^3*b^3*\text{arctanh}(c*x)^2*\ln(c*x-1)+1/2/c^3*b^3*\text{arctanh}(c*x)^2*\ln(c*x+1)-1/c^3*b^3*\text{arctanh}(c*x)^2*\ln((c*x+1)/(-c^2*x^2+1)^(1/2))-1/c^3*b^3*\text{arctanh}(c*x)*\text{polylog}(2, -(c*x+1)^2/(-c^2*x^2+1))+1/2/c*b^3*\text{arctanh}(c*x)^2*x^2-1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1))^3*\text{Pi}-1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*\text{Pi}-1/2*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))^3*\text{Pi}+1/2*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\text{Pi}-1/2*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1))^2*\text{csgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*\text{Pi}+1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1))*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\text{Pi}-1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1))*\text{csgn}(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*\text{Pi}-1/4*I/c^3*b^3*\text{arctanh}(c*x)^2*\text{csgn}(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*\text{csgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))*\text{Pi}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^3 x^3 + \frac{1}{2} \left( 2x^3 \operatorname{artanh}(cx) + c \left( \frac{x^2}{c^2} + \frac{\log(c^2 x^2 - 1)}{c^4} \right) \right) a^2 b - \frac{(b^3 c^3 x^3 - b^3) \log(-cx + 1)^3 - 3(2ab^2 c^3 x^3 + b^3 c^2 x^2 + (b^3 c^3 x^3 - b^3) \log(-cx + 1)^3)}{24 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x))^3,x, algorithm="maxima")

[Out] 1/3\*a^3\*x^3 + 1/2\*(2\*x^3\*arctanh(c\*x) + c\*(x^2/c^2 + log(c^2\*x^2 - 1)/c^4))  
\*a^2\*b - 1/24\*((b^3\*c^3\*x^3 - b^3)\*log(-c\*x + 1)^3 - 3\*(2\*a\*b^2\*c^3\*x^3 + b  
^3\*c^2\*x^2 + (b^3\*c^3\*x^3 + b^3)\*log(c\*x + 1))\*log(-c\*x + 1)^2)/c^3 - integ  
rate(-1/8\*((b^3\*c^3\*x^3 - b^3\*c^2\*x^2)\*log(c\*x + 1)^3 + 6\*(a\*b^2\*c^3\*x^3 -  
a\*b^2\*c^2\*x^2)\*log(c\*x + 1)^2 - (4\*a\*b^2\*c^3\*x^3 + 2\*b^3\*c^2\*x^2 + 3\*(b^3\*c  
^3\*x^3 - b^3\*c^2\*x^2)\*log(c\*x + 1)^2 - 2\*(6\*a\*b^2\*c^2\*x^2 - (6\*a\*b^2\*c^3 +  
b^3\*c^3)\*x^3 - b^3)\*log(c\*x + 1))\*log(-c\*x + 1))/(c^3\*x - c^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3 x^2 \operatorname{artanh}(cx)^3 + 3ab^2 x^2 \operatorname{artanh}(cx)^2 + 3a^2 b x^2 \operatorname{artanh}(cx) + a^3 x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^2\*arctanh(c\*x)^3 + 3\*a\*b^2\*x^2\*arctanh(c\*x)^2 + 3\*a^2\*b\*x^2\*  
arctanh(c\*x) + a^3\*x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x))\*\*3,x)

[Out] Integral(x\*\*2\*(a + b\*atanh(c\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arctanh}(cx) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3\*x^2, x)

### 3.28 $\int x \left( a + b \tanh^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=123

$$\frac{3b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{2c^2} - \frac{3b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^2} + \frac{3b (a + b \tanh^{-1}(cx))^2}{2c^2} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2} x^2$$

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
- (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```

**Rubi [A]** time = 0.250315, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {5916, 5980, 5910, 5984, 5918, 2402, 2315, 5948}

$$\frac{3b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{2c^2} - \frac{3b^2 \log\left(\frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c^2} + \frac{3b (a + b \tanh^{-1}(cx))^2}{2c^2} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2} x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTanh[c*x])^3, x]
```

```
[Out] (3*b*(a + b*ArcTanh[c*x])^2)/(2*c^2) + (3*b*x*(a + b*ArcTanh[c*x])^2)/(2*c)
- (a + b*ArcTanh[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b^2
*(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)]/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 -
c*x)])/(2*c^2)
```

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 5980

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((f_.)*(x_.))^ (m_.))/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*x
```

```
]^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTanh[c*x])^p)/(
d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1
]
```

### Rule 5910

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*A
rcTanh[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTanh[c*x])^(p - 1))/(1 -
c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

### Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rubi steps

$$\begin{aligned}
\int x(a + b \tanh^{-1}(cx))^3 dx &= \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 - \frac{1}{2}(3bc) \int \frac{x^2(a + b \tanh^{-1}(cx))^2}{1 - c^2x^2} dx \\
&= \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 + \frac{(3b) \int (a + b \tanh^{-1}(cx))^2 dx}{2c} - \frac{(3b) \int \frac{(a + b \tanh^{-1}(cx))^2}{1 - c^2x^2} dx}{2c} \\
&= \frac{3bx(a + b \tanh^{-1}(cx))^2}{2c} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 - (3b^2) \int \frac{x(a + b \tanh^{-1}(cx))^2}{1 - c^2x^2} dx \\
&= \frac{3b(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bx(a + b \tanh^{-1}(cx))^2}{2c} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 \\
&= \frac{3b(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bx(a + b \tanh^{-1}(cx))^2}{2c} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 \\
&= \frac{3b(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bx(a + b \tanh^{-1}(cx))^2}{2c} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3 \\
&= \frac{3b(a + b \tanh^{-1}(cx))^2}{2c^2} + \frac{3bx(a + b \tanh^{-1}(cx))^2}{2c} - \frac{(a + b \tanh^{-1}(cx))^3}{2c^2} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx))^3
\end{aligned}$$

**Mathematica [A]** time = 0.28394, size = 161, normalized size = 1.31

$$6b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx)}\right) + a\left(2a^2c^2x^2 + 6abcx + 3ab \log(1 - cx) - 3ab \log(cx + 1) + 6b^2 \log(1 - c^2x^2)\right) + 6b^2(cx - 4c^2x^2)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] (6\*b^2\*(-1 + c\*x)\*(a + b + a\*c\*x)\*ArcTanh[c\*x]^2 + 2\*b^3\*(-1 + c^2\*x^2)\*ArcTanh[c\*x]^3 + 6\*b\*ArcTanh[c\*x]\*(a\*c\*x\*(2\*b + a\*c\*x) - 2\*b^2\*Log[1 + E^(-2\*ArcTanh[c\*x])]) + a\*(6\*a\*b\*c\*x + 2\*a^2\*c^2\*x^2 + 3\*a\*b\*Log[1 - c\*x] - 3\*a\*b\*Log[1 + c\*x] + 6\*b^2\*Log[1 - c^2\*x^2]) + 6\*b^3\*PolyLog[2, -E^(-2\*ArcTanh[c\*x])])/(4\*c^2)

**Maple [C]** time = 0.342, size = 6097, normalized size = 49.6

output too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x))^3,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & \frac{3}{2}ab^2x^2\operatorname{arctanh}(cx)^2 + \frac{1}{2}a^3x^2 + \frac{3}{4}(2x^2\operatorname{arctanh}(cx) + c(2 \\ & \frac{x}{c^2} - \log(cx + 1)/c^3 + \log(cx - 1)/c^3))a^2b + \frac{3}{8}(4c(2x/c^2 - \\ & \log(cx + 1)/c^3 + \log(cx - 1)/c^3)\operatorname{arctanh}(cx) - (2(\log(cx - 1) - 2) \\ & \log(cx + 1) - \log(cx + 1)^2 - \log(cx - 1)^2 - 4\log(cx - 1))/c^2)ab^2 \\ & - \frac{1}{64}(3c^3(x^2/c^3 + \log(c^2x^2 - 1)/c^5) + 21c^2(2x/c^3 - \log(cx \\ & + 1)/c^4 + \log(cx - 1)/c^4) - 576c\operatorname{integrate}(1/4x\log(cx + 1)/(c^3x^2 \\ & - c), x) - 2(12cx\log(cx + 1)^2 + 2(c^2x^2 - 1)\log(cx + 1)^3 - 3(c \\ & ^2x^2 - 2cx - 2(c^2x^2 - 1)\log(cx + 1) + 1)\log(-cx + 1)^2 + 3(c^2 \\ & x^2 - 2(c^2x^2 - 1)\log(cx + 1)^2 + 6cx - 8(cx + 1)\log(cx + 1)) \\ & \log(-cx + 1))/c^2 + ((4\log(-cx + 1)^3 - 6\log(-cx + 1)^2 + 6\log(-cx + \\ & 1) - 3)(cx - 1)^2 + 8(\log(-cx + 1)^3 - 3\log(-cx + 1)^2 + 6\log(-cx + \\ & 1) - 6)(cx - 1))/c^2 + 18\log(4c^3x^2 - 4c)/c^2 - 192\operatorname{integrate}(1/4 \\ & \log(cx + 1)/(c^3x^2 - c), x))b^3 \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3x \operatorname{artanh}(cx)^3 + 3ab^2x \operatorname{artanh}(cx)^2 + 3a^2bx \operatorname{artanh}(cx) + a^3x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="fricas")`

[Out] `integral(b^3*x*arctanh(c*x)^3 + 3*a*b^2*x*arctanh(c*x)^2 + 3*a^2*b*x*arctan  
h(c*x) + a^3*x, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x))\*\*3,x)

[Out] Integral(x\*(a + b\*atanh(c\*x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3\*x, x)

### 3.29 $\int (a + b \tanh^{-1}(cx))^3 dx$

**Optimal.** Leaf size=108

$$\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{2c} + x (a + b \tanh^{-1}(cx))^3 + \frac{(a + b \tanh^{-1}(cx))^3}{c}$$

[Out] (a + b\*ArcTanh[c\*x])^3/c + x\*(a + b\*ArcTanh[c\*x])^3 - (3\*b\*(a + b\*ArcTanh[c\*x])^2\*Log[2/(1 - c\*x)])/c - (3\*b^2\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, 1 - 2/(1 - c\*x)])/c + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c\*x)])/(2\*c)

**Rubi [A]** time = 0.217234, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$ , Rules used = {5910, 5984, 5918, 5948, 6058, 6610}

$$\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right) (a + b \tanh^{-1}(cx))}{c} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)}{2c} + x (a + b \tanh^{-1}(cx))^3 + \frac{(a + b \tanh^{-1}(cx))^3}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^3, x]

[Out] (a + b\*ArcTanh[c\*x])^3/c + x\*(a + b\*ArcTanh[c\*x])^3 - (3\*b\*(a + b\*ArcTanh[c\*x])^2\*Log[2/(1 - c\*x)])/c - (3\*b^2\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, 1 - 2/(1 - c\*x)])/c + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c\*x)])/(2\*c)

#### Rule 5910

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*ArcTanh[c\*x])^p, x] - Dist[b\*c\*p, Int[(x\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]

#### Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

#### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:= -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c^p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_] * ((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_)^2), x_Symbol]
:= -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(cx))^3 dx &= x (a + b \tanh^{-1}(cx))^3 - (3bc) \int \frac{x (a + b \tanh^{-1}(cx))^2}{1 - c^2 x^2} dx \\
&= \frac{(a + b \tanh^{-1}(cx))^3}{c} + x (a + b \tanh^{-1}(cx))^3 - (3b) \int \frac{(a + b \tanh^{-1}(cx))^2}{1 - cx} dx \\
&= \frac{(a + b \tanh^{-1}(cx))^3}{c} + x (a + b \tanh^{-1}(cx))^3 - \frac{3b (a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{c} + (6b^2) \int \frac{(a + b \tanh^{-1}(cx))}{1 - cx} dx \\
&= \frac{(a + b \tanh^{-1}(cx))^3}{c} + x (a + b \tanh^{-1}(cx))^3 - \frac{3b (a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{c} - \frac{3b^2 (a + b \tanh^{-1}(cx))}{c} \\
&= \frac{(a + b \tanh^{-1}(cx))^3}{c} + x (a + b \tanh^{-1}(cx))^3 - \frac{3b (a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1-cx}\right)}{c} - \frac{3b^2 (a + b \tanh^{-1}(cx))}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.260217, size = 161, normalized size = 1.49

$$6ab^2 \left( \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(cx)} \right) + \tanh^{-1}(cx) \left( (cx-1) \tanh^{-1}(cx) - 2 \log \left( e^{-2 \tanh^{-1}(cx)} + 1 \right) \right) \right) + b^3 \left( 6 \tanh^{-1}(cx) \text{Po} \right)$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3, x]

[Out] (2\*a^3\*c\*x + 6\*a^2\*b\*c\*x\*ArcTanh[c\*x] + 3\*a^2\*b\*Log[1 - c^2\*x^2] + 6\*a\*b^2\*(ArcTanh[c\*x]\*((-1 + c\*x)\*ArcTanh[c\*x] - 2\*Log[1 + E^(-2\*ArcTanh[c\*x]))]) + PolyLog[2, -E^(-2\*ArcTanh[c\*x])]) + b^3\*(2\*ArcTanh[c\*x]^2\*(-1 + c\*x)\*ArcTanh[c\*x] - 3\*Log[1 + E^(-2\*ArcTanh[c\*x])]) + 6\*ArcTanh[c\*x]\*PolyLog[2, -E^(-2\*ArcTanh[c\*x])] + 3\*PolyLog[3, -E^(-2\*ArcTanh[c\*x])])/(2\*c)

---

**Maple [B]** time = 0.082, size = 261, normalized size = 2.4

$$xa^3 + b^3x(\text{Artanh}(cx))^3 + \frac{b^3(\text{Artanh}(cx))^3}{c} - 3\frac{b^3(\text{Artanh}(cx))^2}{c} \ln\left(\frac{(cx+1)^2}{-c^2x^2+1} + 1\right) - 3\frac{b^3\text{Artanh}(cx)}{c} \text{polylog}\left(2, \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^3, x)

[Out] x\*a^3+b^3\*x\*arctanh(c\*x)^3+1/c\*b^3\*arctanh(c\*x)^3-3/c\*b^3\*arctanh(c\*x)^2\*ln((c\*x+1)^2/(-c^2\*x^2+1)+1)-3/c\*b^3\*arctanh(c\*x)\*polylog(2, -(c\*x+1)^2/(-c^2\*x^2+1))+3/2/c\*b^3\*polylog(3, -(c\*x+1)^2/(-c^2\*x^2+1))+3\*x\*a\*b^2\*arctanh(c\*x)^2+3/c\*a\*b^2\*arctanh(c\*x)^2-6/c\*arctanh(c\*x)\*ln((c\*x+1)^2/(-c^2\*x^2+1))\*a\*b^2-3/c\*polylog(2, -(c\*x+1)^2/(-c^2\*x^2+1))\*a\*b^2+3\*x\*a^2\*b\*arctanh(c\*x)+3/2/c\*a^2\*b\*ln(-c^2\*x^2+1)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3x + \frac{3(2cx \text{artanh}(cx) + \log(-c^2x^2 + 1))a^2b}{2c} - \frac{(b^3cx - b^3) \log(-cx + 1)^3 - 3(2ab^2cx + (b^3cx + b^3) \log(cx + 1)) \log}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3, x, algorithm="maxima")

```
[Out] a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/c - 1/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c*x + (b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 - 3*(4*a*b^2*c*x + (b^3*c*x - b^3)*log(c*x + 1)^2 - 2*(2*a*b^2 - b^3 - (2*a*b^2*c + b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x - 1), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x) + a^3, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3, x)
```

$$3.30 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{x} dx$$

**Optimal.** Leaf size=184

$$\frac{3}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) - \frac{3}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx)) - \frac{3}{2}b \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) + \frac{3}{2}b \text{PolyLog}\left(2, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx))$$

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - 2/(1 - c*x)]/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -1 + 2/(1 - c*x)]/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 - c*x)]/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)]/2 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x)]/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)]/4
```

---

**Rubi [A]** time = 0.448718, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) - \frac{3}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx)) - \frac{3}{2}b \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)(a+b \tanh^{-1}(cx)) + \frac{3}{2}b \text{PolyLog}\left(2, \frac{2}{1-cx} - 1\right)(a+b \tanh^{-1}(cx))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x, x]
```

```
[Out] 2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, 1 - 2/(1 - c*x)]/2 + (3*b*(a + b*ArcTanh[c*x])^2*PolyLog[2, -1 + 2/(1 - c*x)]/2 + (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, 1 - 2/(1 - c*x)]/2 - (3*b^2*(a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)]/2 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x)]/4 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x)]/4
```

### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_)/(x_), x_Symbol] := Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c^p, Int[((a + b*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

#### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

#### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

#### Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1, u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

#### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^3}{x} dx &= 2(a + b \tanh^{-1}(cx))^3 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - (6bc) \int \frac{(a + b \tanh^{-1}(cx))^2 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right)}{1 - c^2x^2} dx \\
&= 2(a + b \tanh^{-1}(cx))^3 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) + (3bc) \int \frac{(a + b \tanh^{-1}(cx))^2 \log\left(\frac{2}{1 - cx}\right)}{1 - c^2x^2} dx \\
&= 2(a + b \tanh^{-1}(cx))^3 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - \frac{3}{2}b(a + b \tanh^{-1}(cx))^2 \operatorname{Li}_2\left(1 - \frac{2}{1 - cx}\right) + \frac{3}{2}b \\
&= 2(a + b \tanh^{-1}(cx))^3 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - \frac{3}{2}b(a + b \tanh^{-1}(cx))^2 \operatorname{Li}_2\left(1 - \frac{2}{1 - cx}\right) + \frac{3}{2}b \\
&= 2(a + b \tanh^{-1}(cx))^3 \tanh^{-1}\left(1 - \frac{2}{1 - cx}\right) - \frac{3}{2}b(a + b \tanh^{-1}(cx))^2 \operatorname{Li}_2\left(1 - \frac{2}{1 - cx}\right) + \frac{3}{2}b
\end{aligned}$$

**Mathematica [A]** time = 0.127977, size = 178, normalized size = 0.97

$$\frac{3}{4}b \left( 2 \operatorname{PolyLog}\left(2, \frac{cx+1}{1-cx}\right) (a + b \tanh^{-1}(cx))^2 - 2 \operatorname{PolyLog}\left(2, \frac{cx+1}{cx-1}\right) (a + b \tanh^{-1}(cx))^2 + b \left( -2 \operatorname{PolyLog}\left(3, \frac{cx+1}{1-cx}\right) \right. \right.$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3/x, x]

[Out] 2\*(a + b\*ArcTanh[c\*x])^3\*ArcTanh[(1 + c\*x)/(-1 + c\*x)] + (3\*b\*(2\*(a + b\*ArcTanh[c\*x])^2\*PolyLog[2, (1 + c\*x)/(1 - c\*x)] - 2\*(a + b\*ArcTanh[c\*x])^2\*PolyLog[2, (1 + c\*x)/(-1 + c\*x)] + b\*(-2\*(a + b\*ArcTanh[c\*x])\*PolyLog[3, (1 + c\*x)/(1 - c\*x)] + 2\*(a + b\*ArcTanh[c\*x])\*PolyLog[3, (1 + c\*x)/(-1 + c\*x)] + b\*(PolyLog[4, (1 + c\*x)/(1 - c\*x)] - PolyLog[4, (1 + c\*x)/(-1 + c\*x)])))/4

**Maple [C]** time = 0.113, size = 1470, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^3/x, x)

```
[Out] 3/2*a*b^2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-6*a*b^2*polylog(3,(c*x+1)/(-c^
2*x^2+1)^(1/2))-6*a*b^2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*ln(c*x)*
arctanh(c*x)^3-b^3*arctanh(c*x)^3*ln((c*x+1)^2/(-c^2*x^2+1)-1)-3/2*b^3*arct
anh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/2*a^2*b*dilog(c*x)+3/2*b^3*
arctanh(c*x)*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+b^3*arctanh(c*x)^3*ln(1-(c*
x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,(c*x+1)/(-c^2*x^2+1
)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+b^3*arcta
nh(c*x)^3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*b^3*arctanh(c*x)^2*polylog(2,-
(c*x+1)/(-c^2*x^2+1)^(1/2))-6*b^3*arctanh(c*x)*polylog(3,-(c*x+1)/(-c^2*x^2
+1)^(1/2))-3/2*a^2*b*dilog(c*x+1)+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^
2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-
1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-1/2*I*b^3*Pi*csgn(I*((c*x+1)^
2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1
)+1))^2*arctanh(c*x)^3+3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c
*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2-1/2*I*b^3*Pi*csgn(I/((c*x+1)^2/(-
c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1
))^2*arctanh(c*x)^3-3/2*I*a*b^2*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I
*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/
2*I*a*b^2*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2
+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+1/2*I*b^3*Pi*csgn(I*((c
*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^3-3*a*b^2*arcta
nh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3*a*b^2*arctanh(c*x)^2*ln((c*x+1
)^2/(-c^2*x^2+1)-1)+3*a*b^2*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))
+6*a*b^2*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*a*b^2*arctanh
(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+6*a*b^2*arctanh(c*x)*polylog(2,-(c
*x+1)/(-c^2*x^2+1)^(1/2))+3*a^2*b*ln(c*x)*arctanh(c*x)+3*a*b^2*ln(c*x)*arct
anh(c*x)^2+a^3*ln(c*x)-3/4*b^3*polylog(4,-(c*x+1)^2/(-c^2*x^2+1))+6*b^3*pol
ylog(4,(c*x+1)/(-c^2*x^2+1)^(1/2))+6*b^3*polylog(4,-(c*x+1)/(-c^2*x^2+1)^(1
/2))+1/2*I*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1
+1))^3*arctanh(c*x)^3-3/2*a^2*b*ln(c*x)*ln(c*x+1)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3(\log(cx+1) - \log(-cx+1))^3}{8x} + \frac{3ab^2(\log(cx+1) - \log(-cx+1))^2}{4x} + \frac{3a^2b(\log(cx+1) - \log(-cx+1))}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/x + 3/4*a*b
^2*(log(c*x + 1) - log(-c*x + 1))^2/x + 3/2*a^2*b*(log(c*x + 1) - log(-c*x
```

+ 1))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x)^3 + 3\*a\*b^2\*arctanh(c\*x)^2 + 3\*a^2\*b\*arctanh(c\*x) + a^3)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*3/x,x)

[Out] Integral((a + b\*atanh(c\*x))\*\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3/x, x)

$$3.31 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{x^2} dx$$

**Optimal.** Leaf size=102

$$-3b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) (a+b \tanh^{-1}(cx)) - \frac{3}{2}b^3c \operatorname{PolyLog}\left(3, \frac{2}{cx+1} - 1\right) + c(a+b \tanh^{-1}(cx))^3 - \frac{(a+b \tanh^{-1}(cx))^3}{x}$$

[Out] c\*(a + b\*ArcTanh[c\*x])^3 - (a + b\*ArcTanh[c\*x])^3/x + 3\*b\*c\*(a + b\*ArcTanh[c\*x])^2\*Log[2 - 2/(1 + c\*x)] - 3\*b^2\*c\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, -1 + 2/(1 + c\*x)] - (3\*b^3\*c\*PolyLog[3, -1 + 2/(1 + c\*x)])/2

**Rubi [A]** time = 0.268323, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5916, 5988, 5932, 5948, 6056, 6610}

$$-3b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) (a+b \tanh^{-1}(cx)) - \frac{3}{2}b^3c \operatorname{PolyLog}\left(3, \frac{2}{cx+1} - 1\right) + c(a+b \tanh^{-1}(cx))^3 - \frac{(a+b \tanh^{-1}(cx))^3}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^3/x^2, x]

[Out] c\*(a + b\*ArcTanh[c\*x])^3 - (a + b\*ArcTanh[c\*x])^3/x + 3\*b\*c\*(a + b\*ArcTanh[c\*x])^2\*Log[2 - 2/(1 + c\*x)] - 3\*b^2\*c\*(a + b\*ArcTanh[c\*x])\*PolyLog[2, -1 + 2/(1 + c\*x)] - (3\*b^3\*c\*PolyLog[3, -1 + 2/(1 + c\*x)])/2

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)*((d_.)*(x_.))^ (m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^ (p_.)/((x_.)*((d_.) + (e_.)*(x_.)^2)),
  x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
```

}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6056

Int[(Log[u]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^3}{x^2} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{x} + (3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x(1 - c^2x^2)} dx \\
&= c(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{x} + (3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x(1 + cx)} dx \\
&= c(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{x} + 3bc(a + b \tanh^{-1}(cx))^2 \log\left(2 - \frac{2}{1 + cx}\right) - (3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x(1 - cx)} dx \\
&= c(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{x} + 3bc(a + b \tanh^{-1}(cx))^2 \log\left(2 - \frac{2}{1 + cx}\right) - 3bc(a + b \tanh^{-1}(cx))^2 \log\left(2 - \frac{2}{1 - cx}\right) \\
&= c(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{x} + 3bc(a + b \tanh^{-1}(cx))^2 \log\left(2 - \frac{2}{1 + cx}\right) - 3bc(a + b \tanh^{-1}(cx))^2 \log\left(2 - \frac{2}{1 - cx}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.320924, size = 196, normalized size = 1.92

$$3ab^2c \left( \tanh^{-1}(cx) \left( -\frac{\tanh^{-1}(cx)}{cx} + \tanh^{-1}(cx) + 2 \log\left(1 - e^{-2 \tanh^{-1}(cx)}\right) \right) - \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx)}\right) \right) + b^3c \left( 3 \tanh^{-1}(cx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3/x^2,x]

[Out]  $-(a^3/x) - (3a^2b \text{ArcTanh}[c*x])/x + 3a^2b*c \text{Log}[x] - (3a^2b*c \text{Log}[1 - c^2x^2])/2 + 3a*b^2*c*(\text{ArcTanh}[c*x]*(\text{ArcTanh}[c*x] - \text{ArcTanh}[c*x]/(c*x) + 2*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c*x])}]) - \text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c*x])}]) + b^3c*((I/8)*\text{Pi}^3 - \text{ArcTanh}[c*x]^3 - \text{ArcTanh}[c*x]^3/(c*x) + 3*\text{ArcTanh}[c*x]^2*\text{Log}[1 - E^{(2*\text{ArcTanh}[c*x])}] + 3*\text{ArcTanh}[c*x]*\text{PolyLog}[2, E^{(2*\text{ArcTanh}[c*x])}] - (3*\text{PolyLog}[3, E^{(2*\text{ArcTanh}[c*x])}]))/2)$

**Maple [C]** time = 0.183, size = 1583, normalized size = 15.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^3/x^2,x)

```
[Out] 6*c*a*b^2*arctanh(c*x)*ln(c*x)-3*c*a*b^2*arctanh(c*x)*ln(c*x-1)-3*c*a*b^2*ln(c*x)*ln(c*x+1)-3*c*b^3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+6*c*b^3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*c*b^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3/2*c*b^3*arctanh(c*x)^2*ln(c*x-1)-3/2*c*b^3*arctanh(c*x)^2*ln(c*x+1)+3*c*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-3*c*a*b^2*dilog(c*x)-3*c*a*b^2*dilog(c*x+1)-3*a^2*b/x*arctanh(c*x)-3*a*b^2/x*arctanh(c*x)^2+3*c*a^2*b*ln(c*x)-3/2*c*a^2*b*ln(c*x-1)-3/2*c*a^2*b*ln(c*x+1)-3/4*c*a*b^2*ln(c*x-1)^2+3/4*c*a*b^2*ln(c*x+1)^2+3*c*b^3*arctanh(c*x)^2*ln(2)+3*c*a*b^2*dilog(1/2+1/2*c*x)+3*c*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*c*b^3*ln(c*x)*arctanh(c*x)^2-b^3/x*arctanh(c*x)^3-6*c*b^3*polylog(3,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*c*b^3*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))-c*b^3*arctanh(c*x)^3+3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-3/4*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))*arctanh(c*x)^2-a^3/x+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x)^2-3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3*c*a*b^2*arctanh(c*x)*ln(c*x+1)+3/2*c*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-3/2*c*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1)+3/2*c*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)+3/2*I*c*b^3*Pi*arctanh(c*x)^2+3/2*I*c*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2-3/2*I*c*b^3*Pi*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2+3/4*I*c*b^3*Pi*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*arctanh(c*x)^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3}{2} \left( c \left( \log(c^2 x^2 - 1) - \log(x^2) \right) + \frac{2 \operatorname{artanh}(cx)}{x} \right) a^2 b - \frac{a^3}{x} - \frac{(b^3 cx - b^3) \log(-cx + 1)^3 + 3(2ab^2 + (b^3 cx + b^3) \log(cx))}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^2,x, algorithm="maxima")

```
[Out] -3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a^2*b - a^3/x - 1/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 + 3*(2*a*b^2 + (b^3*c*x + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/x - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 + 3*(4*a*b^2*c*x - (b^3*c*x - b^3)*log(c*x + 1)^2 + 2*(b^3*c^2*x^2 + 2*a*b^2 - (2*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x) + a^3)/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^2, x)
```

$$3.32 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{x^3} dx$$

**Optimal.** Leaf size=123

$$-\frac{3}{2}b^3c^2 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + 3b^2c^2 \log\left(2 - \frac{2}{cx+1}\right)(a + b \tanh^{-1}(cx)) + \frac{3}{2}bc^2(a + b \tanh^{-1}(cx))^2 + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3$$

[Out] (3\*b\*c^2\*(a + b\*ArcTanh[c\*x])^2)/2 - (3\*b\*c\*(a + b\*ArcTanh[c\*x])^2)/(2\*x) + (c^2\*(a + b\*ArcTanh[c\*x])^3)/2 - (a + b\*ArcTanh[c\*x])^3/(2\*x^2) + 3\*b^2\*c^2\*(a + b\*ArcTanh[c\*x])\*Log[2 - 2/(1 + c\*x)] - (3\*b^3\*c^2\*PolyLog[2, -1 + 2/(1 + c\*x)])/2

**Rubi [A]** time = 0.294355, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5916, 5982, 5988, 5932, 2447, 5948}

$$-\frac{3}{2}b^3c^2 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) + 3b^2c^2 \log\left(2 - \frac{2}{cx+1}\right)(a + b \tanh^{-1}(cx)) + \frac{3}{2}bc^2(a + b \tanh^{-1}(cx))^2 + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^3/x^3, x]

[Out] (3\*b\*c^2\*(a + b\*ArcTanh[c\*x])^2)/2 - (3\*b\*c\*(a + b\*ArcTanh[c\*x])^2)/(2\*x) + (c^2\*(a + b\*ArcTanh[c\*x])^3)/2 - (a + b\*ArcTanh[c\*x])^3/(2\*x^2) + 3\*b^2\*c^2\*(a + b\*ArcTanh[c\*x])\*Log[2 - 2/(1 + c\*x)] - (3\*b^3\*c^2\*PolyLog[2, -1 + 2/(1 + c\*x)])/2

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (
e_.)*(x_.)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

, x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

### Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^p\_/((x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)), x\_Symbol] :> Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^p\_/((x\_.)\*((d\_.) + (e\_.)\*(x\_.))), x\_Symbol] :> Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] :> With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^p\_/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] :> Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx))^3}{x^3} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{2x^2} + \frac{1}{2}(3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^2(1 - c^2x^2)} dx \\
&= -\frac{(a + b \tanh^{-1}(cx))^3}{2x^2} + \frac{1}{2}(3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^2} dx + \frac{1}{2}(3bc^3) \int \frac{(a + b \tanh^{-1}(cx))}{1 - c^2x^2} dx \\
&= -\frac{3bc(a + b \tanh^{-1}(cx))^2}{2x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{2x^2} + (3b^2c^2) \int \frac{a - b \tanh^{-1}(cx)}{1 - c^2x^2} dx \\
&= \frac{3}{2}bc^2(a + b \tanh^{-1}(cx))^2 - \frac{3bc(a + b \tanh^{-1}(cx))^2}{2x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{2x^2} \\
&= \frac{3}{2}bc^2(a + b \tanh^{-1}(cx))^2 - \frac{3bc(a + b \tanh^{-1}(cx))^2}{2x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{2x^2} \\
&= \frac{3}{2}bc^2(a + b \tanh^{-1}(cx))^2 - \frac{3bc(a + b \tanh^{-1}(cx))^2}{2x} + \frac{1}{2}c^2(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.271884, size = 192, normalized size = 1.56

$$-6b^3c^2x^2\text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx)}\right) + a\left(-2a^2 - 3abc^2x^2 \log(1 - cx) + 3abc^2x^2 \log(cx + 1) - 6abcx + 12b^2c^2x^2 \log\left(\frac{cx}{\sqrt{1-c^2}}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3/x^3, x]

[Out] (6\*b^2\*(-1 + c\*x)\*(a + a\*c\*x + b\*c\*x)\*ArcTanh[c\*x]^2 + 2\*b^3\*(-1 + c^2\*x^2)\*ArcTanh[c\*x]^3 - 6\*b\*ArcTanh[c\*x]\*(a^2 + 2\*a\*b\*c\*x - 2\*b^2\*c^2\*x^2\*Log[1 - E^(-2\*ArcTanh[c\*x])]) + a\*(-2\*a^2 - 6\*a\*b\*c\*x - 3\*a\*b\*c^2\*x^2\*Log[1 - c\*x] + 3\*a\*b\*c^2\*x^2\*Log[1 + c\*x] + 12\*b^2\*c^2\*x^2\*Log[(c\*x)/Sqrt[1 - c^2\*x^2]]) - 6\*b^3\*c^2\*x^2\*PolyLog[2, E^(-2\*ArcTanh[c\*x])])/(4\*x^2)

**Maple [C]** time = 0.316, size = 5098, normalized size = 41.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x))^3/x^3,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{4} \left( \left( c \log(cx+1) - c \log(cx-1) - \frac{2}{x} \right) c - \frac{2 \operatorname{artanh}(cx)}{x^2} \right) a^2 b + \frac{3}{8} \left( (2(\log(cx-1) - 2) \log(cx+1) - \log(cx+1)^2 - \log(cx-1)^2 - 4 \log(cx-1) + 8 \log(x)) c^2 + 4(c \log(cx+1) - c \log(cx-1) - \frac{2}{x}) c \operatorname{artanh}(cx) \right) a b^2 - \frac{1}{16} b^3 \left( (c^2 x^2 - 1) \log(-cx+1)^3 + 3(2c x - (c^2 x^2 - 1) \log(cx+1)) \log(-cx+1)^2 / x^2 + 2 \operatorname{integrate}(-((cx-1) \log(cx+1)^3 + 3(2c^2 x^2 - (cx-1) \log(cx+1)^2 - (c^3 x^3 - cx) \log(cx+1)) \log(-cx+1)) / (c x^4 - x^3), x) - 3/2 a b^2 \operatorname{arctanh}(cx)^2 / x^2 - 1/2 a^3 / x^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="maxima")`

[Out]  $3/4 * ((c * \log(c*x + 1) - c * \log(c*x - 1) - 2/x) * c - 2 * \operatorname{arctanh}(c*x) / x^2) * a^2 * b + 3/8 * ((2 * (\log(c*x - 1) - 2) * \log(c*x + 1) - \log(c*x + 1)^2 - \log(c*x - 1)^2 - 4 * \log(c*x - 1) + 8 * \log(x)) * c^2 + 4 * (c * \log(c*x + 1) - c * \log(c*x - 1) - 2/x) * c * \operatorname{arctanh}(c*x)) * a * b^2 - 1/16 * b^3 * (((c^2 * x^2 - 1) * \log(-c*x + 1)^3 + 3 * (2 * c * x - (c^2 * x^2 - 1) * \log(c*x + 1)) * \log(-c*x + 1)^2) / x^2 + 2 * \operatorname{integrate}(-((c*x - 1) * \log(c*x + 1)^3 + 3 * (2 * c^2 * x^2 - (c*x - 1) * \log(c*x + 1)^2 - (c^3 * x^3 - c*x) * \log(c*x + 1)) * \log(-c*x + 1)) / (c*x^4 - x^3), x)) - 3/2 * a * b^2 * \operatorname{arctanh}(c*x)^2 / x^2 - 1/2 * a^3 / x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="fricas")`

[Out] `integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x) + a^3)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*3/x\*\*3,x)

[Out] Integral((a + b\*atanh(c\*x))\*\*3/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3/x^3, x)

$$3.33 \quad \int \frac{(a + b \tanh^{-1}(cx))^3}{x^4} dx$$

Optimal. Leaf size=200

$$-b^2c^3 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right)(a + b \tanh^{-1}(cx)) - \frac{1}{2}b^3c^3 \text{PolyLog}\left(3, \frac{2}{cx+1} - 1\right) - \frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{3}c^3(a$$

```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a + b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 + b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

**Rubi [A]** time = 0.498341, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$ , Rules used = {5916, 5982, 266, 36, 29, 31, 5948, 5988, 5932, 6056, 6610}

$$-b^2c^3 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right)(a + b \tanh^{-1}(cx)) - \frac{1}{2}b^3c^3 \text{PolyLog}\left(3, \frac{2}{cx+1} - 1\right) - \frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{3}c^3(a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x])^3/x^4, x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTanh[c*x]))/x) + (b*c^3*(a + b*ArcTanh[c*x])^2)/2 - (b*c*(a + b*ArcTanh[c*x])^2)/(2*x^2) + (c^3*(a + b*ArcTanh[c*x])^3)/3 - (a + b*ArcTanh[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 - c^2*x^2])/2 + b*c^3*(a + b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - b^2*c^3*(a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 + c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 + c*x)])/2
```

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^ (p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 5982

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((f\_.)\*(x\_))^(m\_)]/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/d, Int[(f\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x], x] - Dist[e/(d\*f^2), Int[((f\*x)^(m + 2)\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]

Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] -



Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 6056

Int[(Log[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] - Dist[(b\*p)/2, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tanh^{-1}(cx))^3}{x^4} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{3x^3} + (bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^3(1 - c^2x^2)} dx \\
 &= -\frac{(a + b \tanh^{-1}(cx))^3}{3x^3} + (bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^3} dx + (bc^3) \int \frac{(a + b \tanh^{-1}(cx))^2}{x(1 - c^2x^2)} dx \\
 &= -\frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{3x^3} + (b^2c^2) \int \frac{a + b \tanh^{-1}(cx)}{x} dx \\
 &= -\frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{3x^3} + bc^3(a + b \tanh^{-1}(cx)) \\
 &= -\frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx)) \\
 &= -\frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx)) \\
 &= -\frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx)) \\
 &= -\frac{b^2c^2(a + b \tanh^{-1}(cx))}{x} + \frac{1}{2}bc^3(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{2x^2} + \frac{1}{3}c^3(a + b \tanh^{-1}(cx))
 \end{aligned}$$

**Mathematica [C]** time = 0.861358, size = 323, normalized size = 1.62

$$24ab^2 \left( c^3 x^3 \text{PolyLog} \left( 2, e^{-2 \tanh^{-1}(cx)} \right) + c^2 x^2 + (1 - c^3 x^3) \tanh^{-1}(cx)^2 - cx \tanh^{-1}(cx) \left( c^2 x^2 + 2c^2 x^2 \log \left( 1 - e^{-2 \tanh^{-1}(cx)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3/x^4, x]

[Out]  $-(8a^3 + 12a^2 b c x + 24a^2 b \text{ArcTanh}[c x] - 24a^2 b c^3 x^3 \text{Log}[x] + 12a^2 b c^3 x^3 \text{Log}[1 - c^2 x^2] + 24a b^2 (c^2 x^2 + (1 - c^3 x^3) \text{ArcTanh}[c x]^2 - c x \text{ArcTanh}[c x] (-1 + c^2 x^2 + 2c^2 x^2 \text{Log}[1 - E^{(-2 \text{ArcTanh}[c x])}]]) + c^3 x^3 \text{PolyLog}[2, E^{(-2 \text{ArcTanh}[c x])}]]) + b^3 ((-I) c^3 \text{Pi}^3 x^3 + 24c^2 x^2 \text{ArcTanh}[c x] + 12c x \text{ArcTanh}[c x]^2 - 12c^3 x^3 \text{ArcTanh}[c x]^2 + 8 \text{ArcTanh}[c x]^3 + 8c^3 x^3 \text{ArcTanh}[c x]^3 - 24c^3 x^3 \text{ArcTanh}[c x]^2 \text{Log}[1 - E^{(2 \text{ArcTanh}[c x])}] - 24c^3 x^3 \text{Log}[(c x) / \text{Sqrt}[1 - c^2 x^2]] - 24c^3 x^3 \text{ArcTanh}[c x] \text{PolyLog}[2, E^{(2 \text{ArcTanh}[c x])}] + 12c^3 x^3 \text{PolyLog}[3, E^{(2 \text{ArcTanh}[c x])}])) / (24x^3)$

**Maple [C]** time = 0.849, size = 1838, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^3/x^4, x)

[Out]  $-1/2 c^3 a b^2 \ln(c x - 1) + 1/2 c^3 a b^2 \ln(c x + 1) - 1/2 c^3 a^2 b \ln(c x - 1) - 1/2 c^3 a^2 b \ln(c x + 1) - 1/4 c^3 a b^2 \ln(c x - 1)^2 + 1/4 c^3 a b^2 \ln(c x + 1)^2 - a^2 b / x^3 \arctanh(c x) - a b^2 / x^3 \arctanh(c x)^2 - 1/4 I c^3 b^3 \arctanh(c x)^2 * \text{csgn}(I * (c x + 1)^2 / (c^2 x^2 - 1)) * \text{csgn}(I / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1)) * \text{csgn}(I * (c x + 1)^2 / (c^2 x^2 - 1) / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1)) * \text{Pi} + 1/2 I c^3 b^3 \arctanh(c x)^2 * \text{csgn}(I / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1)) * \text{csgn}(I * ((c x + 1)^2 / (-c^2 x^2 + 1) - 1)) * \text{csgn}(I * ((c x + 1)^2 / (-c^2 x^2 + 1) - 1) / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1)) * \text{Pi} - 1/3 a^3 / x^3 - 1/3 b^3 / x^3 \arctanh(c x)^3 + c^3 b^3 \ln((c x + 1) / (-c^2 x^2 + 1)^{(1/2)} - 1) + c^3 b^3 \ln(1 + (c x + 1) / (-c^2 x^2 + 1)^{(1/2)}) - 2 c^3 b^3 \text{polylog}(3, -(c x + 1) / (-c^2 x^2 + 1)^{(1/2)}) - 2 c^3 b^3 \text{polylog}(3, (c x + 1) / (-c^2 x^2 + 1)^{(1/2)}) - c^3 b^3 \arctanh(c x) - 1/3 c^3 b^3 \arctanh(c x)^3 + 1/2 c^3 b^3 \arctanh(c x)^2 + 1/4 I c^3 b^3 \arctanh(c x)^2 * \text{csgn}(I * (c x + 1)^2 / (c^2 x^2 - 1))^3 * \text{Pi} + 1/2 I c^3 b^3 \arctanh(c x)^2 * \text{csgn}(I / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1))^3 * \text{Pi} + 1/2 I c^3 b^3 \arctanh(c x)^2 * \text{csgn}(I * (c x + 1)^2 / (-c^2 x^2 + 1) - 1) / ((c x + 1)^2 / (-c^2 x^2 + 1) + 1))^3 * \text{Pi} + 1/4 I c^3 b^3$

```

*arctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^3*
Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi+1/4
*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)^2/(-c^2*x^2+1)+1))*csgn(I*(c*x+1)
^2/(c^2*x^2-1)/((c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-c^2*a*b^2/x-1/2*c*a^2*b/x^2
-c^2*b^3*arctanh(c*x)/x-1/2*c*b^3*arctanh(c*x)^2/x^2+c^3*b^3*arctanh(c*x)^2
*ln(2)+c^3*a*b^2*dilog(1/2+1/2*c*x)+c^3*b^3*arctanh(c*x)^2*ln(1+(c*x+1)/(-c
^2*x^2+1)^(1/2))+c^3*b^3*ln(c*x)*arctanh(c*x)^2-c^3*b^3*arctanh(c*x)^2*ln((
c*x+1)^2/(-c^2*x^2+1)-1)+2*c^3*b^3*arctanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^
2+1)^(1/2))+c^3*b^3*arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*c^3*b
^3*arctanh(c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-1/2*c^3*b^3*arctanh(c
*x)^2*ln(c*x-1)-1/2*c^3*b^3*arctanh(c*x)^2*ln(c*x+1)+c^3*b^3*arctanh(c*x)^2
*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-c^3*a*b^2*dilog(c*x)-c^3*a*b^2*dilog(c*x+1)
+c^3*a^2*b*ln(c*x)-c^3*a*b^2*arctanh(c*x)*ln(c*x-1)+2*c^3*a*b^2*arctanh(c*x
)*ln(c*x)-c^3*a*b^2*ln(c*x)*ln(c*x+1)-c^3*a*b^2*arctanh(c*x)*ln(c*x+1)+1/2*
c^3*a*b^2*ln(c*x-1)*ln(1/2+1/2*c*x)-1/2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(c*x+1
)+1/2*c^3*a*b^2*ln(-1/2*c*x+1/2)*ln(1/2+1/2*c*x)-c*a*b^2*arctanh(c*x)/x^2+1
/2*I*c^3*b^3*arctanh(c*x)^2*Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*((c*x+1)
^2/(-c^2*x^2+1)-1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*Pi+1/4*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))
^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*Pi+1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I*(c*
x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*Pi-1/4*I*c^3*b^3*a
rctanh(c*x)^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/((
c*x+1)^2/(-c^2*x^2+1)+1))^2*Pi-1/2*I*c^3*b^3*arctanh(c*x)^2*csgn(I/((c*x+1)
^2/(-c^2*x^2+1)+1))*csgn(I*((c*x+1)^2/(-c^2*x^2+1)-1)/((c*x+1)^2/(-c^2*x^2+
1)+1))^2*Pi

```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( c^2 \log(c^2 x^2 - 1) - c^2 \log(x^2) + \frac{1}{x^2} \right) c + \frac{2 \operatorname{artanh}(cx)}{x^3} a^2 b - \frac{a^3}{3x^3} - \frac{(b^3 c^3 x^3 - b^3) \log(-cx + 1)^3 + 3(b^3 cx + 2ab^2)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^4,x, algorithm="maxima")

[Out] 
$$-1/2*((c^2*\log(c^2*x^2 - 1) - c^2*\log(x^2) + 1/x^2)*c + 2*\operatorname{arctanh}(c*x)/x^3) *a^2*b - 1/3*a^3/x^3 - 1/24*((b^3*c^3*x^3 - b^3)*\log(-c*x + 1)^3 + 3*(b^3*c*x + 2*a*b^2 + (b^3*c^3*x^3 + b^3)*\log(c*x + 1))*\log(-c*x + 1)^2)/x^3 - \operatorname{integrate}(-1/8*((b^3*c*x - b^3)*\log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*\log(c*x + 1)^2 + (2*b^3*c^2*x^2 + 4*a*b^2*c*x - 3*(b^3*c*x - b^3)*\log(c*x + 1)^2 + 2*(b^3*c^4*x^4 + 6*a*b^2 - (6*a*b^2*c - b^3*c)*x)*\log(c*x + 1))*\log(-c*x +$$

1))/ (c\*x^5 - x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^4,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x)^3 + 3\*a\*b^2\*arctanh(c\*x)^2 + 3\*a^2\*b\*arctanh(c\*x) + a^3)/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))\*\*3/x\*\*4,x)

[Out] Integral((a + b\*atanh(c\*x))\*\*3/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)^3/x^4, x)

$$3.34 \quad \int \frac{(a+b \tanh^{-1}(cx))^3}{x^5} dx$$

**Optimal.** Leaf size=187

$$-b^3c^4 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{b^2c^2(a+b \tanh^{-1}(cx))}{4x^2} + 2b^2c^4 \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx)) + \frac{1}{4}c^4(a+b \tanh^{-1}(cx))^3$$

[Out]  $-(b^3c^3)/(4*x) + (b^3c^4*ArcTanh[c*x])/4 - (b^2c^2*(a + b*ArcTanh[c*x]))/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]$

**Rubi [A]** time = 0.62579, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5916, 5982, 325, 206, 5988, 5932, 2447, 5948}

$$-b^3c^4 \text{PolyLog}\left(2, \frac{2}{cx+1} - 1\right) - \frac{b^2c^2(a+b \tanh^{-1}(cx))}{4x^2} + 2b^2c^4 \log\left(2 - \frac{2}{cx+1}\right)(a+b \tanh^{-1}(cx)) + \frac{1}{4}c^4(a+b \tanh^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])^3/x^5, x]

[Out]  $-(b^3c^3)/(4*x) + (b^3c^4*ArcTanh[c*x])/4 - (b^2c^2*(a + b*ArcTanh[c*x]))/(4*x^2) + b*c^4*(a + b*ArcTanh[c*x])^2 - (b*c*(a + b*ArcTanh[c*x])^2)/(4*x^3) - (3*b*c^3*(a + b*ArcTanh[c*x])^2)/(4*x) + (c^4*(a + b*ArcTanh[c*x])^3)/4 - (a + b*ArcTanh[c*x])^3/(4*x^4) + 2*b^2*c^4*(a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - b^3*c^4*PolyLog[2, -1 + 2/(1 + c*x)]$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 5982

```
Int[(((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((f_.)*(x_.))^(m_.))/((d_.) + (e_.)*(x_)^2), x_Symbol] := Dist[1/d, Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x], x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

### Rule 325

```
Int[((c_.)*(x_.))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_)^2)), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

### Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_.) + (e_.)*(x_))), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Dist[(b*c*p)/d, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

### Rule 2447

```
Int[Log[u]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
```

, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tanh^{-1}(cx))^3}{x^5} dx &= -\frac{(a + b \tanh^{-1}(cx))^3}{4x^4} + \frac{1}{4}(3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^4(1 - c^2x^2)} dx \\
 &= -\frac{(a + b \tanh^{-1}(cx))^3}{4x^4} + \frac{1}{4}(3bc) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^4} dx + \frac{1}{4}(3bc^3) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^2(1 - c^2x^2)} dx \\
 &= -\frac{bc(a + b \tanh^{-1}(cx))^2}{4x^3} - \frac{(a + b \tanh^{-1}(cx))^3}{4x^4} + \frac{1}{2}(b^2c^2) \int \frac{a + b \tanh^{-1}(cx)}{x^3(1 - c^2x^2)} dx + \frac{1}{4}(3bc^3) \int \frac{(a + b \tanh^{-1}(cx))^2}{x^2(1 - c^2x^2)} dx \\
 &= -\frac{bc(a + b \tanh^{-1}(cx))^2}{4x^3} - \frac{3bc^3(a + b \tanh^{-1}(cx))^2}{4x} + \frac{1}{4}c^4(a + b \tanh^{-1}(cx))^3 - \frac{(a + b \tanh^{-1}(cx))^3}{4x} \\
 &= -\frac{b^2c^2(a + b \tanh^{-1}(cx))}{4x^2} + bc^4(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{4x^3} - \frac{3bc^3(a + b \tanh^{-1}(cx))^2}{4x} \\
 &= -\frac{b^3c^3}{4x} - \frac{b^2c^2(a + b \tanh^{-1}(cx))}{4x^2} + bc^4(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{4x^3} - \frac{3bc^3(a + b \tanh^{-1}(cx))^2}{4x} \\
 &= -\frac{b^3c^3}{4x} + \frac{1}{4}b^3c^4 \tanh^{-1}(cx) - \frac{b^2c^2(a + b \tanh^{-1}(cx))}{4x^2} + bc^4(a + b \tanh^{-1}(cx))^2 - \frac{bc(a + b \tanh^{-1}(cx))^2}{4x^3} - \frac{3bc^3(a + b \tanh^{-1}(cx))^2}{4x}
 \end{aligned}$$

**Mathematica [A]** time = 0.638678, size = 295, normalized size = 1.58

$$8b^3c^4x^4 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx)}\right) + 2b \tanh^{-1}(cx) \left(3a^2 + 2abcx(3c^2x^2 + 1) + b^2c^2x^2(1 - c^2x^2) - 8b^2c^4x^4 \log\left(1 - e^{-2 \tanh^{-1}(cx)}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x])^3/x^5, x]

[Out]  $-(2a^3 + 2a^2bcx + 2ab^2c^2x^2 + 6a^2b^3c^3x^3 + 2b^3c^3x^3 - 2ab^2c^4x^4 + 2b^2(bcx(1 + 3c^2x^2 - 4c^3x^3) + a(3 - 3c^4x^4)) \text{ArcTanh}[cx])^2 - 2b^3(-1 + c^4x^4) \text{ArcTanh}[cx]^3 + 2b \text{ArcTanh}[cx] * (3a^2 + b^2c^2x^2(1 - c^2x^2) + 2ab^2c^3x^3 + 2b^3c^3x^3 - 8b^2c^4x^4 \text{Log}[1 - E^{-2 \text{ArcTanh}[cx]}]) + 3a^2b^3c^4x^4 \text{Log}[1 - cx] - 3a^2b^3c^4x^4 \text{Log}[1 + cx] - 16ab^2c^4x^4 \text{Log}[(cx)/\text{Sqrt}[1 - c^2x^2]] + 8$

$*b^3*c^4*x^4*PolyLog[2, E^{(-2*ArcTanh[c*x])}]/(8*x^4)$

**Maple [C]** time = 0.873, size = 1281, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arctanh}(c*x))^3/x^5, x)$

[Out]  $\frac{1}{4}c^4b^3\text{arctanh}(c*x)^3 - c^4b^3\text{arctanh}(c*x)^2 + 2c^4b^3\text{dilog}(1+(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - 2c^4b^3\text{dilog}((c*x+1)/(-c^2*x^2+1)^{(1/2)}) - 1/4b^3/x^4\text{arctanh}(c*x)^3 - 1/2c^4a*b^2\text{arctanh}(c*x)/x^3 - 3/8I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)/(-c^2*x^2+1)^{(1/2)}) *c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^{2*\text{arctanh}(c*x)^2} - 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1)) *c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))/((c*x+1)^2/(-c^2*x^2+1)+1)^{2*\text{arctanh}(c*x)^2} - 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)/(-c^2*x^2+1)^{(1/2)})^{2*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))*\text{arctanh}(c*x)^2} + 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1)) *c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))/(c*x+1)^2/(-c^2*x^2+1)+1)^{2*\text{arctanh}(c*x)^2} + 1/4b^3c^4\text{arctanh}(c*x) - 1/4a^3/x^4 + 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1)) *c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1)) *c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))/(c*x+1)^2/(-c^2*x^2+1)+1)^{*arctanh}(c*x)^2 - 3/8c^4a^2*b*\ln(c*x-1) + 3/8c^4a^2*b*\ln(c*x+1) - 3/16c^4a*b^2*\ln(c*x-1)^2 - 3/16c^4a*b^2*\ln(c*x+1)^2 - 3/8c^4b^3\text{arctanh}(c*x)^2*\ln(c*x-1) + 3/8c^4b^3\text{arctanh}(c*x)^2*\ln(c*x+1) - 3/4c^4b^3\text{arctanh}(c*x)^2*\ln((c*x+1)/(-c^2*x^2+1)^{(1/2)}) + 2c^4b^3\text{arctanh}(c*x)*\ln(1+(c*x+1)/(-c^2*x^2+1)^{(1/2)}) - 1/4c^4b^3/(c*x+1 - (-c^2*x^2+1)^{(1/2)}) * (-c^2*x^2+1)^{(1/2)} + 1/4c^4b^3/((-c^2*x^2+1)^{(1/2)} + c*x+1) * (-c^2*x^2+1)^{(1/2)} - 1/4c^4b^3\text{arctanh}(c*x)^2/x^3 - 1/4c^2b^3\text{arctanh}(c*x)/x^2 - 3/4c^3b^3\text{arctanh}(c*x)^2/x + 3/8I*c^4b^3\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))^{3*\text{arctanh}(c*x)^2} - 3/4c^3a^2*b/x - 1/4c^2a*b^2/x^2 - 1/4c^3a^2*b/x^3 - 3/4a*b^2/x^4\text{arctanh}(c*x)^2 - 3/4a^2*b/x^4\text{arctanh}(c*x) + 2c^4a*b^2*\ln(c*x) - c^4a*b^2*\ln(c*x-1) - c^4a*b^2*\ln(c*x+1) - 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^{3*\text{arctanh}(c*x)^2} - 3/8I*c^4b^3\text{Pi}*c\text{sgn}(I/((c*x+1)^2/(-c^2*x^2+1)+1))^{2*\text{arctanh}(c*x)^2} - 3/16I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))/((c*x+1)^2/(-c^2*x^2+1)+1))^{3*\text{arctanh}(c*x)^2} - 3/2c^3a*b^2/x*\text{arctanh}(c*x) - 3/4c^4a*b^2\text{arctanh}(c*x)*\ln(c*x-1) + 3/4c^4a*b^2\text{arctanh}(c*x)*\ln(c*x+1) + 3/8c^4a*b^2*\ln(c*x-1)*\ln(1/2+1/2*c*x) + 3/8c^4a*b^2*\ln(-1/2*c*x+1/2)*\ln(c*x+1) - 3/8c^4a*b^2*\ln(-1/2*c*x+1/2)*\ln(1/2+1/2*c*x) + 3/8I*c^4b^3\text{Pi}*c\text{sgn}(I*(c*x+1)^2/(c^2*x^2-1))^{2*\text{arctanh}(c*x)^2}$



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} \left( \left( 3c^3 \log(cx+1) - 3c^3 \log(cx-1) - \frac{2(3c^2x^2+1)}{x^3} \right) c - \frac{6 \operatorname{artanh}(cx)}{x^4} \right) a^2 b + \frac{1}{16} \left( \left( 32c^2 \log(x) - \frac{3c^2x^2 \log(cx+1)}{x^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^5,x, algorithm="maxima")

[Out] 1/8\*((3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) - 2\*(3\*c^2\*x^2 + 1)/x^3)\*c - 6\*arctanh(c\*x)/x^4)\*a^2\*b + 1/16\*((32\*c^2\*log(x) - (3\*c^2\*x^2\*log(c\*x + 1)^2 + 3\*c^2\*x^2\*log(c\*x - 1)^2 + 16\*c^2\*x^2\*log(c\*x - 1) - 2\*(3\*c^2\*x^2\*log(c\*x - 1) - 8\*c^2\*x^2)\*log(c\*x + 1) + 4)/x^2)\*c^2 + 4\*(3\*c^3\*log(c\*x + 1) - 3\*c^3\*log(c\*x - 1) - 2\*(3\*c^2\*x^2 + 1)/x^3)\*c\*arctanh(c\*x))\*a\*b^2 - 1/32\*b^3\*((c^4\*x^4 - 1)\*log(-c\*x + 1)^3 + (6\*c^3\*x^3 + 2\*c\*x - 3\*(c^4\*x^4 - 1)\*log(c\*x + 1))\*log(-c\*x + 1)^2)/x^4 + 4\*integrate(-1/2\*(2\*(c\*x - 1)\*log(c\*x + 1)^3 + (6\*c^4\*x^4 + 2\*c^2\*x^2 - 6\*(c\*x - 1)\*log(c\*x + 1)^2 - 3\*(c^5\*x^5 - c\*x)\*log(c\*x + 1))\*log(-c\*x + 1))/(c\*x^6 - x^5), x) - 3/4\*a\*b^2\*arctanh(c\*x)^2/x^4 - 1/4\*a^3/x^4

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^3/x^5,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x)^3 + 3\*a\*b^2\*arctanh(c\*x)^2 + 3\*a^2\*b\*arctanh(c\*x) + a^3)/x^5, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atanh(c*x))**3/x**5, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx) + a)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3/x^5, x)
```

### 3.35 $\int (dx)^{5/2} (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=124

$$\frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} + \frac{4bd^2 \sqrt{dx}}{7c^3} - \frac{2bd^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}} - \frac{2bd^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}} + \frac{4b(dx)^{5/2}}{35c}$$

[Out]  $(4*b*d^2*\text{Sqrt}[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*\text{ArcTanh}[c*x]))/(7*d) - (2*b*d^(5/2)*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(7*c^(7/2))$

**Rubi [A]** time = 0.0864655, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} + \frac{4bd^2 \sqrt{dx}}{7c^3} - \frac{2bd^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}} - \frac{2bd^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}} + \frac{4b(dx)^{5/2}}{35c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^(5/2)*(a + b*\text{ArcTanh}[c*x]), x]$

[Out]  $(4*b*d^2*\text{Sqrt}[d*x])/(7*c^3) + (4*b*(d*x)^(5/2))/(35*c) - (2*b*d^(5/2)*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(7*c^(7/2)) + (2*(d*x)^(7/2)*(a + b*\text{ArcTanh}[c*x]))/(7*d) - (2*b*d^(5/2)*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(7*c^(7/2))$

#### Rule 5916

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_)]*(b_.)]^(p_)*((d_)*(x_))^(m_), x\_Symbol] \rightarrow \text{Simp}[(d*x)^(m+1)*(a + b*\text{ArcTanh}[c*x])^p]/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^(m+1)*(a + b*\text{ArcTanh}[c*x])^(p-1))/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[p, 0] \&\& (\text{EqQ}[p, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

#### Rule 321

$\text{Int}[(c_)*(x_))^(m_)*((a_.) + (b_.)*(x_))^(n_)]^(p_), x\_Symbol] \rightarrow \text{Simp}[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^(m-n)*(a + b*x^n)^p, x],$

$x] /; \text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 329

$\text{Int}[(c_*)*(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \ :> \ \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m+1)-1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /; \text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 212

$\text{Int}[(a_) + (b_)*(x_)^4)^{-1}, x\_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ !\text{GtQ}[a/b, 0]$

### Rule 208

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

### Rule 205

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

### Rubi steps

$$\begin{aligned}
\int (dx)^{5/2} (a + b \tanh^{-1}(cx)) dx &= \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{(2bc) \int \frac{(dx)^{7/2}}{1-c^2x^2} dx}{7d} \\
&= \frac{4b(dx)^{5/2}}{35c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{(2bd) \int \frac{(dx)^{3/2}}{1-c^2x^2} dx}{7c} \\
&= \frac{4bd^2\sqrt{dx}}{7c^3} + \frac{4b(dx)^{5/2}}{35c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{(2bd^3) \int \frac{1}{\sqrt{dx}(1-c^2x^2)} dx}{7c^3} \\
&= \frac{4bd^2\sqrt{dx}}{7c^3} + \frac{4b(dx)^{5/2}}{35c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{(4bd^2) \text{Subst} \left( \int \frac{1}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx} \right)}{7c^3} \\
&= \frac{4bd^2\sqrt{dx}}{7c^3} + \frac{4b(dx)^{5/2}}{35c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{(2bd^3) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right)}{7c^3} \\
&= \frac{4bd^2\sqrt{dx}}{7c^3} + \frac{4b(dx)^{5/2}}{35c} - \frac{2bd^{5/2} \tan^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{7c^{7/2}} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx))}{7d} - \frac{2bd^{5/2}}{7c^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0731334, size = 128, normalized size = 1.03

$$\frac{(dx)^{5/2} (10ac^{7/2}x^{7/2} + 4bc^{5/2}x^{5/2} + 10bc^{7/2}x^{7/2} \tanh^{-1}(cx) + 20b\sqrt{c}\sqrt{x} + 5b \log(1 - \sqrt{c}\sqrt{x}) - 5b \log(\sqrt{c}\sqrt{x} + 1) - 10b \tanh^{-1}(cx))}{35c^{7/2}x^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*ArcTanh[c\*x]), x]

[Out] ((d\*x)^(5/2)\*(20\*b\*Sqrt[c]\*Sqrt[x] + 4\*b\*c^(5/2)\*x^(5/2) + 10\*a\*c^(7/2)\*x^(7/2) - 10\*b\*ArcTan[Sqrt[c]\*Sqrt[x]] + 10\*b\*c^(7/2)\*x^(7/2)\*ArcTanh[c\*x] + 5\*b\*Log[1 - Sqrt[c]\*Sqrt[x]] - 5\*b\*Log[1 + Sqrt[c]\*Sqrt[x]])/(35\*c^(7/2)\*x^(5/2))

**Maple [A]** time = 0.036, size = 107, normalized size = 0.9

$$\frac{2a}{7d} (dx)^{\frac{7}{2}} + \frac{2b \text{Artanh}(cx)}{7d} (dx)^{\frac{7}{2}} + \frac{4b}{35c} (dx)^{\frac{5}{2}} + \frac{4bd^2}{7c^3} \sqrt{dx} - \frac{2d^3b}{7c^3} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} - \frac{2d^3b}{7c^3} \text{Artanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(5/2)*(a+b*arctanh(c*x)),x)`

[Out]  $\frac{2}{7} \sqrt{d} (d*x)^{7/2} * a + \frac{2}{7} \sqrt{d} * b * (d*x)^{7/2} * \operatorname{arctanh}(c*x) + \frac{4}{35} * b * (d*x)^{5/2} / c + \frac{4}{7} * b * d^2 * (d*x)^{1/2} / c^3 - \frac{2}{7} * d^3 * b / c^3 / (c*d)^{1/2} * \operatorname{arctan}(c * (d*x)^{1/2} / (c*d)^{1/2}) - \frac{2}{7} * d^3 * b / c^3 / (c*d)^{1/2} * \operatorname{arctanh}(c * (d*x)^{1/2} / (c*d)^{1/2})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.33861, size = 660, normalized size = 5.32

$$\left[ \frac{10 b d^2 \sqrt{\frac{d}{c}} \operatorname{arctan}\left(\frac{\sqrt{d x} \sqrt{\frac{d}{c}}}{d}\right) - 5 b d^2 \sqrt{\frac{d}{c}} \log\left(\frac{c d x - 2 \sqrt{d x} \sqrt{\frac{d}{c}} + d}{c x - 1}\right) - \left(5 b c^3 d^2 x^3 \log\left(-\frac{c x + 1}{c x - 1}\right) + 10 a c^3 d^2 x^3 + 4 b c^2 d^2 x^2 + 20 b d^2\right)}{35 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out]  $[-\frac{1}{35} * (10 * b * d^2 * \sqrt{d/c} * \operatorname{arctan}(\sqrt{d*x} * c * \sqrt{d/c} / d) - 5 * b * d^2 * \sqrt{d/c} * \log((c*d*x - 2*\sqrt{d*x})*c*\sqrt{d/c} + d)/(c*x - 1)) - (5*b*c^3*d^2*x^3 * \log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d^2) * \sqrt{d*x}) / c^3, \frac{1}{35} * (10 * b * d^2 * \sqrt{-d/c} * \operatorname{arctan}(\sqrt{d*x} * c * \sqrt{-d/c} / d) + 5 * b * d^2 * \sqrt{-d/c} * \log((c*d*x - 2*\sqrt{d*x})*c*\sqrt{-d/c} - d)/(c*x + 1)) + (5*b*c^3*d^2*x^3 * \log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d^2) * \sqrt{d*x}) / c^3]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(5/2)\*(a+b\*atanh(c\*x)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} (b \operatorname{artanh}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(5/2)\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^(5/2)\*(b\*arctanh(c\*x) + a), x)

### 3.36 $\int (dx)^{3/2} (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=106

$$\frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} + \frac{2bd^{3/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}} - \frac{2bd^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}} + \frac{4b(dx)^{3/2}}{15c}$$

[Out]  $(4*b*(d*x)^{(3/2)})/(15*c) + (2*b*d^{(3/2)}*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^{(5/2)}) + (2*(d*x)^{(5/2)}*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^{(3/2)}*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^{(5/2)})$

**Rubi [A]** time = 0.0672078, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 321, 329, 298, 205, 208}

$$\frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} + \frac{2bd^{3/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}} - \frac{2bd^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}} + \frac{4b(dx)^{3/2}}{15c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^{(3/2)}*(a + b*ArcTanh[c*x]), x]$

[Out]  $(4*b*(d*x)^{(3/2)})/(15*c) + (2*b*d^{(3/2)}*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^{(5/2)}) + (2*(d*x)^{(5/2)}*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*d^{(3/2)}*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*c^{(5/2)})$

#### Rule 5916

$\text{Int}[(a_.) + \text{ArcTanh}[c_.*(x_.)]*(b_.)]^{(p_.)}*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*ArcTanh[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{(m+1)}*(a + b*ArcTanh[c*x])^{(p-1)}]/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{IntegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 321

$\text{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.))^{(n_.)}^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n*(m-n+1))})/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p]$



+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int (dx)^{3/2} (a + b \tanh^{-1}(cx)) dx &= \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} - \frac{(2bc) \int \frac{(dx)^{5/2}}{1-c^2x^2} dx}{5d} \\
 &= \frac{4b(dx)^{3/2}}{15c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} - \frac{(2bd) \int \frac{\sqrt{dx}}{1-c^2x^2} dx}{5c} \\
 &= \frac{4b(dx)^{3/2}}{15c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} - \frac{(4b) \text{Subst} \left( \int \frac{x^2}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx} \right)}{5c} \\
 &= \frac{4b(dx)^{3/2}}{15c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} - \frac{(2bd^2) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right)}{5c^2} + \frac{(2bd)}{5c} \\
 &= \frac{4b(dx)^{3/2}}{15c} + \frac{2bd^{3/2} \tan^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/2}} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx))}{5d} - \frac{2bd^{3/2} \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.055221, size = 115, normalized size = 1.08

$$\frac{(dx)^{3/2} \left( 6ac^{5/2}x^{5/2} + 4bc^{3/2}x^{3/2} + 6bc^{5/2}x^{5/2} \tanh^{-1}(cx) + 3b \log(1 - \sqrt{c}\sqrt{x}) - 3b \log(\sqrt{c}\sqrt{x} + 1) + 6b \tan^{-1}(\sqrt{c}\sqrt{x}) \right)}{15c^{5/2}x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*ArcTanh[c\*x]),x]

[Out] ((d\*x)^(3/2)\*(4\*b\*c^(3/2)\*x^(3/2) + 6\*a\*c^(5/2)\*x^(5/2) + 6\*b\*ArcTan[Sqrt[c]\*Sqrt[x]] + 6\*b\*c^(5/2)\*x^(5/2)\*ArcTanh[c\*x] + 3\*b\*Log[1 - Sqrt[c]\*Sqrt[x]] - 3\*b\*Log[1 + Sqrt[c]\*Sqrt[x]])/(15\*c^(5/2)\*x^(3/2))

**Maple [A]** time = 0.011, size = 93, normalized size = 0.9

$$\frac{2a}{5d} (dx)^{\frac{5}{2}} + \frac{2b \operatorname{Arctanh}(cx)}{5d} (dx)^{\frac{5}{2}} + \frac{4b}{15c} (dx)^{\frac{3}{2}} + \frac{2d^2b}{5c^2} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} - \frac{2d^2b}{5c^2} \operatorname{Arctanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*(a+b\*arctanh(c\*x)),x)

[Out] 2/5/d\*(d\*x)^(5/2)\*a+2/5/d\*b\*(d\*x)^(5/2)\*arctanh(c\*x)+4/15\*b\*(d\*x)^(3/2)/c+2/5\*d^2\*b/c^2/(c\*d)^(1/2)\*arctan(c\*(d\*x)^(1/2)/(c\*d)^(1/2))-2/5\*d^2\*b/c^2/(c\*d)^(1/2)\*arctanh(c\*(d\*x)^(1/2)/(c\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arctanh(c\*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.2526, size = 586, normalized size = 5.53

$$\left[ \frac{6bd\sqrt{\frac{d}{c}} \arctan\left(\frac{\sqrt{dxc}\sqrt{\frac{d}{c}}}{d}\right) + 3bd\sqrt{\frac{d}{c}} \log\left(\frac{cdx-2\sqrt{dxc}\sqrt{\frac{d}{c}}+d}{cx-1}\right) + \left(3bc^2dx^2 \log\left(-\frac{cx+1}{cx-1}\right) + 6ac^2dx^2 + 4bcdx\right)\sqrt{dx} - 6bd\sqrt{-\frac{d}{c}} \arctan\left(\frac{\sqrt{-dxc}\sqrt{\frac{d}{c}}}{d}\right)}{15c^2}, \dots \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] [1/15\*(6\*b\*d\*sqrt(d/c)\*arctan(sqrt(d\*x)\*c\*sqrt(d/c)/d) + 3\*b\*d\*sqrt(d/c)\*log((c\*d\*x - 2\*sqrt(d\*x)\*c\*sqrt(d/c) + d)/(c\*x - 1)) + (3\*b\*c^2\*d\*x^2\*log(-(c\*x + 1)/(c\*x - 1)) + 6\*a\*c^2\*d\*x^2 + 4\*b\*c\*d\*x)\*sqrt(d\*x))/c^2, 1/15\*(6\*b\*d\*sqrt(-d/c)\*arctan(sqrt(d\*x)\*c\*sqrt(-d/c)/d) + 3\*b\*d\*sqrt(-d/c)\*log((c\*d\*x + 2\*sqrt(d\*x)\*c\*sqrt(-d/c) - d)/(c\*x + 1)) + (3\*b\*c^2\*d\*x^2\*log(-(c\*x + 1)/(c\*x - 1)) + 6\*a\*c^2\*d\*x^2 + 4\*b\*c\*d\*x)\*sqrt(d\*x))/c^2]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} (a + b \operatorname{atanh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*(a+b\*atanh(c\*x)),x)

[Out] Integral((d\*x)\*\*(3/2)\*(a + b\*atanh(c\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} (b \operatorname{artanh}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^(3/2)\*(b\*arctanh(c\*x) + a), x)

### 3.37 $\int \sqrt{dx} (a + b \tanh^{-1}(cx)) dx$

**Optimal.** Leaf size=106

$$\frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{2b\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}} - \frac{2b\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}} + \frac{4b\sqrt{dx}}{3c}$$

[Out]  $(4*b*\text{Sqrt}[d*x])/(3*c) - (2*b*\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(3*c^{(3/2)}) + (2*(d*x)^{(3/2)}*(a + b*\text{ArcTanh}[c*x]))/(3*d) - (2*b*\text{Sqrt}[d]*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(3*c^{(3/2)})$

**Rubi [A]** time = 0.0612043, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 321, 329, 212, 208, 205}

$$\frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{2b\sqrt{d} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}} - \frac{2b\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}} + \frac{4b\sqrt{dx}}{3c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d*x]*(a + b*\text{ArcTanh}[c*x]), x]$

[Out]  $(4*b*\text{Sqrt}[d*x])/(3*c) - (2*b*\text{Sqrt}[d]*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(3*c^{(3/2)}) + (2*(d*x)^{(3/2)}*(a + b*\text{ArcTanh}[c*x]))/(3*d) - (2*b*\text{Sqrt}[d]*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(3*c^{(3/2)})$

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 321

```
Int[((c_.)*(x_.))^(m_)*((a_.) + (b_.)*(x_.)^(n_.))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \sqrt{dx} (a + b \tanh^{-1}(cx)) dx &= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{(2bc) \int \frac{(dx)^{3/2}}{1-c^2x^2} dx}{3d} \\
 &= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{(2bd) \int \frac{1}{\sqrt{dx}(1-c^2x^2)} dx}{3c} \\
 &= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{(4b) \text{Subst} \left( \int \frac{1}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx} \right)}{3c} \\
 &= \frac{4b\sqrt{dx}}{3c} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{(2bd) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right)}{3c} - \frac{(2bd) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right)}{3c} \\
 &= \frac{4b\sqrt{dx}}{3c} - \frac{2b\sqrt{d} \tan^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/2}} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx))}{3d} - \frac{2b\sqrt{d} \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0502362, size = 114, normalized size = 1.08

$$\frac{\sqrt{dx} (2ac^{3/2}x^{3/2} + 2bc^{3/2}x^{3/2} \tanh^{-1}(cx) + 4b\sqrt{c}\sqrt{x} + b \log(1 - \sqrt{c}\sqrt{x}) - b \log(\sqrt{c}\sqrt{x} + 1) - 2b \tan^{-1}(\sqrt{c}\sqrt{x}))}{3c^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x]), x]

[Out] (Sqrt[d\*x]\*(4\*b\*Sqrt[c]\*Sqrt[x] + 2\*a\*c^(3/2)\*x^(3/2) - 2\*b\*ArcTan[Sqrt[c]\*Sqrt[x]] + 2\*b\*c^(3/2)\*x^(3/2)\*ArcTanh[c\*x] + b\*Log[1 - Sqrt[c]\*Sqrt[x]] - b\*Log[1 + Sqrt[c]\*Sqrt[x]])/(3\*c^(3/2)\*Sqrt[x])

**Maple [A]** time = 0.012, size = 89, normalized size = 0.8

$$\frac{2a}{3d} (dx)^{\frac{3}{2}} + \frac{2b \operatorname{Artanh}(cx)}{3d} (dx)^{\frac{3}{2}} + \frac{4b}{3c} \sqrt{dx} - \frac{2db}{3c} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} - \frac{2db}{3c} \operatorname{Artanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(a+b\*arctanh(c\*x)), x)

[Out] 2/3/d\*(d\*x)^(3/2)\*a+2/3/d\*b\*(d\*x)^(3/2)\*arctanh(c\*x)+4/3\*b\*(d\*x)^(1/2)/c-2/3\*d\*b/c/(c\*d)^(1/2)\*arctan(c\*(d\*x)^(1/2)/(c\*d)^(1/2))-2/3\*d\*b/c/(c\*d)^(1/2)\*arctanh(c\*(d\*x)^(1/2)/(c\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arctanh(c\*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.22862, size = 509, normalized size = 4.8

$$\left[ \frac{2b\sqrt{\frac{d}{c}} \arctan\left(\frac{\sqrt{dxc}\sqrt{\frac{d}{c}}}{d}\right) - b\sqrt{\frac{d}{c}} \log\left(\frac{cdx-2\sqrt{dxc}\sqrt{\frac{d}{c}}+d}{cx-1}\right) - \left(bcx \log\left(-\frac{cx+1}{cx-1}\right) + 2acx + 4b\right)\sqrt{dx}}{3c}, \frac{2b\sqrt{-\frac{d}{c}} \arctan\left(\frac{\sqrt{dxc}\sqrt{-\frac{d}{c}}}{d}\right)}{3c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] [-1/3\*(2\*b\*sqrt(d/c)\*arctan(sqrt(d\*x)\*c\*sqrt(d/c)/d) - b\*sqrt(d/c)\*log((c\*d\*x - 2\*sqrt(d\*x)\*c\*sqrt(d/c) + d)/(c\*x - 1)) - (b\*c\*x\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a\*c\*x + 4\*b)\*sqrt(d\*x))/c, 1/3\*(2\*b\*sqrt(-d/c)\*arctan(sqrt(d\*x)\*c\*sqrt(-d/c)/d) + b\*sqrt(-d/c)\*log((c\*d\*x - 2\*sqrt(d\*x)\*c\*sqrt(-d/c) - d)/(c\*x + 1)) + (b\*c\*x\*log(-(c\*x + 1)/(c\*x - 1)) + 2\*a\*c\*x + 4\*b)\*sqrt(d\*x))/c]

**Sympy [C]** time = 20.0057, size = 586, normalized size = 5.53

$$\frac{2a(dx)^{\frac{3}{2}}}{3d} + \frac{2b \left\{ \begin{array}{l} -\frac{icd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}} \log\left(-\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}} - \frac{cd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}} \log\left(i\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}} + \frac{icd^{\frac{7}{2}}\left(\frac{1}{c}\right)^{\frac{3}{2}} \log\left(i\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}} + \frac{5id^{\frac{7}{2}}\sqrt{\frac{1}{c}} \log\left(-\sqrt{d}\sqrt{\frac{1}{c}}+\sqrt{dx}\right)}{-\frac{3ic^3d^2}{c^2}+\frac{15ic^2d^2}{c}} \end{array} \right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*(a+b\*atanh(c\*x)),x)

[Out] 2\*a\*(d\*x)\*\*(3/2)/(3\*d) + 2\*b\*Piecewise((-I\*c\*d\*\*(7/2)\*(1/c)\*\*(3/2)\*log(-sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) - c\*d\*(7/2)\*(1/c)\*\*(3/2)\*log(I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) + I\*c\*d\*\*(7/2)\*(1/c)\*\*(3/2)\*log(I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) + 5\*I\*d\*\*(7/2)\*sqrt(1/c)\*log(-sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) - 2\*d\*\*(7/2)\*sqrt(1/c)\*log(-I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) - 2\*I\*d\*\*(7/2)\*sqrt(1/c)\*log(-I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) + 3\*d\*\*(7/2)\*sqrt(1/c)\*log(I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/(-3\*I\*c\*\*3\*d\*\*2/c\*\*2 + 15\*I\*c\*\*2\*d\*\*2/c) - 3\*I\*d\*\*(7/2)\*sqrt(1/c)\*log(I\*sqrt(d)\*sqrt(1/c) + sqrt(d\*x))/

```
(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + 4*I*d**(7/2)*sqrt(1/c)*atanh(c*x)
)/(-3*I*c**3*d**2/c**2 + 15*I*c**2*d**2/c) + (d*x)**(3/2)*atanh(c*x)/3 + 2*
d*sqrt(d*x)/(3*c), Ne(c, 0)), (0, True))/d
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b \operatorname{artanh}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b*arctanh(c*x) + a), x)
```



$$3.38 \quad \int \frac{a+b \tanh^{-1}(cx)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=85

$$\frac{2\sqrt{dx}(a+b \tanh^{-1}(cx))}{d} + \frac{2b \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

[Out] (2\*b\*ArcTan[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/(Sqrt[c]\*Sqrt[d]) + (2\*Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x]))/d - (2\*b\*ArcTanh[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/(Sqrt[c]\*Sqrt[d])

**Rubi [A]** time = 0.0515333, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5916, 329, 298, 205, 208}

$$\frac{2\sqrt{dx}(a+b \tanh^{-1}(cx))}{d} + \frac{2b \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}} - \frac{2b \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/Sqrt[d\*x], x]

[Out] (2\*b\*ArcTan[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/(Sqrt[c]\*Sqrt[d]) + (2\*Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x]))/d - (2\*b\*ArcTanh[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/(Sqrt[c]\*Sqrt[d])

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
  := Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx))}{d} - \frac{(2bc) \int \frac{\sqrt{dx}}{1-c^2x^2} dx}{d} \\
 &= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx))}{d} - \frac{(4bc) \text{Subst} \left( \int \frac{x^2}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx} \right)}{d^2} \\
 &= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx))}{d} - (2b) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right) + (2b) \text{Subst} \left( \int \frac{1}{d+cx^2} dx, x, \sqrt{dx} \right) \\
 &= \frac{2b \tan^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{\sqrt{c}\sqrt{d}} + \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx))}{d} - \frac{2b \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{\sqrt{c}\sqrt{d}}
 \end{aligned}$$

**Mathematica [A]** time = 0.031621, size = 98, normalized size = 1.15

$$\frac{\sqrt{x} (2a\sqrt{c}\sqrt{x} + b \log(1 - \sqrt{c}\sqrt{x}) - b \log(\sqrt{c}\sqrt{x} + 1) + 2b \tan^{-1}(\sqrt{c}\sqrt{x}) + 2b\sqrt{c}\sqrt{x} \tanh^{-1}(cx))}{\sqrt{c}\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/Sqrt[d\*x], x]

[Out] (Sqrt[x]\*(2\*a\*Sqrt[c]\*Sqrt[x] + 2\*b\*ArcTan[Sqrt[c]\*Sqrt[x]] + 2\*b\*Sqrt[c]\*Sqrt[x]\*ArcTanh[c\*x] + b\*Log[1 - Sqrt[c]\*Sqrt[x]] - b\*Log[1 + Sqrt[c]\*Sqrt[x]])/(Sqrt[c]\*Sqrt[d\*x])

**Maple [A]** time = 0.013, size = 70, normalized size = 0.8

$$2 \frac{a\sqrt{dx}}{d} + 2 \frac{b\sqrt{dx}\operatorname{Artanh}(cx)}{d} + 2 \frac{b}{\sqrt{cd}} \arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right) - 2 \frac{b}{\sqrt{cd}} \operatorname{Artanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/(d\*x)^(1/2), x)

[Out] 2/d\*a\*(d\*x)^(1/2)+2/d\*b\*(d\*x)^(1/2)\*arctanh(c\*x)+2\*b/(c\*d)^(1/2)\*arctan(c\*(d\*x)^(1/2)/(c\*d)^(1/2))-2\*b/(c\*d)^(1/2)\*arctanh(c\*(d\*x)^(1/2)/(c\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(1/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.32443, size = 487, normalized size = 5.73

$$\left[ \frac{2\sqrt{cd}b \arctan\left(\frac{\sqrt{cd}\sqrt{dx}}{cdx}\right) - \sqrt{cd}b \log\left(\frac{cdx-2\sqrt{cd}\sqrt{dx}+d}{cdx-1}\right) - \left(bc \log\left(-\frac{cx+1}{cx-1}\right) + 2ac\right)\sqrt{dx}}{cd}, \frac{2\sqrt{-cd}b \arctan\left(\frac{\sqrt{-cd}\sqrt{dx}}{cdx}\right) - \sqrt{-cd}b \log\left(\frac{\sqrt{-cd}\sqrt{dx}+d}{cdx-1}\right) - \left(bc \log\left(-\frac{cx+1}{cx-1}\right) + 2ac\right)\sqrt{dx}}{cd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(1/2),x, algorithm="fricas")

[Out]  $[-(2\sqrt{c*d}*b*\arctan(\sqrt{c*d}*\sqrt{d*x}/(c*d*x)) - \sqrt{c*d}*b*\log((c*d*x - 2\sqrt{c*d}*\sqrt{d*x} + d)/(c*x - 1)) - (b*c*\log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*\sqrt{d*x})/(c*d), (2\sqrt{-c*d}*b*\arctan(\sqrt{-c*d}*\sqrt{d*x}/(c*d*x)) - \sqrt{-c*d}*b*\log((c*d*x - 2\sqrt{-c*d}*\sqrt{d*x} - d)/(c*x + 1)) + (b*c*\log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*\sqrt{d*x})/(c*d)]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/(d\*x)\*\*(1/2),x)

[Out] Integral((a + b\*atanh(c\*x))/sqrt(d\*x), x)

**Giac [A]** time = 1.16206, size = 119, normalized size = 1.4

$$\frac{\left( 2cd \left( \frac{\arctan\left(\frac{\sqrt{dxc}}{\sqrt{cd}}\right)}{\sqrt{cdc}} + \frac{\arctan\left(\frac{\sqrt{dxc}}{\sqrt{-cd}}\right)}{\sqrt{-cdc}} \right) + \sqrt{dx} \log\left(-\frac{cx+1}{cx-1}\right) \right) b + 2\sqrt{dxa}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(1/2),x, algorithm="giac")

[Out]  $((2*c*d*(\arctan(\sqrt{d*x}*c/\sqrt{c*d}))/(\sqrt{c*d}*c) + \arctan(\sqrt{d*x}*c/\sqrt{-c*d}))/(\sqrt{-c*d}*c) + \sqrt{d*x}*\log(-(c*x + 1)/(c*x - 1)))*b + 2*\sqrt{d*x}*a)/d$

$$3.39 \quad \int \frac{a+b \tanh^{-1}(cx)}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=85

$$-\frac{2(a+b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{2b\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

[Out] (2\*b\*Sqrt[c]\*ArcTan[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) - (2\*(a + b\*ArcTanh[c\*x]))/(d\*Sqrt[d\*x]) + (2\*b\*Sqrt[c]\*ArcTanh[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2)

**Rubi [A]** time = 0.0546279, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5916, 329, 212, 208, 205}

$$-\frac{2(a+b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{2b\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/(d\*x)^(3/2), x]

[Out] (2\*b\*Sqrt[c]\*ArcTan[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) - (2\*(a + b\*ArcTanh[c\*x]))/(d\*Sqrt[d\*x]) + (2\*b\*Sqrt[c]\*ArcTanh[(Sqrt[c]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2)

### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
  :> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c
*p)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTanh[c*x])^(p - 1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx)}{(dx)^{3/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{(2bc) \int \frac{1}{\sqrt{dx}(1-c^2x^2)} dx}{d} \\
 &= -\frac{2(a + b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{(4bc) \text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx}\right)}{d^2} \\
 &= -\frac{2(a + b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{(2bc) \text{Subst}\left(\int \frac{1}{d-cx^2} dx, x, \sqrt{dx}\right)}{d} + \frac{(2bc) \text{Subst}\left(\int \frac{1}{d+cx^2} dx, x, \sqrt{dx}\right)}{d} \\
 &= \frac{2b\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2(a + b \tanh^{-1}(cx))}{d\sqrt{dx}} + \frac{2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0429033, size = 99, normalized size = 1.16

$$\frac{x(-2a - b\sqrt{c}\sqrt{x} \log(1 - \sqrt{c}\sqrt{x}) + b\sqrt{c}\sqrt{x} \log(\sqrt{c}\sqrt{x} + 1) + 2b\sqrt{c}\sqrt{x} \tan^{-1}(\sqrt{c}\sqrt{x}) - 2b \tanh^{-1}(cx))}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/(d\*x)^(3/2), x]

[Out] (x\*(-2\*a + 2\*b\*Sqrt[c]\*Sqrt[x]\*ArcTan[Sqrt[c]\*Sqrt[x]] - 2\*b\*ArcTanh[c\*x] - b\*Sqrt[c]\*Sqrt[x]\*Log[1 - Sqrt[c]\*Sqrt[x]] + b\*Sqrt[c]\*Sqrt[x]\*Log[1 + Sqrt[c]\*Sqrt[x]]))/(d\*x)^(3/2)

**Maple [A]** time = 0.013, size = 78, normalized size = 0.9

$$-2 \frac{a}{d\sqrt{dx}} - 2 \frac{b \operatorname{Artanh}(cx)}{d\sqrt{dx}} + 2 \frac{bc}{d\sqrt{cd}} \arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right) + 2 \frac{bc}{d\sqrt{cd}} \operatorname{Artanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/(d\*x)^(3/2), x)

[Out] -2/d\*a/(d\*x)^(1/2)-2/d\*b/(d\*x)^(1/2)\*arctanh(c\*x)+2/d\*b\*c/(c\*d)^(1/2)\*arctan(c\*(d\*x)^(1/2)/(c\*d)^(1/2))+2/d\*b\*c/(c\*d)^(1/2)\*arctanh(c\*(d\*x)^(1/2)/(c\*d)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.16046, size = 494, normalized size = 5.81

$$\left[ \frac{2 b d x \sqrt{\frac{c}{d}} \arctan\left(\frac{\sqrt{d x} \sqrt{\frac{c}{d}}}{c x}\right) - b d x \sqrt{\frac{c}{d}} \log\left(\frac{c x + 2 \sqrt{d x} \sqrt{\frac{c}{d}} + 1}{c x - 1}\right) + \sqrt{d x} \left(b \log\left(-\frac{c x + 1}{c x - 1}\right) + 2 a\right)}{d^2 x}, \frac{2 b d x \sqrt{-\frac{c}{d}} \arctan\left(\frac{\sqrt{d x} \sqrt{-\frac{c}{d}}}{c x}\right) - \dots}{d^2 x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(3/2),x, algorithm="fricas")

[Out]  $[-(2*b*d*x*\sqrt{c/d}*\arctan(\sqrt{d*x}*\sqrt{c/d}/(c*x)) - b*d*x*\sqrt{c/d}*\log((c*x + 2*\sqrt{d*x}*\sqrt{c/d} + 1)/(c*x - 1)) + \sqrt{d*x}*(b*\log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x), -(2*b*d*x*\sqrt{-c/d}*\arctan(\sqrt{d*x}*\sqrt{-c/d}/(c*x)) - b*d*x*\sqrt{-c/d}*\log((c*x + 2*\sqrt{d*x}*\sqrt{-c/d} - 1)/(c*x + 1)) + \sqrt{d*x}*(b*\log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x)]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/(d\*x)\*\*(3/2),x)

[Out] Integral((a + b\*atanh(c\*x))/(d\*x)\*\*(3/2), x)

**Giac [A]** time = 1.27011, size = 127, normalized size = 1.49

$$2bcd \left( \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{cd}}\right)}{\sqrt{c}d^2} - \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{-cd}}\right)}{\sqrt{-c}d^2} \right) - \frac{b \log\left(\frac{-cdx+d}{cdx-d}\right) + \frac{2a}{\sqrt{d}x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(3/2),x, algorithm="giac")

[Out]  $2*b*c*d*(\arctan(\sqrt{d*x}*c/\sqrt{c*d})/(\sqrt{c*d}*d^2) - \arctan(\sqrt{d*x}*c/\sqrt{-c*d})/(\sqrt{-c*d}*d^2)) - (b*\log(-(c*d*x + d)/(c*d*x - d))/\sqrt{d*x} + 2*a/\sqrt{d*x})/d$



$$3.40 \quad \int \frac{a+b \tanh^{-1}(cx)}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=107

$$-\frac{2(a+b \tanh^{-1}(cx))}{3d(dx)^{3/2}} - \frac{2bc^{3/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4bc}{3d^2\sqrt{dx}}$$

[Out]  $(-4*b*c)/(3*d^2*\text{Sqrt}[d*x]) - (2*b*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(3*d^{(5/2)}) - (2*(a + b*\text{ArcTanh}[c*x]))/(3*d*(d*x)^{(3/2)}) + (2*b*c^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(3*d^{(5/2)})$

**Rubi [A]** time = 0.0633332, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2(a+b \tanh^{-1}(cx))}{3d(dx)^{3/2}} - \frac{2bc^{3/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4bc}{3d^2\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/(d\*x)^(5/2), x]

[Out]  $(-4*b*c)/(3*d^2*\text{Sqrt}[d*x]) - (2*b*c^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(3*d^{(5/2)}) - (2*(a + b*\text{ArcTanh}[c*x]))/(3*d*(d*x)^{(3/2)}) + (2*b*c^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(3*d^{(5/2)})$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m+1)), x] - Dist[(b\*c\*p)/(d\*(m+1)), Int[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x])^(p-1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a,

b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx)}{(dx)^{5/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(2bc) \int \frac{1}{(dx)^{3/2}(1-c^2x^2)} dx}{3d} \\
&= -\frac{4bc}{3d^2\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(2bc^3) \int \frac{\sqrt{dx}}{1-c^2x^2} dx}{3d^3} \\
&= -\frac{4bc}{3d^2\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(4bc^3) \text{Subst}\left(\int \frac{x^2}{1-c^2x^4} dx, x, \sqrt{dx}\right)}{3d^4} \\
&= -\frac{4bc}{3d^2\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{3d(dx)^{3/2}} + \frac{(2bc^2) \text{Subst}\left(\int \frac{1}{d-cx^2} dx, x, \sqrt{dx}\right)}{3d^2} - \frac{(2bc^2) \text{Subst}\left(\int \frac{1}{d+cx^2} dx, x, \sqrt{dx}\right)}{3d^2} \\
&= -\frac{4bc}{3d^2\sqrt{dx}} - \frac{2bc^{3/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2(a + b \tanh^{-1}(cx))}{3d(dx)^{3/2}} + \frac{2bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0604537, size = 107, normalized size = 1.

$$\frac{x(2a + bc^{3/2}x^{3/2} \log(1 - \sqrt{c}\sqrt{x}) - bc^{3/2}x^{3/2} \log(\sqrt{c}\sqrt{x} + 1) + 2bc^{3/2}x^{3/2} \tan^{-1}(\sqrt{c}\sqrt{x}) + 4bcx + 2b \tanh^{-1}(cx))}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/(d\*x)^(5/2), x]

[Out] -(x\*(2\*a + 4\*b\*c\*x + 2\*b\*c^(3/2)\*x^(3/2)\*ArcTan[Sqrt[c]\*Sqrt[x]] + 2\*b\*ArcTanh[c\*x] + b\*c^(3/2)\*x^(3/2)\*Log[1 - Sqrt[c]\*Sqrt[x]] - b\*c^(3/2)\*x^(3/2)\*Log[1 + Sqrt[c]\*Sqrt[x]])/(3\*(d\*x)^(5/2))

**Maple [A]** time = 0.014, size = 94, normalized size = 0.9

$$-\frac{2a}{3d}(dx)^{-\frac{3}{2}} - \frac{2b \text{Artanh}(cx)}{3d}(dx)^{-\frac{3}{2}} - \frac{4bc}{3d^2} \frac{1}{\sqrt{dx}} - \frac{2bc^2}{3d^2} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} + \frac{2bc^2}{3d^2} \text{Artanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/(d\*x)^(5/2), x)

```
[Out] -2/3/d*a/(d*x)^(3/2)-2/3/d*b/(d*x)^(3/2)*arctanh(c*x)-4/3*b*c/d^2/(d*x)^(1/2)-2/3/d^2*b*c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))+2/3/d^2*b*c^2/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.2123, size = 558, normalized size = 5.21

$$\left[ \frac{2bcdx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right) + bcdx^2\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right) - \left(4bcx + b\log\left(-\frac{cx+1}{cx-1}\right) + 2a\right)\sqrt{dx}}{3d^3x^2}, -\frac{2bcdx^2\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right) + bcdx^2\sqrt{-\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{-\frac{c}{d}}+1}{cx-1}\right) - \left(4bcx + b\log\left(-\frac{cx+1}{cx-1}\right) + 2a\right)\sqrt{dx}}{3d^3x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(2*b*c*d*x^2*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + b*c*d*x^2*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (4*b*c*x + b*log(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2), -1/3*(2*b*c*d*x^2*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - b*c*d*x^2*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + b*log(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/(d\*x)\*\*(5/2),x)

[Out] Integral((a + b\*atanh(c\*x))/(d\*x)\*\*(5/2), x)

**Giac [A]** time = 1.34339, size = 163, normalized size = 1.52

$$-\frac{2}{3}bc^3\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}cd^2} + \frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}cd^2}\right) - \frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}dx} + \frac{2(2bcdx+ad)}{\sqrt{dx}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(5/2),x, algorithm="giac")

[Out]  $-\frac{2}{3}b*c^3*\left(\frac{\arctan(\sqrt{d*x}*c/\sqrt{c*d})}{(\sqrt{c*d}*c*d^2)} + \frac{\arctan(\sqrt{d*x}*c/\sqrt{-c*d})}{(\sqrt{-c*d}*c*d^2)}\right) - \frac{1}{3}*(b*\log(-(c*d*x + d)/(c*d*x - d)))/(\sqrt{d*x}*d*x) + \frac{2*(2*b*c*d*x + a*d)}{(\sqrt{d*x}*d^2*x)}/d$

$$3.41 \quad \int \frac{a+b \tanh^{-1}(cx)}{(dx)^{7/2}} dx$$

**Optimal.** Leaf size=107

$$-\frac{2(a+b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{2bc^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{2bc^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{4bc}{15d^2(dx)^{3/2}}$$

[Out]  $(-4*b*c)/(15*d^2*(d*x)^{(3/2)}) + (2*b*c^{(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^{(7/2)}) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^{(5/2)}) + (2*b*c^{(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^{(7/2)})$

**Rubi [A]** time = 0.0662016, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 325, 329, 212, 208, 205}

$$-\frac{2(a+b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{2bc^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{2bc^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{4bc}{15d^2(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x])/(d\*x)^(7/2), x]

[Out]  $(-4*b*c)/(15*d^2*(d*x)^{(3/2)}) + (2*b*c^{(5/2)*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^{(7/2)}) - (2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^{(5/2)}) + (2*b*c^{(5/2)*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(5*d^{(7/2)})$

### Rule 5916

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^p)/(d\*(m + 1)), x] - Dist[(b\*c\*p)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x])^(p - 1))/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a,

b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx)}{(dx)^{7/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{(2bc) \int \frac{1}{(dx)^{5/2}(1-c^2x^2)} dx}{5d} \\
&= -\frac{4bc}{15d^2(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{(2bc^3) \int \frac{1}{\sqrt{dx}(1-c^2x^2)} dx}{5d^3} \\
&= -\frac{4bc}{15d^2(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{(4bc^3) \text{Subst}\left(\int \frac{1}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx}\right)}{5d^4} \\
&= -\frac{4bc}{15d^2(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{(2bc^3) \text{Subst}\left(\int \frac{1}{d-cx^2} dx, x, \sqrt{dx}\right)}{5d^3} + \frac{(2bc^3) \text{Subst}\left(\int \frac{1}{d-cx^2} dx, x, \sqrt{dx}\right)}{5d^3} \\
&= -\frac{4bc}{15d^2(dx)^{3/2}} + \frac{2bc^{5/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2(a + b \tanh^{-1}(cx))}{5d(dx)^{5/2}} + \frac{2bc^{5/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0480971, size = 108, normalized size = 1.01

$$\frac{x(-6a - 3bc^{5/2}x^{5/2} \log(1 - \sqrt{c}\sqrt{x}) + 3bc^{5/2}x^{5/2} \log(\sqrt{c}\sqrt{x} + 1) + 6bc^{5/2}x^{5/2} \tan^{-1}(\sqrt{c}\sqrt{x}) - 4bcx - 6b \tanh^{-1}(cx))}{15(dx)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/(d\*x)^(7/2), x]

[Out] (x\*(-6\*a - 4\*b\*c\*x + 6\*b\*c^(5/2)\*x^(5/2)\*ArcTan[Sqrt[c]\*Sqrt[x]] - 6\*b\*ArcTanh[c\*x] - 3\*b\*c^(5/2)\*x^(5/2)\*Log[1 - Sqrt[c]\*Sqrt[x]] + 3\*b\*c^(5/2)\*x^(5/2)\*Log[1 + Sqrt[c]\*Sqrt[x]])/(15\*(d\*x)^(7/2))

**Maple [A]** time = 0.014, size = 94, normalized size = 0.9

$$-\frac{2a}{5d}(dx)^{-\frac{5}{2}} - \frac{2b \text{Artanh}(cx)}{5d}(dx)^{-\frac{5}{2}} + \frac{2bc^3}{5d^3} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} + \frac{2bc^3}{5d^3} \text{Artanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} - \frac{4bc}{15d^2}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))/(d\*x)^(7/2), x)



[Out]  $-2/5/d*a/(d*x)^{(5/2)}-2/5/d*b/(d*x)^{(5/2)}*\operatorname{arctanh}(c*x)+2/5/d^3*b*c^3/(c*d)^{(1/2)}*\operatorname{arctan}(c*(d*x)^{(1/2)}/(c*d)^{(1/2)})+2/5/d^3*b*c^3/(c*d)^{(1/2)}*\operatorname{arctanh}(c*(d*x)^{(1/2)}/(c*d)^{(1/2)})-4/15*b*c/d^2/(d*x)^{(3/2)}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.27957, size = 583, normalized size = 5.45

$$\left[ \frac{6bc^2dx^3\sqrt{\frac{c}{d}}\operatorname{arctan}\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right) - 3bc^2dx^3\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right) + \left(4bcx + 3b\log\left(-\frac{cx+1}{cx-1}\right) + 6a\right)\sqrt{dx}}{15d^4x^3}, - \frac{6bc^2dx^3\sqrt{-\frac{c}{d}}}{15d^4x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="fricas")`

[Out]  $[-1/15*(6*b*c^2*d*x^3*\sqrt{c/d}*\operatorname{arctan}(\sqrt{d*x}*\sqrt{c/d}/(c*x)) - 3*b*c^2*d*x^3*\sqrt{c/d}*\log((c*x + 2*\sqrt{d*x}*\sqrt{c/d} + 1)/(c*x - 1)) + (4*b*c*x + 3*b*\log(-(c*x + 1)/(c*x - 1)) + 6*a)*\sqrt{d*x})/(d^4*x^3), -1/15*(6*b*c^2*d*x^3*\sqrt{-c/d}*\operatorname{arctan}(\sqrt{d*x}*\sqrt{-c/d}/(c*x)) - 3*b*c^2*d*x^3*\sqrt{-c/d}*\log((c*x + 2*\sqrt{d*x}*\sqrt{-c/d} - 1)/(c*x + 1)) + (4*b*c*x + 3*b*\log(-(c*x + 1)/(c*x - 1)) + 6*a)*\sqrt{d*x})/(d^4*x^3)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/(d\*x)\*\*(7/2),x)

[Out] Timed out

**Giac [A]** time = 1.26523, size = 159, normalized size = 1.49

$$\frac{2}{5}bc^3 \left( \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} - \frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{-cd}}\right)}{\sqrt{-cd}d^3} \right) - \frac{3b \log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{d}xd^2x^2} + \frac{2(2bcdx+3ad)}{\sqrt{d}xd^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(7/2),x, algorithm="giac")

[Out]  $\frac{2}{5}b*c^3*(\arctan(\sqrt{d*x}*c/\sqrt{c*d})/(\sqrt{c*d}*d^3) - \arctan(\sqrt{d*x}*c/\sqrt{-c*d})/(\sqrt{-c*d}*d^3)) - \frac{1}{15}*(3*b*\log(-(c*d*x + d)/(c*d*x - d)))/(\sqrt{d*x}*d^2*x^2) + \frac{2*(2*b*c*d*x + 3*a*d)}{\sqrt{d*x}*d^3*x^2})/d$

$$3.42 \quad \int \frac{a+b \tanh^{-1}(cx)}{(dx)^{9/2}} dx$$

**Optimal.** Leaf size=125

$$-\frac{2(a+b \tanh^{-1}(cx))}{7d(dx)^{7/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2bc^{7/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \frac{2bc^{7/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{4bc}{35d^2(dx)^{5/2}}$$

[Out]  $(-4*b*c)/(35*d^2*(d*x)^{(5/2)}) - (4*b*c^3)/(7*d^4*\text{Sqrt}[d*x]) - (2*b*c^{(7/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(7*d^{(9/2)}) - (2*(a + b*\text{ArcTanh}[c*x]))/(7*d*(d*x)^{(7/2)}) + (2*b*c^{(7/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(7*d^{(9/2)})$

**Rubi [A]** time = 0.078228, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5916, 325, 329, 298, 205, 208}

$$-\frac{2(a+b \tanh^{-1}(cx))}{7d(dx)^{7/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2bc^{7/2} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \frac{2bc^{7/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{4bc}{35d^2(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c*x])/(d*x)^{(9/2)}, x]$

[Out]  $(-4*b*c)/(35*d^2*(d*x)^{(5/2)}) - (4*b*c^3)/(7*d^4*\text{Sqrt}[d*x]) - (2*b*c^{(7/2)}*\text{ArcTan}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(7*d^{(9/2)}) - (2*(a + b*\text{ArcTanh}[c*x]))/(7*d*(d*x)^{(7/2)}) + (2*b*c^{(7/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(7*d^{(9/2)})$

### Rule 5916

$\text{Int}[(a + \text{ArcTanh}[c*x])*(b*x)^p*(d*x)^m, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x])^p/(d*(m+1)), x] - \text{Dist}[(b*c*p)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x])^{p-1}/(1 - c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{IntegerQ}[m]) \ \&\& \ \text{NeQ}[m, -1]$

### Rule 325

$\text{Int}[(c*x)^m*(a + (b*x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c*x)^{m+1}*(a + b*x^n)^{p+1}/(a*c*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1))$

+ 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx)}{(dx)^{9/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{(2bc) \int \frac{1}{(dx)^{7/2}(1-c^2x^2)} dx}{7d} \\
&= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{(2bc^3) \int \frac{1}{(dx)^{3/2}(1-c^2x^2)} dx}{7d^3} \\
&= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{(2bc^5) \int \frac{\sqrt{dx}}{1-c^2x^2} dx}{7d^5} \\
&= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{(4bc^5) \text{Subst} \left( \int \frac{x^2}{1-\frac{c^2x^4}{d^2}} dx, x, \sqrt{dx} \right)}{7d^6} \\
&= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{(2bc^4) \text{Subst} \left( \int \frac{1}{d-cx^2} dx, x, \sqrt{dx} \right)}{7d^4} - \frac{(2bc^4)}{7d^4} \\
&= -\frac{4bc}{35d^2(dx)^{5/2}} - \frac{4bc^3}{7d^4\sqrt{dx}} - \frac{2bc^{7/2} \tan^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{7d^{9/2}} - \frac{2(a + b \tanh^{-1}(cx))}{7d(dx)^{7/2}} + \frac{2bc^{7/2} \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}} \right)}{7d^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0661833, size = 122, normalized size = 0.98

$$\frac{\sqrt{dx} (10a + 20bc^3x^3 + 5bc^{7/2}x^{7/2} \log(1 - \sqrt{c}\sqrt{x}) - 5bc^{7/2}x^{7/2} \log(\sqrt{c}\sqrt{x} + 1) + 10bc^{7/2}x^{7/2} \tan^{-1}(\sqrt{c}\sqrt{x}) + 4bcx + 10)}{35d^5x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x])/(d\*x)^(9/2), x]

[Out] -(Sqrt[d\*x]\*(10\*a + 4\*b\*c\*x + 20\*b\*c^3\*x^3 + 10\*b\*c^(7/2)\*x^(7/2)\*ArcTan[Sqrt[c]\*Sqrt[x]] + 10\*b\*ArcTanh[c\*x] + 5\*b\*c^(7/2)\*x^(7/2)\*Log[1 - Sqrt[c]\*Sqrt[x]] - 5\*b\*c^(7/2)\*x^(7/2)\*Log[1 + Sqrt[c]\*Sqrt[x]])/(35\*d^5\*x^4)

**Maple [A]** time = 0.015, size = 108, normalized size = 0.9

$$-\frac{2a}{7d}(dx)^{-\frac{7}{2}} - \frac{2b \operatorname{Artanh}(cx)}{7d}(dx)^{-\frac{7}{2}} - \frac{2bc^4}{7d^4} \arctan\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right) \frac{1}{\sqrt{cd}} - \frac{4bc}{35d^2}(dx)^{-\frac{5}{2}} - \frac{4bc^3}{7d^4} \frac{1}{\sqrt{dx}} + \frac{2bc^4}{7d^4} \operatorname{Artanh}\left(c\sqrt{dx} \frac{1}{\sqrt{cd}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x))/(d*x)^(9/2),x)
```

```
[Out] -2/7/d*a/(d*x)^(7/2)-2/7/d*b/(d*x)^(7/2)*arctanh(c*x)-2/7/d^4*b*c^4/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-4/35*b*c/d^2/(d*x)^(5/2)-4/7*b*c^3/d^4/(d*x)^(1/2)+2/7/d^4*b*c^4/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.45178, size = 628, normalized size = 5.02

$$\left[ \frac{10bc^3dx^4\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right) + 5bc^3dx^4\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right) - \left(20bc^3x^3 + 4bcx + 5b\log\left(-\frac{cx+1}{cx-1}\right) + 10a\right)\sqrt{dx}}{35d^5x^4}, -10a\sqrt{dx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/35*(10*b*c^3*d*x^4*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + 5*b*c^3*d*x^4*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5*x^4), -1/35*(10*b*c^3*d*x^4*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 5*b*c^3*d*x^4*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5*x^4)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x))/(d\*x)\*\*(9/2),x)

[Out] Timed out

**Giac [A]** time = 1.4231, size = 188, normalized size = 1.5

$$-\frac{2}{7}bc^5\left(\frac{\arctan\left(\frac{\sqrt{d}xc}{\sqrt{cd}}\right)}{\sqrt{cd}cd^4} + \frac{\arctan\left(\frac{\sqrt{d}xc}{\sqrt{-cd}}\right)}{\sqrt{-cd}cd^4}\right) - \frac{5b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{d}xd^3x^3} + \frac{2(10bc^3d^3x^3+2bcd^3x+5ad^3)}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))/(d\*x)^(9/2),x, algorithm="giac")

[Out]  $-\frac{2}{7}b*c^5*\left(\frac{\arctan(\sqrt{d}*x)*c/\sqrt{c*d}}{(\sqrt{c*d})*c*d^4} + \frac{\arctan(\sqrt{d}*x)*c/\sqrt{-c*d}}{(\sqrt{-c*d})*c*d^4}\right) - \frac{1}{35}*(5*b*\log(-(c*d*x + d)/(c*d*x - d)))/(\sqrt{d}*x)*d^3*x^3 + \frac{2*(10*b*c^3*d^3*x^3 + 2*b*c*d^3*x + 5*a*d^3)}{(\sqrt{d}*x)*d^6*x^3}/d$

$$3.43 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3 dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^3, x]

**Rubi [A]** time = 0.0244612, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3 dx$$

**Mathematica [A]** time = 3.51876, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^3,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^3, x]



**Maple [A]** time = 1.024, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x))^3,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{artanh}(cx)^3 + 3ab^2 \operatorname{artanh}(cx)^2 + 3a^2b \operatorname{artanh}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^3,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x)^3 + 3\*a\*b^2\*arctanh(c\*x)^2 + 3\*a^2\*b\*arctanh(c\*x) + a^3)\*(d\*x)^m, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{atanh}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**3, x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^3*(d*x)^m, x)
```

$$3.44 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2 dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^2, x]

**Rubi [A]** time = 0.0234099, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2 dx$$

**Mathematica [A]** time = 2.32125, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^2, x]

**Maple [A]** time = 0.914, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x))^2,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x)^2 + 2\*a\*b\*arctanh(c\*x) + a^2)\*(d\*x)^m, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{atanh}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atanh(c*x))**2, x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^2*(d*x)^m, x)
```

### 3.45 $\int (dx)^m \left( a + b \tanh^{-1}(cx) \right) dx$

**Optimal.** Leaf size=72

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx) \right)}{d(m+1)} - \frac{bc(dx)^{m+2} \text{Hypergeometric2F1} \left( 1, \frac{m+2}{2}, \frac{m+4}{2}, c^2 x^2 \right)}{d^2(m+1)(m+2)}$$

[Out]  $((d*x)^{(1+m)}*(a + b*ArcTanh[c*x]))/(d*(1+m)) - (b*c*(d*x)^{(2+m)}*Hypergeometric2F1[1, (2+m)/2, (4+m)/2, c^2*x^2])/(d^2*(1+m)*(2+m))$

**Rubi [A]** time = 0.032328, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {5916, 364}

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx) \right)}{d(m+1)} - \frac{bc(dx)^{m+2} {}_2F_1 \left( 1, \frac{m+2}{2}; \frac{m+4}{2}; c^2 x^2 \right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m*(a + b*ArcTanh[c*x]), x]$

[Out]  $((d*x)^{(1+m)}*(a + b*ArcTanh[c*x]))/(d*(1+m)) - (b*c*(d*x)^{(2+m)}*Hypergeometric2F1[1, (2+m)/2, (4+m)/2, c^2*x^2])/(d^2*(1+m)*(2+m))$

#### Rule 5916

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*((d_.)*(x_.))^(m_.), x_Symbol]
  :> Simp[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^p)/(d*(m+1)), x] - Dist[(b*c
*p)/(d*(m+1)), Int[((d*x)^(m+1)*(a + b*ArcTanh[c*x])^(p-1))/(1 - c^2*
x^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || In
tegerQ[m]) && NeQ[m, -1]
```

#### Rule 364

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[(a^
p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a
])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

#### Rubi steps

$$\int (dx)^m (a + b \tanh^{-1}(cx)) dx = \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx))}{d(1+m)} - \frac{(bc) \int \frac{(dx)^{1+m}}{1-c^2x^2} dx}{d(1+m)}$$

$$= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx))}{d(1+m)} - \frac{bc(dx)^{2+m} {}_2F_1\left(1, \frac{2+m}{2}; \frac{4+m}{2}; c^2x^2\right)}{d^2(1+m)(2+m)}$$

**Mathematica [A]** time = 0.0667753, size = 59, normalized size = 0.82

$$\frac{x(dx)^m (bcx \text{Hypergeometric2F1}\left(1, \frac{m}{2} + 1, \frac{m}{2} + 2, c^2x^2\right) - (m+2)(a + b \tanh^{-1}(cx)))}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x]), x]

[Out] -((x\*(d\*x)^m\*(-((2 + m)\*(a + b\*ArcTanh[c\*x])) + b\*c\*x\*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, c^2\*x^2])))/((1 + m)\*(2 + m))

**Maple [F]** time = 0.925, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \text{Artanh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x)), x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \operatorname{artanh}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x)),x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x) + a)\*(d\*x)^m, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{atanh}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*atanh(c\*x)),x)

[Out] Integral((d\*x)\*\*m\*(a + b\*atanh(c\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)(dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x) + a)\*(d\*x)^m, x)



$$3.46 \quad \int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b \tanh^{-1}(cx)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x]), x]

**Rubi [A]** time = 0.0271058, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx$$

**Mathematica [A]** time = 0.253179, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x]), x]

---

**Maple [A]** time = 0.654, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{Arctanh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m/(a+b*arctanh(c*x)),x)`

[Out] `int((d*x)^m/(a+b*arctanh(c*x)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="maxima")`

[Out] `integrate((d*x)^m/(b*arctanh(c*x) + a), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="fricas")`

[Out] `integral((d*x)^m/(b*arctanh(c*x) + a), x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{atanh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*atanh(c\*x)),x)

[Out] Integral((d\*x)\*\*m/(a + b\*atanh(c\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x) + a), x)

$$3.47 \quad \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2}, x \right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x])^2, x]

**Rubi [A]** time = 0.0263568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x])^2,x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2} dx$$

**Mathematica [A]** time = 0.499509, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x])^2,x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x])^2, x]

**Maple [A]** time = 0.658, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{Arctanh}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x))^2,x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2(c^2 d^m x^2 - d^m)x^m}{b^2 c \log(cx + 1) - b^2 c \log(-cx + 1) + 2abc} + \int -\frac{2(c^2 d^m (m + 2)x^2 - d^m m)x^m}{b^2 cx \log(cx + 1) - b^2 cx \log(-cx + 1) + 2abcx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x))^2,x, algorithm="maxima")

[Out] 2\*(c^2\*d^m\*x^2 - d^m)\*x^m/(b^2\*c\*log(c\*x + 1) - b^2\*c\*log(-c\*x + 1) + 2\*a\*b\*c) + integrate(-2\*(c^2\*d^m\*(m + 2)\*x^2 - d^m\*m)\*x^m/(b^2\*c\*x\*log(c\*x + 1) - b^2\*c\*x\*log(-c\*x + 1) + 2\*a\*b\*c\*x), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}(cx)^2 + 2ab \operatorname{artanh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x))^2,x, algorithm="fricas")

[Out] integral((d\*x)^m/(b^2\*arctanh(c\*x)^2 + 2\*a\*b\*arctanh(c\*x) + a^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{atanh}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*atanh(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*m/(a + b\*atanh(c\*x))\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{artanh}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x))^2,x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x) + a)^2, x)

$$3.48 \quad \int (a + b \tanh^{-1}(cx))^p dx$$

**Optimal.** Leaf size=12

$$\text{Unintegrable}\left((a + b \tanh^{-1}(cx))^p, x\right)$$

[Out] Unintegrable[(a + b\*ArcTanh[c\*x])^p, x]

**Rubi [A]** time = 0.0053112, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (a + b \tanh^{-1}(cx))^p dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x])^p, x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x])^p, x]

Rubi steps

$$\int (a + b \tanh^{-1}(cx))^p dx = \int (a + b \tanh^{-1}(cx))^p dx$$

**Mathematica [A]** time = 0.441212, size = 0, normalized size = 0.

$$\int (a + b \tanh^{-1}(cx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x])^p, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x])^p, x]

**Maple [A]** time = 0.201, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Arctanh}(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x))^p,x)

[Out] int((a+b\*arctanh(c\*x))^p,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^p,x, algorithm="maxima")

[Out] integrate((b\*arctanh(c\*x) + a)^p, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((b \operatorname{artanh}(cx) + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x))^p,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x) + a)^p, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b*atanh(c*x))**p,x)
```

```
[Out] Integral((a + b*atanh(c*x))**p, x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x) + a)^p, x)
```

$$3.49 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x]

**Rubi [A]** time = 0.0236748, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^p,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p dx = \int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p dx$$

**Mathematica [A]** time = 0.326303, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx) \right)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^p,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x])^p, x]

**Maple [A]** time = 0.378, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx))^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x))^p,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x))^p,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \operatorname{artanh}(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^p,x, algorithm="maxima")

[Out] integrate((d\*x)^m\*(b\*arctanh(c\*x) + a)^p, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x))^p,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x))**p,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (b \operatorname{artanh}(cx) + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

### 3.50 $\int x^7 (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=54

$$\frac{1}{8}x^8 (a + b \tanh^{-1}(cx^2)) + \frac{bx^2}{8c^3} - \frac{b \tanh^{-1}(cx^2)}{8c^4} + \frac{bx^6}{24c}$$

[Out] (b\*x^2)/(8\*c^3) + (b\*x^6)/(24\*c) - (b\*ArcTanh[c\*x^2])/(8\*c^4) + (x^8\*(a + b\*ArcTanh[c\*x^2]))/8

**Rubi [A]** time = 0.03915, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 302, 206}

$$\frac{1}{8}x^8 (a + b \tanh^{-1}(cx^2)) + \frac{bx^2}{8c^3} - \frac{b \tanh^{-1}(cx^2)}{8c^4} + \frac{bx^6}{24c}$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (b\*x^2)/(8\*c^3) + (b\*x^6)/(24\*c) - (b\*ArcTanh[c\*x^2])/(8\*c^4) + (x^8\*(a + b\*ArcTanh[c\*x^2]))/8

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :> Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt

$Q[m, 2*n - 1]$

### Rule 206

$\text{Int}[(a + b \cdot (x^2)^{-1}), x\_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] / ; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int x^7 (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{8} x^8 (a + b \tanh^{-1}(cx^2)) - \frac{1}{4} (bc) \int \frac{x^9}{1 - c^2 x^4} dx \\ &= \frac{1}{8} x^8 (a + b \tanh^{-1}(cx^2)) - \frac{1}{8} (bc) \text{Subst} \left( \int \frac{x^4}{1 - c^2 x^2} dx, x, x^2 \right) \\ &= \frac{1}{8} x^8 (a + b \tanh^{-1}(cx^2)) - \frac{1}{8} (bc) \text{Subst} \left( \int \left( -\frac{1}{c^4} - \frac{x^2}{c^2} + \frac{1}{c^4 (1 - c^2 x^2)} \right) dx, x, x^2 \right) \\ &= \frac{bx^2}{8c^3} + \frac{bx^6}{24c} + \frac{1}{8} x^8 (a + b \tanh^{-1}(cx^2)) - \frac{b \text{Subst} \left( \int \frac{1}{1 - c^2 x^2} dx, x, x^2 \right)}{8c^3} \\ &= \frac{bx^2}{8c^3} + \frac{bx^6}{24c} - \frac{b \tanh^{-1}(cx^2)}{8c^4} + \frac{1}{8} x^8 (a + b \tanh^{-1}(cx^2)) \end{aligned}$$

**Mathematica [A]** time = 0.016156, size = 78, normalized size = 1.44

$$\frac{ax^8}{8} + \frac{bx^2}{8c^3} + \frac{b \log(1 - cx^2)}{16c^4} - \frac{b \log(cx^2 + 1)}{16c^4} + \frac{bx^6}{24c} + \frac{1}{8} bx^8 \tanh^{-1}(cx^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (b\*x^2)/(8\*c^3) + (b\*x^6)/(24\*c) + (a\*x^8)/8 + (b\*x^8\*ArcTanh[c\*x^2])/8 + (b\*Log[1 - c\*x^2])/(16\*c^4) - (b\*Log[1 + c\*x^2])/(16\*c^4)

**Maple [A]** time = 0.022, size = 66, normalized size = 1.2

$$\frac{x^8 a}{8} + \frac{bx^8 \text{Artanh}(cx^2)}{8} + \frac{bx^6}{24c} + \frac{bx^2}{8c^3} + \frac{b \ln(cx^2 - 1)}{16c^4} - \frac{b \ln(cx^2 + 1)}{16c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a+b*arctanh(c*x^2)),x)`

[Out]  $\frac{1}{8}x^8a + \frac{1}{8}bx^8\operatorname{arctanh}(cx^2) + \frac{1}{24}bx^6/c + \frac{1}{8}bx^2/c^3 + \frac{1}{16}b/c^4 \ln(c^2x^2 - 1) - \frac{1}{16}b/c^4 \ln(c^2x^2 + 1)$

**Maxima [A]** time = 0.952859, size = 93, normalized size = 1.72

$$\frac{1}{8}ax^8 + \frac{1}{48} \left( 6x^8 \operatorname{artanh}(cx^2) + c \left( \frac{2(c^2x^6 + 3x^2)}{c^4} - \frac{3 \log(cx^2 + 1)}{c^5} + \frac{3 \log(cx^2 - 1)}{c^5} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out]  $\frac{1}{8}ax^8 + \frac{1}{48}(6x^8\operatorname{arctanh}(cx^2) + c(2(c^2x^6 + 3x^2)/c^4 - 3\log(c^2x^2 + 1)/c^5 + 3\log(c^2x^2 - 1)/c^5))b$

**Fricas [A]** time = 1.99897, size = 135, normalized size = 2.5

$$\frac{6ac^4x^8 + 2bc^3x^6 + 6bcx^2 + 3(bc^4x^8 - b) \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{48c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out]  $\frac{1}{48}(6a*c^4*x^8 + 2*b*c^3*x^6 + 6*b*c*x^2 + 3*(b*c^4*x^8 - b)*\log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4$

**Sympy [A]** time = 29.8785, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8 \operatorname{atanh}(cx^2)}{8} + \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b \operatorname{atanh}(cx^2)}{8c^4} & \text{for } c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Piecewise((a\*x\*\*8/8 + b\*x\*\*8\*atanh(c\*x\*\*2)/8 + b\*x\*\*6/(24\*c) + b\*x\*\*2/(8\*c\*  
\*3) - b\*atanh(c\*x\*\*2)/(8\*c\*\*4), Ne(c, 0)), (a\*x\*\*8/8, True))

**Giac [A]** time = 1.20634, size = 105, normalized size = 1.94

$$\frac{1}{16} b x^8 \log\left(-\frac{c x^2 + 1}{c x^2 - 1}\right) + \frac{1}{8} a x^8 + \frac{b x^6}{24 c} + \frac{b x^2}{8 c^3} - \frac{b \log(c x^2 + 1)}{16 c^4} + \frac{b \log(c x^2 - 1)}{16 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] 1/16\*b\*x^8\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 1/8\*a\*x^8 + 1/24\*b\*x^6/c + 1/8\*b  
\*x^2/c^3 - 1/16\*b\*log(c\*x^2 + 1)/c^4 + 1/16\*b\*log(c\*x^2 - 1)/c^4



### 3.51 $\int x^5 (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=48

$$\frac{1}{6}x^6(a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2x^4)}{12c^3} + \frac{bx^4}{12c}$$

[Out] (b\*x^4)/(12\*c) + (x^6\*(a + b\*ArcTanh[c\*x^2]))/6 + (b\*Log[1 - c^2\*x^4])/(12\*c^3)

**Rubi [A]** time = 0.0347672, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6097, 266, 43}

$$\frac{1}{6}x^6(a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2x^4)}{12c^3} + \frac{bx^4}{12c}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (b\*x^4)/(12\*c) + (x^6\*(a + b\*ArcTanh[c\*x^2]))/6 + (b\*Log[1 - c^2\*x^4])/(12\*c^3)

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned}
 \int x^5 (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^2)) - \frac{1}{3} (bc) \int \frac{x^7}{1 - c^2 x^4} dx \\
 &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^2)) - \frac{1}{12} (bc) \text{Subst} \left( \int \frac{x}{1 - c^2 x} dx, x, x^4 \right) \\
 &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^2)) - \frac{1}{12} (bc) \text{Subst} \left( \int \left( -\frac{1}{c^2} - \frac{1}{c^2 (-1 + c^2 x)} \right) dx, x, x^4 \right) \\
 &= \frac{bx^4}{12c} + \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2 x^4)}{12c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0139929, size = 53, normalized size = 1.1

$$\frac{ax^6}{6} + \frac{b \log(1 - c^2 x^4)}{12c^3} + \frac{bx^4}{12c} + \frac{1}{6} bx^6 \tanh^{-1}(cx^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (b\*x^4)/(12\*c) + (a\*x^6)/6 + (b\*x^6\*ArcTanh[c\*x^2])/6 + (b\*Log[1 - c^2\*x^4])/(12\*c^3)

**Maple [A]** time = 0.009, size = 45, normalized size = 0.9

$$\frac{x^6 a}{6} + \frac{bx^6 \text{Artanh}(cx^2)}{6} + \frac{bx^4}{12c} + \frac{b \ln(c^2 x^4 - 1)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arctanh(c\*x^2)),x)

[Out] 1/6\*x^6\*a+1/6\*b\*x^6\*arctanh(c\*x^2)+1/12\*b\*x^4/c+1/12\*b/c^3\*ln(c^2\*x^4-1)

**Maxima [A]** time = 0.98235, size = 62, normalized size = 1.29

$$\frac{1}{6}ax^6 + \frac{1}{12}\left(2x^6 \operatorname{artanh}(cx^2) + \left(\frac{x^4}{c^2} + \frac{\log(c^2x^4 - 1)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^2)),x, algorithm="maxima")

[Out] 1/6\*a\*x^6 + 1/12\*(2\*x^6\*arctanh(c\*x^2) + (x^4/c^2 + log(c^2\*x^4 - 1)/c^4)\*c)\*b

**Fricas [A]** time = 2.05896, size = 134, normalized size = 2.79

$$\frac{bc^3x^6 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2ac^3x^6 + bc^2x^4 + b \log(c^2x^4 - 1)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^2)),x, algorithm="fricas")

[Out] 1/12\*(b\*c^3\*x^6\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 2\*a\*c^3\*x^6 + b\*c^2\*x^4 + b\*log(c^2\*x^4 - 1))/c^3

**Sympy [A]** time = 21.0352, size = 85, normalized size = 1.77

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atanh}(cx^2)}{6} + \frac{bx^4}{12c} + \frac{b \log\left(x - i\sqrt{\frac{1}{c}}\right)}{6c^3} + \frac{b \log\left(x + i\sqrt{\frac{1}{c}}\right)}{6c^3} - \frac{b \operatorname{atanh}(cx^2)}{6c^3} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Piecewise((a\*x\*\*6/6 + b\*x\*\*6\*atanh(c\*x\*\*2)/6 + b\*x\*\*4/(12\*c) + b\*log(x - I\*sqrt(1/c))/(6\*c\*\*3) + b\*log(x + I\*sqrt(1/c))/(6\*c\*\*3) - b\*atanh(c\*x\*\*2)/(6\*c\*\*3), Ne(c, 0)), (a\*x\*\*6/6, True))

---

**Giac [A]** time = 1.17006, size = 77, normalized size = 1.6

$$\frac{1}{12} b x^6 \log\left(-\frac{c x^2 + 1}{c x^2 - 1}\right) + \frac{1}{6} a x^6 + \frac{b x^4}{12 c} + \frac{b \log(c^2 x^4 - 1)}{12 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] 1/12\*b\*x^6\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 1/6\*a\*x^6 + 1/12\*b\*x^4/c + 1/12\*b\*log(c^2\*x^4 - 1)/c^3

### 3.52 $\int x^3 (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=43

$$\frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) - \frac{b \tanh^{-1}(cx^2)}{4c^2} + \frac{bx^2}{4c}$$

[Out]  $(b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4$

**Rubi [A]** time = 0.0297085, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 321, 206}

$$\frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) - \frac{b \tanh^{-1}(cx^2)}{4c^2} + \frac{bx^2}{4c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*ArcTanh[c*x^2]),x]$

[Out]  $(b*x^2)/(4*c) - (b*ArcTanh[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2]))/4$

#### Rule 6097

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Simp}[(d*x)^{(m+1)}*(a + b*ArcTanh[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 275

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :\> \text{With}\{k = \text{GCD}[m + 1, n]\}, \text{Dist}[1/k, \text{Subst}[\text{Int}[x^{((m+1)/k - 1)}*(a + b*x^{(n/k)})^p}, x], x, x^{k}], x] /;$   
 $k \neq 1] /;$   
 $\text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 321

$\text{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] :\> \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m+n*p]$

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int x^3 (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) - \frac{1}{2}(bc) \int \frac{x^5}{1 - c^2x^4} dx \\ &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) - \frac{1}{4}(bc) \text{Subst}\left(\int \frac{x^2}{1 - c^2x^2} dx, x, x^2\right) \\ &= \frac{bx^2}{4c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) - \frac{b \text{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, x^2\right)}{4c} \\ &= \frac{bx^2}{4c} - \frac{b \tanh^{-1}(cx^2)}{4c^2} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^2)) \end{aligned}$$

**Mathematica [A]** time = 0.0129561, size = 67, normalized size = 1.56

$$\frac{ax^4}{4} + \frac{b \log(1 - cx^2)}{8c^2} - \frac{b \log(cx^2 + 1)}{8c^2} + \frac{bx^2}{4c} + \frac{1}{4}bx^4 \tanh^{-1}(cx^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (b\*x^2)/(4\*c) + (a\*x^4)/4 + (b\*x^4\*ArcTanh[c\*x^2])/4 + (b\*Log[1 - c\*x^2])/(8\*c^2) - (b\*Log[1 + c\*x^2])/(8\*c^2)

**Maple [A]** time = 0.012, size = 57, normalized size = 1.3

$$\frac{x^4 a}{4} + \frac{bx^4 \text{Artanh}(cx^2)}{4} + \frac{bx^2}{4c} + \frac{b \ln(cx^2 - 1)}{8c^2} - \frac{b \ln(cx^2 + 1)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arctanh(c*x^2)),x)`

[Out]  $\frac{1}{4}ax^4 + \frac{1}{4}bx^4 \operatorname{arctanh}(cx^2) + \frac{1}{4}bx^2/c + \frac{1}{8}b/c^2 \ln(cx^2-1) - \frac{1}{8}b/c^2 \ln(cx^2+1)$

**Maxima [A]** time = 0.955376, size = 78, normalized size = 1.81

$$\frac{1}{4}ax^4 + \frac{1}{8} \left( 2x^4 \operatorname{artanh}(cx^2) + c \left( \frac{2x^2}{c^2} - \frac{\log(cx^2+1)}{c^3} + \frac{\log(cx^2-1)}{c^3} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out]  $\frac{1}{4}ax^4 + \frac{1}{8}(2x^4 \operatorname{arctanh}(cx^2) + c(2x^2/c^2 - \log(cx^2+1)/c^3 + \log(cx^2-1)/c^3))b$

**Fricas [A]** time = 2.02008, size = 112, normalized size = 2.6

$$\frac{2ac^2x^4 + 2bcx^2 + (bc^2x^4 - b) \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out]  $\frac{1}{8}(2ac^2x^4 + 2bcx^2 + (bc^2x^4 - b) \log(-(cx^2+1)/(cx^2-1)))/c^2$

**Sympy [A]** time = 14.0799, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atanh}(cx^2)}{4} + \frac{bx^2}{4c} - \frac{b \operatorname{atanh}(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Piecewise((a\*x\*\*4/4 + b\*x\*\*4\*atanh(c\*x\*\*2)/4 + b\*x\*\*2/(4\*c) - b\*atanh(c\*x\*\*2)/(4\*c\*\*2), Ne(c, 0)), (a\*x\*\*4/4, True))

**Giac [A]** time = 1.18379, size = 93, normalized size = 2.16

$$\frac{1}{8}bx^4 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{1}{4}ax^4 + \frac{bx^2}{4c} - \frac{b \log(cx^2+1)}{8c^2} + \frac{b \log(cx^2-1)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] 1/8\*b\*x^4\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 1/4\*a\*x^4 + 1/4\*b\*x^2/c - 1/8\*b\*log(c\*x^2 + 1)/c^2 + 1/8\*b\*log(c\*x^2 - 1)/c^2



### 3.53 $\int x (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=37

$$\frac{1}{2}x^2 (a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2x^4)}{4c}$$

[Out]  $(x^2(a + b \operatorname{ArcTanh}[c x^2]))/2 + (b \operatorname{Log}[1 - c^2 x^4])/(4 c)$

**Rubi [A]** time = 0.0149314, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6097, 260}

$$\frac{1}{2}x^2 (a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2x^4)}{4c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x(a + b \operatorname{ArcTanh}[c x^2]), x]$

[Out]  $(x^2(a + b \operatorname{ArcTanh}[c x^2]))/2 + (b \operatorname{Log}[1 - c^2 x^4])/(4 c)$

#### Rule 6097

$\operatorname{Int}[(a + \operatorname{ArcTanh}[c x^n]) x^m, x]$   
 $\rightarrow \operatorname{Simp}[(d x)^{m+1} (a + b \operatorname{ArcTanh}[c x^n]) / (d(m+1)), x] - \operatorname{Dist}[(b c^n) / (d(m+1)), \operatorname{Int}[(x^{n-1} (d x)^{m+1}) / (1 - c^2 x^{2n}), x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, d, m, n, x\} \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rule 260

$\operatorname{Int}[x^m / (a + b x^n), x]$   
 $\rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b x^n, x]] / (b n), x] /;$   
 $\operatorname{FreeQ}\{a, b, m, n, x\} \ \&\& \ \operatorname{EqQ}[m, n - 1]$

#### Rubi steps

$$\begin{aligned} \int x (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^2)) - (bc) \int \frac{x^3}{1 - c^2x^4} dx \\ &= \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^2)) + \frac{b \log(1 - c^2x^4)}{4c} \end{aligned}$$

**Mathematica [A]** time = 0.0069701, size = 42, normalized size = 1.14

$$\frac{ax^2}{2} + \frac{b \log(1 - c^2x^4)}{4c} + \frac{1}{2}bx^2 \tanh^{-1}(cx^2)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (a\*x^2)/2 + (b\*x^2\*ArcTanh[c\*x^2])/2 + (b\*Log[1 - c^2\*x^4])/(4\*c)

**Maple [A]** time = 0.003, size = 37, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{Artanh}(cx^2)}{2} + \frac{b \ln(-c^2x^4 + 1)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c\*x^2)),x)

[Out] 1/2\*a\*x^2+1/2\*b\*x^2\*arctanh(c\*x^2)+1/4\*b\*ln(-c^2\*x^4+1)/c

**Maxima [A]** time = 0.955621, size = 50, normalized size = 1.35

$$\frac{1}{2}ax^2 + \frac{(2cx^2 \operatorname{artanh}(cx^2) + \log(-c^2x^4 + 1))b}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^2)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/4\*(2\*c\*x^2\*arctanh(c\*x^2) + log(-c^2\*x^4 + 1))\*b/c

**Fricas [A]** time = 2.06531, size = 108, normalized size = 2.92

$$\frac{bcx^2 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2acx^2 + b \log(c^2x^4 - 1)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}*(b*c*x^2*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2 + b*\log(c^2*x^4 - 1))/c$

**Sympy [A]** time = 10.0682, size = 71, normalized size = 1.92

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atanh}(cx^2)}{2} + \frac{b \log\left(x - i\sqrt{\frac{1}{c}}\right)}{2c} + \frac{b \log\left(x + i\sqrt{\frac{1}{c}}\right)}{2c} - \frac{b \operatorname{atanh}(cx^2)}{2c} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c*x**2)),x)`

[Out] `Piecewise((a*x**2/2 + b*x**2*atanh(c*x**2)/2 + b*log(x - I*sqrt(1/c))/(2*c) + b*log(x + I*sqrt(1/c))/(2*c) - b*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*x**2/2, True))`

**Giac [A]** time = 1.18776, size = 66, normalized size = 1.78

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(x^2 \log\left(-\frac{cx^2 + 1}{cx^2 - 1}\right) + \frac{\log(|c^2x^4 - 1|)}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="giac")`

[Out]  $\frac{1}{2}*a*x^2 + \frac{1}{4}*(x^2*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + \log(\operatorname{abs}(c^2*x^4 - 1)))/c)*b$

$$3.54 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x} dx$$

**Optimal.** Leaf size=30

$$-\frac{1}{4}b\text{PolyLog}(2, -cx^2) + \frac{1}{4}b\text{PolyLog}(2, cx^2) + a \log(x)$$

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^2)])/4 + (b\*PolyLog[2, c\*x^2])/4

**Rubi [A]** time = 0.0331072, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6095, 5912}

$$-\frac{1}{4}b\text{PolyLog}(2, -cx^2) + \frac{1}{4}b\text{PolyLog}(2, cx^2) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x, x]

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^2)])/4 + (b\*PolyLog[2, c\*x^2])/4

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}(cx^2)}{x} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, x^2 \right) \\ &= a \log(x) - \frac{1}{4}b\text{Li}_2(-cx^2) + \frac{1}{4}b\text{Li}_2(cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0133833, size = 28, normalized size = 0.93

$$\frac{1}{4}b \left( \text{PolyLog} \left( 2, cx^2 \right) - \text{PolyLog} \left( 2, -cx^2 \right) \right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/x,x]

[Out] a\*Log[x] + (b\*(-PolyLog[2, -(c\*x^2)] + PolyLog[2, c\*x^2]))/4

**Maple [B]** time = 0.029, size = 124, normalized size = 4.1

$$a \ln(x) + b \ln(x) \text{Artanh}(cx^2) - \frac{b \ln(x)}{2} \ln(1 + x\sqrt{-c}) - \frac{b \ln(x)}{2} \ln(1 - x\sqrt{-c}) - \frac{b}{2} \text{dilog}(1 + x\sqrt{-c}) - \frac{b}{2} \text{dilog}(1 - x\sqrt{-c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/x,x)

[Out] a\*ln(x)+b\*ln(x)\*arctanh(c\*x^2)-1/2\*b\*ln(x)\*ln(1+x\*(-c)^(1/2))-1/2\*b\*ln(x)\*ln(1-x\*(-c)^(1/2))-1/2\*b\*dilog(1+x\*(-c)^(1/2))-1/2\*b\*dilog(1-x\*(-c)^(1/2))+1/2\*b\*ln(x)\*ln(1-x\*c^(1/2))+1/2\*b\*ln(x)\*ln(1+x\*c^(1/2))+1/2\*b\*dilog(1-x\*c^(1/2))+1/2\*b\*dilog(1+x\*c^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b \int \frac{\log(cx^2 + 1) - \log(-cx^2 + 1)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x,x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(c\*x^2 + 1) - log(-c\*x^2 + 1))/x, x) + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx^2) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^2) + a)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/x,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^2) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)/x, x)

$$3.55 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^3} dx$$

**Optimal.** Leaf size=40

$$-\frac{a+b \tanh^{-1}(cx^2)}{2x^2} - \frac{1}{4}bc \log(1-c^2x^4) + bc \log(x)$$

[Out]  $-(a + b*\text{ArcTanh}[c*x^2])/(2*x^2) + b*c*\text{Log}[x] - (b*c*\text{Log}[1 - c^2*x^4])/4$

**Rubi [A]** time = 0.0255877, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b \tanh^{-1}(cx^2)}{2x^2} - \frac{1}{4}bc \log(1-c^2x^4) + bc \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c*x^2])/x^3, x]$

[Out]  $-(a + b*\text{ArcTanh}[c*x^2])/(2*x^2) + b*c*\text{Log}[x] - (b*c*\text{Log}[1 - c^2*x^4])/4$

### Rule 6097

$\text{Int}[(a + \text{ArcTanh}[c*(x)^n])*(d*(x))^m, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{n-1}*(d*x)^{m+1})/(1 - c^2*x^{2*n}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

### Rule 266

$\text{Int}[(x)^m*(a + (b*(x)^n)^p), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /;$   
 $\text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

### Rule 36

$\text{Int}[1/(((a + (b*(x))^c) + (d*(x))^d)), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 29

`Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]`

Rule 31

`Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]`

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^2)}{x^3} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{2x^2} + (bc) \int \frac{1}{x(1 - c^2x^4)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{2x^2} + \frac{1}{4}(bc) \operatorname{Subst} \left( \int \frac{1}{x(1 - c^2x)} dx, x, x^4 \right) \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{2x^2} + \frac{1}{4}(bc) \operatorname{Subst} \left( \int \frac{1}{x} dx, x, x^4 \right) + \frac{1}{4}(bc^3) \operatorname{Subst} \left( \int \frac{1}{1 - c^2x} dx, x, x^4 \right) \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{2x^2} + bc \log(x) - \frac{1}{4}bc \log(1 - c^2x^4)
 \end{aligned}$$

**Mathematica [A]** time = 0.0110482, size = 45, normalized size = 1.12

$$-\frac{a}{2x^2} - \frac{1}{4}bc \log(1 - c^2x^4) - \frac{b \tanh^{-1}(cx^2)}{2x^2} + bc \log(x)$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*ArcTanh[c*x^2])/x^3, x]`

[Out] `-a/(2*x^2) - (b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^4])/4`

**Maple [A]** time = 0.013, size = 49, normalized size = 1.2

$$-\frac{a}{2x^2} - \frac{b \operatorname{Artanh}(cx^2)}{2x^2} - \frac{bc \ln(cx^2 + 1)}{4} + bc \ln(x) - \frac{bc \ln(cx^2 - 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a+b*arctanh(c*x^2))/x^3,x)`

[Out]  $-1/2*a/x^2 - 1/2*b/x^2*arctanh(c*x^2) - 1/4*b*c*\ln(c*x^2+1) + b*c*\ln(x) - 1/4*b*c*\ln(c*x^2-1)$

**Maxima [A]** time = 0.980565, size = 55, normalized size = 1.38

$$-\frac{1}{4} \left( c \left( \log(c^2 x^4 - 1) - \log(x^4) \right) + \frac{2 \operatorname{artanh}(cx^2)}{x^2} \right) b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="maxima")`

[Out]  $-1/4*(c*(\log(c^2*x^4 - 1) - \log(x^4)) + 2*arctanh(c*x^2)/x^2)*b - 1/2*a/x^2$

**Fricas [A]** time = 2.00197, size = 130, normalized size = 3.25

$$\frac{bcx^2 \log(c^2 x^4 - 1) - 4bcx^2 \log(x) + b \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="fricas")`

[Out]  $-1/4*(b*c*x^2*\log(c^2*x^4 - 1) - 4*b*c*x^2*\log(x) + b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^2$

**Sympy [A]** time = 17.2699, size = 80, normalized size = 2.

$$\begin{cases} -\frac{a}{2x^2} + bc \log(x) - \frac{bc \log\left(x - i\sqrt{\frac{1}{c}}\right)}{2} - \frac{bc \log\left(x + i\sqrt{\frac{1}{c}}\right)}{2} + \frac{bc \operatorname{atanh}(cx^2)}{2} - \frac{b \operatorname{atanh}(cx^2)}{2x^2} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/x\*\*3,x)

[Out] Piecewise((-a/(2\*x\*\*2) + b\*c\*log(x) - b\*c\*log(x - I\*sqrt(1/c)))/2 - b\*c\*log(x + I\*sqrt(1/c))/2 + b\*c\*atanh(c\*x\*\*2)/2 - b\*atanh(c\*x\*\*2)/(2\*x\*\*2), Ne(c, 0)), (-a/(2\*x\*\*2), True))

**Giac [A]** time = 1.1719, size = 69, normalized size = 1.72

$$-\frac{1}{4}bc \log(c^2x^4 - 1) + bc \log(x) - \frac{b \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{4x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^3,x, algorithm="giac")

[Out] -1/4\*b\*c\*log(c^2\*x^4 - 1) + b\*c\*log(x) - 1/4\*b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1))/x^2 - 1/2\*a/x^2

$$3.56 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^5} dx$$

**Optimal.** Leaf size=41

$$-\frac{a+b \tanh^{-1}(cx^2)}{4x^4} + \frac{1}{4}bc^2 \tanh^{-1}(cx^2) - \frac{bc}{4x^2}$$

[Out]  $-(b*c)/(4*x^2) + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)$

**Rubi [A]** time = 0.0266015, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 325, 206}

$$-\frac{a+b \tanh^{-1}(cx^2)}{4x^4} + \frac{1}{4}bc^2 \tanh^{-1}(cx^2) - \frac{bc}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x^5, x]

[Out]  $-(b*c)/(4*x^2) + (b*c^2*ArcTanh[c*x^2])/4 - (a + b*ArcTanh[c*x^2])/(4*x^4)$

### Rule 6097

Int[((a\_) + ArcTanh[(c\_)\*(x\_)^(n\_)])\*(b\_))\*((d\_)\*(x\_)^(m\_)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 275

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = GCD[m+1, n]}, Dist[1/k, Subst[Int[x^((m+1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 325

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,

x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{x^5} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{4x^4} + \frac{1}{2}(bc) \int \frac{1}{x^3(1 - c^2x^4)} dx \\
&= -\frac{a + b \tanh^{-1}(cx^2)}{4x^4} + \frac{1}{4}(bc) \operatorname{Subst}\left(\int \frac{1}{x^2(1 - c^2x^2)} dx, x, x^2\right) \\
&= -\frac{bc}{4x^2} - \frac{a + b \tanh^{-1}(cx^2)}{4x^4} + \frac{1}{4}(bc^3) \operatorname{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, x^2\right) \\
&= -\frac{bc}{4x^2} + \frac{1}{4}bc^2 \tanh^{-1}(cx^2) - \frac{a + b \tanh^{-1}(cx^2)}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0113402, size = 65, normalized size = 1.59

$$-\frac{a}{4x^4} - \frac{1}{8}bc^2 \log(1 - cx^2) + \frac{1}{8}bc^2 \log(cx^2 + 1) - \frac{bc}{4x^2} - \frac{b \tanh^{-1}(cx^2)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/x^5, x]

[Out] -a/(4\*x^4) - (b\*c)/(4\*x^2) - (b\*ArcTanh[c\*x^2])/(4\*x^4) - (b\*c^2\*Log[1 - c\*x^2])/8 + (b\*c^2\*Log[1 + c\*x^2])/8

**Maple [A]** time = 0.013, size = 55, normalized size = 1.3

$$-\frac{a}{4x^4} - \frac{b \operatorname{Artanh}(cx^2)}{4x^4} + \frac{bc^2 \ln(cx^2 + 1)}{8} - \frac{bc^2 \ln(cx^2 - 1)}{8} - \frac{bc}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))/x^5,x)`

[Out]  $-1/4*a/x^4 - 1/4*b/x^4*arctanh(c*x^2) + 1/8*b*c^2*\ln(c*x^2+1) - 1/8*b*c^2*\ln(c*x^2-1) - 1/4*b*c/x^2$

**Maxima [A]** time = 0.958714, size = 69, normalized size = 1.68

$$\frac{1}{8} \left( \left( c \log(cx^2 + 1) - c \log(cx^2 - 1) - \frac{2}{x^2} \right) c - \frac{2 \operatorname{artanh}(cx^2)}{x^4} \right) b - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="maxima")`

[Out]  $1/8*((c*\log(c*x^2 + 1) - c*\log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4)*b - 1/4*a/x^4$

**Fricas [A]** time = 1.93919, size = 103, normalized size = 2.51

$$\frac{2bcx^2 - (bc^2x^4 - b) \log\left(\frac{-cx^2+1}{cx^2-1}\right) + 2a}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="fricas")`

[Out]  $-1/8*(2*b*c*x^2 - (b*c^2*x^4 - b)*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^4$

**Sympy [A]** time = 12.4636, size = 41, normalized size = 1.

$$-\frac{a}{4x^4} + \frac{bc^2 \operatorname{atanh}(cx^2)}{4} - \frac{bc}{4x^2} - \frac{b \operatorname{atanh}(cx^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))/x**5,x)`

[Out]  $-a/(4*x**4) + b*c**2*atanh(c*x**2)/4 - b*c/(4*x**2) - b*atanh(c*x**2)/(4*x**4)$

---

**Giac [A]** time = 1.19515, size = 90, normalized size = 2.2

$$\frac{1}{8}bc^2 \log(cx^2 + 1) - \frac{1}{8}bc^2 \log(cx^2 - 1) - \frac{b \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8x^4} - \frac{bcx^2 + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="giac")`

[Out]  $1/8*b*c^2*\log(c*x^2 + 1) - 1/8*b*c^2*\log(c*x^2 - 1) - 1/8*b*\log(-(c*x^2 + 1)/(c*x^2 - 1))/x^4 - 1/4*(b*c*x^2 + a)/x^4$

$$3.57 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^7} dx$$

**Optimal.** Leaf size=56

$$-\frac{a+b \tanh^{-1}(cx^2)}{6x^6} - \frac{1}{12}bc^3 \log(1-c^2x^4) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{12x^4}$$

[Out]  $-(b*c)/(12*x^4) - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^4])/12$

**Rubi [A]** time = 0.0340155, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6097, 266, 44}

$$-\frac{a+b \tanh^{-1}(cx^2)}{6x^6} - \frac{1}{12}bc^3 \log(1-c^2x^4) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{12x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x^7, x]

[Out]  $-(b*c)/(12*x^4) - (a + b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^4])/12$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m+1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^2)}{x^7} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{6x^6} + \frac{1}{3}(bc) \int \frac{1}{x^5(1 - c^2x^4)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{6x^6} + \frac{1}{12}(bc) \text{Subst} \left( \int \frac{1}{x^2(1 - c^2x)} dx, x, x^4 \right) \\
 &= -\frac{a + b \tanh^{-1}(cx^2)}{6x^6} + \frac{1}{12}(bc) \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{c^2}{x} - \frac{c^4}{-1 + c^2x} \right) dx, x, x^4 \right) \\
 &= -\frac{bc}{12x^4} - \frac{a + b \tanh^{-1}(cx^2)}{6x^6} + \frac{1}{3}bc^3 \log(x) - \frac{1}{12}bc^3 \log(1 - c^2x^4)
 \end{aligned}$$

**Mathematica [A]** time = 0.0112476, size = 61, normalized size = 1.09

$$-\frac{a}{6x^6} - \frac{1}{12}bc^3 \log(1 - c^2x^4) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{12x^4} - \frac{b \tanh^{-1}(cx^2)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/x^7, x]

[Out] -a/(6\*x^6) - (b\*c)/(12\*x^4) - (b\*ArcTanh[c\*x^2])/(6\*x^6) + (b\*c^3\*Log[x])/3 - (b\*c^3\*Log[1 - c^2\*x^4])/12

**Maple [A]** time = 0.015, size = 63, normalized size = 1.1

$$-\frac{a}{6x^6} - \frac{b \text{Artanh}(cx^2)}{6x^6} - \frac{bc^3 \ln(cx^2 + 1)}{12} - \frac{bc}{12x^4} + \frac{bc^3 \ln(x)}{3} - \frac{bc^3 \ln(cx^2 - 1)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/x^7, x)



[Out]  $-1/6*a/x^6 - 1/6*b/x^6*\operatorname{arctanh}(c*x^2) - 1/12*b*c^3*\ln(c*x^2+1) - 1/12*b*c/x^4 + 1/3*b*c^3*\ln(x) - 1/12*b*c^3*\ln(c*x^2-1)$

**Maxima [A]** time = 0.975046, size = 69, normalized size = 1.23

$$-\frac{1}{12} \left( \left( c^2 \log(c^2 x^4 - 1) - c^2 \log(x^4) + \frac{1}{x^4} \right) c + \frac{2 \operatorname{artanh}(cx^2)}{x^6} \right) b - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="maxima")`

[Out]  $-1/12*((c^2*\log(c^2*x^4 - 1) - c^2*\log(x^4) + 1/x^4)*c + 2*\operatorname{arctanh}(c*x^2)/x^6)*b - 1/6*a/x^6$

**Fricas [A]** time = 2.11026, size = 150, normalized size = 2.68

$$\frac{bc^3x^6 \log(c^2x^4 - 1) - 4bc^3x^6 \log(x) + bcx^2 + b \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="fricas")`

[Out]  $-1/12*(b*c^3*x^6*\log(c^2*x^4 - 1) - 4*b*c^3*x^6*\log(x) + b*c*x^2 + b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^6$

**Sympy [A]** time = 34.4314, size = 97, normalized size = 1.73

$$\begin{cases} -\frac{a}{6x^6} + \frac{bc^3 \log(x)}{3} - \frac{bc^3 \log\left(x-i\sqrt{\frac{1}{c}}\right)}{6} - \frac{bc^3 \log\left(x+i\sqrt{\frac{1}{c}}\right)}{6} + \frac{bc^3 \operatorname{atanh}(cx^2)}{6} - \frac{bc}{12x^4} - \frac{b \operatorname{atanh}(cx^2)}{6x^6} & \text{for } c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))/x**7,x)`

```
[Out] Piecewise((-a/(6*x**6) + b*c**3*log(x)/3 - b*c**3*log(x - I*sqrt(1/c))/6 -
b*c**3*log(x + I*sqrt(1/c))/6 + b*c**3*atanh(c*x**2)/6 - b*c/(12*x**4) - b*
atanh(c*x**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

**Giac [A]** time = 1.33286, size = 88, normalized size = 1.57

$$-\frac{1}{12}bc^3 \log(c^2x^4 - 1) + \frac{1}{3}bc^3 \log(x) - \frac{b \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{12x^6} - \frac{bcx^2 + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*log(c^2*x^4 - 1) + 1/3*b*c^3*log(x) - 1/12*b*log(-(c*x^2 + 1)/(
c*x^2 - 1))/x^6 - 1/12*(b*c*x^2 + 2*a)/x^6
```

### 3.58 $\int x^4 (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=65

$$\frac{1}{5}x^5(a + b \tanh^{-1}(cx^2)) + \frac{b \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{5c^{5/2}} + \frac{2bx^3}{15c}$$

[Out] (2\*b\*x^3)/(15\*c) + (b\*ArcTan[Sqrt[c]\*x])/(5\*c^(5/2)) - (b\*ArcTanh[Sqrt[c]\*x])/(5\*c^(5/2)) + (x^5\*(a + b\*ArcTanh[c\*x^2]))/5

**Rubi [A]** time = 0.0352452, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 321, 298, 203, 206}

$$\frac{1}{5}x^5(a + b \tanh^{-1}(cx^2)) + \frac{b \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{5c^{5/2}} + \frac{2bx^3}{15c}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (2\*b\*x^3)/(15\*c) + (b\*ArcTan[Sqrt[c]\*x])/(5\*c^(5/2)) - (b\*ArcTanh[Sqrt[c]\*x])/(5\*c^(5/2)) + (x^5\*(a + b\*ArcTanh[c\*x^2]))/5

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 298

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

### Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^2)) - \frac{1}{5}(2bc) \int \frac{x^6}{1 - c^2x^4} dx \\
 &= \frac{2bx^3}{15c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^2)) - \frac{(2b) \int \frac{x^2}{1 - c^2x^4} dx}{5c} \\
 &= \frac{2bx^3}{15c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^2)) - \frac{b \int \frac{1}{1 - cx^2} dx}{5c^2} + \frac{b \int \frac{1}{1 + cx^2} dx}{5c^2} \\
 &= \frac{2bx^3}{15c} + \frac{b \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{5c^{5/2}} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^2))
 \end{aligned}$$

**Mathematica [A]** time = 0.0239303, size = 93, normalized size = 1.43

$$\frac{ax^5}{5} + \frac{b \log(1 - \sqrt{cx})}{10c^{5/2}} - \frac{b \log(\sqrt{cx} + 1)}{10c^{5/2}} + \frac{b \tan^{-1}(\sqrt{cx})}{5c^{5/2}} + \frac{2bx^3}{15c} + \frac{1}{5}bx^5 \tanh^{-1}(cx^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x^3)/(15*c) + (a*x^5)/5 + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) + (b*x^5*ArcTanh[c*x^2])/5 + (b*Log[1 - Sqrt[c]*x])/(10*c^(5/2)) - (b*Log[1 + Sqrt[c]*x])/(10*c^(5/2))
```

---

**Maple [A]** time = 0.011, size = 53, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{x^5 b \operatorname{Artanh}(cx^2)}{5} + \frac{2bx^3}{15c} + \frac{b}{5} \arctan(x\sqrt{c})c^{-\frac{5}{2}} - \frac{b}{5} \operatorname{Artanh}(x\sqrt{c})c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*arctanh(c*x^2)),x)`

[Out] `1/5*a*x^5+1/5*x^5*b*arctanh(c*x^2)+2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(5/2)-1/5*b*arctanh(x*c^(1/2))/c^(5/2)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.00913, size = 473, normalized size = 7.28

$$\left[ \frac{3bc^3x^5 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6ac^3x^5 + 4b^2cx^3 + 6b\sqrt{c} \arctan(\sqrt{cx}) + 3b\sqrt{c} \log\left(\frac{cx^2-2\sqrt{cx}+1}{cx^2-1}\right)}{30c^3}, \frac{3bc^3x^5 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6ac^3x^5}{30c^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out] `[1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(c)*arctan(sqrt(c)*x) + 3*b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c^3, 1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(-c)*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^3]`

---

**Sympy [A]** time = 19.3632, size = 185, normalized size = 2.85

$$\left\{ \begin{array}{l} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atanh}(cx^2)}{5} + \frac{2bx^3}{15c} - \frac{b \log\left(x - i\sqrt{\frac{1}{c}}\right)}{10c^3 \sqrt{\frac{1}{c}}} - \frac{ib \log\left(x - i\sqrt{\frac{1}{c}}\right)}{10c^3 \sqrt{\frac{1}{c}}} - \frac{b \log\left(x + i\sqrt{\frac{1}{c}}\right)}{10c^3 \sqrt{\frac{1}{c}}} + \frac{ib \log\left(x + i\sqrt{\frac{1}{c}}\right)}{10c^3 \sqrt{\frac{1}{c}}} + \frac{b \log\left(x - \sqrt{\frac{1}{c}}\right)}{5c^3 \sqrt{\frac{1}{c}}} + \frac{b \operatorname{atanh}(cx^2)}{5c^3 \sqrt{\frac{1}{c}}} \\ \frac{ax^5}{5} \end{array} \right. \quad \begin{array}{l} \text{for } c < 0 \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Piecewise((a\*x\*\*5/5 + b\*x\*\*5\*atanh(c\*x\*\*2)/5 + 2\*b\*x\*\*3/(15\*c) - b\*log(x - I\*sqrt(1/c))/(10\*c\*\*3\*sqrt(1/c)) - I\*b\*log(x - I\*sqrt(1/c))/(10\*c\*\*3\*sqrt(1/c)) - b\*log(x + I\*sqrt(1/c))/(10\*c\*\*3\*sqrt(1/c)) + I\*b\*log(x + I\*sqrt(1/c))/(10\*c\*\*3\*sqrt(1/c)) + b\*log(x - sqrt(1/c))/(5\*c\*\*3\*sqrt(1/c)) + b\*atanh(c\*x\*\*2)/(5\*c\*\*3\*sqrt(1/c)), Ne(c, 0)), (a\*x\*\*5/5, True))

---

**Giac [A]** time = 1.27424, size = 103, normalized size = 1.58

$$\frac{1}{5} bc^9 \left( \frac{\arctan(\sqrt{cx})}{c^{\frac{23}{2}}} + \frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}c^{11}} \right) + \frac{1}{10} bx^5 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{1}{5} ax^5 + \frac{2bx^3}{15c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] 1/5\*b\*c^9\*(arctan(sqrt(c)\*x)/c^(23/2) + arctan(c\*x/sqrt(-c))/(sqrt(-c)\*c^11)) + 1/10\*b\*x^5\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 1/5\*a\*x^5 + 2/15\*b\*x^3/c

### 3.59 $\int x^2 (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=63

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2)) - \frac{b \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{2bx}{3c}$$

[Out]  $(2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^{(3/2)}) - (b*ArcTanh[Sqrt[c]*x])/(3*c^{(3/2)}) + (x^3*(a + b*ArcTanh[c*x^2]))/3$

**Rubi [A]** time = 0.0330533, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 321, 212, 206, 203}

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2)) - \frac{b \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{2bx}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^2]),x]

[Out]  $(2*b*x)/(3*c) - (b*ArcTan[Sqrt[c]*x])/(3*c^{(3/2)}) - (b*ArcTanh[Sqrt[c]*x])/(3*c^{(3/2)}) + (x^3*(a + b*ArcTanh[c*x^2]))/3$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
  [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
  , 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
 \int x^2 (a + b \tanh^{-1}(cx^2)) dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2)) - \frac{1}{3}(2bc) \int \frac{x^4}{1 - c^2x^4} dx \\
 &= \frac{2bx}{3c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2)) - \frac{(2b) \int \frac{1}{1 - c^2x^4} dx}{3c} \\
 &= \frac{2bx}{3c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2)) - \frac{b \int \frac{1}{1 - cx^2} dx}{3c} - \frac{b \int \frac{1}{1 + cx^2} dx}{3c} \\
 &= \frac{2bx}{3c} - \frac{b \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{b \tanh^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^2))
 \end{aligned}$$

**Mathematica [A]** time = 0.0188546, size = 91, normalized size = 1.44

$$\frac{ax^3}{3} + \frac{b \log(1 - \sqrt{cx})}{6c^{3/2}} - \frac{b \log(\sqrt{cx} + 1)}{6c^{3/2}} - \frac{b \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{1}{3}bx^3 \tanh^{-1}(cx^2) + \frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^2]),x]
```

```
[Out] (2*b*x)/(3*c) + (a*x^3)/3 - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (b*x^3*ArcT
anh[c*x^2])/3 + (b*Log[1 - Sqrt[c]*x])/(6*c^(3/2)) - (b*Log[1 + Sqrt[c]*x]
)/(6*c^(3/2))
```



---

**Maple [A]** time = 0.01, size = 51, normalized size = 0.8

$$\frac{x^3 a}{3} + \frac{x^3 b \operatorname{Artanh}(cx^2)}{3} + \frac{2bx}{3c} - \frac{b}{3} \arctan(x\sqrt{c})c^{-\frac{3}{2}} - \frac{b}{3} \operatorname{Artanh}(x\sqrt{c})c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^2)),x)`

[Out] `1/3*x^3*a+1/3*x^3*b*arctanh(c*x^2)+2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2)-1/3*b*arctanh(x*c^(1/2))/c^(3/2)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.22894, size = 448, normalized size = 7.11

$$\left[ \frac{bc^2x^3 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2ac^2x^3 + 4bcx - 2b\sqrt{c} \arctan(\sqrt{cx}) + b\sqrt{c} \log\left(\frac{cx^2-2\sqrt{cx}+1}{cx^2-1}\right)}{6c^2}, \frac{bc^2x^3 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2ac^2x^3 + 4bcx - 2b\sqrt{-c} \arctan(\sqrt{-c}x) - b\sqrt{-c} \log\left(\frac{cx^2+2\sqrt{-c}x-1}{cx^2+1}\right)}{6c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out] `[1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x - 2*b*sqrt(c)*arctan(sqrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c^2, 1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*x) - b*sqrt(-c)*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^2]`

**Sympy [A]** time = 12.9636, size = 581, normalized size = 9.22

$$\left\{ \begin{array}{l} -\frac{4ac^2x^3\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}} - \frac{4iac^2x^3\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}} - \frac{4bc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}(cx^2)}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}} - \frac{4ibc^2x^3\sqrt{\frac{1}{c}}\operatorname{atanh}(cx^2)}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}} - \frac{2ibc^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{-12c^4\sqrt{\frac{1}{c}}-12ic^4\sqrt{\frac{1}{c}}} - \frac{8bcx\sqrt{\frac{1}{c}}}{-12c^2\sqrt{\frac{1}{c}}-12ic^2\sqrt{\frac{1}{c}}} - \frac{ax^3}{3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Piecewise((-4\*a\*c\*\*2\*x\*\*3\*sqrt(1/c)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 4\*I\*a\*c\*\*2\*x\*\*3\*sqrt(1/c)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 4\*b\*c\*\*2\*x\*\*3\*sqrt(1/c)\*atanh(c\*x\*\*2)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 4\*I\*b\*c\*\*2\*x\*\*3\*sqrt(1/c)\*atanh(c\*x\*\*2)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 2\*I\*b\*c\*\*2\*log(x + I\*sqrt(1/c))/(-12\*c\*\*4\*sqrt(1/c) - 12\*I\*c\*\*4\*sqrt(1/c)) - 8\*b\*c\*x\*sqrt(1/c)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 8\*I\*b\*c\*x\*sqrt(1/c)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) + 6\*I\*b\*c\*log(x + I\*sqrt(1/c))/(-12\*c\*\*3\*sqrt(1/c) - 12\*I\*c\*\*3\*sqrt(1/c)) - 4\*I\*b\*c\*log(x - sqrt(1/c))/(-12\*c\*\*3\*sqrt(1/c) - 12\*I\*c\*\*3\*sqrt(1/c)) - 4\*I\*b\*c\*atanh(c\*x\*\*2)/(-12\*c\*\*3\*sqrt(1/c) - 12\*I\*c\*\*3\*sqrt(1/c)) + 4\*b\*log(x - I\*sqrt(1/c))/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)) - 4\*b\*atanh(c\*x\*\*2)/(-12\*c\*\*2\*sqrt(1/c) - 12\*I\*c\*\*2\*sqrt(1/c)), Ne(c, 0)), (a\*x\*\*3/3, True))

**Giac [A]** time = 1.27659, size = 101, normalized size = 1.6

$$-\frac{1}{3}bc^5\left(\frac{\arctan(\sqrt{c}x)}{c^{\frac{13}{2}}}-\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}c^6}\right)+\frac{1}{6}bx^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{3}ax^3+\frac{2bx}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] -1/3\*b\*c^5\*(arctan(sqrt(c)\*x)/c^(13/2) - arctan(c\*x/sqrt(-c))/(sqrt(-c)\*c^6)) + 1/6\*b\*x^3\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 1/3\*a\*x^3 + 2/3\*b\*x/c

### 3.60 $\int (a + b \tanh^{-1}(cx^2)) dx$

**Optimal.** Leaf size=44

$$ax + bx \tanh^{-1}(cx^2) + \frac{b \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b \tanh^{-1}(\sqrt{cx})}{\sqrt{c}}$$

[Out] a\*x + (b\*ArcTan[Sqrt[c]\*x])/Sqrt[c] - (b\*ArcTanh[Sqrt[c]\*x])/Sqrt[c] + b\*x\*ArcTanh[c\*x^2]

**Rubi [A]** time = 0.0244469, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6091, 298, 203, 206}

$$ax + bx \tanh^{-1}(cx^2) + \frac{b \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b \tanh^{-1}(\sqrt{cx})}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*x^2], x]

[Out] a\*x + (b\*ArcTan[Sqrt[c]\*x])/Sqrt[c] - (b\*ArcTanh[Sqrt[c]\*x])/Sqrt[c] + b\*x\*ArcTanh[c\*x^2]

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int (a + b \tanh^{-1}(cx^2)) dx &= ax + b \int \tanh^{-1}(cx^2) dx \\ &= ax + bx \tanh^{-1}(cx^2) - (2bc) \int \frac{x^2}{1 - c^2x^4} dx \\ &= ax + bx \tanh^{-1}(cx^2) - b \int \frac{1}{1 - cx^2} dx + b \int \frac{1}{1 + cx^2} dx \\ &= ax + \frac{b \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} + bx \tanh^{-1}(cx^2) \end{aligned}$$

**Mathematica [A]** time = 0.0196384, size = 57, normalized size = 1.3

$$ax + bx \tanh^{-1}(cx^2) + \frac{b(\log(1 - \sqrt{cx}) - \log(\sqrt{cx} + 1) + 2 \tan^{-1}(\sqrt{cx}))}{2\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*x^2], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x^2] + (b\*(2\*ArcTan[Sqrt[c]\*x] + Log[1 - Sqrt[c]\*x] - Log[1 + Sqrt[c]\*x]))/(2\*Sqrt[c])

**Maple [A]** time = 0.007, size = 37, normalized size = 0.8

$$ax + bx \operatorname{Artanh}(cx^2) + b \arctan(x\sqrt{c}) \frac{1}{\sqrt{c}} - b \operatorname{Artanh}(x\sqrt{c}) \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctanh(c\*x^2), x)

[Out]  $a*x+b*x*\operatorname{arctanh}(c*x^2)+b*\operatorname{arctan}(x*c^{(1/2)})/c^{(1/2)}-b*\operatorname{arctanh}(x*c^{(1/2)})/c^{(1/2)}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.07371, size = 394, normalized size = 8.95

$$\left[ \frac{bcx \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2acx + 2b\sqrt{c} \operatorname{arctan}(\sqrt{cx}) + b\sqrt{c} \log\left(\frac{cx^2-2\sqrt{cx}+1}{cx^2-1}\right)}{2c}, \frac{bcx \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2acx + 2b\sqrt{-c} \operatorname{arctan}(\sqrt{-c})}{2c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c*x^2),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{2}*(b*c*x*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*\sqrt{c}*\operatorname{arctan}(\sqrt{c}*x) + b*\sqrt{c}*\log((c*x^2 - 2*\sqrt{c}*x + 1)/(c*x^2 - 1)))/c, \frac{1}{2}*(b*c*x*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*\sqrt{-c}*\operatorname{arctan}(\sqrt{-c}*x) - b*\sqrt{-c}*\log((c*x^2 - 2*\sqrt{-c}*x - 1)/(c*x^2 + 1)))/c \right]$

**Sympy [A]** time = 8.39401, size = 139, normalized size = 3.16

$$ax + b \left( \begin{cases} x \operatorname{atanh}(cx^2) - \frac{\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} - \frac{i\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} - \frac{\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} + \frac{i\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c\sqrt{\frac{1}{c}}} + \frac{\log\left(x-\sqrt{\frac{1}{c}}\right)}{c\sqrt{\frac{1}{c}}} + \frac{\operatorname{atanh}(cx^2)}{c\sqrt{\frac{1}{c}}} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atanh(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((x*atanh(c*x**2) - log(x - I*sqrt(1/c))/(2*c*sqrt(1/c)) -
  I*log(x - I*sqrt(1/c))/(2*c*sqrt(1/c)) - log(x + I*sqrt(1/c))/(2*c*sqrt(1/
  c)) + I*log(x + I*sqrt(1/c))/(2*c*sqrt(1/c)) + log(x - sqrt(1/c))/(c*sqrt(1
  /c)) + atanh(c*x**2)/(c*sqrt(1/c)), Ne(c, 0)), (0, True))
```

**Giac [B]** time = 1.19516, size = 112, normalized size = 2.55

$$\frac{1}{2} \left( c \left( \frac{2\sqrt{|c|} \arctan(x\sqrt{|c|})}{c^2} - \frac{\sqrt{|c|} \log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{c^2} + \frac{\sqrt{|c|} \log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{c^2} \right) + x \log\left(-\frac{cx^2 + 1}{cx^2 - 1}\right) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x
+ 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) + x
*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + a*x
```

$$3.61 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^2} dx$$

**Optimal.** Leaf size=46

$$-\frac{a+b \tanh^{-1}(cx^2)}{x} + b\sqrt{c} \tan^{-1}(\sqrt{cx}) + b\sqrt{c} \tanh^{-1}(\sqrt{cx})$$

[Out] b\*Sqrt[c]\*ArcTan[Sqrt[c]\*x] + b\*Sqrt[c]\*ArcTanh[Sqrt[c]\*x] - (a + b\*ArcTanh[c\*x^2])/x

**Rubi [A]** time = 0.0256217, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 212, 206, 203}

$$-\frac{a+b \tanh^{-1}(cx^2)}{x} + b\sqrt{c} \tan^{-1}(\sqrt{cx}) + b\sqrt{c} \tanh^{-1}(\sqrt{cx})$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x^2,x]

[Out] b\*Sqrt[c]\*ArcTan[Sqrt[c]\*x] + b\*Sqrt[c]\*ArcTanh[Sqrt[c]\*x] - (a + b\*ArcTanh[c\*x^2])/x

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

#### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx^2)}{x^2} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{x} + (2bc) \int \frac{1}{1 - c^2x^4} dx \\ &= -\frac{a + b \tanh^{-1}(cx^2)}{x} + (bc) \int \frac{1}{1 - cx^2} dx + (bc) \int \frac{1}{1 + cx^2} dx \\ &= b\sqrt{c} \tan^{-1}(\sqrt{cx}) + b\sqrt{c} \tanh^{-1}(\sqrt{cx}) - \frac{a + b \tanh^{-1}(cx^2)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.0170497, size = 75, normalized size = 1.63

$$-\frac{a}{x} - \frac{b \tanh^{-1}(cx^2)}{x} - \frac{1}{2}b\sqrt{c} \log(1 - \sqrt{cx}) + \frac{1}{2}b\sqrt{c} \log(\sqrt{cx} + 1) + b\sqrt{c} \tan^{-1}(\sqrt{cx})$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^2, x]
```

```
[Out] -(a/x) + b*Sqrt[c]*ArcTan[Sqrt[c]*x] - (b*ArcTanh[c*x^2])/x - (b*Sqrt[c]*Lo
g[1 - Sqrt[c]*x])/2 + (b*Sqrt[c]*Log[1 + Sqrt[c]*x])/2
```

**Maple [A]** time = 0.013, size = 42, normalized size = 0.9

$$-\frac{a}{x} - \frac{b \operatorname{Arctanh}(cx^2)}{x} + b \arctan(x\sqrt{c})\sqrt{c} + b \operatorname{Arctanh}(x\sqrt{c})\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((a+b*arctanh(c*x^2))/x^2,x)`

[Out]  $-a/x - b/x \operatorname{arctanh}(c x^2) + b \operatorname{arctan}(x \sqrt{c}) \sqrt{c} + b \operatorname{arctanh}(x \sqrt{c}) \sqrt{c}$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.12528, size = 385, normalized size = 8.37

$$\left[ \frac{2b\sqrt{cx} \arctan(\sqrt{cx}) + b\sqrt{cx} \log\left(\frac{cx^2+2\sqrt{cx}+1}{cx^2-1}\right) - b \log\left(-\frac{cx^2+1}{cx^2-1}\right) - 2a}{2x}, -\frac{2b\sqrt{-cx} \arctan(\sqrt{-cx}) - b\sqrt{-cx} \log\left(\frac{cx^2+2\sqrt{-cx}+1}{cx^2+1}\right) - b \log\left(\frac{cx^2+1}{cx^2-1}\right) + 2a}{2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="fricas")`

[Out]  $[1/2*(2*b*\sqrt{c}*x*\arctan(\sqrt{c}*x) + b*\sqrt{c}*x*\log((c*x^2 + 2*\sqrt{c})*x + 1)/(c*x^2 - 1)) - b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) - 2*a]/x, -1/2*(2*b*\sqrt{-c}*x*\arctan(\sqrt{-c}*x) - b*\sqrt{-c}*x*\log((c*x^2 + 2*\sqrt{-c})*x - 1)/(c*x^2 + 1)) + b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a]/x]$

**Sympy [A]** time = 15.1448, size = 699, normalized size = 15.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/x\*\*2,x)

[Out] Piecewise((-a - oo\*b)/x, Eq(c, -1/x\*\*2)), (-a + oo\*b)/x, Eq(c, x\*\*(-2))),  
 (-a/x, Eq(c, 0)), (-4\*a\*x\*\*4/(4\*x\*\*5 - 4\*x/c\*\*2) + 4\*a/(4\*c\*\*2\*x\*\*5 - 4\*x)  
 - b\*c\*\*2\*x\*\*5\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) - I\*b\*c  
 \*\*2\*x\*\*5\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) + 2\*b\*c\*x\*\*  
 5\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) - 2\*I\*b\*c\*x\*\*5\*sqrt(1/  
 c)\*log(x - I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) + 3\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x +  
 I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) + 3\*I\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x + I\*sqrt(1/  
 c))/(4\*x\*\*5 - 4\*x/c\*\*2) - 4\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x - sqrt(1/c))/(4\*x\*\*5 -  
 4\*x/c\*\*2) - 4\*b\*c\*x\*\*5\*sqrt(1/c)\*atanh(c\*x\*\*2)/(4\*x\*\*5 - 4\*x/c\*\*2) - 4\*b\*x  
 \*\*4\*atanh(c\*x\*\*2)/(4\*x\*\*5 - 4\*x/c\*\*2) - 2\*b\*x\*sqrt(1/c)\*log(x - I\*sqrt(1/c)  
 )/(4\*c\*x\*\*5 - 4\*x/c) + 2\*I\*b\*x\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(4\*c\*x\*\*5 - 4  
 \*x/c) - 3\*b\*x\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(4\*c\*x\*\*5 - 4\*x/c) - 3\*I\*b\*x\*s  
 qrt(1/c)\*log(x + I\*sqrt(1/c))/(4\*c\*x\*\*5 - 4\*x/c) + 4\*b\*x\*sqrt(1/c)\*log(x -  
 sqrt(1/c))/(4\*c\*x\*\*5 - 4\*x/c) + 4\*b\*x\*sqrt(1/c)\*atanh(c\*x\*\*2)/(4\*c\*x\*\*5 - 4  
 \*x/c) + b\*x\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) + I\*b\*x\*(  
 1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(4\*x\*\*5 - 4\*x/c\*\*2) + 4\*b\*atanh(c\*x\*\*2)/(4  
 \*c\*\*2\*x\*\*5 - 4\*x), True))

**Giac [B]** time = 1.22457, size = 107, normalized size = 2.33

$$\frac{1}{2}bc \left( \frac{2 \arctan(x\sqrt{|c|})}{\sqrt{|c|}} + \frac{\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}} - \frac{\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}} \right) - \frac{b \log\left(\frac{-cx^2+1}{cx^2-1}\right)}{2x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^2,x, algorithm="giac")

[Out] 1/2\*b\*c\*(2\*arctan(x\*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c))  
 ))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/2\*b\*log(-(  
 c\*x^2 + 1)/(c\*x^2 - 1))/x - a/x

$$3.62 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^4} dx$$

**Optimal.** Leaf size=63

$$-\frac{a+b \tanh^{-1}(cx^2)}{3x^3} - \frac{1}{3}bc^{3/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{3}bc^{3/2} \tanh^{-1}(\sqrt{cx}) - \frac{2bc}{3x}$$

[Out]  $(-2*b*c)/(3*x) - (b*c^{(3/2)*ArcTan[Sqrt[c]*x]})/3 + (b*c^{(3/2)*ArcTanh[Sqrt[c]*x]})/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)$

**Rubi [A]** time = 0.0334463, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 325, 298, 203, 206}

$$-\frac{a+b \tanh^{-1}(cx^2)}{3x^3} - \frac{1}{3}bc^{3/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{3}bc^{3/2} \tanh^{-1}(\sqrt{cx}) - \frac{2bc}{3x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x^4,x]

[Out]  $(-2*b*c)/(3*x) - (b*c^{(3/2)*ArcTan[Sqrt[c]*x]})/3 + (b*c^{(3/2)*ArcTanh[Sqrt[c]*x]})/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

### Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx^2)}{x^4} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{3x^3} + \frac{1}{3}(2bc) \int \frac{1}{x^2(1 - c^2x^4)} dx \\ &= -\frac{2bc}{3x} - \frac{a + b \tanh^{-1}(cx^2)}{3x^3} + \frac{1}{3}(2bc^3) \int \frac{x^2}{1 - c^2x^4} dx \\ &= -\frac{2bc}{3x} - \frac{a + b \tanh^{-1}(cx^2)}{3x^3} + \frac{1}{3}(bc^2) \int \frac{1}{1 - cx^2} dx - \frac{1}{3}(bc^2) \int \frac{1}{1 + cx^2} dx \\ &= -\frac{2bc}{3x} - \frac{1}{3}bc^{3/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{3}bc^{3/2} \tanh^{-1}(\sqrt{cx}) - \frac{a + b \tanh^{-1}(cx^2)}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.0273203, size = 91, normalized size = 1.44

$$-\frac{a}{3x^3} - \frac{1}{6}bc^{3/2} \log(1 - \sqrt{cx}) + \frac{1}{6}bc^{3/2} \log(\sqrt{cx} + 1) - \frac{1}{3}bc^{3/2} \tan^{-1}(\sqrt{cx}) - \frac{b \tanh^{-1}(cx^2)}{3x^3} - \frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^4, x]
```

```
[Out] -a/(3*x^3) - (2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 - (b*ArcTanh[c*x^2])/(3*x^3) - (b*c^(3/2)*Log[1 - Sqrt[c]*x])/6 + (b*c^(3/2)*Log[1 + Sqrt
```

[c]\*x))/6

**Maple [A]** time = 0.014, size = 51, normalized size = 0.8

$$-\frac{a}{3x^3} - \frac{b \operatorname{Arctanh}(cx^2)}{3x^3} - \frac{b}{3} c^{\frac{3}{2}} \arctan(x\sqrt{c}) - \frac{2bc}{3x} + \frac{b}{3} c^{\frac{3}{2}} \operatorname{Arctanh}(x\sqrt{c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/x^4,x)

[Out]  $-\frac{1}{3} \frac{a}{x^3} - \frac{1}{3} \frac{b}{x^3} \operatorname{arctanh}(cx^2) - \frac{1}{3} b c^{\frac{3}{2}} \arctan(xc^{\frac{1}{2}}) - \frac{2}{3} \frac{bc}{x} + \frac{1}{3} b c^{\frac{3}{2}} \operatorname{arctanh}(xc^{\frac{1}{2}})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.13394, size = 440, normalized size = 6.98

$$\left[ \frac{2bc^{\frac{3}{2}}x^3 \arctan(\sqrt{cx}) - bc^{\frac{3}{2}}x^3 \log\left(\frac{cx^2+2\sqrt{cx}+1}{cx^2-1}\right) + 4bcx^2 + b \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2a}{6x^3}, -\frac{2b\sqrt{-c}cx^3 \arctan(\sqrt{-c}x) - b\sqrt{-c}}{6x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^4,x, algorithm="fricas")

[Out]  $[-\frac{1}{6} \frac{(2bc^{\frac{3}{2}}x^3 \arctan(\sqrt{c}x) - bc^{\frac{3}{2}}x^3 \log((cx^2 + 2\sqrt{c}x + 1)/(cx^2 - 1)) + 4bcx^2 + b \log(-(cx^2 + 1)/(cx^2 - 1)) + 2a)}{x^3}, -\frac{1}{6} \frac{(2b\sqrt{-c}cx^3 \arctan(\sqrt{-c}x) - b\sqrt{-c})}{x^3}]$

$$(c*x^2 - 2*\sqrt{-c}*x - 1)/(c*x^2 + 1) + 4*b*c*x^2 + b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^3]$$

**Sympy [A]** time = 21.2908, size = 813, normalized size = 12.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/x\*\*4,x)

[Out] Piecewise((-a/(3\*x\*\*3), Eq(c, 0)), (-a - oo\*b)/(3\*x\*\*3), Eq(c, -1/x\*\*2)), (-a + oo\*b)/(3\*x\*\*3), Eq(c, x\*\*(-2))), (-4\*a\*x\*\*4/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 4\*a/(12\*c\*\*2\*x\*\*7 - 12\*x\*\*3) - b\*c\*\*3\*x\*\*7\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + I\*b\*c\*\*3\*x\*\*7\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 2\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 2\*I\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 3\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 3\*I\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 4\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*log(x - sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 4\*b\*c\*\*2\*x\*\*7\*sqrt(1/c)\*atanh(c\*x\*\*2)/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 8\*b\*c\*x\*\*6/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + b\*c\*x\*\*3\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - I\*b\*c\*x\*\*3\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 4\*b\*x\*\*4\*atanh(c\*x\*\*2)/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 2\*b\*x\*\*3\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 2\*I\*b\*x\*\*3\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) - 3\*b\*x\*\*3\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 3\*I\*b\*x\*\*3\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 4\*b\*x\*\*3\*sqrt(1/c)\*log(x - sqrt(1/c))/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 4\*b\*x\*\*3\*sqrt(1/c)\*atanh(c\*x\*\*2)/(12\*x\*\*7 - 12\*x\*\*3/c\*\*2) + 8\*b\*x\*\*2/(12\*c\*x\*\*7 - 12\*x\*\*3/c) + 4\*b\*atanh(c\*x\*\*2)/(12\*c\*\*2\*x\*\*7 - 12\*x\*\*3), True))

**Giac [B]** time = 1.35212, size = 132, normalized size = 2.1

$$-\frac{1}{6}bc^3 \left( \frac{2\sqrt{|c|} \arctan(x\sqrt{|c|})}{c^2} - \frac{\sqrt{|c|} \log\left(x + \frac{1}{\sqrt{|c|}}\right)}{c^2} + \frac{\sqrt{|c|} \log\left(x - \frac{1}{\sqrt{|c|}}\right)}{c^2} \right) - \frac{b \log\left(-\frac{cx^2+1}{cx^2-1}\right)}{6x^3} - \frac{2bcx^2+a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*b*c^3*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x + 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) - 1/6*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^3 - 1/3*(2*b*c*x^2 + a)/x^3
```

$$3.63 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{x^6} dx$$

**Optimal.** Leaf size=63

$$-\frac{a+b \tanh^{-1}(cx^2)}{5x^5} + \frac{1}{5}bc^{5/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{5}bc^{5/2} \tanh^{-1}(\sqrt{cx}) - \frac{2bc}{15x^3}$$

[Out]  $(-2*b*c)/(15*x^3) + (b*c^{(5/2)*ArcTan[Sqrt[c]*x]})/5 + (b*c^{(5/2)*ArcTanh[Sqrt[c]*x]})/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)$

**Rubi [A]** time = 0.0319471, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 325, 212, 206, 203}

$$-\frac{a+b \tanh^{-1}(cx^2)}{5x^5} + \frac{1}{5}bc^{5/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{5}bc^{5/2} \tanh^{-1}(\sqrt{cx}) - \frac{2bc}{15x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/x^6,x]

[Out]  $(-2*b*c)/(15*x^3) + (b*c^{(5/2)*ArcTan[Sqrt[c]*x]})/5 + (b*c^{(5/2)*ArcTanh[Sqrt[c]*x]})/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n])/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 325

Int[((c\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*c^n\*(m+1)), Int[(c\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 212



```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
  2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
  x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
  Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
  [a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
  , 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}(cx^2)}{x^6} dx &= -\frac{a + b \tanh^{-1}(cx^2)}{5x^5} + \frac{1}{5}(2bc) \int \frac{1}{x^4(1 - c^2x^4)} dx \\ &= -\frac{2bc}{15x^3} - \frac{a + b \tanh^{-1}(cx^2)}{5x^5} + \frac{1}{5}(2bc^3) \int \frac{1}{1 - c^2x^4} dx \\ &= -\frac{2bc}{15x^3} - \frac{a + b \tanh^{-1}(cx^2)}{5x^5} + \frac{1}{5}(bc^3) \int \frac{1}{1 - cx^2} dx + \frac{1}{5}(bc^3) \int \frac{1}{1 + cx^2} dx \\ &= -\frac{2bc}{15x^3} + \frac{1}{5}bc^{5/2} \tan^{-1}(\sqrt{cx}) + \frac{1}{5}bc^{5/2} \tanh^{-1}(\sqrt{cx}) - \frac{a + b \tanh^{-1}(cx^2)}{5x^5} \end{aligned}$$

**Mathematica [A]** time = 0.0272086, size = 91, normalized size = 1.44

$$-\frac{a}{5x^5} - \frac{1}{10}bc^{5/2} \log(1 - \sqrt{cx}) + \frac{1}{10}bc^{5/2} \log(\sqrt{cx} + 1) + \frac{1}{5}bc^{5/2} \tan^{-1}(\sqrt{cx}) - \frac{2bc}{15x^3} - \frac{b \tanh^{-1}(cx^2)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])/x^6, x]
```

```
[Out] -a/(5*x^5) - (2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (b*ArcTan
h[c*x^2])/(5*x^5) - (b*c^(5/2)*Log[1 - Sqrt[c]*x])/10 + (b*c^(5/2)*Log[1 +
```

Sqrt[c]\*x))/10

**Maple [A]** time = 0.013, size = 51, normalized size = 0.8

$$-\frac{a}{5x^5} - \frac{b \operatorname{Arctanh}(cx^2)}{5x^5} + \frac{b}{5} c^{\frac{5}{2}} \arctan(x\sqrt{c}) + \frac{b}{5} c^{\frac{5}{2}} \operatorname{Arctanh}(x\sqrt{c}) - \frac{2bc}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/x^6,x)

[Out] -1/5\*a/x^5-1/5\*b/x^5\*arctanh(c\*x^2)+1/5\*b\*c^(5/2)\*arctan(x\*c^(1/2))+1/5\*b\*c^(5/2)\*arctanh(x\*c^(1/2))-2/15\*b\*c/x^3

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.14109, size = 458, normalized size = 7.27

$$\left[ \frac{6bc^{\frac{5}{2}}x^5 \arctan(\sqrt{cx}) + 3bc^{\frac{5}{2}}x^5 \log\left(\frac{cx^2+2\sqrt{cx}+1}{cx^2-1}\right) - 4bcx^2 - 3b \log\left(-\frac{cx^2+1}{cx^2-1}\right) - 6a}{30x^5}, -\frac{6b\sqrt{-c}c^2x^5 \arctan(\sqrt{-cx}) - 3b\sqrt{-c}}{30x^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/x^6,x, algorithm="fricas")

[Out] [1/30\*(6\*b\*c^(5/2)\*x^5\*arctan(sqrt(c)\*x) + 3\*b\*c^(5/2)\*x^5\*log((c\*x^2 + 2\*sqrt(c)\*x + 1)/(c\*x^2 - 1)) - 4\*b\*c\*x^2 - 3\*b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) - 6\*a)/x^5, -1/30\*(6\*b\*sqrt(-c)\*c^2\*x^5\*arctan(sqrt(-c)\*x) - 3\*b\*sqrt(-c)\*c

$$^2*x^5*\log((c*x^2 + 2*\sqrt{-c}*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + 3*b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)/x^5]$$

**Sympy [A]** time = 30.8841, size = 833, normalized size = 13.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/x\*\*6,x)

[Out] Piecewise((-a/(5\*x\*\*5), Eq(c, 0)), (-a - oo\*b)/(5\*x\*\*5), Eq(c, -1/x\*\*2)), (-a + oo\*b)/(5\*x\*\*5), Eq(c, x\*\*(-2))), (-12\*a\*x\*\*4/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 12\*a/(60\*c\*\*2\*x\*\*9 - 60\*x\*\*5) - 3\*b\*c\*\*4\*x\*\*9\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 3\*I\*b\*c\*\*4\*x\*\*9\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 6\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 6\*I\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 9\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 9\*I\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 12\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*log(x - sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 12\*b\*c\*\*3\*x\*\*9\*sqrt(1/c)\*atanh(c\*x\*\*2)/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 3\*b\*c\*\*2\*x\*\*5\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 3\*I\*b\*c\*\*2\*x\*\*5\*(1/c)\*\*(3/2)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 8\*b\*c\*x\*\*6/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 6\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 6\*I\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x - I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 9\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 9\*I\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x + I\*sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 12\*b\*c\*x\*\*5\*sqrt(1/c)\*log(x - sqrt(1/c))/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 12\*b\*c\*x\*\*5\*sqrt(1/c)\*atanh(c\*x\*\*2)/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) - 12\*b\*x\*\*4\*atanh(c\*x\*\*2)/(60\*x\*\*9 - 60\*x\*\*5/c\*\*2) + 8\*b\*x\*\*2/(60\*c\*x\*\*9 - 60\*x\*\*5/c) + 12\*b\*atanh(c\*x\*\*2)/(60\*c\*\*2\*x\*\*9 - 60\*x\*\*5), True))

**Giac [A]** time = 1.53314, size = 123, normalized size = 1.95

$$\frac{1}{10}bc^3\left(\frac{2\arctan(x\sqrt{|c|})}{\sqrt{|c|}} + \frac{\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}} - \frac{\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{10x^5} - \frac{2bcx^2+3a}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b*c^3*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c))))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/10*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^5 - 1/15*(2*b*c*x^2 + 3*a)/x^5
```

### 3.64 $\int x^7 \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$

**Optimal.** Leaf size=125

$$\frac{abx^2}{4c^3} - \frac{(a + b \tanh^{-1}(cx^2))^2}{8c^4} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^2))^2 + \frac{bx^6 (a + b \tanh^{-1}(cx^2))}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2 \log(1 - c^2x^4)}{6c^4} + \frac{b^2}{6c^4}$$

[Out] (a\*b\*x^2)/(4\*c^3) + (b^2\*x^4)/(24\*c^2) + (b^2\*x^2\*ArcTanh[c\*x^2])/(4\*c^3) + (b\*x^6\*(a + b\*ArcTanh[c\*x^2]))/(12\*c) - (a + b\*ArcTanh[c\*x^2])^2/(8\*c^4) + (x^8\*(a + b\*ArcTanh[c\*x^2])^2)/8 + (b^2\*Log[1 - c^2\*x^4])/(6\*c^4)

**Rubi [C]** time = 1.54731, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.187$ , Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right)}{16c^4} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right)}{16c^4} + \frac{abx^2}{8c^3} - \frac{bx^4(2a - b \log(1 - cx^2))}{32c^2} - \frac{1}{192}b \left( -\frac{3(1 - cx^2)^4}{c^4} + \dots \right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^7\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (a\*b\*x^2)/(8\*c^3) + (23\*b^2\*x^2)/(192\*c^3) + (b^2\*x^4)/(128\*c^2) - (7\*b^2\*x^6)/(576\*c) - (b^2\*x^8)/256 + (3\*b^2\*(1 - c\*x^2)^2)/(32\*c^4) - (b^2\*(1 - c\*x^2)^3)/(36\*c^4) + (b^2\*(1 - c\*x^2)^4)/(256\*c^4) - (5\*b^2\*Log[1 - c\*x^2])/(192\*c^4) + (b^2\*(1 - c\*x^2)\*Log[1 - c\*x^2])/(16\*c^4) + (b^2\*Log[1 - c\*x^2]^2)/(32\*c^4) - (b\*x^4\*(2\*a - b\*Log[1 - c\*x^2]))/(32\*c^2) + (b\*x^6\*(2\*a - b\*Log[1 - c\*x^2]))/(48\*c) - (b\*x^8\*(2\*a - b\*Log[1 - c\*x^2]))/64 + (x^8\*(2\*a - b\*Log[1 - c\*x^2])^2)/32 - (b\*(2\*a - b\*Log[1 - c\*x^2]))\*((48\*(1 - c\*x^2))/c^4 - (36\*(1 - c\*x^2)^2)/c^4 + (16\*(1 - c\*x^2)^3)/c^4 - (3\*(1 - c\*x^2)^4)/c^4 - (12\*Log[1 - c\*x^2])/c^4)/192 - (b\*(2\*a - b\*Log[1 - c\*x^2]))\*Log[(1 + c\*x^2)/2]/(16\*c^4) + (b^2\*Log[1 + c\*x^2])/(24\*c^4) + (b^2\*x^6\*Log[1 + c\*x^2])/(24\*c) + (b^2\*(1 + c\*x^2)\*Log[1 + c\*x^2])/(8\*c^4) + (b^2\*Log[(1 - c\*x^2)/2]\*Log[1 + c\*x^2])/(16\*c^4) + (b\*x^8\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2])/16 - (b^2\*Log[1 + c\*x^2]^2)/(32\*c^4) + (b^2\*x^8\*Log[1 + c\*x^2]^2)/32 + (b^2\*PolyLog[2, (1 - c\*x^2)/2])/(16\*c^4) + (b^2\*PolyLog[2, (1 + c\*x^2)/2])/(16\*c^4)

**Rule 6099**

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Int[ExpandIntegrand[(d*x)^(m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && ! (EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

#### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol]
:= Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol]
:= Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:= Int[ExpandIntegrand[(a + b*x)^(m*(c + d*x)^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:= With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && ! (EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 14

$\text{Int}[(u_*)((c_*)(x_))^{(m_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /; \text{FreeQ}\{c, m\}, x] \ \&\& \ \text{SumQ}[u] \ \&\& \ !\text{LinearQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (a_*) + (b_*)(v_)] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{InverseFunctionQ}[v]$

Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_*)(x_)^{(n_)}] * (b_.)] / (x_), x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^{2/(2*b*n)}, x] /; \text{FreeQ}\{a, b, c, n\}, x]$

Rule 2395

$\text{Int}[(a_.) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_.)] * ((f_.) + (g_*)(x_))^{(q_)}, x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{(q+1)} * (a + b*\text{Log}[c*(d + e*x)^n]) / (g*(q+1)), x] - \text{Dist}[(b*e^n) / (g*(q+1)), \text{Int}[(f + g*x)^{(q+1)} / (d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[q, -1]$

Rule 2439

$\text{Int}[(a_.) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_.)]^{(p_)} * ((f_.) + \text{Log}[(h_*)((i_*) + (j_*)(x_))^{(m_)}] * (g_*)(x_))^{(r_)}], x\_Symbol] \rightarrow \text{Simp}[(x^{(r+1)} * (a + b*\text{Log}[c*(d + e*x)^n])^p * (f + g*\text{Log}[h*(i + j*x)^m])) / (r+1), x] + (-\text{Dist}[(g*j*m) / (r+1), \text{Int}[(x^{(r+1)} * (a + b*\text{Log}[c*(d + e*x)^n])^p] / (i + j*x), x], x] - \text{Dist}[(b*e^n*p) / (r+1), \text{Int}[(x^{(r+1)} * (a + b*\text{Log}[c*(d + e*x)^n])^{(p-1)} * (f + g*\text{Log}[h*(i + j*x)^m])) / (d + e*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, m, n\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IntegerQ}[r] \ \&\& \ (\text{EqQ}[p, 1] \ || \ \text{GtQ}[r, 0]) \ \&\& \ \text{NeQ}[r, -1]$

Rule 2416

$\text{Int}[(a_.) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_.)]^{(p_)} * ((h_*)(x_))^{(m_)} * ((f_.) + (g_*)(x_))^{(r_)}]^{(q_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p * (h*x)^m * (f + g*x^r)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, n, p, q, r\}, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[q]$

Rule 2389

$\text{Int}[(a_.) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_)}] * (b_.)]^{(p_)}, x\_Symbol] :$

```
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rubi steps



$$\begin{aligned}
\int x^7 (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4} x^7 (2a - b \log(1 - cx^2))^2 - \frac{1}{2} b x^7 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4} b^2 x^7 \log^2(1 + cx^2) \right) dx \\
&= \frac{1}{4} \int x^7 (2a - b \log(1 - cx^2))^2 dx - \frac{1}{2} b \int x^7 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} \int b^2 x^7 \log^2(1 + cx^2) dx \\
&= \frac{1}{8} \text{Subst} \left( \int x^3 (2a - b \log(1 - cx))^2 dx, x, x^2 \right) - \frac{1}{4} b \text{Subst} \left( \int x^3 (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^2 \right) + \frac{1}{4} \int b^2 x^7 \log^2(1 + cx^2) dx \\
&= \frac{1}{32} x^8 (2a - b \log(1 - cx^2))^2 + \frac{1}{16} b x^8 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{32} b^2 x^8 \log^2(1 + cx^2) \\
&= \frac{1}{32} x^8 (2a - b \log(1 - cx^2))^2 + \frac{1}{16} b x^8 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{32} b^2 x^8 \log^2(1 + cx^2) \\
&= \frac{1}{32} x^8 (2a - b \log(1 - cx^2))^2 - \frac{1}{192} b (2a - b \log(1 - cx^2)) \left( \frac{48(1 - cx^2)}{c^4} - \frac{36(1 - cx^2)}{c^4} \right) \\
&= \frac{abx^2}{8c^3} - \frac{bx^4(2a - b \log(1 - cx^2))}{32c^2} + \frac{bx^6(2a - b \log(1 - cx^2))}{48c} - \frac{1}{64} bx^8 (2a - b \log(1 - cx^2)) \\
&= \frac{abx^2}{8c^3} - \frac{bx^4(2a - b \log(1 - cx^2))}{32c^2} + \frac{bx^6(2a - b \log(1 - cx^2))}{48c} - \frac{1}{64} bx^8 (2a - b \log(1 - cx^2)) \\
&= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2(1 - cx^2)^2}{32c^4} - \frac{b^2(1 - cx^2)^3}{36c^4} + \frac{b^2(1 - cx^2)^4}{256c^4} \\
&= \frac{abx^2}{8c^3} + \frac{55b^2x^2}{192c^3} - \frac{5b^2x^4}{384c^2} + \frac{b^2x^6}{576c} - \frac{b^2x^8}{256} + \frac{3b^2(1 - cx^2)^2}{32c^4} - \frac{b^2(1 - cx^2)^3}{36c^4} + \frac{b^2(1 - cx^2)^4}{256c^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0734534, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^8 + 2abc^3x^6 + 2bcx^2 \tanh^{-1}(cx^2) (3ac^3x^6 + b(c^2x^4 + 3)) + 6abcx^2 + b(3a + 4b) \log(1 - cx^2) - 3ab \log(cx^2 + 1)}{24c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (6\*a\*b\*c\*x^2 + b^2\*c^2\*x^4 + 2\*a\*b\*c^3\*x^6 + 3\*a^2\*c^4\*x^8 + 2\*b\*c\*x^2\*(3\*a\*c^3\*x^6 + b\*(3 + c^2\*x^4))\*ArcTanh[c\*x^2] + 3\*b^2\*(-1 + c^4\*x^8)\*ArcTanh[c

$$\frac{x^2 + b(3a + 4b)\text{Log}[1 - cx^2] - 3ab\text{Log}[1 + cx^2] + 4b^2\text{Log}[1 + cx^2]}{(24c^4)}$$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x^7 (a + b \operatorname{Artanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] int(x^7\*(a+b\*arctanh(c\*x^2))^2,x)

**Maxima [A]** time = 1.00696, size = 293, normalized size = 2.34

$$\frac{1}{8} b^2 x^8 \operatorname{artanh}(cx^2)^2 + \frac{1}{8} a^2 x^8 + \frac{1}{24} \left( 6 x^8 \operatorname{artanh}(cx^2) + c \left( \frac{2(c^2 x^6 + 3x^2)}{c^4} - \frac{3 \log(cx^2 + 1)}{c^5} + \frac{3 \log(cx^2 - 1)}{c^5} \right) \right) ab + \frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] 1/8\*b^2\*x^8\*arctanh(c\*x^2)^2 + 1/8\*a^2\*x^8 + 1/24\*(6\*x^8\*arctanh(c\*x^2) + c\*(2\*(c^2\*x^6 + 3\*x^2)/c^4 - 3\*log(c\*x^2 + 1)/c^5 + 3\*log(c\*x^2 - 1)/c^5))\*a\*b + 1/96\*(4\*c\*(2\*(c^2\*x^6 + 3\*x^2)/c^4 - 3\*log(c\*x^2 + 1)/c^5 + 3\*log(c\*x^2 - 1)/c^5)\*arctanh(c\*x^2) + (4\*c^2\*x^4 - 2\*(3\*log(c\*x^2 - 1) - 8)\*log(c\*x^2 + 1) + 3\*log(c\*x^2 + 1)^2 + 3\*log(c\*x^2 - 1)^2 + 16\*log(c\*x^2 - 1))/c^4)\*b^2

**Fricas [A]** time = 2.0428, size = 375, normalized size = 3.

$$\frac{12 a^2 c^4 x^8 + 8 a b c^3 x^6 + 4 b^2 c^2 x^4 + 24 a b c x^2 + 3 (b^2 c^4 x^8 - b^2) \log\left(-\frac{c x^2 + 1}{c x^2 - 1}\right)^2 - 4 (3 a b - 4 b^2) \log(cx^2 + 1) + 4 (3 a b + 4 b^2) \log(cx^2 - 1)}{96 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out]  $\frac{1}{96}*(12*a^2*c^4*x^8 + 8*a*b*c^3*x^6 + 4*b^2*c^2*x^4 + 24*a*b*c*x^2 + 3*(b^2*c^4*x^8 - b^2)*\log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(3*a*b - 4*b^2)*\log(c*x^2 + 1) + 4*(3*a*b + 4*b^2)*\log(c*x^2 - 1) + 4*(3*a*b*c^4*x^8 + b^2*c^3*x^6 + 3*b^2*c*x^2)*\log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4$

**Sympy [A]** time = 39.731, size = 206, normalized size = 1.65

$$\left\{ \begin{array}{l} \frac{a^2x^8}{8} + \frac{abx^8 \operatorname{atanh}(cx^2)}{4} + \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab \operatorname{atanh}(cx^2)}{4c^4} + \frac{b^2x^8 \operatorname{atanh}^2(cx^2)}{8} + \frac{b^2x^6 \operatorname{atanh}(cx^2)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2 \operatorname{atanh}(cx^2)}{4c^3} + \frac{b^2 \log\left(x - i\sqrt{\frac{1}{c}}\right)}{3c^4} \\ \frac{a^2x^8}{8} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(a+b\*atanh(c\*x\*\*2))\*\*2,x)

[Out] Piecewise((a\*\*2\*x\*\*8/8 + a\*b\*x\*\*8\*atanh(c\*x\*\*2)/4 + a\*b\*x\*\*6/(12\*c) + a\*b\*x\*\*2/(4\*c\*\*3) - a\*b\*atanh(c\*x\*\*2)/(4\*c\*\*4) + b\*\*2\*x\*\*8\*atanh(c\*x\*\*2)\*\*2/8 + b\*\*2\*x\*\*6\*atanh(c\*x\*\*2)/(12\*c) + b\*\*2\*x\*\*4/(24\*c\*\*2) + b\*\*2\*x\*\*2\*atanh(c\*x\*\*2)/(4\*c\*\*3) + b\*\*2\*log(x - I\*sqrt(1/c))/(3\*c\*\*4) + b\*\*2\*log(x + I\*sqrt(1/c))/(3\*c\*\*4) - b\*\*2\*atanh(c\*x\*\*2)\*\*2/(8\*c\*\*4) - b\*\*2\*atanh(c\*x\*\*2)/(3\*c\*\*4), Ne(c, 0)), (a\*\*2\*x\*\*8/8, True))

**Giac [A]** time = 1.50282, size = 236, normalized size = 1.89

$$\frac{1}{8} a^2 x^8 + \frac{abx^6}{12c} + \frac{b^2x^4}{24c^2} + \frac{1}{32} \left( b^2x^8 - \frac{b^2}{c^4} \right) \log\left( \frac{-cx^2 + 1}{cx^2 - 1} \right)^2 + \frac{1}{24} \left( 3abx^8 + \frac{b^2x^6}{c} + \frac{3b^2x^2}{c^3} \right) \log\left( \frac{-cx^2 + 1}{cx^2 - 1} \right) + \frac{abx^2}{4c^3} - \frac{(3a^2 - b^2) \log\left( x - i\sqrt{\frac{1}{c}} \right)}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="giac")

[Out]  $\frac{1}{8}a^2x^8 + \frac{1}{12}a*b*x^6/c + \frac{1}{24}b^2*x^4/c^2 + \frac{1}{32}*(b^2*x^8 - b^2/c^4)*\log(-(c*x^2 + 1)/(c*x^2 - 1))^2 + \frac{1}{24}*(3*a*b*x^8 + b^2*x^6/c + 3*b^2*x^2/c^3)*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + \frac{1}{4}a*b*x^2/c^3 - \frac{1}{24}*(3*a*b - 4*b^2)*\log(c*x^2 + 1)/c^4 + \frac{1}{24}*(3*a*b + 4*b^2)*\log(c*x^2 - 1)/c^4$

### 3.65 $\int x^5 \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$

**Optimal.** Leaf size=146

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^2}\right)}{6c^3} + \frac{\left(a + b \tanh^{-1}(cx^2)\right)^2}{6c^3} - \frac{b \log\left(\frac{2}{1-cx^2}\right) \left(a + b \tanh^{-1}(cx^2)\right)}{3c^3} + \frac{1}{6} x^6 \left(a + b \tanh^{-1}(cx^2)\right)^2 + \dots$$

[Out]  $(b^2 x^2)/(6c^2) - (b^2 \text{ArcTanh}[cx^2])/(6c^3) + (b x^4 (a + b \text{ArcTanh}[cx^2]))/(6c) + (a + b \text{ArcTanh}[cx^2])^2/(6c^3) + (x^6 (a + b \text{ArcTanh}[cx^2])^2)/6 - (b (a + b \text{ArcTanh}[cx^2]) \text{Log}[2/(1 - cx^2)])/(3c^3) - (b^2 \text{PolyLog}[2, 1 - 2/(1 - cx^2)])/(6c^3)$

**Rubi [B]** time = 1.29604, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.187$ , Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right)}{12c^3} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right)}{12c^3} - \frac{abx^2}{6c^2} - \frac{1}{72} b \left( \frac{2(1 - cx^2)^3}{c^3} - \frac{9(1 - cx^2)^2}{c^3} + \frac{18(1 - cx^2)}{c^3} - \dots \right)$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[x^5 (a + b \text{ArcTanh}[cx^2])^2, x]$

[Out]  $-(a b x^2)/(6c^2) + (19 b^2 x^2)/(72c^2) - (5 b^2 x^4)/(144c) - (b^2 x^6)/108 + (b^2 (1 - cx^2)^2)/(16c^3) - (b^2 (1 - cx^2)^3)/(108c^3) + (b^2 \text{Log}[1 - cx^2])/(72c^3) - (b^2 (1 - cx^2) \text{Log}[1 - cx^2])/(12c^3) + (b^2 \text{Log}[1 - cx^2]^2)/(24c^3) + (b x^4 (2a - b \text{Log}[1 - cx^2]))/(24c) - (b x^6 (2a - b \text{Log}[1 - cx^2]))/36 + (x^6 (2a - b \text{Log}[1 - cx^2])^2)/24 - (b (2a - b \text{Log}[1 - cx^2]) ((18(1 - cx^2))/c^3 - (9(1 - cx^2)^2)/c^3 + (2(1 - cx^2)^3)/c^3 - (6 \text{Log}[1 - cx^2])/c^3))/72 + (b (2a - b \text{Log}[1 - cx^2]) \text{Log}[(1 + cx^2)/2])/(12c^3) - (b^2 \text{Log}[1 + cx^2])/(12c^3) + (b^2 x^4 \text{Log}[1 + cx^2])/(12c) + (b^2 \text{Log}[(1 - cx^2)/2] \text{Log}[1 + cx^2])/(12c^3) + (b x^6 (2a - b \text{Log}[1 - cx^2]) \text{Log}[1 + cx^2])/12 + (b^2 \text{Log}[1 + cx^2]^2)/(24c^3) + (b^2 x^6 \text{Log}[1 + cx^2]^2)/24 - (b^2 \text{PolyLog}[2, (1 - cx^2)/2])/(12c^3) + (b^2 \text{PolyLog}[2, (1 + cx^2)/2])/(12c^3)$

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

#### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol]
:> Dist[1/e, Subst[Int[((g*x)/e)^q*(e*h - d*i)/e + (i*x)/e]^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*(x\_)^(r\_.), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m])/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m])/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :

```
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :=> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol] :=> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p)/((f_) + (g_.)*(x_))^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4} x^5 (2a - b \log(1 - cx^2))^2 - \frac{1}{2} b x^5 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4} b^2 x^5 \log^2(1 - cx^2) \right) dx \\
&= \frac{1}{4} \int x^5 (2a - b \log(1 - cx^2))^2 dx - \frac{1}{2} b \int x^5 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} \int b^2 x^5 \log^2(1 - cx^2) dx \\
&= \frac{1}{8} \text{Subst} \left( \int x^2 (2a - b \log(1 - cx))^2 dx, x, x^2 \right) - \frac{1}{4} b \text{Subst} \left( \int x^2 (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^2 \right) + \frac{1}{4} \int b^2 x^5 \log^2(1 - cx^2) dx \\
&= \frac{1}{24} x^6 (2a - b \log(1 - cx^2))^2 + \frac{1}{12} b x^6 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{24} b^2 x^6 \log^2(1 - cx^2) \\
&= \frac{1}{24} x^6 (2a - b \log(1 - cx^2))^2 + \frac{1}{12} b x^6 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{24} b^2 x^6 \log^2(1 - cx^2) \\
&= \frac{1}{24} x^6 (2a - b \log(1 - cx^2))^2 - \frac{1}{72} b (2a - b \log(1 - cx^2)) \left( \frac{18(1 - cx^2)}{c^3} - \frac{9(1 - cx^2)^2}{c^3} \right) + \frac{1}{24} b^2 x^6 \log^2(1 - cx^2) \\
&= -\frac{abx^2}{6c^2} + \frac{bx^4(2a - b \log(1 - cx^2))}{24c} - \frac{1}{36} b x^6 (2a - b \log(1 - cx^2)) + \frac{1}{24} x^6 (2a - b \log(1 - cx^2))^2 \\
&= -\frac{abx^2}{6c^2} + \frac{bx^4(2a - b \log(1 - cx^2))}{24c} - \frac{1}{36} b x^6 (2a - b \log(1 - cx^2)) + \frac{1}{24} x^6 (2a - b \log(1 - cx^2))^2 \\
&= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2(1 - cx^2)^2}{16c^3} - \frac{b^2(1 - cx^2)^3}{108c^3} + \frac{b^2 \log(1 - cx^2)}{72c^3} - \frac{b^2 \log^2(1 - cx^2)}{72c^3} \\
&= -\frac{abx^2}{6c^2} + \frac{13b^2x^2}{72c^2} + \frac{b^2x^4}{144c} - \frac{b^2x^6}{108} + \frac{b^2(1 - cx^2)^2}{16c^3} - \frac{b^2(1 - cx^2)^3}{108c^3} + \frac{b^2 \log(1 - cx^2)}{72c^3} - \frac{b^2 \log^2(1 - cx^2)}{72c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.277859, size = 132, normalized size = 0.9

$$\frac{b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^2)}\right) + a^2 c^3 x^6 + abc^2 x^4 + ab \log(c^2 x^4 - 1) + b \tanh^{-1}(cx^2) \left(2ac^3 x^6 + bc^2 x^4 - 2b \log\left(e^{-2 \tanh^{-1}(cx^2)}\right)\right)}{6c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (b^2\*c\*x^2 + a\*b\*c^2\*x^4 + a^2\*c^3\*x^6 + b^2\*(-1 + c^3\*x^6)\*ArcTanh[c\*x^2]^2 + b\*ArcTanh[c\*x^2]\*(-b + b\*c^2\*x^4 + 2\*a\*c^3\*x^6 - 2\*b\*Log[1 + E^(-2\*ArcT



$\text{anh}[c*x^2])) + a*b*\text{Log}[-1 + c^2*x^4] + b^2*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x^2])}]]/(6*c^3)$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x^5 (a + b \text{Artanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arctanh(c*x^2))^2,x)`

[Out] `int(x^5*(a+b*arctanh(c*x^2))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} a^2 x^6 + \frac{1}{6} \left( 2x^6 \text{artanh}(cx^2) + \left( \frac{x^4}{c^2} + \frac{\log(c^2 x^4 - 1)}{c^4} \right) c \right) ab + \frac{1}{432} \left( 18x^6 \log(-cx^2 + 1)^2 - 2c^4 \left( \frac{2(c^2 x^6 + 3x^2)}{c^6} - \frac{3 \log}{c^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")`

[Out] `1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*c)*a*b + 1/432*(18*x^6*log(-c*x^2 + 1)^2 - 2*c^4*(2*(c^2*x^6 + 3*x^2)/c^6 - 3*log(c*x^2 + 1)/c^7 + 3*log(c*x^2 - 1)/c^7) + 3*c^3*(x^4/c^4 + log(c^2*x^4 - 1)/c^6) + 1296*c^3*integrate(1/9*x^7*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) - 9*c^2*(2*x^2/c^4 - log(c*x^2 + 1)/c^5 + log(c*x^2 - 1)/c^5) - 6*c*((2*c^2*x^6 + 3*c*x^4 + 6*x^2)/c^3 + 6*log(c*x^2 - 1)/c^4)*log(-c*x^2 + 1) + 648*c*integrate(1/9*x^3*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) + 6*(3*c^3*x^6*log(c*x^2 + 1)^2 + (2*c^3*x^6 - 3*c^2*x^4 + 6*c*x^2 - 6*(c^3*x^6 + 1))*log(c*x^2 + 1))*log(-c*x^2 + 1)/c^3 + (4*c^3*x^6 + 15*c^2*x^4 + 66*c*x^2 + 18*log(c*x^2 - 1)^2 + 66*log(c*x^2 - 1))/c^3 - 18*log(9*c^4*x^4 - 9*c^2)/c^3 + 648*integrate(1/9*x*log(c*x^2 + 1)/(c^4*x^4 - c^2), x))*b^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^2 x^5 \text{artanh}(cx^2)^2 + 2 ab x^5 \text{artanh}(cx^2) + a^2 x^5, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^5\*arctanh(c\*x^2)^2 + 2\*a\*b\*x^5\*arctanh(c\*x^2) + a^2\*x^5, x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{atanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*atanh(c\*x\*\*2))\*\*2,x)

[Out] Integral(x\*\*5\*(a + b\*atanh(c\*x\*\*2))\*\*2, x)

**Giac** [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2\*x^5, x)

### 3.66 $\int x^3 \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$

**Optimal.** Leaf size=91

$$-\frac{(a + b \tanh^{-1}(cx^2))^2}{4c^2} + \frac{abx^2}{2c} + \frac{1}{4}x^4(a + b \tanh^{-1}(cx^2))^2 + \frac{b^2 \log(1 - c^2x^4)}{4c^2} + \frac{b^2x^2 \tanh^{-1}(cx^2)}{2c}$$

[Out] (a\*b\*x^2)/(2\*c) + (b^2\*x^2\*ArcTanh[c\*x^2])/(2\*c) - (a + b\*ArcTanh[c\*x^2])^2/(4\*c^2) + (x^4\*(a + b\*ArcTanh[c\*x^2])^2)/4 + (b^2\*Log[1 - c^2\*x^4])/(4\*c^2)

**Rubi [C]** time = 0.96549, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right)}{8c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right)}{8c^2} + \frac{(1 - cx^2)^2 (2a - b \log(1 - cx^2))^2}{16c^2} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))}{8c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (3\*a\*b\*x^2)/(4\*c) - (b^2\*x^4)/16 + (b^2\*(1 - c\*x^2)^2)/(32\*c^2) + (b^2\*(1 + c\*x^2)^2)/(32\*c^2) - (b^2\*Log[1 - c\*x^2])/(16\*c^2) + (3\*b^2\*(1 - c\*x^2)\*Log[1 - c\*x^2])/(8\*c^2) - (b\*x^4\*(2\*a - b\*Log[1 - c\*x^2]))/16 + (b\*(1 - c\*x^2)^2\*(2\*a - b\*Log[1 - c\*x^2]))/(16\*c^2) - ((1 - c\*x^2)\*(2\*a - b\*Log[1 - c\*x^2])^2)/(8\*c^2) + ((1 - c\*x^2)^2\*(2\*a - b\*Log[1 - c\*x^2])^2)/(16\*c^2) - (b\*(2\*a - b\*Log[1 - c\*x^2])\*Log[(1 + c\*x^2)/2])/(8\*c^2) - (b^2\*Log[1 + c\*x^2])/(16\*c^2) + (b^2\*x^4\*Log[1 + c\*x^2])/16 + (3\*b^2\*(1 + c\*x^2)\*Log[1 + c\*x^2])/(8\*c^2) - (b^2\*(1 + c\*x^2)^2\*Log[1 + c\*x^2])/(16\*c^2) + (b^2\*Log[(1 - c\*x^2)/2]\*Log[1 + c\*x^2])/(8\*c^2) + (b\*x^4\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2])/8 - (b^2\*(1 + c\*x^2)\*Log[1 + c\*x^2]^2)/(8\*c^2) + (b^2\*(1 + c\*x^2)^2\*Log[1 + c\*x^2]^2)/(16\*c^2) + (b^2\*PolyLog[2, (1 - c\*x^2)/2])/(8\*c^2) + (b^2\*PolyLog[2, (1 + c\*x^2)/2])/(8\*c^2)

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte

gerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)]^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]^(p\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n

\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

#### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((x\_)^(r\_.)), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4} x^3 (2a - b \log(1 - cx^2))^2 - \frac{1}{2} bx^3 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4} b^2 x^3 \log^2(1 - cx^2) \right) dx \\
 &= \frac{1}{4} \int x^3 (2a - b \log(1 - cx^2))^2 dx - \frac{1}{2} b \int x^3 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} b^2 \int x^3 \log^2(1 - cx^2) dx \\
 &= \frac{1}{8} \text{Subst} \left( \int x (2a - b \log(1 - cx))^2 dx, x, x^2 \right) - \frac{1}{4} b \text{Subst} \left( \int x (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^2 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \log^2(1 - cx) dx, x, x^2 \right) \\
 &= \frac{1}{8} bx^4 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{8} \text{Subst} \left( \int \left( \frac{(2a - b \log(1 - cx))^2}{c} - \frac{(1 - cx) \log^2(1 - cx)}{c} \right) dx, x, x^2 \right) \\
 &= \frac{1}{8} bx^4 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{\text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, x^2 \right)}{8c} - \frac{\text{Subst} \left( \int (1 - cx) \log^2(1 - cx) dx, x, x^2 \right)}{8c} \\
 &= \frac{1}{8} bx^4 (2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{8} b \text{Subst} \left( \int x (-2a + b \log(1 - cx)) dx, x, x^2 \right) + \frac{1}{8} b^2 \text{Subst} \left( \int \log^2(1 - cx) dx, x, x^2 \right) \\
 &= \frac{abx^2}{4c} - \frac{1}{16} bx^4 (2a - b \log(1 - cx^2)) - \frac{(1 - cx^2) (2a - b \log(1 - cx^2))^2}{8c^2} + \frac{(1 - cx^2)^2 (2a - b \log(1 - cx^2))}{8c^2} \\
 &= \frac{3abx^2}{4c} - \frac{3b^2x^2}{8c} + \frac{b^2(1 - cx^2)^2}{32c^2} + \frac{b^2(1 + cx^2)^2}{32c^2} - \frac{1}{16} bx^4 (2a - b \log(1 - cx^2)) + \frac{b(1 - cx^2) \log^2(1 - cx^2)}{8c^2} \\
 &= \frac{3abx^2}{4c} - \frac{b^2x^4}{16} + \frac{b^2(1 - cx^2)^2}{32c^2} + \frac{b^2(1 + cx^2)^2}{32c^2} - \frac{b^2 \log(1 - cx^2)}{16c^2} + \frac{3b^2(1 - cx^2) \log(1 - cx^2)}{8c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0603613, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^4 + 2abcx^2 + b(a+b)\log(1-cx^2) - ab\log(cx^2+1) + 2bcx^2 \tanh^{-1}(cx^2)(acx^2+b) + b^2(c^2x^4-1)\tanh^{-1}(cx^2)}{4c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (2\*a\*b\*c\*x^2 + a^2\*c^2\*x^4 + 2\*b\*c\*x^2\*(b + a\*c\*x^2)\*ArcTanh[c\*x^2] + b^2\*(-1 + c^2\*x^4)\*ArcTanh[c\*x^2]^2 + b\*(a + b)\*Log[1 - c\*x^2] - a\*b\*Log[1 + c\*x^2] + b^2\*Log[1 + c\*x^2])/(4\*c^2)

**Maple [B]** time = 0.163, size = 247, normalized size = 2.7

$$\frac{b^2(c^2x^4-1)(\ln(cx^2+1))^2}{16c^2} + \frac{b(-x^4b\ln(-cx^2+1)c^2 + 2ac^2x^4 + 2bcx^2 + b\ln(-cx^2+1))\ln(cx^2+1)}{8c^2} + \frac{b^2x^4(\ln(-cx^2+1))^2}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] 1/16\*b^2\*(c^2\*x^4-1)/c^2\*ln(c\*x^2+1)^2+1/8\*b\*(-x^4\*b\*ln(-c\*x^2+1)\*c^2+2\*a\*c^2\*x^4+2\*b\*c\*x^2+b\*ln(-c\*x^2+1))/c^2\*ln(c\*x^2+1)+1/16\*b^2\*x^4\*ln(-c\*x^2+1)^2-1/4\*a\*b\*x^4\*ln(-c\*x^2+1)+1/4\*a^2\*x^4-1/4/c\*b^2\*x^2\*ln(-c\*x^2+1)+1/2\*a\*b\*x^2/c-1/16/c^2\*b^2\*ln(-c\*x^2+1)^2+1/4/c^2\*b\*ln(-c\*x^2+1)\*a+1/4/c^2\*b^2\*ln(-c\*x^2+1)-1/4/c^2\*b\*ln(-c\*x^2-1)\*a+1/4/c^2\*b^2\*ln(-c\*x^2-1)

**Maxima [B]** time = 0.976063, size = 251, normalized size = 2.76

$$\frac{1}{4}b^2x^4 \operatorname{artanh}(cx^2)^2 + \frac{1}{4}a^2x^4 + \frac{1}{4}\left(2x^4 \operatorname{artanh}(cx^2) + c\left(\frac{2x^2}{c^2} - \frac{\log(cx^2+1)}{c^3} + \frac{\log(cx^2-1)}{c^3}\right)\right)ab + \frac{1}{16}\left(4c\left(\frac{2x^2}{c^2} - \frac{\log(cx^2+1)}{c^3} + \frac{\log(cx^2-1)}{c^3}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}b^2x^4\operatorname{arctanh}(cx^2)^2 + \frac{1}{4}a^2x^4 + \frac{1}{4}(2x^4\operatorname{arctanh}(cx^2) + c(2x^2/c^2 - \log(cx^2 + 1)/c^3 + \log(cx^2 - 1)/c^3))ab + \frac{1}{16}(4c(2x^2/c^2 - \log(cx^2 + 1)/c^3 + \log(cx^2 - 1)/c^3)\operatorname{arctanh}(cx^2) - (2(\log(cx^2 - 1) - 2)\log(cx^2 + 1) - \log(cx^2 + 1)^2 - \log(cx^2 - 1)^2 - 4\log(cx^2 - 1))/c^2)b^2$

**Fricas [A]** time = 2.11289, size = 292, normalized size = 3.21

$$\frac{4a^2c^2x^4 + 8abcx^2 + (b^2c^2x^4 - b^2)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 - 4(ab - b^2)\log(cx^2 + 1) + 4(ab + b^2)\log(cx^2 - 1) + 4(abc^2x^4 + b^2cx^2)}{16c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{16}(4a^2c^2x^4 + 8a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*\log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(a*b - b^2)*\log(c*x^2 + 1) + 4*(a*b + b^2)*\log(c*x^2 - 1) + 4*(a*b*c^2*x^4 + b^2*c*x^2)*\log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2$

**Sympy [A]** time = 19.1199, size = 163, normalized size = 1.79

$$\left\{ \begin{array}{l} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atanh}(cx^2)}{2} + \frac{abx^2}{2c} - \frac{ab\operatorname{atanh}(cx^2)}{2c^2} + \frac{b^2x^4\operatorname{atanh}^2(cx^2)}{4} + \frac{b^2x^2\operatorname{atanh}(cx^2)}{2c} + \frac{b^2\log\left(x-i\sqrt{\frac{1}{c}}\right)}{2c^2} + \frac{b^2\log\left(x+i\sqrt{\frac{1}{c}}\right)}{2c^2} - \frac{b^2\operatorname{atanh}^2(cx^2)}{4c^2} \\ \frac{a^2x^4}{4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*atanh(c*x**2))**2,x)`

[Out] `Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x**2)/2 + a*b*x**2/(2*c) - a*b*atanh(c*x**2)/(2*c**2) + b**2*x**4*atanh(c*x**2)**2/4 + b**2*x**2*atanh(c*x**2)/(2*c) + b**2*log(x - I*sqrt(1/c))/(2*c**2) + b**2*log(x + I*sqrt(1/c))/(2*c**2) - b**2*atanh(c*x**2)**2/(4*c**2) - b**2*atanh(c*x**2)/(2*c**2), Ne(c, 0)), (a**2*x**4/4, True))`



**Giac [A]** time = 1.38177, size = 186, normalized size = 2.04

$$\frac{1}{4} a^2 x^4 + \frac{abx^2}{2c} + \frac{1}{16} \left( b^2 x^4 - \frac{b^2}{c^2} \right) \log \left( -\frac{cx^2 + 1}{cx^2 - 1} \right)^2 + \frac{1}{4} \left( abx^4 + \frac{b^2 x^2}{c} \right) \log \left( -\frac{cx^2 + 1}{cx^2 - 1} \right) - \frac{(ab - b^2) \log(cx^2 + 1)}{4c^2} + \frac{(ab + b^2) \log(cx^2 - 1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="giac")

[Out] 1/4\*a^2\*x^4 + 1/2\*a\*b\*x^2/c + 1/16\*(b^2\*x^4 - b^2/c^2)\*log(-(c\*x^2 + 1)/(c\*x^2 - 1))^2 + 1/4\*(a\*b\*x^4 + b^2\*x^2/c)\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) - 1/4\*(a\*b - b^2)\*log(c\*x^2 + 1)/c^2 + 1/4\*(a\*b + b^2)\*log(c\*x^2 - 1)/c^2

### 3.67 $\int x \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$

**Optimal.** Leaf size=94

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^2}\right)}{2c} + \frac{1}{2}x^2 \left( a + b \tanh^{-1}(cx^2) \right)^2 + \frac{\left( a + b \tanh^{-1}(cx^2) \right)^2}{2c} - \frac{b \log\left(\frac{2}{1-cx^2}\right) \left( a + b \tanh^{-1}(cx^2) \right)}{c}$$

[Out] (a + b\*ArcTanh[c\*x^2])^2/(2\*c) + (x^2\*(a + b\*ArcTanh[c\*x^2])^2)/2 - (b\*(a + b\*ArcTanh[c\*x^2])\*Log[2/(1 - c\*x^2)]/c - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x^2)]))/(2\*c)

**Rubi [B]** time = 0.513899, antiderivative size = 207, normalized size of antiderivative = 2.2, number of steps used = 28, number of rules used = 12, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right)}{4c} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right)}{4c} + \frac{b \log\left(\frac{1}{2}(cx^2 + 1)\right) \left( 2a - b \log(1 - cx^2) \right)}{4c} + \frac{1}{4}bx^2 \log(cx^2)$$

Warning: Unable to verify antiderivative.

[In] Int[x\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] -((1 - c\*x^2)\*(2\*a - b\*Log[1 - c\*x^2])^2)/(8\*c) + (b\*(2\*a - b\*Log[1 - c\*x^2])\*Log[(1 + c\*x^2)/2])/(4\*c) + (b^2\*Log[(1 - c\*x^2)/2]\*Log[1 + c\*x^2])/(4\*c) + (b\*x^2\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2])/4 + (b^2\*(1 + c\*x^2)\*Log[1 + c\*x^2]^2)/(8\*c) - (b^2\*PolyLog[2, (1 - c\*x^2)/2])/(4\*c) + (b^2\*PolyLog[2, (1 + c\*x^2)/2])/(4\*c)

#### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)])\*(b\_.)^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},

$x]$  && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.)), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m)))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int x(a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4}x(2a - b \log(1 - cx^2))^2 - \frac{1}{2}bx(-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4}b^2x \log^2 \right. \\
&= \frac{1}{4} \int x(2a - b \log(1 - cx^2))^2 dx - \frac{1}{2}b \int x(-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} \int b^2x \log^2 \\
&= \frac{1}{8} \text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, x^2 \right) - \frac{1}{4}b \text{Subst} \left( \int (-2a + b \log(1 - cx)) \log(1 - cx) dx, x, x^2 \right) \\
&= \frac{1}{4}bx^2(2a - b \log(1 - cx^2)) \log(1 + cx^2) - \frac{\text{Subst} \left( \int (2a - b \log(x))^2 dx, x, 1 - cx^2 \right)}{8c} + \frac{b^2 \log^2(1 - cx^2)}{4c} \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8c} + \frac{1}{4}bx^2(2a - b \log(1 - cx^2)) \log(1 + cx^2) + \frac{b^2(1 + cx^2) \log(1 + cx^2)}{4c} \\
&= \frac{1}{2}abx^2 + \frac{b^2x^2}{4} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8c} - \frac{b^2(1 + cx^2) \log(1 + cx^2)}{4c} + \frac{1}{4}bx^2 \log^2(1 + cx^2) \\
&= \frac{b^2x^2}{2} + \frac{b^2(1 - cx^2) \log(1 - cx^2)}{4c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8c} + \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{4c} \\
&= \frac{b^2x^2}{4} + \frac{b^2(1 - cx^2) \log(1 - cx^2)}{4c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8c} + \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{4c} \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8c} + \frac{b(2a - b \log(1 - cx^2)) \log\left(\frac{1}{2}(1 + cx^2)\right)}{4c} + \frac{b^2 \log^2(1 + cx^2)}{4c}
\end{aligned}$$

**Mathematica [A]** time = 0.0656213, size = 99, normalized size = 1.05

$$\frac{b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^2)}\right) + a(acx^2 + b \log(1 - c^2x^4)) + 2b \tanh^{-1}(cx^2)(acx^2 - b \log(e^{-2 \tanh^{-1}(cx^2)} + 1)) + b^2 \log^2(1 + cx^2)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (b^2\*(-1 + c\*x^2)\*ArcTanh[c\*x^2]^2 + 2\*b\*ArcTanh[c\*x^2]\*(a\*c\*x^2 - b\*Log[1 + E^(-2\*ArcTanh[c\*x^2])]) + a\*(a\*c\*x^2 + b\*Log[1 - c^2\*x^4]) + b^2\*PolyLog[2, -E^(-2\*ArcTanh[c\*x^2])])/(2\*c)

**Maple [A]** time = 0.035, size = 144, normalized size = 1.5

$$\frac{(\operatorname{Artanh}(cx^2))^2 x^2 b^2}{2} + abx^2 \operatorname{Artanh}(cx^2) + \frac{a^2 x^2}{2} + \frac{b^2 (\operatorname{Artanh}(cx^2))^2}{2c} - \frac{\operatorname{Artanh}(cx^2) b^2}{c} \ln \left( \frac{(cx^2 + 1)^2}{-c^2 x^4 + 1} + 1 \right) - \frac{b^2}{2c} \operatorname{polylog}(2, -\frac{(cx^2 + 1)^2}{-c^2 x^4 + 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^2))^2,x)`

[Out] `1/2*arctanh(c*x^2)^2*x^2*b^2+a*b*x^2*arctanh(c*x^2)+1/2*a^2*x^2+1/2/c*b^2*arctanh(c*x^2)^2-1/c*arctanh(c*x^2)*ln((c*x^2+1)^2/(-c^2*x^4+1)+1)*b^2-1/2/c*polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1))*b^2+1/2/c*a*b*ln(-c^2*x^4+1)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 x^2 + \frac{1}{8} \left( x^2 \log(-cx^2 + 1)^2 - c^2 \left( \frac{2x^2}{c^2} - \frac{\log(cx^2 + 1)}{c^3} + \frac{\log(cx^2 - 1)}{c^3} \right) - 2c \left( \frac{x^2}{c} + \frac{\log(cx^2 - 1)}{c^2} \right) \log(-cx^2 + 1) + 12 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")`

[Out] `1/2*a^2*x^2 + 1/8*(x^2*log(-c*x^2 + 1)^2 - c^2*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3) - 2*c*(x^2/c + log(c*x^2 - 1)/c^2)*log(-c*x^2 + 1) + 12*c*integrate(x^3*log(c*x^2 + 1)/(c^2*x^4 - 1), x) + (c*x^2*log(c*x^2 + 1)^2 + 2*(c*x^2 - (c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c + (2*c*x^2 + log(c*x^2 - 1)^2 + 2*log(c*x^2 - 1))/c - log(c^2*x^4 - 1)/c + 4*integrate(x*log(c*x^2 + 1)/(c^2*x^4 - 1), x))*b^2 + 1/2*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*a*b/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( b^2 x \operatorname{artanh}(cx^2)^2 + 2 abx \operatorname{artanh}(cx^2) + a^2 x, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x*arctanh(c*x^2)^2 + 2*a*b*x*arctanh(c*x^2) + a^2*x, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{atanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c*x**2))**2,x)`

[Out] `Integral(x*(a + b*atanh(c*x**2))**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^2*x, x)`

$$3.68 \quad \int \frac{(a + b \tanh^{-1}(cx^2))^2}{x} dx$$

**Optimal.** Leaf size=137

$$-\frac{1}{2}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^2}\right)(a + b \tanh^{-1}(cx^2)) + \frac{1}{2}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^2} - 1\right)(a + b \tanh^{-1}(cx^2)) + \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - cx^2}\right) - \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1 - cx^2}\right)$$

[Out] (a + b\*ArcTanh[c\*x^2])^2\*ArcTanh[1 - 2/(1 - c\*x^2)] - (b\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[2, 1 - 2/(1 - c\*x^2)]/2 + (b\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[2, -1 + 2/(1 - c\*x^2)]/2 + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^2)]/4 - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^2)]/4

**Rubi [A]** time = 0.335539, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{2}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^2}\right)(a + b \tanh^{-1}(cx^2)) + \frac{1}{2}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^2} - 1\right)(a + b \tanh^{-1}(cx^2)) + \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - cx^2}\right) - \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1 - cx^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x, x]

[Out] (a + b\*ArcTanh[c\*x^2])^2\*ArcTanh[1 - 2/(1 - c\*x^2)] - (b\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[2, 1 - 2/(1 - c\*x^2)]/2 + (b\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[2, -1 + 2/(1 - c\*x^2)]/2 + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^2)]/4 - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^2)]/4

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5914

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcTanh[c\*x])^p\*ArcTanh[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[(a + b\*ArcTanh[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 - c\*x)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]



Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^2))^2}{x} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx, x, x^2 \right) \\
&= (a + b \tanh^{-1}(cx^2))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - (2bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \tanh^{-1} \left( \frac{2}{1 - cx} \right)}{1 - c^2 x^2} dx, x, x^2 \right) \\
&= (a + b \tanh^{-1}(cx^2))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) + (bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \log \left( \frac{2}{1 - cx} \right)}{1 - c^2 x^2} dx, x, x^2 \right) \\
&= (a + b \tanh^{-1}(cx^2))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - \frac{1}{2} b (a + b \tanh^{-1}(cx^2)) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^2} \right) + \\
&= (a + b \tanh^{-1}(cx^2))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - \frac{1}{2} b (a + b \tanh^{-1}(cx^2)) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^2} \right) +
\end{aligned}$$

**Mathematica [A]** time = 0.100625, size = 141, normalized size = 1.03

$$\frac{1}{4} b \left( 2 \text{PolyLog} \left( 2, \frac{cx^2 + 1}{1 - cx^2} \right) (a + b \tanh^{-1}(cx^2)) - 2 \text{PolyLog} \left( 2, \frac{cx^2 + 1}{cx^2 - 1} \right) (a + b \tanh^{-1}(cx^2)) + b \left( \text{PolyLog} \left( 3, \frac{cx^2 + 1}{cx^2 - 1} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^2*ArcTanh[1 + 2/(-1 + c*x^2)] + (b*(2*(a + b*ArcTanh[c*x^2])*PolyLog[2, (1 + c*x^2)/(1 - c*x^2)] - 2*(a + b*ArcTanh[c*x^2])*PolyLog[2, (1 + c*x^2)/(-1 + c*x^2)] + b*(-PolyLog[3, (1 + c*x^2)/(1 - c*x^2)] + PolyLog[3, (1 + c*x^2)/(-1 + c*x^2)]))/4
```

**Maple [F]** time = 0.133, size = 0, normalized size = 0.

$$\int \frac{(a + b \text{Artanh}(cx^2))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^2/x,x)
```

[Out] `int((a+b*arctanh(c*x^2))^2/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2(\log(cx^2 + 1) - \log(-cx^2 + 1))^2}{4x} + \frac{ab(\log(cx^2 + 1) - \log(-cx^2 + 1))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="maxima")`

[Out] `a^2*log(x) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + a*b*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))**2/x,x)`

[Out] `Integral((a + b*atanh(c*x**2))**2/x, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x, x)
```

$$3.69 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^3} dx$$

**Optimal.** Leaf size=87

$$-\frac{1}{2}b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx^2+1}-1\right) + \frac{1}{2}c(a+b \tanh^{-1}(cx^2))^2 - \frac{(a+b \tanh^{-1}(cx^2))^2}{2x^2} + bc \log\left(2 - \frac{2}{cx^2+1}\right)(a+b \tanh^{-1}(cx^2))$$

[Out] (c\*(a + b\*ArcTanh[c\*x^2])^2)/2 - (a + b\*ArcTanh[c\*x^2])^2/(2\*x^2) + b\*c\*(a + b\*ArcTanh[c\*x^2])\*Log[2 - 2/(1 + c\*x^2)] - (b^2\*c\*PolyLog[2, -1 + 2/(1 + c\*x^2)])/2

**Rubi [B]** time = 0.63159, antiderivative size = 237, normalized size of antiderivative = 2.72, number of steps used = 24, number of rules used = 13, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$ , Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{2}b^2c \operatorname{PolyLog}(2, -cx^2) + \frac{1}{2}b^2c \operatorname{PolyLog}(2, cx^2) + \frac{1}{4}b^2c \operatorname{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right) - \frac{1}{4}b^2c \operatorname{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right) -$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x^3, x]

[Out] 2\*a\*b\*c\*Log[x] - ((1 - c\*x^2)\*(2\*a - b\*Log[1 - c\*x^2])^2)/(8\*x^2) - (b\*c\*(2\*a - b\*Log[1 - c\*x^2])\*Log[(1 + c\*x^2)/2])/4 - (b^2\*c\*Log[(1 - c\*x^2)/2]\*Log[1 + c\*x^2])/4 - (b\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2])/(4\*x^2) - (b^2\*(1 + c\*x^2)\*Log[1 + c\*x^2]^2)/(8\*x^2) - (b^2\*c\*PolyLog[2, -(c\*x^2)])/2 + (b^2\*c\*PolyLog[2, c\*x^2])/2 + (b^2\*c\*PolyLog[2, (1 - c\*x^2)/2])/4 - (b^2\*c\*PolyLog[2, (1 + c\*x^2)/2])/4

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Lo

```
g[c*(d + e*x)^p]^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^(q_), x_Symbol] := Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
- d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

### Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/(x_), x_Symbol] := Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p]/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^2))^2}{x^3} dx &= \int \left( \frac{(2a - b \log(1 - cx^2))^2}{4x^3} - \frac{b(-2a + b \log(1 - cx^2)) \log(1 + cx^2)}{2x^3} + \frac{b^2 \log^2(1 + cx^2)}{4x^3} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - cx^2))^2}{x^3} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - cx^2)) \log(1 + cx^2)}{x^3} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + cx^2)}{x^3} dx \\
&= \frac{1}{8} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^2} dx, x, x^2 \right) - \frac{1}{4} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log(1 + cx)}{x^2} dx, x, x^2 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x^2} dx, x, x^2 \right) \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{4x^2} - \frac{b^2(1 + cx^2) \log^2(1 + cx^2)}{8x^2} \\
&= abc \log(x) - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{4x^2} - \frac{b^2(1 + cx^2) \log^2(1 + cx^2)}{8x^2} \\
&= abc \log(x) - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{4x^2} - \frac{b^2(1 + cx^2) \log^2(1 + cx^2)}{8x^2} \\
&= 2abc \log(x) - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{1}{4} bc(2a - b \log(1 - cx^2)) \log\left(\frac{1}{2}(1 + cx^2)\right) - \frac{1}{8} b^2(1 + cx^2) \log^2\left(\frac{1}{2}(1 + cx^2)\right) \\
&= 2abc \log(x) - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{1}{4} bc(2a - b \log(1 - cx^2)) \log\left(\frac{1}{2}(1 + cx^2)\right) - \frac{1}{8} b^2(1 + cx^2) \log^2\left(\frac{1}{2}(1 + cx^2)\right) \\
&= 2abc \log(x) - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^2}{8x^2} - \frac{1}{4} bc(2a - b \log(1 - cx^2)) \log\left(\frac{1}{2}(1 + cx^2)\right) - \frac{1}{8} b^2(1 + cx^2) \log^2\left(\frac{1}{2}(1 + cx^2)\right)
\end{aligned}$$

**Mathematica [A]** time = 0.156982, size = 119, normalized size = 1.37

$$\frac{1}{2} b^2 c \left( \tanh^{-1}(cx^2) \left( -\frac{\tanh^{-1}(cx^2)}{cx^2} + \tanh^{-1}(cx^2) + 2 \log\left(1 - e^{-2 \tanh^{-1}(cx^2)}\right) \right) - \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx^2)}\right) \right) - \frac{a^2}{2x^2} + c$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^3, x]

[Out] -a^2/(2\*x^2) + a\*b\*c\*(-(ArcTanh[c\*x^2]/(c\*x^2)) + Log[c\*x^2] - Log[1 - c^2\*x^4/2] + (b^2\*c\*(ArcTanh[c\*x^2]\*(ArcTanh[c\*x^2] - ArcTanh[c\*x^2]/(c\*x^2) + 2\*Log[1 - E^(-2\*ArcTanh[c\*x^2])]) - PolyLog[2, E^(-2\*ArcTanh[c\*x^2])]))/2



**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^2))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))^2/x^3,x)`

[Out] `int((a+b*arctanh(c*x^2))^2/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( c(\log(c^2x^4 - 1) - \log(x^4)) + \frac{2 \operatorname{artanh}(cx^2)}{x^2} \right) ab - \frac{1}{8} b^2 \left( \frac{\log(-cx^2 + 1)^2}{x^2} + 2 \int -\frac{(cx^2 - 1) \log(cx^2 + 1)^2 + 2(cx^2 - 1) \log(cx^2 + 1)}{(cx^2 + 1)^3} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="maxima")`

[Out] `-1/2*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a*b - 1/8*b^2*(log(-c*x^2 + 1)^2/x^2 + 2*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(c*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3), x)) - 1/2*a^2/x^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/x\*\*3,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*2/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2/x^3, x)

$$3.70 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^5} dx$$

**Optimal.** Leaf size=88

$$\frac{1}{4}c^2(a+b \tanh^{-1}(cx^2))^2 - \frac{bc(a+b \tanh^{-1}(cx^2))}{2x^2} - \frac{(a+b \tanh^{-1}(cx^2))^2}{4x^4} - \frac{1}{4}b^2c^2 \log(1-c^2x^4) + b^2c^2 \log(x)$$

[Out]  $-(b*c*(a + b*ArcTanh[c*x^2]))/(2*x^2) + (c^2*(a + b*ArcTanh[c*x^2])^2)/4 - (a + b*ArcTanh[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^4])/4$

**Rubi [C]** time = 1.05672, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{8}b^2c^2 \text{PolyLog}\left(2, \frac{1}{2}(1-cx^2)\right) - \frac{1}{8}b^2c^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^2+1)\right) + \frac{1}{8}bc^2 \log\left(\frac{1}{2}(cx^2+1)\right)(2a - b \log(1-cx^2)) + \frac{1}{16}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x^5, x]

[Out]  $b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^2])/8 - (b*c*(2*a - b*Log[1 - c*x^2]))/(8*x^2) - (b*c*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2]))/(8*x^2) + (c^2*(2*a - b*Log[1 - c*x^2])^2)/16 - (2*a - b*Log[1 - c*x^2])^2/(16*x^4) + (b*c^2*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/8 - (b^2*c^2*Log[1 + c*x^2])/4 - (b^2*c*Log[1 + c*x^2])/(4*x^2) - (b^2*c^2*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/8 - (b*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(8*x^4) + (b^2*c^2*Log[1 + c*x^2]^2)/16 - (b^2*Log[1 + c*x^2]^2)/(16*x^4) - (b^2*c^2*PolyLog[2, (1 - c*x^2)/2])/8 - (b^2*c^2*PolyLog[2, (1 + c*x^2)/2])/8$

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.)), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_)]^(p_.))*((d_) + (e_.)*(x_)^(q_)))/(x_), x_Symbol] := Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]^(p_.))/(x_)*((d_) + (e_.)*(x_)), x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]^(p_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] := Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p]/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
```

[p, 1] || GtQ[r, 0] && NeQ[r, -1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

### Rule 2392

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*d])\*Log[x], x] + Dist[b, Int[Log[1 + (e\*x)/d]/x, x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[c\*d, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.)))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.)))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] :> Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \tanh^{-1}(cx^2))^2}{x^5} dx &= \int \left( \frac{(2a - b \log(1 - cx^2))^2}{4x^5} - \frac{b(-2a + b \log(1 - cx^2)) \log(1 + cx^2)}{2x^5} + \frac{b^2 \log^2(1 + cx^2)}{4x^5} \right) dx \\
 &= \frac{1}{4} \int \frac{(2a - b \log(1 - cx^2))^2}{x^5} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - cx^2)) \log(1 + cx^2)}{x^5} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + cx^2)}{x^5} dx \\
 &= \frac{1}{8} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^3} dx, x, x^2 \right) - \frac{1}{4} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log(1 + cx)}{x^3} dx, x, x^2 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x^3} dx, x, x^2 \right) \\
 &= -\frac{(2a - b \log(1 - cx^2))^2}{16x^4} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{8x^4} - \frac{b^2 \log^2(1 + cx^2)}{16x^4} + \frac{1}{8} \log(x) \\
 &= -\frac{(2a - b \log(1 - cx^2))^2}{16x^4} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{8x^4} - \frac{b^2 \log^2(1 + cx^2)}{16x^4} - \frac{1}{8} \log(x) \\
 &= -\frac{(2a - b \log(1 - cx^2))^2}{16x^4} - \frac{b(2a - b \log(1 - cx^2)) \log(1 + cx^2)}{8x^4} - \frac{b^2 \log^2(1 + cx^2)}{16x^4} - \frac{1}{8} \log(x) \\
 &= -\frac{1}{2} abc^2 \log(x) - \frac{bc(2a - b \log(1 - cx^2))}{8x^2} - \frac{bc(1 - cx^2)(2a - b \log(1 - cx^2))}{8x^2} - \frac{(2a - b \log(1 - cx^2))^2}{8x^2} \\
 &= \frac{1}{4} b^2 c^2 \log(x) - \frac{bc(2a - b \log(1 - cx^2))}{8x^2} - \frac{bc(1 - cx^2)(2a - b \log(1 - cx^2))}{8x^2} + \frac{1}{16} c^2 (2a - b \log(1 - cx^2))^2 \\
 &= \frac{1}{2} b^2 c^2 \log(x) - \frac{1}{8} b^2 c^2 \log(1 - cx^2) - \frac{bc(2a - b \log(1 - cx^2))}{8x^2} - \frac{bc(1 - cx^2)(2a - b \log(1 - cx^2))}{8x^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.083499, size = 111, normalized size = 1.26

$$\frac{1}{4} \left( -\frac{a^2}{x^4} - bc^2(a+b) \log(1-cx^2) + bc^2(a-b) \log(cx^2+1) - \frac{2abc}{x^2} - \frac{2b \tanh^{-1}(cx^2)(a+bcx^2)}{x^4} + \frac{b^2(c^2x^4-1) \tanh^{-1}(cx^2)}{x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^5, x]

[Out]  $-(a^2/x^4) - (2*a*b*c)/x^2 - (2*b*(a + b*c*x^2)*ArcTanh[c*x^2])/x^4 + (b^2*(-1 + c^2*x^4)*ArcTanh[c*x^2]^2)/x^4 + 4*b^2*c^2*Log[x] - b*(a + b)*c^2*Log[1 - c*x^2] + (a - b)*b*c^2*Log[1 + c*x^2])/4$

**Maple [B]** time = 0.188, size = 257, normalized size = 2.9

$$\frac{b^2(c^2x^4-1)(\ln(cx^2+1))^2}{16x^4} - \frac{b(x^4b \ln(-cx^2+1)c^2 + 2bcx^2 - b \ln(-cx^2+1) + 2a) \ln(cx^2+1)}{8x^4} + \frac{b^2c^2x^4(\ln(-cx^2+1))^2}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/x^5, x)

[Out]  $1/16*b^2*(c^2*x^4-1)/x^4*\ln(cx^2+1)^2-1/8*b*(x^4*b*\ln(-cx^2+1)*c^2+2*b*c*x^2-b*\ln(-cx^2+1)+2*a)/x^4*\ln(cx^2+1)+1/16*(b^2*c^2*x^4*\ln(-cx^2+1)^2+4*b*c^2*\ln(cx^2+1)*x^4*a-4*b^2*c^2*\ln(cx^2+1)*x^4-4*b*c^2*\ln(cx^2-1)*x^4*a-4*b^2*c^2*\ln(cx^2-1)*x^4+16*b^2*c^2*\ln(x)*x^4+4*b^2*c*x^2*\ln(-cx^2+1)-8*a*b*c*x^2-b^2*\ln(-cx^2+1)^2+4*b*\ln(-cx^2+1)*a-4*a^2)/x^4$

**Maxima [B]** time = 1.00288, size = 236, normalized size = 2.68

$$\frac{1}{4} \left( \left( c \log(cx^2+1) - c \log(cx^2-1) - \frac{2}{x^2} \right) c - \frac{2 \operatorname{artanh}(cx^2)}{x^4} \right) ab + \frac{1}{16} \left( \left( 2(\log(cx^2-1) - 2) \log(cx^2+1) - \log(cx^2+1) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^5, x, algorithm="maxima")



```
[Out] 1/4*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a*b + 1/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - 1
og(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1) -
c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*b^2 - 1/4*b^2*arctanh(c*x^2)^2
/x^4 - 1/4*a^2/x^4
```

**Fricas [A]** time = 2.16351, size = 324, normalized size = 3.68

$$\frac{16 b^2 c^2 x^4 \log(x) + 4 (ab - b^2) c^2 x^4 \log(cx^2 + 1) - 4 (ab + b^2) c^2 x^4 \log(cx^2 - 1) - 8 abc x^2 + (b^2 c^2 x^4 - b^2) \log\left(-\frac{cx^2+1}{cx^2-1}\right)^2}{16 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c^2*x^4*log(x) + 4*(a*b - b^2)*c^2*x^4*log(c*x^2 + 1) - 4*(a*b
+ b^2)*c^2*x^4*log(c*x^2 - 1) - 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*
x^2 + 1)/(c*x^2 - 1))^2 - 4*a^2 - 4*(b^2*c*x^2 + a*b)*log(-(c*x^2 + 1)/(c*x
^2 - 1)))/x^4
```

**Sympy [A]** time = 26.0661, size = 175, normalized size = 1.99

$$\left\{ \begin{array}{l} -\frac{a^2}{4x^4} + \frac{abc^2 \operatorname{atanh}(cx^2)}{2} - \frac{abc}{2x^2} - \frac{ab \operatorname{atanh}(cx^2)}{2x^4} + b^2 c^2 \log(x) - \frac{b^2 c^2 \log\left(x - i\sqrt{\frac{1}{c}}\right)}{2} - \frac{b^2 c^2 \log\left(x + i\sqrt{\frac{1}{c}}\right)}{2} + \frac{b^2 c^2 \operatorname{atanh}^2(cx^2)}{4} + \frac{b^2 c^2 \operatorname{atanh}(cx^2)}{2} \\ -\frac{a^2}{4x^4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**2*atanh(c*x**2)/2 - a*b*c/(2*x**2) - a*b
*atanh(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x - I*sqrt(1/c))
/2 - b**2*c**2*log(x + I*sqrt(1/c))/2 + b**2*c**2*atanh(c*x**2)**2/4 + b**2
*c**2*atanh(c*x**2)/2 - b**2*c*atanh(c*x**2)/(2*x**2) - b**2*atanh(c*x**2)*
*2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^5, x)
```

$$3.71 \quad \int x^4 \left( a + b \tanh^{-1} (cx^2) \right)^2 dx$$

**Optimal.** Leaf size=1173

result too large to display

```
[Out] (8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[Sq
rt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b^2
*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2)) - (
b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1
- Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*
x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1
- I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c
]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(5*c
^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[
-c] - Sqrt[c])*(1 + Sqrt[c]*x)))]/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x)))]/(5*c
^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*
x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2)) + (b*x^3*(2*a
- b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*Arc
Tanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^5*(2*a - b*Log[1
- c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x
^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*
x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[
c]*x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/(5*c
^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x
)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/(5*c^(5/2)) + (b^2
*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 + (2*Sq
rt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x)))]/(10*c^(5/2
)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])
*(1 + Sqrt[c]*x)))]/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 +
Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(5*c^(5/2))
```

---

**Rubi [A]** time = 2.33394, antiderivative size = 1173, normalized size of antiderivative = 1., number of steps used = 102, number of rules used = 26, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$ , Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out]  $(8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[\sqrt{c}*x])/(5*c^{(5/2)}) - (4*b^2*ArcTan[\sqrt{c}*x])/(15*c^{(5/2)}) + ((I/5)*b^2*ArcTan[\sqrt{c}*x]^2)/c^{(5/2)} - (4*b^2*ArcTanh[\sqrt{c}*x])/(15*c^{(5/2)}) - (b^2*ArcTanh[\sqrt{c}*x]^2)/(5*c^{(5/2)}) + (2*b^2*ArcTanh[\sqrt{c}*x]*Log[2/(1 - \sqrt{c}*x)])/(5*c^{(5/2)}) - (2*b^2*ArcTan[\sqrt{c}*x]*Log[2/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)}) + (b^2*ArcTan[\sqrt{c}*x]*Log[((1 + I)*(1 - \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)}) + (2*b^2*ArcTan[\sqrt{c}*x]*Log[2/(1 + I*\sqrt{c}*x)])/(5*c^{(5/2)}) - (2*b^2*ArcTanh[\sqrt{c}*x]*Log[2/(1 + \sqrt{c}*x)])/(5*c^{(5/2)}) + (b^2*ArcTanh[\sqrt{c}*x]*Log[(-2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \sqrt{c}*x))])/(5*c^{(5/2)}) + (b^2*ArcTanh[\sqrt{c}*x]*Log[(2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \sqrt{c})*(1 + \sqrt{c}*x))])/(5*c^{(5/2)}) + (b^2*ArcTan[\sqrt{c}*x]*Log[((1 - I)*(1 + \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)}) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x^5*Log[1 - c*x^2])/25 - (b^2*ArcTan[\sqrt{c}*x]*Log[1 - c*x^2])/(5*c^{(5/2)}) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2]))/25 - (b*ArcTanh[\sqrt{c}*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^{(5/2)}) + (x^5*(2*a - b*Log[1 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*b*x^5*Log[1 + c*x^2])/5 + (b^2*ArcTan[\sqrt{c}*x]*Log[1 + c*x^2])/(5*c^{(5/2)}) - (b^2*ArcTanh[\sqrt{c}*x]*Log[1 + c*x^2])/(5*c^{(5/2)}) - (b^2*x^5*Log[1 - c*x^2]*Log[1 + c*x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*PolyLog[2, 1 - 2/(1 - \sqrt{c}*x)])/(5*c^{(5/2)}) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)}) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)}) + ((I/5)*b^2*PolyLog[2, 1 - 2/(1 + I*\sqrt{c}*x)])/(5*c^{(5/2)}) + (b^2*PolyLog[2, 1 - 2/(1 + \sqrt{c}*x)])/(5*c^{(5/2)}) - (b^2*PolyLog[2, 1 + (2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \sqrt{c}*x))])/(10*c^{(5/2)}) - (b^2*PolyLog[2, 1 - (2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \sqrt{c})*(1 + \sqrt{c}*x))])/(10*c^{(5/2)}) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(5*c^{(5/2)})$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a +

$b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]^{(q-1)} / (d + e \cdot x^n), x, x] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

### Rule 2476

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x^n)^p] \cdot (b \cdot x^m) \cdot (f + g \cdot x^s)^r), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]^q, x^m \cdot (f + g \cdot x^s)^r), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

### Rule 2448

$\text{Int}[\text{Log}[c \cdot (d + e \cdot x^n)^p], x\_Symbol] :> \text{Simp}[x \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p], x] - \text{Dist}[e \cdot n \cdot p, \text{Int}[x^n / (d + e \cdot x^n), x], x] /;$  FreeQ[{c, d, e, n, p}, x]

### Rule 321

$\text{Int}[(c \cdot x)^m \cdot (a + b \cdot x^n)^p, x\_Symbol] :> \text{Simp}[(c^{n-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1}) / (b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^{n-1} \cdot (c \cdot x)^{m-n+1}) / (b \cdot (m + n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n \cdot p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

$\text{Int}[(a + b \cdot x^2)^{-1}, x\_Symbol] :> \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 2455

$\text{Int}[(a + \text{Log}[c \cdot (d + e \cdot x^n)^p] \cdot (b \cdot x^m) \cdot (f \cdot x)^{m+1}), x\_Symbol] :> \text{Simp}[(f \cdot x)^{m+1} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]) / (f \cdot (m + 1)), x] - \text{Dist}[(b \cdot e \cdot n \cdot p) / (f \cdot (m + 1)), \text{Int}[(x^{n-1} \cdot (f \cdot x)^{m+1}) / (d + e \cdot x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 302

$\text{Int}[x^m / (a + b \cdot x^n), x\_Symbol] :> \text{Int}[\text{PolynomialDivide}[x^m, a + b \cdot x^n, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2 \cdot n - 1]

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] := With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)
, x), x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_))], x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

### Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

### Rule 5992

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))*(x_)^(m_)/((d_) + (e_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 5920

```
Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
)))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps



$$\begin{aligned}
\int x^4 (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4}x^4 (2a - b \log(1 - cx^2))^2 - \frac{1}{2}bx^4 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4}b^2x^4 \log^2(1 + cx^2) \right) dx \\
&= \frac{1}{4} \int x^4 (2a - b \log(1 - cx^2))^2 dx - \frac{1}{2}b \int x^4 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} \int b^2x^4 \log^2(1 + cx^2) dx \\
&= \frac{1}{20}x^5 (2a - b \log(1 - cx^2))^2 + \frac{1}{20}b^2x^5 \log^2(1 + cx^2) - \frac{1}{2}b \int (-2ax^4 \log(1 + cx^2) + bx^4 \log(1 - cx^2) \log(1 + cx^2)) dx \\
&= \frac{1}{20}x^5 (2a - b \log(1 - cx^2))^2 + \frac{1}{20}b^2x^5 \log^2(1 + cx^2) + (ab) \int x^4 \log(1 + cx^2) dx - \frac{1}{2} \int bx^4 \log(1 - cx^2) \log(1 + cx^2) dx \\
&= \frac{1}{20}x^5 (2a - b \log(1 - cx^2))^2 + \frac{1}{5}abx^5 \log(1 + cx^2) - \frac{1}{10}b^2x^5 \log(1 - cx^2) \log(1 + cx^2) \\
&= \frac{2abx}{5c^2} + \frac{bx^3(2a - b \log(1 - cx^2))}{15c} + \frac{1}{25}bx^5(2a - b \log(1 - cx^2)) - \frac{b \tanh^{-1}(\sqrt{cx})}{5c^{5/2}} (2a - b \log(1 - cx^2)) \\
&= \frac{2b^2x}{5c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 - \frac{b^2x \log(1 - cx^2)}{5c^2} + \frac{bx^3(2a - b \log(1 - cx^2))}{15c} + \frac{1}{25}bx^5(2a - b \log(1 - cx^2)) \\
&= \frac{92b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{2b^2 \tan^{-1}(\sqrt{cx})}{5c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})}{5c^{5/2}} \\
&= \frac{92b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{46b^2 \tan^{-1}(\sqrt{cx})}{75c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})}{5c^{5/2}} \\
&= \frac{32b^2x}{75c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{4b^2x^5}{125} + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{46b^2 \tan^{-1}(\sqrt{cx})}{75c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})}{5c^{5/2}} \\
&= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{16b^2 \tan^{-1}(\sqrt{cx})}{75c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{5c^{5/2}} \\
&= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{4b^2 \tan^{-1}(\sqrt{cx})}{15c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{5c^{5/2}} \\
&= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{4b^2 \tan^{-1}(\sqrt{cx})}{15c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{5c^{5/2}} \\
&= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{4b^2 \tan^{-1}(\sqrt{cx})}{15c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{5c^{5/2}} \\
&= \frac{8b^2x}{15c^2} + \frac{2abx^3}{15c} - \frac{2}{25}abx^5 + \frac{2ab \tan^{-1}(\sqrt{cx})}{5c^{5/2}} - \frac{4b^2 \tan^{-1}(\sqrt{cx})}{15c^{5/2}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{5c^{5/2}}
\end{aligned}$$

**Mathematica [F]** time = 9.30766, size = 0, normalized size = 0.

$$\int x^4 (a + b \tanh^{-1}(cx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] Integrate[x^4\*(a + b\*ArcTanh[c\*x^2])^2, x]

---

**Maple [F]** time = 0.216, size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{Artanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] int(x^4\*(a+b\*arctanh(c\*x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2x^4 \operatorname{artanh}(cx^2)^2 + 2abx^4 \operatorname{artanh}(cx^2) + a^2x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c*x^2)^2 + 2*a*b*x^4*arctanh(c*x^2) + a^2*x^4, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{atanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c*x**2))**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^4, x)
```

### 3.72 $\int x^2 \left( a + b \tanh^{-1} (cx^2) \right)^2 dx$

**Optimal.** Leaf size=1129

result too large to display

```
[Out] (4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (
4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^(3
/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2)/(
3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/2))
+ (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*Arc
Tan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^2
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]
*x)))]/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x
))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x)))]/(3*c^(3/2)) - (b^2*ArcTan[Sqrt[
c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (2*b^
2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[Sqrt[c
]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*
ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (x^3*(2*a - b*Lo
g[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x
^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*ArcTanh[
Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*
x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c
]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)
) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])
/c^(3/2) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*P
olyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 + (2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x)))]/(6*c^(3/2))
- (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x)))]/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt
[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3/2))
```

---

**Rubi [A]** time = 2.06081, antiderivative size = 1129, normalized size of antiderivative = 1., number of steps used = 86, number of rules used = 26, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$ , Rules used = {6099, 2457, 2476, 2448, 321, 206, 2455, 302, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 207, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$\frac{1}{12} (2a - b \log(1 - cx^2))^2 x^3 + \frac{1}{12} b^2 \log^2(cx^2 + 1) x^3 - \frac{2}{9} abx^3 + \frac{1}{9} b^2 \log(1 - cx^2) x^3 + \frac{1}{9} b (2a - b \log(1 - cx^2)) x^3 + \frac{1}{3} a$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out]  $(4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[\sqrt{c}*x])/(3*c^{3/2}) + (4*b^2*ArcTan[\sqrt{c}*x])/(3*c^{3/2}) - ((1/3)*b^2*ArcTan[\sqrt{c}*x]^2)/c^{3/2} - (4*b^2*ArcTanh[\sqrt{c}*x])/(3*c^{3/2}) - (b^2*ArcTanh[\sqrt{c}*x]^2)/(3*c^{3/2}) + (2*b^2*ArcTanh[\sqrt{c}*x]*Log[2/(1 - \sqrt{c}*x)])/(3*c^{3/2}) + (2*b^2*ArcTan[\sqrt{c}*x]*Log[2/(1 - I*\sqrt{c}*x)])/(3*c^{3/2}) - (b^2*ArcTan[\sqrt{c}*x]*Log[((1 + I)*(1 - \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(3*c^{3/2}) - (2*b^2*ArcTan[\sqrt{c}*x]*Log[2/(1 + I*\sqrt{c}*x)])/(3*c^{3/2}) - (2*b^2*ArcTanh[\sqrt{c}*x]*Log[2/(1 + \sqrt{c}*x)])/(3*c^{3/2}) + (b^2*ArcTanh[\sqrt{c}*x]*Log[(-2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \sqrt{c}*x))])/(3*c^{3/2}) + (b^2*ArcTanh[\sqrt{c}*x]*Log[(2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \sqrt{c})*(1 + \sqrt{c}*x))])/(3*c^{3/2}) - (b^2*ArcTan[\sqrt{c}*x]*Log[((1 - I)*(1 + \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(3*c^{3/2}) - (2*b^2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*ArcTan[\sqrt{c}*x]*Log[1 - c*x^2])/(3*c^{3/2}) + (b*x^3*(2*a - b*Log[1 - c*x^2]))/9 - (b*ArcTanh[\sqrt{c}*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^{3/2}) + (x^3*(2*a - b*Log[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b*x^3*Log[1 + c*x^2])/3 - (b^2*ArcTan[\sqrt{c}*x]*Log[1 + c*x^2])/(3*c^{3/2}) - (b^2*ArcTanh[\sqrt{c}*x]*Log[1 + c*x^2])/(3*c^{3/2}) - (b^2*x^3*Log[1 - c*x^2]*Log[1 + c*x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2, 1 - 2/(1 - \sqrt{c}*x)])/(3*c^{3/2}) - ((1/3)*b^2*PolyLog[2, 1 - 2/(1 - I*\sqrt{c}*x)])/(c^{3/2}) + ((1/6)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(c^{3/2}) - ((1/3)*b^2*PolyLog[2, 1 - 2/(1 + I*\sqrt{c}*x)])/(c^{3/2}) + (b^2*PolyLog[2, 1 - 2/(1 + \sqrt{c}*x)])/(3*c^{3/2}) - (b^2*PolyLog[2, 1 + (2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \sqrt{c}*x))])/(6*c^{3/2}) - (b^2*PolyLog[2, 1 - (2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \sqrt{c})*(1 + \sqrt{c}*x))])/(6*c^{3/2}) + ((1/6)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + \sqrt{c}*x))/(1 - I*\sqrt{c}*x)])/(c^{3/2})$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a + b\*Log[c\*(d + e\*x^n)^p])^(q - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d,

e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

### Rule 2476

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p])^q, x^m\*(f + g\*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 302

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Int[PolynomialDivide[x^m, a + b\*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2\*n - 1]

### Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] := With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 5984

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

### Rule 5918

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*
p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 - c^2*x^2)
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0
]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)/((d_) + (e_.)*(x_))], x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

, 0] || GtQ[b, 0])

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rule 2557

Int[Log[v\_]\*Log[w\_]\*(u\_), x\_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]\*Log[w], z, x] + (-Int[SimplifyIntegrand[(z\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 5992

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))\*(x\_)^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[a + b\*ArcTanh[c\*x], x^m/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

### Rule 5920

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcTanh[c\*x])\*Log[(2\*c\*(d + e\*x))/(c\*d + e)\*(1 + c\*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 4928



```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))*(x_)^(m_.)/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
;/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))]/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

### Rule 4920

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[(a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]]/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( \frac{1}{4}x^2 (2a - b \log(1 - cx^2))^2 - \frac{1}{2}bx^2 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) + \frac{1}{4}b^2x^2 \log^2(1 + cx^2) \right) dx \\
&= \frac{1}{4} \int x^2 (2a - b \log(1 - cx^2))^2 dx - \frac{1}{2}b \int x^2 (-2a + b \log(1 - cx^2)) \log(1 + cx^2) dx + \frac{1}{4} \int b^2x^2 \log^2(1 + cx^2) dx \\
&= \frac{1}{12}x^3 (2a - b \log(1 - cx^2))^2 + \frac{1}{12}b^2x^3 \log^2(1 + cx^2) - \frac{1}{2}b \int (-2ax^2 \log(1 + cx^2) + bx^2 \log^2(1 + cx^2)) dx \\
&= \frac{1}{12}x^3 (2a - b \log(1 - cx^2))^2 + \frac{1}{12}b^2x^3 \log^2(1 + cx^2) + (ab) \int x^2 \log(1 + cx^2) dx - \frac{1}{2}b \int bx^2 \log^2(1 + cx^2) dx \\
&= \frac{1}{12}x^3 (2a - b \log(1 - cx^2))^2 + \frac{1}{3}abx^3 \log(1 + cx^2) - \frac{1}{6}b^2x^3 \log(1 - cx^2) \log(1 + cx^2) + \frac{1}{12}b^2x^3 \log^2(1 + cx^2) \\
&= \frac{2abx}{3c} + \frac{1}{9}bx^3 (2a - b \log(1 - cx^2)) - \frac{b \tanh^{-1}(\sqrt{cx}) (2a - b \log(1 - cx^2))}{3c^{3/2}} + \frac{1}{12}x^3 (2a - b \log(1 - cx^2))^2 \\
&= \frac{4abx}{3c} - \frac{2b^2x}{3c} - \frac{2}{9}abx^3 - \frac{b^2x \log(1 - cx^2)}{3c} + \frac{1}{9}bx^3 (2a - b \log(1 - cx^2)) - \frac{b \tanh^{-1}(\sqrt{cx}) (2a - b \log(1 - cx^2))}{3c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 + \frac{4b^2x^3}{27} - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{2b^2 \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})}{9c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 + \frac{4b^2x^3}{27} - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{8b^2 \tan^{-1}(\sqrt{cx})}{9c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{8b^2 \tanh^{-1}(\sqrt{cx})}{9c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 + \frac{4b^2x^3}{27} - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{8b^2 \tan^{-1}(\sqrt{cx})}{9c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{8b^2 \tanh^{-1}(\sqrt{cx})}{9c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{14b^2 \tan^{-1}(\sqrt{cx})}{9c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{14b^2 \tanh^{-1}(\sqrt{cx})}{9c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{4b^2 \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{4b^2 \tanh^{-1}(\sqrt{cx})}{3c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{4b^2 \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{4b^2 \tanh^{-1}(\sqrt{cx})}{3c^{3/2}} \\
&= \frac{4abx}{3c} - \frac{2}{9}abx^3 - \frac{2ab \tan^{-1}(\sqrt{cx})}{3c^{3/2}} + \frac{4b^2 \tan^{-1}(\sqrt{cx})}{3c^{3/2}} - \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{3c^{3/2}} - \frac{4b^2 \tanh^{-1}(\sqrt{cx})}{3c^{3/2}}
\end{aligned}$$

**Mathematica [F]** time = 9.0584, size = 0, normalized size = 0.

$$\int x^2 (a + b \tanh^{-1}(cx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] Integrate[x^2\*(a + b\*ArcTanh[c\*x^2])^2, x]

---

**Maple [F]** time = 0.184, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{Arctanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] int(x^2\*(a+b\*arctanh(c\*x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2x^2 \operatorname{artanh}(cx^2)^2 + 2abx^2 \operatorname{artanh}(cx^2) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c*x^2)^2 + 2*a*b*x^2*arctanh(c*x^2) + a^2*x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c*x**2))**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*x^2, x)
```

### 3.73 $\int (a + b \tanh^{-1}(cx^2))^2 dx$

**Optimal.** Leaf size=958

result too large to display

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
+ (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c]
- (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/((2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/((2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

---

**Rubi [A]** time = 1.46813, antiderivative size = 958, normalized size of antiderivative = 1., number of steps used = 69, number of rules used = 21, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.75$ , Rules used = {6093, 2448, 321, 206, 2450, 2476, 2470, 12, 5984, 5918, 2402, 2315, 203, 2556, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$xa^2 + \frac{2b \tan^{-1}(\sqrt{cx}) a}{\sqrt{c}} - \frac{2b \tanh^{-1}(\sqrt{cx}) a}{\sqrt{c}} - bx \log(1 - cx^2) a + bx \log(cx^2 + 1) a + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2,x]

```
[Out] a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sqr
t[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sqrt
[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2*Ar
cTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*
Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*ArcTan[S
qrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*x]*Log
[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTa
nh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + S
qrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTan[Sqrt[c]*x
]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c]
+ (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*ArcTan[Sqrt[c]*
x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c]
- (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 + c*x^2]^2)/4 +
(b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(
1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*
x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/Sqrt[c] - (b^2*PolyL
og[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x
))])/((2*Sqrt[c]) - (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[
-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/((2*Sqrt[c]) - ((I/2)*b^2*PolyLog[2, 1 - (
(1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c]
```

### Rule 6093

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.), x_Symbol] := Int[Expand
Integrand[(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; Fr
eeQ[{a, b, c, n}, x] && IGtQ[p, 0] && IntegerQ[n]
```

### Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.), x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2450

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)]\*(b\_))^(q\_), x\_Symbol] := Simp[x\*(a + b\*Log[c\*(d + e\*x^n)^p])^q, x] - Dist[b\*e\*n\*p\*q, Int[(x^n\*(a + b\*Log[c\*(d + e\*x^n)^p])^(q - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])

Rule 2476

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)]\*(b\_))^(q\_)\*(x\_)^(m\_) \* ((f\_) + (g\_)\*(x\_)^(s\_))^(r\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p])^q, x^m\*(f + g\*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

Rule 2470

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)]\*(b\_))/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 5984

Int[((a\_) + ArcTanh[(c\_)\*(x\_)]\*(b\_))^(p\_)\*(x\_)/((d\_) + (e\_)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rule 5918

Int[((a\_) + ArcTanh[(c\_)\*(x\_)]\*(b\_))^(p\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c^p)/e, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)]/(1 - c^2\*x^2)

, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 2556

Int[Log[v\_]\*Log[w\_], x\_Symbol] :> Simp[x\*Log[v]\*Log[w], x] + (-Int[SimplifyIntegrand[(x\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 5992

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))\*(x\_)^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] :> Int[ExpandIntegrand[a + b\*ArcTanh[c\*x], x^m/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

### Rule 5920

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))/((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> -Simp[((a + b\*ArcTanh[c\*x])\*Log[2/(1 + c\*x)])/e, x] + (Dist[(b\*c)/e, Int[Log[2/(1 + c\*x)]/(1 - c^2\*x^2), x], x] - Dist[(b\*c)/e, Int[Log[(2\*c\*(d + e\*x))/(c\*d + e\*(1 + c\*x))]/(1 - c^2\*x^2), x], x] + Simp[((a + b\*ArcTanh[c\*x])\*Log[(2\*c\*(d + e\*x))/(c\*d + e\*(1 + c\*x))])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 - e^2, 0]

### Rule 2447



```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

### Rule 4928

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))*(x_)^(m_)/((d_) + (e_)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4856

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_)/((d_) + (e_)*(x_)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
)))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

### Rule 4920

```
Int[(((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)*(x_)/((d_) + (e_)*(x_)^2),
x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### Rule 4854

```
Int[((a_) + ArcTan[(c_)*(x_)])*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol]
:= -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(cx^2))^2 dx &= \int \left( a^2 - ab \log(1 - cx^2) + \frac{1}{4}b^2 \log^2(1 - cx^2) + ab \log(1 + cx^2) - \frac{1}{2}b^2 \log(1 - cx^2) \log(1 + cx^2) \right) dx \\
&= a^2x - (ab) \int \log(1 - cx^2) dx + (ab) \int \log(1 + cx^2) dx + \frac{1}{4}b^2 \int \log^2(1 - cx^2) dx + \frac{1}{4}b^2 \int \log^2(1 + cx^2) dx \\
&= a^2x - abx \log(1 - cx^2) + \frac{1}{4}b^2x \log^2(1 - cx^2) + abx \log(1 + cx^2) - \frac{1}{2}b^2x \log(1 - cx^2) \log(1 + cx^2) \\
&= a^2x - abx \log(1 - cx^2) + \frac{1}{4}b^2x \log^2(1 - cx^2) + abx \log(1 + cx^2) - \frac{1}{2}b^2x \log(1 - cx^2) \log(1 + cx^2) \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - abx \log(1 - cx^2) + \frac{1}{4}b^2x \log^2(1 - cx^2) + abx \log(1 + cx^2) \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - abx \log(1 - cx^2) - b^2x \log(1 - cx^2) + \frac{b^2 \tan^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + 4b^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - abx \log(1 - cx^2) - \frac{b^2 \tan^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{2b^2 \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}} \\
&= a^2x + \frac{2ab \tan^{-1}(\sqrt{cx})}{\sqrt{c}} + \frac{ib^2 \tan^{-1}(\sqrt{cx})^2}{\sqrt{c}} - \frac{2ab \tanh^{-1}(\sqrt{cx})}{\sqrt{c}} - \frac{b^2 \tanh^{-1}(\sqrt{cx})^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}(\sqrt{cx}) \log(1 + cx^2)}{\sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 2.56231, size = 566, normalized size = 0.59

$$\frac{1}{2}x \left( \frac{b^2 \left( \text{PolyLog} \left( 2, \frac{1}{2} (1 - \sqrt{cx^2}) \right) - \text{PolyLog} \left( 2, \left( -\frac{1}{2} - \frac{i}{2} \right) (\sqrt{cx^2} - 1) \right) - \text{PolyLog} \left( 2, \left( -\frac{1}{2} + \frac{i}{2} \right) (\sqrt{cx^2} - 1) \right) - \text{PolyLog} \left( 2, \frac{1}{2} (1 + \sqrt{cx^2}) \right) - \text{PolyLog} \left( 2, \left( -\frac{1}{2} - \frac{i}{2} \right) (\sqrt{cx^2} + 1) \right) - \text{PolyLog} \left( 2, \left( -\frac{1}{2} + \frac{i}{2} \right) (\sqrt{cx^2} + 1) \right) - \text{PolyLog} \left( 2, \frac{1}{2} (1 + \sqrt{cx^2}) \right) \right)}{\dots} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] (x\*(2\*a^2 + 4\*a\*b\*ArcTanh[c\*x^2] + (4\*a\*b\*(ArcTan[Sqrt[c\*x^2]] - ArcTanh[Sqrt[c\*x^2]]))/Sqrt[c\*x^2] + (b^2\*((-2\*I)\*ArcTan[Sqrt[c\*x^2]]^2 + 4\*ArcTan[Sqrt[c\*x^2]]\*ArcTanh[c\*x^2] + 2\*Sqrt[c\*x^2]\*ArcTanh[c\*x^2]^2 + 2\*ArcTan[Sqrt[c\*x^2]]\*Log[1 + E^((4\*I)\*ArcTan[Sqrt[c\*x^2]])] + 2\*ArcTanh[c\*x^2]\*Log[1 - Sqrt[c\*x^2]] - Log[2]\*Log[1 - Sqrt[c\*x^2]] + Log[1 - Sqrt[c\*x^2]]^2/2 - Log[1 - Sqrt[c\*x^2]]\*Log[(1/2 + I/2)\*(-I + Sqrt[c\*x^2])] - 2\*ArcTanh[c\*x^2]\*Log[1 + Sqrt[c\*x^2]] + Log[2]\*Log[1 + Sqrt[c\*x^2]] + Log[((1 + I) - (1 - I)\*Sqrt[c\*x^2])/2]\*Log[1 + Sqrt[c\*x^2]] + Log[(-1/2 - I/2)\*(I + Sqrt[c\*x^2])]\*Log[1 + Sqrt[c\*x^2]] - Log[1 + Sqrt[c\*x^2]]^2/2 - Log[1 - Sqrt[c\*x^2]]\*Log[((1 + I) + (1 - I)\*Sqrt[c\*x^2])/2] - (I/2)\*PolyLog[2, -E^((4\*I)\*ArcTan[Sqrt[c\*x^2]])] + PolyLog[2, (1 - Sqrt[c\*x^2])/2] - PolyLog[2, (-1/2 - I/2)\*(-1 + Sqrt[c\*x^2])] - PolyLog[2, (-1/2 + I/2)\*(-1 + Sqrt[c\*x^2])] - PolyLog[2, (1 + Sqrt[c\*x^2])/2] + PolyLog[2, (1/2 - I/2)\*(1 + Sqrt[c\*x^2])] + PolyLog[2, (1/2 + I/2)\*(1 + Sqrt[c\*x^2])]))/Sqrt[c\*x^2])/2

**Maple [F]** time = 0.125, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Arctanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2,x)

[Out] int((a+b\*arctanh(c\*x^2))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out] integral(b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2, x)

$$3.74 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^2}{x^2} dx$$

Optimal. Leaf size=942

result too large to display

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
+ 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x)))]/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x)))]/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
+ Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

---

**Rubi [A]** time = 1.33509, antiderivative size = 942, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 21, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.313$ , Rules used = {6099, 2457, 206, 2470, 12, 5984, 5918, 2402, 2315, 2455, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$i\sqrt{c} \tan^{-1}(\sqrt{cx})^2 b^2 + \sqrt{c} \tanh^{-1}(\sqrt{cx})^2 b^2 - \frac{\log^2(cx^2 + 1) b^2}{4x} - 2\sqrt{c} \tanh^{-1}(\sqrt{cx}) \log\left(\frac{2}{1 - \sqrt{cx}}\right) b^2 - 2\sqrt{c} \tan^{-1}(\sqrt{cx})$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x^2,x]

```
[Out] 2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*S
qrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 - S
qrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] + b^2
*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
+ 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sqrt[c]
*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]
*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))]
- b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c
] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 - I)
*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[1
- c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]) - (2*a - b
*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]*ArcTan[Sqrt
[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2] + (b^
2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^2)/(4*x) - b^2
*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c
]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)
] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*Sqrt[c]*PolyLog[2,
1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x)))]/
2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + S
qrt[c])*(1 + Sqrt[c]*x)))]/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(1
+ Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

### Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)*((d_.)*(x_)^(m_.), x_Sy
mbol] :=> Int[ExpandIntegrand[(d*x)^(m)*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]
```

### Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.)^(q_.)*((f_.)*(
x_)^(m_.), x_Symbol] :=> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q
)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a +
b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

Rule 5918

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c^p)/e, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2402

Int[Log[(c\_.)/((d\_) + (e\_.)\*(x\_))]/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :=> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :=> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :=> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :=> -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)]/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x)) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :=> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
```



x][[2]], Expon[Pq, x]]

### Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
;/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x)
)/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

### Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

### Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

### Rubi steps



**Mathematica [A]** time = 3.36907, size = 566, normalized size = 0.6

$$b^2\sqrt{cx^2}\left(-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{cx^2}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{cx^2}-1\right)\right)+\text{PolyLog}\left(2,\frac{1}{2}\left(1+\sqrt{cx^2}\right)\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^2,x]

[Out]  $(-2*a^2 - 4*a*b*ArcTanh[c*x^2] + 4*a*b*Sqrt[c*x^2]*(ArcTan[Sqrt[c*x^2]] + ArcTanh[Sqrt[c*x^2]]) + b^2*Sqrt[c*x^2]*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*ArcTan[Sqrt[c*x^2]]*ArcTanh[c*x^2] - (2*ArcTanh[c*x^2]^2)/Sqrt[c*x^2] + 2*ArcTan[Sqrt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x^2]*Log[1 - Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*ArcTanh[c*x^2]*Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 + I) - (1 - I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c*x^2])]*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - Sqrt[c*x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(1 + Sqrt[c*x^2])] + PolyLog[2, (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])])/(2*x)$

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^2))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/x^2,x)

[Out] int((a+b\*arctanh(c\*x^2))^2/x^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*2/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^2,x, algorithm="giac")

```
[Out] integrate((b*arctanh(c*x^2) + a)^2/x^2, x)
```

$$3.75 \quad \int \frac{\left(a+b \tanh^{-1}(cx^2)\right)^2}{x^4} dx$$

Optimal. Leaf size=1102

result too large to display

```
[Out] (-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Arc
Tan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2)*
ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^(3/
2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan[Sqr
t[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((
1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan[Sqrt[
c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[2
/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 -
Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*Arc
Tanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 +
Sqrt[c]*x))])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]
*x))/(1 - I*Sqrt[c]*x)])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) +
(b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (2*a - b*Log[1
- c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2*c*Log[1 + c*x^
2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*c^(3/2)
*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])
/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)*PolyLog[2, 1 - 2/
(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)]
+ (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c
]*x)] - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(3/2)
)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/3 + (b^2*c^(3/2)*PolyLog[2, 1 + (2*Sqr
t[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/6 + (b^2*c^
(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1
+ Sqrt[c]*x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*
x))/(1 - I*Sqrt[c]*x)]
```

---

**Rubi [A]** time = 1.84194, antiderivative size = 1102, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 24, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.5$ , Rules used = {6099, 2457, 2476, 2455, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 325, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

$$-\frac{1}{3}ic^{3/2} \tan^{-1}(\sqrt{cx})^2 b^2 + \frac{1}{3}c^{3/2} \tanh^{-1}(\sqrt{cx})^2 b^2 - \frac{\log^2(cx^2 + 1)b^2}{12x^3} + \frac{4}{3}c^{3/2} \tan^{-1}(\sqrt{cx}) b^2 + \frac{4}{3}c^{3/2} \tanh^{-1}(\sqrt{cx}) b^2 -$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x^4,x]

[Out] 
$$\begin{aligned} & (-2*a*b*c)/(3*x) - (2*a*b*c^{(3/2)}*ArcTan[\sqrt{c}*x])/3 + (4*b^2*c^{(3/2)}*Arc \\ & Tan[\sqrt{c}*x])/3 - (1/3)*b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]^2 + (4*b^2*c^{(3/2)}* \\ & ArcTanh[\sqrt{c}*x])/3 + (b^2*c^{(3/2)}*ArcTanh[\sqrt{c}*x]^2)/3 - (2*b^2*c^{(3/2)}* \\ & ArcTanh[\sqrt{c}*x]*Log[2/(1 - \sqrt{c}*x)]/3 + (2*b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]* \\ & Log[2/(1 - I*\sqrt{c}*x)]/3 - (b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]*Log[((1 + I)*(1 - \\ & \sqrt{c}*x))/(1 - I*\sqrt{c}*x)]/3 - (2*b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]*Log[2/(1 + \\ & \sqrt{c}*x)]/3 + (2*b^2*c^{(3/2)}*ArcTanh[\sqrt{c}*x]*Log[2/(1 + \sqrt{c}*x)]/3 - \\ & (b^2*c^{(3/2)}*ArcTanh[\sqrt{c}*x]*Log[(-2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \\ & \sqrt{c}*x))]/3 - (b^2*c^{(3/2)}*ArcTanh[\sqrt{c}*x]*Log[(2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \\ & \sqrt{c})*(1 + \sqrt{c}*x))]/3 - (b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]*Log[((1 - I)*(1 + \sqrt{c}*x))/ \\ & (1 - I*\sqrt{c}*x)]/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]* \\ & Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^2]))/(3*x) + (b*c^{(3/2)}*ArcTanh[\sqrt{c}*x]* \\ & (2*a - b*Log[1 - c*x^2])/3 - (2*a - b*Log[1 - c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/ \\ & (3*x^3) - (2*b^2*c*Log[1 + c*x^2])/(3*x) - (b^2*c^{(3/2)}*ArcTan[\sqrt{c}*x]*Log[1 + c*x^2])/3 + \\ & (b^2*c^{(3/2)}*ArcTanh[\sqrt{c}*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/ \\ & (6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^{(3/2)}*PolyLog[2, 1 - 2/(1 - \\ & \sqrt{c}*x)]/3 - (1/3)*b^2*c^{(3/2)}*PolyLog[2, 1 - 2/(1 - I*\sqrt{c}*x)] + (1/6)*b^2*c^{(3/2)}* \\ & PolyLog[2, 1 - ((1 + I)*(1 - \sqrt{c}*x))/(1 - I*\sqrt{c}*x)] - (1/3)*b^2*c^{(3/2)}*PolyLog[2, \\ & 1 - 2/(1 + \sqrt{c}*x)] - (b^2*c^{(3/2)}*PolyLog[2, 1 - 2/(1 + \sqrt{c}*x)]/3 + (b^2*c^{(3/2)}*PolyLog[2, \\ & 1 + (2*\sqrt{c}*(1 - \sqrt{-c}*x))/((\sqrt{-c} - \sqrt{c})*(1 + \sqrt{c}*x))]/6 + (b^2*c^{(3/2)}*PolyLog[2, \\ & 1 - (2*\sqrt{c}*(1 + \sqrt{-c}*x))/((\sqrt{-c} + \sqrt{c})*(1 + \sqrt{c}*x))]/6 + (1/6)*b^2*c^{(3/2)}*PolyLog[2, \\ & 1 - ((1 - I)*(1 + \sqrt{c}*x))/(1 - I*\sqrt{c}*x)] \end{aligned}$$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*(b\_.))^(q\_.)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a + b\*Log[c\*(d + e\*x^n)^p])^(q - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

Rule 2476

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p]]^q, x^m\*(f + g\*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 5984

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)\*(x\_)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*e\*(p + 1)), x] + Dist[1/(c\*d), Int[(a + b\*ArcTanh[c\*x])^p/(1 - c\*x), x], x] /; FreeQ[{a, b, c, d, e



}, x] && EqQ[c^2\*d + e, 0] && IGtQ[p, 0]

### Rule 5918

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2402

Int[Log[(c\_.)/((d\_.) + (e\_.)\*(x\_))]/((f\_.) + (g\_.)\*(x\_)^2), x\_Symbol] :> -Dist[e/g, Subst[Int[Log[2\*d\*x]/(1 - 2\*d\*x), x], x, 1/(d + e\*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2\*d] && EqQ[e^2\*f + d^2\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v_]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)*(x_)/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)]/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rubi steps



**Mathematica [F]** time = 2.5538, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^4, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^4, x]

**Maple [F]** time = 0.175, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Artanh}(cx^2))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/x^4, x)

[Out] int((a+b\*arctanh(c\*x^2))^2/x^4, x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^4,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/x\*\*4,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*2/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2/x^4, x)

$$3.76 \quad \int \frac{\left(a+b \tanh^{-1}(cx^2)\right)^2}{x^6} dx$$

**Optimal.** Leaf size=1176

result too large to display

```
[Out] (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/5 + (b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*x) - (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[1 - c*x^2]))/(15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2])^2/(20*x^5) - (a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15*x^3) + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(10*x^5) - (b^2*Log[1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + I*Sqrt[c]*x)] - (b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/5 + (b^2*c^(5/2)*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/10 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)]
```

---

**Rubi [A]** time = 1.98966, antiderivative size = 1176, normalized size of antiderivative = 1., number of steps used = 77, number of rules used = 24, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.5$ , Rules used = {6099, 2457, 2476, 2455, 325, 206, 207, 2470, 12, 5984, 5918, 2402, 2315, 6742, 203, 30, 2557, 5992, 5920, 2447, 4928, 4856, 4920, 4854}

result too large to display

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/x^6,x]

[Out] 
$$\begin{aligned} & (-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^{5/2} \\ & \text{ArcTan}[\text{Sqrt}[c]*x])/5 - (4*b^2*c^{5/2}*\text{ArcTan}[\text{Sqrt}[c]*x])/15 + (I/5)*b^2*c^{5/2} \\ & \text{ArcTan}[\text{Sqrt}[c]*x]^2 + (4*b^2*c^{5/2}*\text{ArcTanh}[\text{Sqrt}[c]*x])/15 + (b^2*c^{5/2} \\ & \text{ArcTanh}[\text{Sqrt}[c]*x]^2)/5 - (2*b^2*c^{5/2}*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 - \\ & \text{Sqrt}[c]*x)])/5 - (2*b^2*c^{5/2}*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 - I*\text{Sqrt}[c]* \\ & x)])/5 + (b^2*c^{5/2}*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[(1 + I)*(1 - \text{Sqrt}[c]*x)/(1 - \\ & I*\text{Sqrt}[c]*x)])/5 + (2*b^2*c^{5/2}*\text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 + I*\text{Sqrt}[c]*x) \\ & ])/5 + (2*b^2*c^{5/2}*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[2/(1 + \text{Sqrt}[c]*x)])/5 - (b^2*c^{5/2} \\ & \text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[(-2*\text{Sqrt}[c]*(1 - \text{Sqrt}[-c]*x))/((\text{Sqrt}[-c] - \text{Sqrt} \\ & [c])*(1 + \text{Sqrt}[c]*x))])/5 - (b^2*c^{5/2}*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{Log}[(2*\text{Sqrt}[c] \\ & ]*(1 + \text{Sqrt}[-c]*x))/((\text{Sqrt}[-c] + \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x))])/5 + (b^2*c^{5/2} \\ & \text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[(1 - I)*(1 + \text{Sqrt}[c]*x)/(1 - I*\text{Sqrt}[c]*x)])/5 + \\ & (b^2*c*\text{Log}[1 - c*x^2])/(15*x^3) - (b^2*c^2*\text{Log}[1 - c*x^2])/(5*x) - (b^2*c^{5/2} \\ & \text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[1 - c*x^2])/5 - (b*c*(2*a - b*\text{Log}[1 - c*x^2]))/( \\ & 15*x^3) - (b*c^2*(2*a - b*\text{Log}[1 - c*x^2]))/(5*x) + (b*c^{5/2}*\text{ArcTanh}[\text{Sqrt} \\ & [c]*x]*(2*a - b*\text{Log}[1 - c*x^2]))/5 - (2*a - b*\text{Log}[1 - c*x^2])^2/(20*x^5) - ( \\ & a*b*\text{Log}[1 + c*x^2])/(5*x^5) - (2*b^2*c*\text{Log}[1 + c*x^2])/(15*x^3) + (b^2*c^{5/2} \\ & \text{ArcTan}[\text{Sqrt}[c]*x]*\text{Log}[1 + c*x^2])/5 + (b^2*c^{5/2}*\text{ArcTanh}[\text{Sqrt}[c]*x]*\text{L} \\ & \text{og}[1 + c*x^2])/5 + (b^2*\text{Log}[1 - c*x^2]*\text{Log}[1 + c*x^2])/(10*x^5) - (b^2*\text{Log}[ \\ & 1 + c*x^2]^2)/(20*x^5) - (b^2*c^{5/2}*\text{PolyLog}[2, 1 - 2/(1 - \text{Sqrt}[c]*x)])/5 \\ & + (I/5)*b^2*c^{5/2}*\text{PolyLog}[2, 1 - 2/(1 - I*\text{Sqrt}[c]*x)] - (I/10)*b^2*c^{5/2} \\ & *\text{PolyLog}[2, 1 - ((1 + I)*(1 - \text{Sqrt}[c]*x))/(1 - I*\text{Sqrt}[c]*x)] + (I/5)*b^2*c^{5/2} \\ & *\text{PolyLog}[2, 1 - 2/(1 + I*\text{Sqrt}[c]*x)] - (b^2*c^{5/2}*\text{PolyLog}[2, 1 - 2/ \\ & (1 + \text{Sqrt}[c]*x)])/5 + (b^2*c^{5/2}*\text{PolyLog}[2, 1 + (2*\text{Sqrt}[c]*(1 - \text{Sqrt}[-c]* \\ & x))/((\text{Sqrt}[-c] - \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x))])/10 + (b^2*c^{5/2}*\text{PolyLog}[2, 1 \\ & - (2*\text{Sqrt}[c]*(1 + \text{Sqrt}[-c]*x))/((\text{Sqrt}[-c] + \text{Sqrt}[c])*(1 + \text{Sqrt}[c]*x))])/10 \\ & - (I/10)*b^2*c^{5/2}*\text{PolyLog}[2, 1 - ((1 - I)*(1 + \text{Sqrt}[c]*x))/(1 - I*\text{Sqrt} \\ & [c]*x)] \end{aligned}$$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a +



$b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]^{(q-1)} / (d + e \cdot x^n), x, x] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

### Rule 2476

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_)^{(n_)})^{(p_.)}] \cdot (b_.)]^{(q_.)} \cdot (x_)^{(m_.)} \cdot ((f_.) + (g_.) \cdot (x_)^{(s_)})^{(r_.)}, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]^{(q)}, x^m \cdot (f + g \cdot x^s)^r, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

### Rule 2455

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_)^{(n_)})^{(p_.)}] \cdot (b_.)] \cdot ((f_.) \cdot (x_))^{(m_.)}, x\_Symbol] := \text{Simp}[(f \cdot x)^{(m+1)} \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]) / (f \cdot (m+1)), x] - \text{Dist}[(b \cdot e \cdot n \cdot p) / (f \cdot (m+1)), \text{Int}[(x^{(n-1)} \cdot (f \cdot x)^{(m+1)}) / (d + e \cdot x^n), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 325

$\text{Int}[(c_.) \cdot (x_)^{(m_.)} \cdot ((a_.) + (b_.) \cdot (x_)^{(n_)})^{(p_.)}, x\_Symbol] := \text{Simp}[(c \cdot x)^{(m+1)} \cdot (a + b \cdot x^n)^{(p+1)} / (a \cdot c \cdot (m+1)), x] - \text{Dist}[(b \cdot (m+n \cdot (p+1) + 1)) / (a \cdot c^n \cdot (m+1)), \text{Int}[(c \cdot x)^{(m+n)} \cdot (a + b \cdot x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

$\text{Int}[(a_.) + (b_.) \cdot (x_)^2]^{(-1)}, x\_Symbol] := \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 207

$\text{Int}[(a_.) + (b_.) \cdot (x_)^2]^{(-1)}, x\_Symbol] := -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 2470

$\text{Int}[(a_.) + \text{Log}[(c_.) \cdot ((d_.) + (e_.) \cdot (x_)^{(n_)})^{(p_.)}] \cdot (b_.)] / ((f_.) + (g_.) \cdot (x_)^2), x\_Symbol] := \text{With}[\{u = \text{IntHide}[1 / (f + g \cdot x^2), x]\}, \text{Simp}[u \cdot (a + b \cdot \text{Log}[c \cdot (d + e \cdot x^n)^p]), x] - \text{Dist}[b \cdot e \cdot n \cdot p, \text{Int}[(u \cdot x^{(n-1)}) / (d + e \cdot x^n), x], x]] /;$  FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 5984

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))^(p_)*(x_)/((d_) + (e_)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5918

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))^(p_)/((d_) + (e_)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)/e, Int[((a + b*ArcTanh[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/((1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_)/((d_) + (e_)*(x_))]/((f_) + (g_)*(x_)^2), x_Symbol] := -Dist[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_)*(x_)/((d_) + (e_)*(x_))], x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 30

`Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]`

### Rule 2557

`Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]`

### Rule 5992

`Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])`

### Rule 5920

`Int[((a_) + ArcTanh[(c_)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*Log[(2*c*(d + e*x))/(c*d + e)*(1 + c*x)])/e, x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 - e^2, 0]`

### Rule 2447

`Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]`

### Rule 4928

`Int[(((a_) + ArcTan[(c_)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])`

### Rule 4856

`Int[((a_) + ArcTan[(c_)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c`

\*x])\*Log[(2\*c\*(d + e\*x))/((c\*d + I\*e)\*(1 - I\*c\*x))]/e, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2\*d^2 + e^2, 0]

#### Rule 4920

Int[(((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)\*(x\_.))/((d\_.) + (e\_.)\*(x\_.)^2), x\_Symbol] := -Simp[(I\*(a + b\*ArcTan[c\*x])^(p + 1))/(b\*e\*(p + 1)), x] - Dist[1/(c\*d), Int[(a + b\*ArcTan[c\*x])^p/(I - c\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2\*d] && IGtQ[p, 0]

#### Rule 4854

Int[((a\_.) + ArcTan[(c\_.)\*(x\_.)]\*(b\_.))^(p\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[((a + b\*ArcTan[c\*x])^p\*Log[2/(1 + (e\*x)/d)])/e, x] + Dist[(b\*c\*p)/e, Int[((a + b\*ArcTan[c\*x])^(p - 1)\*Log[2/(1 + (e\*x)/d)])/((1 + c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 + e^2, 0]

#### Rubi steps



**Mathematica [F]** time = 3.01188, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^6, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^2])^2/x^6, x]

**Maple [F]** time = 0.167, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^2))^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/x^6, x)

[Out] int((a+b\*arctanh(c\*x^2))^2/x^6, x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^6,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)/x^6, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/x\*\*6,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*2/x\*\*6, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/x^6,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2/x^6, x)

### 3.77 $\int x^3 \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$

**Optimal.** Leaf size=141

$$-\frac{3b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^2}\right)}{4c^2} - \frac{3b^2 \log\left(\frac{2}{1-cx^2}\right) \left(a + b \tanh^{-1}(cx^2)\right)}{2c^2} - \frac{\left(a + b \tanh^{-1}(cx^2)\right)^3}{4c^2} + \frac{3b \left(a + b \tanh^{-1}(cx^2)\right)^2}{4c^2}$$

[Out]  $(3*b*(a + b*ArcTanh[c*x^2])^2)/(4*c^2) + (3*b*x^2*(a + b*ArcTanh[c*x^2])^2)/(4*c) - (a + b*ArcTanh[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTanh[c*x^2])^3)/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)]/(2*c^2) - (3*b^3*PolyLog[2, 1 - 2/(1 - c*x^2)])/(4*c^2)$

**Rubi [B]** time = 4.21552, antiderivative size = 479, normalized size of antiderivative = 3.4, number of steps used = 155, number of rules used = 30, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.875$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$-\frac{3b^3 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right)}{8c^2} + \frac{3b^3 \text{PolyLog}\left(2, \frac{1}{2}(cx^2 + 1)\right)}{8c^2} - \frac{3b^2 \log^2(cx^2 + 1) (2a - b \log(1 - cx^2))}{32c^2} + \frac{3b^2 \log\left(\frac{1}{2}(cx^2 + 1)\right) (a + b \tanh^{-1}(cx^2))}{4c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x^2])^3,x]

[Out]  $(-3*b*(1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^2)/(16*c^2) - ((1 - c*x^2)*(2*a - b*Log[1 - c*x^2])^3)/(16*c^2) + ((1 - c*x^2)^2*(2*a - b*Log[1 - c*x^2])^3)/(32*c^2) + (3*b^2*(2*a - b*Log[1 - c*x^2])*Log[(1 + c*x^2)/2])/(8*c^2) + (3*b^3*Log[(1 - c*x^2)/2]*Log[1 + c*x^2])/(8*c^2) + (3*b^2*x^2*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2])/(8*c) - (3*b*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/(32*c^2) + (3*b*x^4*(2*a - b*Log[1 - c*x^2])^2*Log[1 + c*x^2])/32 + (3*b^3*(1 + c*x^2)*Log[1 + c*x^2]^2)/(16*c^2) - (3*b^2*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2]^2)/(32*c^2) + (3*b^2*x^4*(2*a - b*Log[1 - c*x^2])*Log[1 + c*x^2]^2)/32 - (b^3*(1 + c*x^2)*Log[1 + c*x^2]^3)/(16*c^2) + (b^3*(1 + c*x^2)^2*Log[1 + c*x^2]^3)/(32*c^2) - (3*b^3*PolyLog[2, (1 - c*x^2)/2])/(8*c^2) + (3*b^3*PolyLog[2, (1 + c*x^2)/2])/(8*c^2)$

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 -



$c*x^n)/2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

#### Rule 2454

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*(x_)^{(m_.)}, x\_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{GtQ}[(m+1)/n, 0] \|\| \text{IGtQ}[q, 0]) \&\& !(EqQ[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

#### Rule 2401

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_)^{(q_.)}), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(f + g*x)^q*(a + b*\text{Log}[c*(d + e*x)^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{IGtQ}[q, 0]$

#### Rule 2389

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}, x\_Symbol] :> \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\}$

#### Rule 2296

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))^{(p_.)}, x\_Symbol] :> \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{(p-1)}, x], x] /; \text{FreeQ}\{a, b, c, n\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{IntegerQ}[2*p]$

#### Rule 2295

$\text{Int}[\text{Log}[(c_.)*(x_)^{(n_.)}], x\_Symbol] :> \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n\}, x\}$

#### Rule 2390

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_)^{(q_.)}), x\_Symbol] :> \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \&\& \text{EqQ}[e*f - d*g, 0]$

#### Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

#### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.))*(x_)^(r_.), x_Symbol]
:> Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

#### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((h_.)*(x_)
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.))*((k_.) + (l_.)*(x_)^(r_.), x_Sym
bol]
:> Dist[1/e, Subst[Int[(k*x)/d]^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.)), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
```

```
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.)), x_Symbol] := Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

$(g*(q + 1)), x] - \text{Dist}[(b*e*n)/(g*(q + 1)), \text{Int}[(f + g*x)^(q + 1)/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 2375

$\text{Int}[(\text{Log}[(d_.)*((e_.) + (f_.)*(x_.)^(m_.))]^(r_.))*((a_.) + \text{Log}[(c_.)*(x_.)^(n_.)])*(b_.)^(p_.)]/(x_.), x\_Symbol] := \text{Simp}[(\text{Log}[d*(e + f*x^m)^r]*(a + b*\text{Log}[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - \text{Dist}[(f*m*r)/(b*n*(p + 1)), \text{Int}[(x^(m - 1)*(a + b*\text{Log}[c*x^n])^(p + 1))/(e + f*x^m), x], x] /;$  FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2317

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^(n_.)]*(b_.)^(p_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] := \text{Simp}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^p)/e, x] - \text{Dist}[(b*n*p)/e, \text{Int}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^(p - 1))/x, x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2425

$\text{Int}[(\text{Log}[(f_.)*(x_.)^(m_.)]*((a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.)))/(x_.), x\_Symbol] := \text{Simp}[(\text{Log}[f*x^m]^2*(a + b*\text{Log}[c*(d + e*x)^n]))/(2*m), x] - \text{Dist}[(b*e*n)/(2*m), \text{Int}[\text{Log}[f*x^m]^2/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rubi steps

$$\begin{aligned}
\int x^3 (a + b \tanh^{-1}(cx^2))^3 dx &= \int \left( \frac{1}{8} x^3 (2a - b \log(1 - cx^2))^3 + \frac{3}{8} bx^3 (-2a + b \log(1 - cx^2))^2 \log(1 + cx^2) - \frac{3}{8} b^2 x^3 \log(1 + cx^2) \right) dx \\
&= \frac{1}{8} \int x^3 (2a - b \log(1 - cx^2))^3 dx + \frac{1}{8} (3b) \int x^3 (-2a + b \log(1 - cx^2))^2 \log(1 + cx^2) dx - \frac{3}{8} b^2 \int x^3 \log(1 + cx^2) dx \\
&= \frac{1}{16} \text{Subst} \left( \int x (2a - b \log(1 - cx))^3 dx, x, x^2 \right) + \frac{1}{16} (3b) \text{Subst} \left( \int x (-2a + b \log(1 - cx))^2 \log(1 + cx) dx, x, x^2 \right) - \frac{3}{16} b^2 \text{Subst} \left( \int x \log(1 + cx) dx, x, x^2 \right) \\
&= \frac{3}{32} bx^4 (2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{32} b^2 x^4 (2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{32} b^2 x^4 \log^3(1 + cx^2) \\
&= \frac{3}{32} bx^4 (2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{32} b^2 x^4 (2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{32} b^2 x^4 \log^3(1 + cx^2) \\
&= \frac{3}{32} bx^4 (2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{32} b^2 x^4 (2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{32} b^2 x^4 \log^3(1 + cx^2) \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c^2} + \frac{(1 - cx^2)^2(2a - b \log(1 - cx^2))^3}{32c^2} - \frac{3b(2a - b \log(1 - cx^2))^3}{64c^2} \\
&= -\frac{9b(1 - cx^2)(2a - b \log(1 - cx^2))^2}{32c^2} + \frac{3b(1 - cx^2)^2(2a - b \log(1 - cx^2))^2}{64c^2} - \frac{(1 - cx^2)^3}{64c^2} \\
&= \frac{9ab^2x^2}{8c} + \frac{9b^3x^2}{16c} + \frac{3b^3(1 - cx^2)^2}{128c^2} - \frac{3b^3(1 + cx^2)^2}{128c^2} + \frac{3b^2(1 - cx^2)^2(2a - b \log(1 - cx^2))}{64c^2} \\
&= \frac{9ab^2x^2}{8c} + \frac{9b^3x^2}{8c} + \frac{3b^3(1 - cx^2)^2}{128c^2} - \frac{3b^3(1 + cx^2)^2}{128c^2} + \frac{9b^3(1 - cx^2) \log(1 - cx^2)}{16c^2} + \frac{3b^2(1 - cx^2)^2(2a - b \log(1 - cx^2))}{64c^2} \\
&= \frac{3ab^2x^2}{4c} + \frac{15b^3x^2}{16c} + \frac{9b^3(1 - cx^2) \log(1 - cx^2)}{16c^2} - \frac{3b(1 - cx^2)(2a - b \log(1 - cx^2))^2}{16c^2} \\
&= \frac{3ab^2x^2}{4c} + \frac{3b^3x^2}{4c} + \frac{3b^3(1 - cx^2) \log(1 - cx^2)}{8c^2} - \frac{3b(1 - cx^2)(2a - b \log(1 - cx^2))^2}{16c^2} - \frac{3b^2(1 - cx^2)^3}{64c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.464082, size = 185, normalized size = 1.31

$$6b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^2)}\right) + a\left(2a^2c^2x^4 + 6abcx^2 + 3ab \log(1 - cx^2) - 3ab \log(cx^2 + 1) + 6b^2 \log(1 - c^2x^4)\right) + 6$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x^2])^3,x]

[Out]  $(6b^2(-1 + cx^2)(a + b + acx^2)\text{ArcTanh}[cx^2]^2 + 2b^3(-1 + c^2x^4)\text{ArcTanh}[cx^2]^3 + 6b\text{ArcTanh}[cx^2](a^2cx^2(2b + acx^2) - 2b^2\text{Log}[1 + E^{(-2\text{ArcTanh}[cx^2])}]) + a(6ab^2cx^2 + 2a^2c^2x^4 + 3ab\text{Log}[1 - cx^2] - 3ab\text{Log}[1 + cx^2] + 6b^2\text{Log}[1 - c^2x^4]) + 6b^3\text{PolyLog}[2, -E^{(-2\text{ArcTanh}[cx^2])}])/(8c^2)$

**Maple [C]** time = 0.269, size = 751, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x^2))^3,x)

[Out]  $1/32b^3(c^2x^4-1)/c^2\ln(cx^2+1)^3+3/32b^2(-x^4b\ln(-cx^2+1)*c^2+2*ac^2x^4+2*b*cx^2+b\ln(-cx^2+1)-2*a+2*b)/c^2\ln(cx^2+1)^2+(3/32b^3(c^2x^4-1)/c^2\ln(-cx^2+1)^2-3/8b^2x^2*(acx^2+b)/c\ln(-cx^2+1)-3/8b*(-a^2c^2x^4-2*ab*cx^2-b\ln(-cx^2+1)*a-b^2\ln(-cx^2+1))/c^2)\ln(cx^2+1)+3/16b^3/cx^2\ln(-cx^2+1)^2-3/8a^2b*x^4\ln(-cx^2+1)+3/8a^2b/c^2\ln(cx^2-1)-3/16ab^2/c^2\ln(-cx^2+1)^2+3/8/c^2b^3\ln(-cx^2+1)-3/16b^3/c^2-3/8/c^2b^3\ln(cx^2-1)-1/32b^3x^4\ln(-cx^2+1)^3-3/16b^3/c^2\ln(-cx^2+1)^2+1/32b^3/c^2\ln(-cx^2+1)^3+3/4/ca^2b*x^2+3/16b^2/c^2a*\ln(cx^2-1)-3/8a^2b/c^2\ln(cx^2+1)+3/4ab^2/c^2\ln(cx^2+1)+3/4b^2/c*\text{Sum}(-\ln(x-\alpha)*\ln(-cx^2+1)+2*c*(-1/2*\ln(x-\alpha))*(\ln(\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=1)-x+\alpha)/\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=1))+\ln(\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=2)-x+\alpha)/\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=2)))/c-1/2*(\text{dilog}(\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=1)-x+\alpha)/\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=1))+\text{dilog}(\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=2)-x+\alpha)/\text{RootOf}(\_Z^2*c+2*\_Z*\alpha*c-2,\text{index}=2))/c)*b/c,\alpha=\text{RootOf}(\_Z^2*c+1))+1/4*x^4*a^3+3/16*a*b^2*x^4*\ln(-cx^2+1)^2+9/16*a*b^2/c^2*\ln(-cx^2+1)-3/4*a*b^2/cx^2*\ln(-cx^2+1)$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="maxima")

[Out]  $\frac{3}{4}ab^2x^4\operatorname{arctanh}(cx^2)^2 + \frac{1}{4}a^3x^4 + \frac{3}{8}(2x^4\operatorname{arctanh}(cx^2) + c(2x^2/c^2 - \log(cx^2 + 1)/c^3 + \log(cx^2 - 1)/c^3))a^2b + \frac{3}{16}(4c(2x^2/c^2 - \log(cx^2 + 1)/c^3 + \log(cx^2 - 1)/c^3)\operatorname{arctanh}(cx^2) - (2(\log(cx^2 - 1) - 2)\log(cx^2 + 1) - \log(cx^2 + 1)^2 - \log(cx^2 - 1)^2 - 4\log(cx^2 - 1))/c^2)a^2b^2 - \frac{1}{128}(4x^4\log(-cx^2 + 1)^3 + 3c^3(x^4/c^3 + \log(c^2x^4 - 1)/c^5) - 6c((cx^4 + 2x^2)/c^2 + 2\log(cx^2 - 1)/c^3)\log(-cx^2 + 1)^2 + 21c^2(2x^2/c^3 - \log(cx^2 + 1)/c^4 + \log(cx^2 - 1)/c^4) + c(6(c^2x^4 + 6cx^2 + 2\log(cx^2 - 1)^2 + 6\log(cx^2 - 1))\log(-cx^2 + 1)/c^3 - (3c^2x^4 + 42cx^2 + 4\log(cx^2 - 1)^3 + 18\log(cx^2 - 1)^2 + 42\log(cx^2 - 1))/c^3) - 1152c\operatorname{integrate}(1/4x^3\log(cx^2 + 1)/(c^3x^4 - c), x) - 2(12cx^2\log(cx^2 + 1)^2 + 2(c^2x^4 - 1)\log(cx^2 + 1)^3 - 3(c^2x^4 - 2cx^2 - 2(c^2x^4 - 1)\log(cx^2 + 1) + 1)\log(-cx^2 + 1)^2 + 3(c^2x^4 + 6cx^2 - 2(c^2x^4 - 1)\log(cx^2 + 1)^2 - 8(cx^2 + 1)\log(cx^2 + 1)\log(-cx^2 + 1))/c^2 + 18\log(4c^3x^4 - 4c)/c^2 - 384\operatorname{integrate}(1/4x\log(cx^2 + 1)/(c^3x^4 - c), x))b^3$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x^3\operatorname{artanh}(cx^2)^3 + 3ab^2x^3\operatorname{artanh}(cx^2)^2 + 3a^2bx^3\operatorname{artanh}(cx^2) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="fricas")

[Out]  $\operatorname{integral}(b^3x^3\operatorname{arctanh}(cx^2)^3 + 3a^2bx^3\operatorname{arctanh}(cx^2)^2 + 3a^2bx^3\operatorname{arctanh}(cx^2) + a^3x^3, x)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atanh}(cx^2))^3 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*2))\*\*3,x)

[Out] Integral(x\*\*3\*(a + b\*atanh(c\*x\*\*2))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^3\*x^3, x)

### 3.78 $\int x \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$

**Optimal.** Leaf size=134

$$-\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^2}\right) (a + b \tanh^{-1}(cx^2))}{2c} + \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^2}\right)}{4c} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^2))^3 + \frac{(a + b \tanh^{-1}(cx^2))^3}{2c}$$

[Out] (a + b\*ArcTanh[c\*x^2])^3/(2\*c) + (x^2\*(a + b\*ArcTanh[c\*x^2])^3)/2 - (3\*b\*(a + b\*ArcTanh[c\*x^2])^2\*Log[2/(1 - c\*x^2)])/(2\*c) - (3\*b^2\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[2, 1 - 2/(1 - c\*x^2)])/(2\*c) + (3\*b^3\*PolyLog[3, 1 - 2/(1 - c\*x^2)])/(4\*c)

**Rubi [B]** time = 2.32313, antiderivative size = 390, normalized size of antiderivative = 2.91, number of steps used = 82, number of rules used = 23, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.643$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$-\frac{3b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^2)\right) (2a - b \log(1 - cx^2))}{4c} - \frac{3b^3 \text{PolyLog}\left(3, \frac{1}{2}(1 - cx^2)\right)}{4c} - \frac{3b^3 \text{PolyLog}\left(3, \frac{1}{2}(cx^2 + 1)\right)}{4c} + \frac{3b^3 \text{PolyLog}\left(3, \frac{1}{2}(cx^2 + 1)\right)}{4c}$$

Warning: Unable to verify antiderivative.

[In] Int[x\*(a + b\*ArcTanh[c\*x^2])^3, x]

[Out] -((1 - c\*x^2)\*(2\*a - b\*Log[1 - c\*x^2])^3)/(16\*c) + (3\*b\*(2\*a - b\*Log[1 - c\*x^2])^2\*Log[(1 + c\*x^2)/2])/(8\*c) - (3\*b\*(2\*a - b\*Log[1 - c\*x^2])^2\*Log[1 + c\*x^2])/(16\*c) + (3\*b\*x^2\*(2\*a - b\*Log[1 - c\*x^2])^2\*Log[1 + c\*x^2])/16 + (3\*b^3\*Log[(1 - c\*x^2)/2]\*Log[1 + c\*x^2]^2)/(8\*c) + (3\*b^2\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2]^2)/(16\*c) + (3\*b^2\*x^2\*(2\*a - b\*Log[1 - c\*x^2])\*Log[1 + c\*x^2]^2)/16 + (b^3\*(1 + c\*x^2)\*Log[1 + c\*x^2]^3)/(16\*c) - (3\*b^2\*(2\*a - b\*Log[1 - c\*x^2])\*PolyLog[2, (1 - c\*x^2)/2])/(4\*c) + (3\*b^3\*Log[1 + c\*x^2]^2)\*PolyLog[2, (1 + c\*x^2)/2])/(4\*c) - (3\*b^3\*PolyLog[3, (1 - c\*x^2)/2])/(4\*c) - (3\*b^3\*PolyLog[3, (1 + c\*x^2)/2])/(4\*c)

#### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function0 fQ[x^(m + 1), u, x]

Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.)]\*(g\_.)), x\_Symbol] := Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m)))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c

, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m)], x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2346

Int[(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.))/(x\_), x\_Symbol] := Dist[d, Int[((d + e\*x)^(q - 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] + Dist[e, Int[(d + e\*x)^(q - 1)\*(a + b\*Log[c\*x^n])^p, x], x] /; Fre

$eQ[\{a, b, c, d, e, n\}, x] \&\& IGtQ[p, 0] \&\& GtQ[q, 0] \&\& IntegerQ[2*q]$

### Rule 2301

$Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x\_Symbol] \rightarrow Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[\{a, b, c, n\}, x]$

### Rule 6742

$Int[u_, x\_Symbol] \rightarrow With[\{v = ExpandIntegrand[u, x]\}, Int[v, x] /; SumQ[v]]$

### Rule 43

$Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x\_Symbol] \rightarrow Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[\{a, b, c, d, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& IGtQ[m, 0] \&\& (!IntegerQ[n] || (EqQ[c, 0] \&\& LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])$

### Rule 2394

$Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[\{a, b, c, d, e, f, g, n\}, x] \&\& NeQ[e*f - d*g, 0]$

### Rule 2393

$Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[\{a, b, c, d, e, f, g\}, x] \&\& NeQ[e*f - d*g, 0] \&\& EqQ[g + c*(e*f - d*g), 0]$

### Rule 2391

$Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x\_Symbol] \rightarrow -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[\{c, d, e, n\}, x] \&\& EqQ[c*d, 1]$

### Rule 2375

$Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.)]^(r_.))*((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)^(p_.)/(x_), x\_Symbol] \rightarrow Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m - 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[\{a, b, c, d,$

$e, f, r, m, n\}, x]$  && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2425

Int[(Log[(f\_.)\*(x\_)^(m\_.)]\*((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.)))/(x\_), x\_Symbol] :> Simp[(Log[f\*x^m]^2\*(a + b\*Log[c\*(d + e\*x)^n])]/(2\*m), x] - Dist[(b\*e\*n)/(2\*m), Int[Log[f\*x^m]^2/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rubi steps

$$\begin{aligned}
\int x(a + b \tanh^{-1}(cx^2))^3 dx &= \int \left( \frac{1}{8}x(2a - b \log(1 - cx^2))^3 + \frac{3}{8}bx(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2) - \frac{3}{8}b^2x(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2) \right) dx \\
&= \frac{1}{8} \int x(2a - b \log(1 - cx^2))^3 dx + \frac{1}{8}(3b) \int x(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2) dx - \frac{3}{8}b^2 \int x(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2) dx \\
&= \frac{1}{16} \text{Subst} \left( \int (2a - b \log(1 - cx))^3 dx, x, x^2 \right) + \frac{1}{16}(3b) \text{Subst} \left( \int (-2a + b \log(1 - cx))^2 \log(1 + cx) dx, x, x^2 \right) - \frac{3}{16}b^2 \text{Subst} \left( \int (-2a + b \log(1 - cx)) \log^2(1 + cx) dx, x, x^2 \right) \\
&= \frac{3}{16}bx^2(2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{16}b^2x^2(2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{16}b^3x^2 \log^3(1 + cx^2) \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3}{16}bx^2(2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{16}b^2x^2(2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{16}b^3x^2 \log^3(1 + cx^2) \\
&= -\frac{3b(1 - cx^2)(2a - b \log(1 - cx^2))^2}{16c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3}{16}bx^2(2a - b \log(1 - cx^2))^2 \log(1 + cx^2) + \frac{3}{16}b^2x^2(2a - b \log(1 - cx^2)) \log^2(1 + cx^2) - \frac{3}{16}b^3x^2 \log^3(1 + cx^2) \\
&= \frac{3}{4}ab^2x^2 - \frac{3b^3x^2}{8} - \frac{3b(1 - cx^2)(2a - b \log(1 - cx^2))^2}{16c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3}{4}ab^2x^2 + \frac{3b^3(1 - cx^2) \log(1 - cx^2)}{8c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3b(2a - b \log(1 - cx^2))^2 \log\left(\frac{1}{2}(1 + cx^2)\right)}{8c} - \frac{3b(2a - b \log(1 - cx^2)) \log^2\left(\frac{1}{2}(1 + cx^2)\right)}{8c} \\
&= \frac{3b^3x^2}{8} + \frac{3b^3(1 - cx^2) \log(1 - cx^2)}{8c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3b(2a - b \log(1 - cx^2))^2 \log\left(\frac{1}{2}(1 + cx^2)\right)}{8c} - \frac{3b(2a - b \log(1 - cx^2)) \log^2\left(\frac{1}{2}(1 + cx^2)\right)}{8c} \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3b(2a - b \log(1 - cx^2))^2 \log\left(\frac{1}{2}(1 + cx^2)\right)}{8c} - \frac{3b(2a - b \log(1 - cx^2)) \log^2\left(\frac{1}{2}(1 + cx^2)\right)}{8c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3b(2a - b \log(1 - cx^2))^2 \log\left(\frac{1}{2}(1 + cx^2)\right)}{8c} - \frac{3b(2a - b \log(1 - cx^2)) \log^2\left(\frac{1}{2}(1 + cx^2)\right)}{8c}
\end{aligned}$$

**Mathematica [A]** time = 0.222373, size = 213, normalized size = 1.59

$$\frac{3ab^2 \left( \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(cx^2)} \right) + \tanh^{-1}(cx^2) \left( cx^2 \tanh^{-1}(cx^2) - \tanh^{-1}(cx^2) - 2 \log \left( e^{-2 \tanh^{-1}(cx^2)} + 1 \right) \right) \right)}{2c} + \frac{3b^3 \log^3 \left( \frac{1}{2}(1 + cx^2) \right)}{8c} - \frac{3b^3 \log^2 \left( \frac{1}{2}(1 + cx^2) \right)}{8c} - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16c} + \frac{3b(2a - b \log(1 - cx^2))^2 \log \left( \frac{1}{2}(1 + cx^2) \right)}{8c} - \frac{3b(2a - b \log(1 - cx^2)) \log^2 \left( \frac{1}{2}(1 + cx^2) \right)}{8c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^2])^3,x]

[Out] (a^3\*x^2)/2 + (3\*a^2\*b\*x^2\*ArcTanh[c\*x^2])/2 + (3\*a^2\*b\*Log[1 - c^2\*x^4])/(4\*c) + (3\*a\*b^2\*(ArcTanh[c\*x^2]\*(-ArcTanh[c\*x^2] + c\*x^2\*ArcTanh[c\*x^2] - 2

\*Log[1 + E^(-2\*ArcTanh[c\*x^2])] + PolyLog[2, -E^(-2\*ArcTanh[c\*x^2])])/(2\*c) + (b^3\*(ArcTanh[c\*x^2]^2\*(-ArcTanh[c\*x^2] + c\*x^2\*ArcTanh[c\*x^2] - 3\*Log[1 + E^(-2\*ArcTanh[c\*x^2])]) + 3\*ArcTanh[c\*x^2]\*PolyLog[2, -E^(-2\*ArcTanh[c\*x^2])]) + (3\*PolyLog[3, -E^(-2\*ArcTanh[c\*x^2])])/(2)))/(2\*c)

**Maple [B]** time = 0.007, size = 298, normalized size = 2.2

$$\frac{x^2 a^3}{2} + \frac{b^3 x^2 (\operatorname{Artanh}(cx^2))^3}{2} + \frac{b^3 (\operatorname{Artanh}(cx^2))^3}{2c} - \frac{3b^3 (\operatorname{Artanh}(cx^2))^2}{2c} \ln \left( \frac{(cx^2 + 1)^2}{-c^2 x^4 + 1} + 1 \right) - \frac{3b^3 \operatorname{Artanh}(cx^2)}{2c} \operatorname{polylog} \left( 2, -\frac{(cx^2 + 1)^2}{-c^2 x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c\*x^2))^3,x)

[Out] 1/2\*x^2\*a^3+1/2\*b^3\*x^2\*arctanh(c\*x^2)^3+1/2/c\*b^3\*arctanh(c\*x^2)^3-3/2/c\*b^3\*arctanh(c\*x^2)^2\*ln((c\*x^2+1)^2/(-c^2\*x^4+1)+1)-3/2/c\*b^3\*arctanh(c\*x^2)\*polylog(2,-(c\*x^2+1)^2/(-c^2\*x^4+1))+3/4/c\*b^3\*polylog(3,-(c\*x^2+1)^2/(-c^2\*x^4+1))+3/2\*arctanh(c\*x^2)^2\*x^2\*a\*b^2-3/c\*ln((c\*x^2+1)^2/(-c^2\*x^4+1)+1)\*arctanh(c\*x^2)\*a\*b^2+3/2/c\*a\*b^2\*arctanh(c\*x^2)^2-3/2/c\*polylog(2,-(c\*x^2+1)^2/(-c^2\*x^4+1))\*a\*b^2+3/2\*x^2\*a^2\*b\*arctanh(c\*x^2)+3/4/c\*a^2\*b\*ln(-c^2\*x^4+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^3 x^2 + \frac{3(2cx^2 \operatorname{artanh}(cx^2) + \log(-c^2x^4 + 1))a^2b}{4c} - \frac{(b^3cx^2 - b^3) \log(-cx^2 + 1)^3 - 3(2ab^2cx^2 + (b^3cx^2 + b^3) \log(cx^2 + 1))}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="maxima")

[Out] 1/2\*a^3\*x^2 + 3/4\*(2\*c\*x^2\*arctanh(c\*x^2) + log(-c^2\*x^4 + 1))\*a^2\*b/c - 1/16\*((b^3\*c\*x^2 - b^3)\*log(-c\*x^2 + 1)^3 - 3\*(2\*a\*b^2\*c\*x^2 + (b^3\*c\*x^2 + b^3)\*log(c\*x^2 + 1))\*log(-c\*x^2 + 1)^2)/c - integrate(-1/8\*((b^3\*c\*x^3 - b^3\*x)\*log(c\*x^2 + 1)^3 + 6\*(a\*b^2\*c\*x^3 - a\*b^2\*x)\*log(c\*x^2 + 1)^2 - 3\*(4\*a\*b^2\*c\*x^3 + (b^3\*c\*x^3 - b^3\*x)\*log(c\*x^2 + 1)^2 + 2\*((2\*a\*b^2\*c + b^3\*c)\*x^3 - (2\*a\*b^2 - b^3)\*x)\*log(c\*x^2 + 1))\*log(-c\*x^2 + 1))/(c\*x^2 - 1), x)



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x \operatorname{artanh}(cx^2)^3 + 3ab^2x \operatorname{artanh}(cx^2)^2 + 3a^2bx \operatorname{artanh}(cx^2) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")`

[Out] `integral(b^3*x*arctanh(c*x^2)^3 + 3*a*b^2*x*arctanh(c*x^2)^2 + 3*a^2*b*x*arctanh(c*x^2) + a^3*x, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(a + b \operatorname{atanh}(cx^2))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c*x**2))**3,x)`

[Out] `Integral(x*(a + b*atanh(c*x**2))**3, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^3*x, x)`

$$3.79 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^3}{x} dx$$

**Optimal.** Leaf size=207

$$\frac{3}{4}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^2}\right)(a+b \tanh^{-1}(cx^2)) - \frac{3}{4}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx^2} - 1\right)(a+b \tanh^{-1}(cx^2)) - \frac{3}{4}b \text{PolyLog}\left(2, \frac{2}{1-cx^2}\right)(a+b \tanh^{-1}(cx^2))$$

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*PolyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x^2)])/8
```

**Rubi [A]** time = 0.559731, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{3}{4}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^2}\right)(a+b \tanh^{-1}(cx^2)) - \frac{3}{4}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx^2} - 1\right)(a+b \tanh^{-1}(cx^2)) - \frac{3}{4}b \text{PolyLog}\left(2, \frac{2}{1-cx^2}\right)(a+b \tanh^{-1}(cx^2))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])^3/x, x]
```

```
[Out] (a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - (3*b*(a + b*ArcTanh[c*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/4 + (3*b*(a + b*ArcTanh[c*x^2])^2*PolyLog[2, -1 + 2/(1 - c*x^2)])/4 + (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, 1 - 2/(1 - c*x^2)])/4 - (3*b^2*(a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1 - c*x^2)])/4 - (3*b^3*PolyLog[4, 1 - 2/(1 - c*x^2)])/8 + (3*b^3*PolyLog[4, -1 + 2/(1 - c*x^2)])/8
```

### Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] := Simp[2*(a +
b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x))]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6062

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^2))^3}{x} dx &= \frac{1}{2} \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^3}{x} dx, x, x^2 \right) \\
&= (a + b \tanh^{-1}(cx^2))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - (3bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \tanh^{-1}}{1 - c^2x^2} \right) \\
&= (a + b \tanh^{-1}(cx^2))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) + \frac{1}{2} (3bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \log}{1 - c^2x^2} \right) \\
&= (a + b \tanh^{-1}(cx^2))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - \frac{3}{4} b (a + b \tanh^{-1}(cx^2))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^2} \right) + \\
&= (a + b \tanh^{-1}(cx^2))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - \frac{3}{4} b (a + b \tanh^{-1}(cx^2))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^2} \right) + \\
&= (a + b \tanh^{-1}(cx^2))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^2} \right) - \frac{3}{4} b (a + b \tanh^{-1}(cx^2))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^2} \right) +
\end{aligned}$$

**Mathematica [A]** time = 0.191135, size = 211, normalized size = 1.02

$$\frac{3}{8} b \left( 2 \text{PolyLog} \left( 2, \frac{cx^2 + 1}{1 - cx^2} \right) (a + b \tanh^{-1}(cx^2))^2 - 2 \text{PolyLog} \left( 2, \frac{cx^2 + 1}{cx^2 - 1} \right) (a + b \tanh^{-1}(cx^2))^2 + b \left( -2 \text{PolyLog} \left( 3, \frac{cx^2 + 1}{1 - cx^2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^3/x,x]

[Out] (a + b\*ArcTanh[c\*x^2])^3\*ArcTanh[1 + 2/(-1 + c\*x^2)] + (3\*b\*(2\*(a + b\*ArcTanh[c\*x^2])^2\*PolyLog[2, (1 + c\*x^2)/(1 - c\*x^2)] - 2\*(a + b\*ArcTanh[c\*x^2])^2\*PolyLog[2, (1 + c\*x^2)/(-1 + c\*x^2)] + b\*(-2\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[3, (1 + c\*x^2)/(1 - c\*x^2)] + 2\*(a + b\*ArcTanh[c\*x^2])\*PolyLog[3, (1 + c\*x^2)/(-1 + c\*x^2)] + b\*(PolyLog[4, (1 + c\*x^2)/(1 - c\*x^2)] - PolyLog[4, (1 + c\*x^2)/(-1 + c\*x^2)])))/8

**Maple [F]** time = 0.138, size = 0, normalized size = 0.

$$\int \frac{(a + b \text{Artanh}(cx^2))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))^3/x,x)`

[Out] `int((a+b*arctanh(c*x^2))^3/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3(\log(cx^2 + 1) - \log(-cx^2 + 1))^3}{8x} + \frac{3ab^2(\log(cx^2 + 1) - \log(-cx^2 + 1))^2}{4x} + \frac{3a^2b(\log(cx^2 + 1) - \log(-cx^2 + 1))}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="maxima")`

[Out] `a^3*log(x) + integrate(1/8*b^3*(log(c*x^2 + 1) - log(-c*x^2 + 1))^3/x + 3/4*a*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + 3/2*a^2*b*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx^2)^3 + 3ab^2 \operatorname{artanh}(cx^2)^2 + 3a^2b \operatorname{artanh}(cx^2) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="fricas")`

[Out] `integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh(c*x^2) + a^3)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*3/x,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^3/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^3/x, x)

$$3.80 \quad \int \frac{(a+b \tanh^{-1}(cx^2))^3}{x^3} dx$$

**Optimal.** Leaf size=125

$$-\frac{3}{2}b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx^2+1} - 1\right)(a+b \tanh^{-1}(cx^2)) - \frac{3}{4}b^3c \operatorname{PolyLog}\left(3, \frac{2}{cx^2+1} - 1\right) + \frac{1}{2}c(a+b \tanh^{-1}(cx^2))^3 - \frac{(a+b \tanh^{-1}(cx^2))^3}{x^3}$$

[Out]  $(c*(a + b*\operatorname{ArcTanh}[c*x^2])^3)/2 - (a + b*\operatorname{ArcTanh}[c*x^2])^3/(2*x^2) + (3*b*c*(a + b*\operatorname{ArcTanh}[c*x^2])^2*\operatorname{Log}[2 - 2/(1 + c*x^2)])/2 - (3*b^2*c*(a + b*\operatorname{ArcTanh}[c*x^2])* \operatorname{PolyLog}[2, -1 + 2/(1 + c*x^2)])/2 - (3*b^3*c*\operatorname{PolyLog}[3, -1 + 2/(1 + c*x^2)])/4$

**Rubi [F]** time = 0.780071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(cx^2))^3}{x^3} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcTanh}[c*x^2])^3/x^3, x]$

[Out]  $(3*b*c*\operatorname{Log}[c*x^2]*(2*a - b*\operatorname{Log}[1 - c*x^2])^2)/16 - ((1 - c*x^2)*(2*a - b*\operatorname{Log}[1 - c*x^2])^3)/(16*x^2) + (3*b^3*c*\operatorname{Log}[-(c*x^2)]*\operatorname{Log}[1 + c*x^2]^2)/16 - (b^3*(1 + c*x^2)*\operatorname{Log}[1 + c*x^2]^3)/(16*x^2) - (3*b^2*c*(2*a - b*\operatorname{Log}[1 - c*x^2])* \operatorname{PolyLog}[2, 1 - c*x^2])/8 + (3*b^3*c*\operatorname{Log}[1 + c*x^2]*\operatorname{PolyLog}[2, 1 + c*x^2])/8 - (3*b^3*c*\operatorname{PolyLog}[3, 1 - c*x^2])/8 - (3*b^3*c*\operatorname{PolyLog}[3, 1 + c*x^2])/8 + (3*b*\operatorname{Defer}[\operatorname{Subst}][\operatorname{Defer}[\operatorname{Int}]][((-2*a + b*\operatorname{Log}[1 - c*x])^2*\operatorname{Log}[1 + c*x])/x^2, x], x, x^2))/16 - (3*b^2*\operatorname{Defer}[\operatorname{Subst}][\operatorname{Defer}[\operatorname{Int}]][((-2*a + b*\operatorname{Log}[1 - c*x])*\operatorname{Log}[1 + c*x]^2)/x^2, x], x, x^2))/16$

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^2))^3}{x^3} dx &= \int \left( \frac{(2a - b \log(1 - cx^2))^3}{8x^3} + \frac{3b(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2)}{8x^3} - \frac{3b^2(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2)}{8x^3} + \frac{b^3 \log^3(1 + cx^2)}{8x^3} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - cx^2))^3}{x^3} dx + \frac{1}{8}(3b) \int \frac{(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2)}{x^3} dx - \frac{1}{8}(3b^2) \int \frac{(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2)}{x^3} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^2)}{x^3} dx \\
&= \frac{1}{16} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^3}{x^2} dx, x, x^2 \right) + \frac{1}{16}(3b) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^2} dx, x, x^2 \right) - \frac{1}{16}(3b^2) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^2} dx, x, x^2 \right) + \frac{1}{16} \int \frac{b^3 \log^3(1 + cx^2)}{x^3} dx \\
&= -\frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16x^2} - \frac{b^3(1 + cx^2) \log^3(1 + cx^2)}{16x^2} + \frac{1}{16}(3b) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^2} dx, x, x^2 \right) - \frac{1}{16}(3b^2) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^2} dx, x, x^2 \right) + \frac{1}{16} \int \frac{b^3 \log^3(1 + cx^2)}{x^3} dx \\
&= \frac{3}{16} bc \log(cx^2) (2a - b \log(1 - cx^2))^2 - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16x^2} + \frac{3}{16} b^3 c \log(-cx^2) \log(1 + cx^2) \\
&= \frac{3}{16} bc \log(cx^2) (2a - b \log(1 - cx^2))^2 - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16x^2} + \frac{3}{16} b^3 c \log(-cx^2) \log(1 + cx^2) \\
&= \frac{3}{16} bc \log(cx^2) (2a - b \log(1 - cx^2))^2 - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16x^2} + \frac{3}{16} b^3 c \log(-cx^2) \log(1 + cx^2) \\
&= \frac{3}{16} bc \log(cx^2) (2a - b \log(1 - cx^2))^2 - \frac{(1 - cx^2)(2a - b \log(1 - cx^2))^3}{16x^2} + \frac{3}{16} b^3 c \log(-cx^2) \log(1 + cx^2)
\end{aligned}$$

**Mathematica [C]** time = 0.414398, size = 222, normalized size = 1.78

$$\frac{1}{4} \left( 6ab^2c \left( \tanh^{-1}(cx^2) \left( \left( 1 - \frac{1}{cx^2} \right) \tanh^{-1}(cx^2) + 2 \log \left( 1 - e^{-2 \tanh^{-1}(cx^2)} \right) \right) - \text{PolyLog} \left( 2, e^{-2 \tanh^{-1}(cx^2)} \right) \right) + 2b^3c \left( 3 \tan^{-1}(cx^2) \log(1 + cx^2) - \log(1 - cx^2) \log(1 + cx^2) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^3/x^3, x]

[Out] ((-2\*a^3)/x^2 - (6\*a^2\*b\*ArcTanh[c\*x^2])/x^2 + 12\*a^2\*b\*c\*Log[x] - 3\*a^2\*b\*c\*Log[1 - c^2\*x^4] + 6\*a\*b^2\*c\*(ArcTanh[c\*x^2]\*((1 - 1/(c\*x^2))\*ArcTanh[c\*x^2] + 2\*Log[1 - E^(-2\*ArcTanh[c\*x^2])]) - PolyLog[2, E^(-2\*ArcTanh[c\*x^2])]) + 2\*b^3\*c\*((I/8)\*Pi^3 - ArcTanh[c\*x^2]^3 - ArcTanh[c\*x^2]^3/(c\*x^2) + 3\*ArcTanh[c\*x^2]^2\*Log[1 - E^(2\*ArcTanh[c\*x^2])] + 3\*ArcTanh[c\*x^2]\*PolyLog[2, E^(2\*ArcTanh[c\*x^2])] - (3\*PolyLog[3, E^(2\*ArcTanh[c\*x^2])])/2))/4



**Maple [F]** time = 0.194, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Artanh}(cx^2))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^3/x^3,x)

[Out] int((a+b\*arctanh(c\*x^2))^3/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3}{4} \left( c(\log(c^2x^4 - 1) - \log(x^4)) + \frac{2 \operatorname{artanh}(cx^2)}{x^2} \right) a^2 b - \frac{a^3}{2x^2} - \frac{(b^3cx^2 - b^3) \log(-cx^2 + 1)^3 + 3(2ab^2 + (b^3cx^2 + b^3))}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^3/x^3,x, algorithm="maxima")

[Out] -3/4\*(c\*(log(c^2\*x^4 - 1) - log(x^4)) + 2\*arctanh(c\*x^2)/x^2)\*a^2\*b - 1/2\*a^3/x^2 - 1/16\*((b^3\*c\*x^2 - b^3)\*log(-c\*x^2 + 1)^3 + 3\*(2\*a\*b^2 + (b^3\*c\*x^2 + b^3)\*log(c\*x^2 + 1))\*log(-c\*x^2 + 1)^2)/x^2 - integrate(-1/8\*((b^3\*c\*x^2 - b^3)\*log(c\*x^2 + 1)^3 + 6\*(a\*b^2\*c\*x^2 - a\*b^2)\*log(c\*x^2 + 1)^2 + 3\*(4\*a\*b^2\*c\*x^2 - (b^3\*c\*x^2 - b^3)\*log(c\*x^2 + 1)^2 + 2\*(b^3\*c^2\*x^4 + 2\*a\*b^2 - (2\*a\*b^2\*c - b^3\*c)\*x^2)\*log(c\*x^2 + 1))\*log(-c\*x^2 + 1))/(c\*x^5 - x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^3 \operatorname{artanh}(cx^2)^3 + 3ab^2 \operatorname{artanh}(cx^2)^2 + 3a^2b \operatorname{artanh}(cx^2) + a^3}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^3/x^3,x, algorithm="fricas")

[Out] `integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctanh(c*x^2) + a^3)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))**3/x**3,x)`

[Out] `Integral((a + b*atanh(c*x**2))**3/x**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^3/x^3, x)`

$$3.81 \quad \int \frac{(a + b \tanh^{-1}(cx^2))^3}{x^5} dx$$

**Optimal.** Leaf size=139

$$-\frac{3}{4}b^3c^2\text{PolyLog}\left(2, \frac{2}{cx^2+1} - 1\right) + \frac{3}{2}b^2c^2 \log\left(2 - \frac{2}{cx^2+1}\right)(a + b \tanh^{-1}(cx^2)) + \frac{3}{4}bc^2(a + b \tanh^{-1}(cx^2))^2 + \frac{1}{4}c^2(a + b \tanh^{-1}(cx^2))^3$$

[Out] (3\*b\*c^2\*(a + b\*ArcTanh[c\*x^2])^2)/4 - (3\*b\*c\*(a + b\*ArcTanh[c\*x^2])^2)/(4\*x^2) + (c^2\*(a + b\*ArcTanh[c\*x^2])^3)/4 - (a + b\*ArcTanh[c\*x^2])^3/(4\*x^4) + (3\*b^2\*c^2\*(a + b\*ArcTanh[c\*x^2])\*Log[2 - 2/(1 + c\*x^2)])/2 - (3\*b^3\*c^2\*PolyLog[2, -1 + 2/(1 + c\*x^2)])/4

**Rubi [F]** time = 1.55794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^2))^3}{x^5} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^2])^3/x^5,x]

[Out] (3\*a\*b^2\*c^2\*Log[x])/4 - (3\*b\*c\*(1 - c\*x^2)\*(2\*a - b\*Log[1 - c\*x^2])^2)/(32\*x^2) + (3\*b\*c^2\*Log[c\*x^2]\*(2\*a - b\*Log[1 - c\*x^2])^2)/32 + (c^2\*(2\*a - b\*Log[1 - c\*x^2])^3)/32 - (2\*a - b\*Log[1 - c\*x^2])^3/(32\*x^4) - (3\*b^3\*c\*(1 + c\*x^2)\*Log[1 + c\*x^2]^2)/(32\*x^2) - (3\*b^3\*c^2\*Log[-(c\*x^2)]\*Log[1 + c\*x^2]^2)/32 + (b^3\*c^2\*Log[1 + c\*x^2]^3)/32 - (b^3\*Log[1 + c\*x^2]^3)/(32\*x^4) - (3\*b^3\*c^2\*PolyLog[2, -(c\*x^2)])/16 + (3\*b^3\*c^2\*PolyLog[2, c\*x^2])/16 - (3\*b^2\*c^2\*(2\*a - b\*Log[1 - c\*x^2])\*PolyLog[2, 1 - c\*x^2])/16 - (3\*b^3\*c^2\*Log[1 + c\*x^2]\*PolyLog[2, 1 + c\*x^2])/16 - (3\*b^3\*c^2\*PolyLog[3, 1 - c\*x^2])/16 + (3\*b^3\*c^2\*PolyLog[3, 1 + c\*x^2])/16 + (3\*b\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])^2\*Log[1 + c\*x])/x^3, x], x, x^2])/16 - (3\*b^2\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])\*Log[1 + c\*x]^2)/x^3, x], x, x^2])/16

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^2))^3}{x^5} dx &= \int \left( \frac{(2a - b \log(1 - cx^2))^3}{8x^5} + \frac{3b(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2)}{8x^5} - \frac{3b^2(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2)}{8x^5} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - cx^2))^3}{x^5} dx + \frac{1}{8}(3b) \int \frac{(-2a + b \log(1 - cx^2))^2 \log(1 + cx^2)}{x^5} dx - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^2)) \log^2(1 + cx^2)}{x^5} dx \\
&= \frac{1}{16} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^3}{x^3} dx, x, x^2 \right) + \frac{1}{16}(3b) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^3} dx, x, x^2 \right) - \frac{1}{16} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^3} dx \\
&= -\frac{(2a - b \log(1 - cx^2))^3}{32x^4} - \frac{b^3 \log^3(1 + cx^2)}{32x^4} + \frac{1}{16}(3b) \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^3} dx, x, x^2 \right) - \frac{1}{16} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^3} dx \\
&= -\frac{(2a - b \log(1 - cx^2))^3}{32x^4} - \frac{b^3 \log^3(1 + cx^2)}{32x^4} - \frac{1}{32}(3b) \text{Subst} \left( \int \frac{(2a - b \log(x))^2}{x \left(\frac{1}{c} - \frac{x}{c}\right)^2} dx, x, 1 - cx^2 \right) + \frac{1}{32} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^3} dx \\
&= -\frac{(2a - b \log(1 - cx^2))^3}{32x^4} - \frac{b^3 \log^3(1 + cx^2)}{32x^4} - \frac{1}{32}(3b) \text{Subst} \left( \int \frac{(2a - b \log(x))^2}{\left(\frac{1}{c} - \frac{x}{c}\right)^2} dx, x, 1 - cx^2 \right) + \frac{1}{32} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^3} dx \\
&= -\frac{3bc(1 - cx^2)(2a - b \log(1 - cx^2))^2}{32x^2} - \frac{(2a - b \log(1 - cx^2))^3}{32x^4} - \frac{3b^3c(1 + cx^2) \log^2(1 + cx^2)}{32x^2} + \frac{1}{32} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^3} dx \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{3bc(1 - cx^2)(2a - b \log(1 - cx^2))^2}{32x^2} + \frac{3}{32} bc^2 \log(cx^2)(2a - b \log(1 - cx^2))^2 \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{3bc(1 - cx^2)(2a - b \log(1 - cx^2))^2}{32x^2} + \frac{3}{32} bc^2 \log(cx^2)(2a - b \log(1 - cx^2))^2 \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{3bc(1 - cx^2)(2a - b \log(1 - cx^2))^2}{32x^2} + \frac{3}{32} bc^2 \log(cx^2)(2a - b \log(1 - cx^2))^2
\end{aligned}$$

**Mathematica [A]** time = 0.290196, size = 218, normalized size = 1.57

$$\frac{-6b^3c^2x^4 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx^2)}\right) + a\left(-2a^2 - 3abc^2x^4 \log(1 - cx^2) + 3abc^2x^4 \log(cx^2 + 1) - 6abcx^2 + 12b^2c^2x^4 \log\left(\frac{1}{c} - \frac{x}{c}\right)\right)}{32x^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^3/x^5, x]

```
[Out] (6*b^2*(-1 + c*x^2)*(a + a*c*x^2 + b*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 +
c^2*x^4)*ArcTanh[c*x^2]^3 - 6*b*ArcTanh[c*x^2]*(a^2 + 2*a*b*c*x^2 - 2*b^2*c
^2*x^4*Log[1 - E^(-2*ArcTanh[c*x^2])]) + a*(-2*a^2 - 6*a*b*c*x^2 - 3*a*b*c^
2*x^4*Log[1 - c*x^2] + 3*a*b*c^2*x^4*Log[1 + c*x^2] + 12*b^2*c^2*x^4*Log[(c
*x^2)/Sqrt[1 - c^2*x^4]]) - 6*b^3*c^2*x^4*PolyLog[2, E^(-2*ArcTanh[c*x^2])])
)/(8*x^4)
```

**Maple [F]** time = 0.231, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^2))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctanh(c*x^2))^3/x^5,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{8} \left( \left( c \log(cx^2 + 1) - c \log(cx^2 - 1) - \frac{2}{x^2} \right) c - \frac{2 \operatorname{artanh}(cx^2)}{x^4} \right) a^2 b + \frac{3}{16} \left( \left( 2 (\log(cx^2 - 1) - 2) \log(cx^2 + 1) - \log(cx^2 - 1)^2 - \log(cx^2 + 1)^2 - 4 \log(cx^2 - 1) + 16 \log(x) \right) c^2 + 4 (c \log(cx^2 + 1) - c \log(cx^2 - 1) - 2/x^2) c \operatorname{arctanh}(cx^2) \right) a b^2 - 1/32 b^3 \left( (c^2 x^4 - 1) \log(-c x^2 + 1)^3 + 3 (2 c x^2 - (c^2 x^4 - 1) \log(cx^2 + 1)) \log(-c x^2 + 1)^2 \right) / x^4 + 4 \operatorname{integrate}(-((c x^2 - 1) \log(cx^2 + 1)^3 + 3 (2 c^2 x^4 - (c x^2 - 1) \log(cx^2 + 1)^2 - (c^3 x^6 - c x^2) \log(cx^2 + 1)) \log(-c x^2 + 1)) / (c x^7 - x^5), x) - 3/4 a b^2 \operatorname{arctanh}(cx^2)^2 / x^4 - 1/4 a^3 / x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^4
)*a^2*b + 3/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 -
log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1)
- c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*a*b^2 - 1/32*b^3*(((c^2*x^4
- 1)*log(-c*x^2 + 1)^3 + 3*(2*c*x^2 - (c^2*x^4 - 1)*log(c*x^2 + 1))*log(-c
*x^2 + 1)^2)/x^4 + 4*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^3 + 3*(2*c^2*x^4
- (c*x^2 - 1)*log(c*x^2 + 1)^2 - (c^3*x^6 - c*x^2)*log(c*x^2 + 1))*log(-c
*x^2 + 1))/(c*x^7 - x^5), x)) - 3/4*a*b^2*arctanh(c*x^2)^2/x^4 - 1/4*a^3/x^4
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx^2)^3 + 3ab^2 \operatorname{artanh}(cx^2)^2 + 3a^2b \operatorname{artanh}(cx^2) + a^3}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^3/x^5,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x^2)^3 + 3\*a\*b^2\*arctanh(c\*x^2)^2 + 3\*a^2\*b\*arctanh(c\*x^2) + a^3)/x^5, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*3/x\*\*5,x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))\*\*3/x\*\*5, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^3/x^5,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^3/x^5, x)

### 3.82 $\int (dx)^{5/2} \left( a + b \tanh^{-1} (cx^2) \right) dx$

**Optimal.** Leaf size=317

$$\frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{bd^{5/2} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}c^{7/4}} + \frac{bd^{5/2} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}c^{7/4}} + \frac{2bd^{5/2}}{7\sqrt{2}c^{7/4}}$$

```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```

**Rubi [A]** time = 0.334361, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6097, 16, 321, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{bd^{5/2} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}c^{7/4}} + \frac{bd^{5/2} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}c^{7/4}} + \frac{2bd^{5/2}}{7\sqrt{2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]), x]
```

```
[Out] (8*b*d*(d*x)^(3/2))/(21*c) + (2*b*d^(5/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) + (Sqrt[2]*b*d^(5/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (Sqrt[2]*b*d^(5/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) + (2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d) - (2*b*d^(5/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*c^(7/4)) - (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4)) + (b*d^(5/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*c^(7/4))
```

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_)*(v_)^{(m_)}*((b_)*(v_)^{(n_)}), x\_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}\{b, n\}, x\} \&\& \text{IntegerQ}[m]$

### Rule 321

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_))^{(p_)}), x\_Symbol] := \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 329

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_))^{(p_)}), x\_Symbol] := \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)}))/c^n]^p, x], x, (c*x)^{(1/k)}, x]] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 300

$\text{Int}[(x_)^{(m_)} / ((a_) + (b_)*(x_)^{(n_)}), x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[x^m/(r + s*x^{(n/2)}), x], x] + \text{Dist}[r/(2*a), \text{Int}[x^m/(r - s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{IGtQ}[n/4, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LtQ}[m, n/2] \&\& !\text{GtQ}[a/b, 0]$

### Rule 297

$\text{Int}[(x_)^2 / ((a_) + (b_)*(x_)^4), x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& (\text{GtQ}[a/b, 0] \mid\mid (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 1162

$\text{Int}[((d_) + (e_)*(x_)^2) / ((a_) + (c_)*(x_)^4), x\_Symbol] := \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /;$   
 $\text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$



Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int (dx)^{5/2} (a + b \tanh^{-1}(cx^2)) dx &= \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(4bc) \int \frac{x(dx)^{7/2}}{1-c^2x^4} dx}{7d} \\
&= \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(4bc) \int \frac{(dx)^{9/2}}{1-c^2x^4} dx}{7d^2} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(4bd^2) \int \frac{\sqrt{dx}}{1-c^2x^4} dx}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(8bd) \text{Subst} \left( \int \frac{x^2}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx} \right)}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(4bd^3) \text{Subst} \left( \int \frac{x^2}{d^2-cx^4} dx, x, \sqrt{dx} \right)}{7c} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{(2bd^3) \text{Subst} \left( \int \frac{1}{d-\sqrt{cx^2}} dx, x, \sqrt{dx} \right)}{7c^{3/2}} + \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{7c^{7/4}} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{2bd^{5/2} \tanh^{-1}}{7c^{7/4}} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{7c^{7/4}} + \frac{2(dx)^{7/2} (a + b \tanh^{-1}(cx^2))}{7d} - \frac{2bd^{5/2} \tanh^{-1}}{7c^{7/4}} \\
&= \frac{8bd(dx)^{3/2}}{21c} + \frac{2bd^{5/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{7c^{7/4}} + \frac{\sqrt{2}bd^{5/2} \tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{7c^{7/4}} - \frac{\sqrt{2}bd^{5/2} \tan^{-1}}{7c}
\end{aligned}$$

**Mathematica [A]** time = 0.125296, size = 241, normalized size = 0.76

$$\frac{(dx)^{5/2} \left( 12ac^{7/4}x^{7/2} + 16bc^{3/4}x^{3/2} + 12bc^{7/4}x^{7/2} \tanh^{-1}(cx^2) + 6b \log(1 - \sqrt[4]{c}\sqrt{x}) - 6b \log(\sqrt[4]{c}\sqrt{x} + 1) - 3\sqrt{2}b \log(\sqrt{cx} - \dots) \right)}{7c^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(5/2)\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] ((d\*x)^(5/2)\*(16\*b\*c^(3/4)\*x^(3/2) + 12\*a\*c^(7/4)\*x^(7/2) + 6\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 6\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 12\*b\*ArcTan[c^(1/4)\*Sqrt[x]] + 12\*b\*c^(7/4)\*x^(7/2)\*ArcTanh[c\*x^2] + 6\*b\*Log[1 - c^(1/4)\*Sqrt[x]] - 6\*b\*Log[1 + c^(1/4)\*Sqrt[x]] - 3\*Sqrt[2]

$*b*\text{Log}[1 - \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[c]*x] + 3*\text{Sqrt}[2]*b*\text{Log}[1 + \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[c]*x])]/(42*c^{(7/4)}*x^{(5/2)})$

**Maple [A]** time = 0.022, size = 302, normalized size = 1.

$$\frac{2a}{7d}(dx)^{\frac{7}{2}} + \frac{2b\text{Artanh}(cx^2)}{7d}(dx)^{\frac{7}{2}} + \frac{8bd}{21c}(dx)^{\frac{3}{2}} - \frac{d^3b\sqrt{2}}{14c^2} \ln \left( \left( dx - \sqrt{\frac{d^2}{c}}\sqrt{dx}\sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx + \sqrt{\frac{d^2}{c}}\sqrt{dx}\sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x)`

[Out]  $2/7/d*(d*x)^{(7/2)}*a+2/7/d*b*(d*x)^{(7/2)}*\text{arctanh}(c*x^2)+8/21*b*d*(d*x)^{(3/2)}/c-1/14*d^3*b/c^2/(d^2/c)^{(1/4)}*2^{(1/2)}*\ln((d*x-(d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)}+(d^2/c)^{(1/2)})/(d*x+(d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)}+(d^2/c)^{(1/2)}))-1/7*d^3*b/c^2/(d^2/c)^{(1/4)}*2^{(1/2)}*\text{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)}+1)-1/7*d^3*b/c^2/(d^2/c)^{(1/4)}*2^{(1/2)}*\text{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)}-1)+2/7*d^3*b/c^2/(d^2/c)^{(1/4)}*\text{arctan}((d*x)^{(1/2)}/(d^2/c)^{(1/4)})-1/7*d^3*b/c^2/(d^2/c)^{(1/4)}*\ln(((d*x)^{(1/2)}+(d^2/c)^{(1/4)})/((d*x)^{(1/2)}-(d^2/c)^{(1/4)}))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.2931, size = 124, normalized size = 0.39

$$\frac{\left(3bcd^2x^3 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6acd^2x^3 + 8bd^2x\right)\sqrt{dx}}{21c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/21*(3*b*c*d^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x^3 + 8*b*d^2*x)*sqrt(d*x)/c
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{5}{2}} (b \operatorname{artanh}(cx^2) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arctanh(c*x^2) + a), x)
```

### 3.83 $\int (dx)^{3/2} \left( a + b \tanh^{-1} \left( cx^2 \right) \right) dx$

**Optimal.** Leaf size=317

$$\frac{2(dx)^{5/2} \left( a + b \tanh^{-1} \left( cx^2 \right) \right)}{5d} + \frac{bd^{3/2} \log \left( \sqrt{c} \sqrt{dx} - \sqrt{2} \sqrt[4]{c} \sqrt{dx} + \sqrt{d} \right)}{5\sqrt{2}c^{5/4}} - \frac{bd^{3/2} \log \left( \sqrt{c} \sqrt{dx} + \sqrt{2} \sqrt[4]{c} \sqrt{dx} + \sqrt{d} \right)}{5\sqrt{2}c^{5/4}} - \frac{2bd^{3/2}}{5\sqrt{2}c^{5/4}}$$

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) - (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*c^(5/4))
```

**Rubi [A]** time = 0.299853, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6097, 16, 321, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(dx)^{5/2} \left( a + b \tanh^{-1} \left( cx^2 \right) \right)}{5d} + \frac{bd^{3/2} \log \left( \sqrt{c} \sqrt{dx} - \sqrt{2} \sqrt[4]{c} \sqrt{dx} + \sqrt{d} \right)}{5\sqrt{2}c^{5/4}} - \frac{bd^{3/2} \log \left( \sqrt{c} \sqrt{dx} + \sqrt{2} \sqrt[4]{c} \sqrt{dx} + \sqrt{d} \right)}{5\sqrt{2}c^{5/4}} - \frac{2bd^{3/2}}{5\sqrt{2}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]), x]
```

```
[Out] (8*b*d*Sqrt[d*x])/(5*c) - (2*b*d^(3/2)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (Sqrt[2]*b*d^(3/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) - (Sqrt[2]*b*d^(3/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) - (2*b*d^(3/2)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(5*c^(5/4)) + (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*c^(5/4)) - (b*d^(3/2)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(5*Sqrt[2]*c^(5/4))
```

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_)*(v_)^{(m_)}*((b_)*(v_))^{(n_)}, x\_Symbol] \ :> \ \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

### Rule 321

$\text{Int}[((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_)^{(n_))^{(p_)}, x\_Symbol] \ :> \ \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 329

$\text{Int}[((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_)^{(n_))^{(p_)}, x\_Symbol] \ :> \ \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)}))/c^n]^p, x], x, (c*x)^{(1/k)}, x]] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 214

$\text{Int}[((a_)+(b_)*(x_)^{(n_))^{(-1)}, x\_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^{(n/2)}), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n/4, 1] \ \&\& \ \text{!GtQ}[a/b, 0]$

### Rule 212

$\text{Int}[((a_)+(b_)*(x_)^4)^{(-1)}, x\_Symbol] \ :> \ \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{!GtQ}[a/b, 0]$

### Rule 208

$\text{Int}[((a_)+(b_)*(x_)^2)^{(-1)}, x\_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int (dx)^{3/2} (a + b \tanh^{-1}(cx^2)) dx &= \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(4bc) \int \frac{x(dx)^{5/2}}{1-c^2x^4} dx}{5d} \\
&= \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(4bc) \int \frac{(dx)^{7/2}}{1-c^2x^4} dx}{5d^2} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(4bd^2) \int \frac{1}{\sqrt{dx}(1-c^2x^4)} dx}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(8bd) \text{Subst} \left( \int \frac{1}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx} \right)}{5c} \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(4bd^3) \text{Subst} \left( \int \frac{1}{d^2-cx^4} dx, x, \sqrt{dx} \right)}{5c} - (4b) \\
&= \frac{8bd\sqrt{dx}}{5c} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{(2bd^2) \text{Subst} \left( \int \frac{1}{d-\sqrt{c}x^2} dx, x, \sqrt{dx} \right)}{5c} - (2) \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{2bd^{3/2} \tanh^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} + \frac{2(dx)^{5/2} (a + b \tanh^{-1}(cx^2))}{5d} - \frac{2bd^{3/2} \tanh^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} \\
&= \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} + \frac{\sqrt{2}bd^{3/2} \tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}} - \frac{\sqrt{2}bd^{3/2} \tan^{-1} \left( 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{5c^{5/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.0933528, size = 240, normalized size = 0.76

$$(dx)^{3/2} (4ac^{5/4}x^{5/2} + 4bc^{5/4}x^{5/2} \tanh^{-1}(cx^2) + 16b\sqrt[4]{c}\sqrt{x} + 2b \log(1 - \sqrt[4]{c}\sqrt{x}) - 2b \log(\sqrt[4]{c}\sqrt{x} + 1) + \sqrt{2}b \log(\sqrt{cx} - \sqrt{2}))$$

10

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*(a + b\*ArcTanh[c\*x^2]), x]

[Out] ((d\*x)^(3/2)\*(16\*b\*c^(1/4)\*Sqrt[x] + 4\*a\*c^(5/4)\*x^(5/2) + 2\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 2\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]])



$$\text{rt}[x]] - 4*b*\text{ArcTan}[c^{(1/4)}*\text{Sqrt}[x]] + 4*b*c^{(5/4)}*x^{(5/2)}*\text{ArcTanh}[c*x^2] + 2*b*\text{Log}[1 - c^{(1/4)}*\text{Sqrt}[x]] - 2*b*\text{Log}[1 + c^{(1/4)}*\text{Sqrt}[x]] + \text{Sqrt}[2]*b*\text{Log}[1 - \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[c]*x] - \text{Sqrt}[2]*b*\text{Log}[1 + \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[x] + \text{Sqrt}[c]*x))/(10*c^{(5/4)}*x^{(3/2)})$$

**Maple [A]** time = 0.016, size = 292, normalized size = 0.9

$$\frac{2a}{5d}(dx)^{\frac{5}{2}} + \frac{2b\text{Artanh}(cx^2)}{5d}(dx)^{\frac{5}{2}} + \frac{8bd}{5c}\sqrt{dx} - \frac{bd\sqrt{2}}{10c}\sqrt[4]{\frac{d^2}{c}}\ln\left(\left(dx + \sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2} + \sqrt{\frac{d^2}{c}}\right)\left(dx - \sqrt[4]{\frac{d^2}{c}}\sqrt{dx}\sqrt{2} + \sqrt{\frac{d^2}{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x)`

[Out]  $\frac{2}{5}d*(d*x)^{(5/2)}*a + \frac{2}{5}d*b*(d*x)^{(5/2)}*\text{arctanh}(c*x^2) + \frac{8}{5}b*d*(d*x)^{(1/2)}/c - \frac{1}{10}d*b/c*(d^2/c)^{(1/4)}*2^{(1/2)}*\ln((d*x + (d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)} + (d^2/c)^{(1/2)})/(d*x - (d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)} + (d^2/c)^{(1/2)})) - \frac{1}{5}d*b/c*(d^2/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)} + 1) - \frac{1}{5}d*b/c*(d^2/c)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)} - 1) - \frac{1}{5}d*b/c*(d^2/c)^{(1/4)}*\ln(((d*x)^{(1/2)} + (d^2/c)^{(1/4)})/((d*x)^{(1/2)} - (d^2/c)^{(1/4)})) - \frac{2}{5}d*b/c*(d^2/c)^{(1/4)}*\arctan((d*x)^{(1/2)}/(d^2/c)^{(1/4)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.27177, size = 109, normalized size = 0.34

$$\frac{(bcdx^2 \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2acdx^2 + 8bd)\sqrt{dx}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/5*(b*c*d*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*d*x^2 + 8*b*d)*sqrt(d*x)/c
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(a+b*atanh(c*x**2)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} (b \operatorname{artanh}(cx^2) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b*arctanh(c*x^2) + a), x)
```

### 3.84 $\int \sqrt{dx} \left( a + b \tanh^{-1} \left( cx^2 \right) \right) dx$

**Optimal.** Leaf size=301

$$\frac{2(dx)^{3/2} \left( a + b \tanh^{-1} \left( cx^2 \right) \right)}{3d} + \frac{b\sqrt{d} \log \left( \sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d} \right)}{3\sqrt{2}c^{3/4}} - \frac{b\sqrt{d} \log \left( \sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d} \right)}{3\sqrt{2}c^{3/4}} + \frac{2b\sqrt{d}t}{3\sqrt{2}c^{3/4}}$$

```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4))
+ (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```

**Rubi [A]** time = 0.250824, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {6097, 16, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(dx)^{3/2} \left( a + b \tanh^{-1} \left( cx^2 \right) \right)}{3d} + \frac{b\sqrt{d} \log \left( \sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d} \right)}{3\sqrt{2}c^{3/4}} - \frac{b\sqrt{d} \log \left( \sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d} \right)}{3\sqrt{2}c^{3/4}} + \frac{2b\sqrt{d}t}{3\sqrt{2}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]), x]
```

```
[Out] (2*b*Sqrt[d]*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) - (Sqrt[2]*b*
Sqrt[d]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) + (Sqr
t[2]*b*Sqrt[d]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4))
+ (2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (2*b*Sqrt[d]*ArcTanh[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]]/(3*c^(3/4)) + (b*Sqrt[d]*Log[Sqrt[d] + Sqrt[c]*Sq
rt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4)) - (b*Sqrt[d]*Log[
Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*c^(3/4
))
```

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*)^{(n_*)}), x\_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}\{b, n\}, x\} \&\& \text{IntegerQ}[m]$

### Rule 329

$\text{Int}[((c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}), x\_Symbol] := \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 301

$\text{Int}[(x_*)^{(m_*)}/((a_*) + (b_*)*(x_*)^{(n_*)}), x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)}/(r + s*x^{(n/2)}), x], x] - \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)}/(r - s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{IGtQ}[n/4, 0] \&\& \text{IGtQ}[m, 0] \&\& \text{LeQ}[n/2, m] \&\& \text{LtQ}[m, n] \&\& \text{!GtQ}[a/b, 0]$

### Rule 297

$\text{Int}[(x_*)^2/((a_*) + (b_*)*(x_*)^4), x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& (\text{GtQ}[a/b, 0] \mid\mid (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 1162

$\text{Int}[((d_*) + (e_*)*(x_*)^2)/((a_*) + (c_*)*(x_*)^4), x\_Symbol] := \text{With}\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /;$   
 $\text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

### Rule 617

$\text{Int}[((a_*) + (b_*)*(x_*) + (c_*)*(x_*)^2)^{-1}, x\_Symbol] := \text{With}\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /;$   
 $\text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \mid\mid \text{!RationalQ}[b^2 - 4*a*c]) /;$   
 $\text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} (a + b \tanh^{-1}(cx^2)) dx &= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{(4bc) \int \frac{x(dx)^{3/2}}{1-c^2x^4} dx}{3d} \\
&= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{(4bc) \int \frac{(dx)^{5/2}}{1-c^2x^4} dx}{3d^2} \\
&= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{(8bc) \text{Subst} \left( \int \frac{x^6}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx} \right)}{3d^3} \\
&= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{1}{3}(4bd) \text{Subst} \left( \int \frac{x^2}{d^2 - cx^4} dx, x, \sqrt{dx} \right) + \frac{1}{3}(4bd) \text{Subst} \left( \int \frac{1}{d - \sqrt{cx^2}} dx, x, \sqrt{dx} \right) \\
&= \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{(2bd) \text{Subst} \left( \int \frac{1}{d - \sqrt{cx^2}} dx, x, \sqrt{dx} \right)}{3\sqrt{c}} + \frac{(2bd) \text{Subst} \left( \int \frac{1}{d - \sqrt{cx^2}} dx, x, \sqrt{dx} \right)}{3\sqrt{c}} \\
&= \frac{2b\sqrt{d} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{2b\sqrt{d} \tanh^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{b\sqrt{d}}{3c^{3/4}} \\
&= \frac{2b\sqrt{d} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{2(dx)^{3/2} (a + b \tanh^{-1}(cx^2))}{3d} - \frac{2b\sqrt{d} \tanh^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{b\sqrt{d}}{3c^{3/4}} \\
&= \frac{2b\sqrt{d} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} - \frac{\sqrt{2}b\sqrt{d} \tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{\sqrt{2}b\sqrt{d} \tan^{-1} \left( 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{3c^{3/4}} + \frac{b\sqrt{d}}{3c^{3/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.071632, size = 227, normalized size = 0.75

$$\frac{\sqrt{dx} (4ac^{3/4}x^{3/2} + 4bc^{3/4}x^{3/2} \tanh^{-1}(cx^2) + 2b \log(1 - \sqrt[4]{c}\sqrt{x}) - 2b \log(\sqrt[4]{c}\sqrt{x} + 1) + \sqrt{2}b \log(\sqrt{cx} - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1) - \sqrt{2}b \log(\sqrt{cx} + \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1))}{6c^{3/4}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x^2]),x]

[Out] (Sqrt[d\*x]\*(4\*a\*c^(3/4)\*x^(3/2) - 2\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 2\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 4\*b\*ArcTan[c^(1/4)\*Sqrt[x]] + 4\*b\*c^(3/4)\*x^(3/2)\*ArcTanh[c\*x^2] + 2\*b\*Log[1 - c^(1/4)\*Sqrt[x]] - 2\*b\*Log[1 + c^(1/4)\*Sqrt[x]] + Sqrt[2]\*b\*Log[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x] - Sqrt[2]\*b\*Log[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x])/(6\*c^(3/4)\*Sqrt[x])

---

**Maple [A]** time = 0.011, size = 280, normalized size = 0.9

$$\frac{2a}{3d} (dx)^{\frac{3}{2}} + \frac{2b \operatorname{Artanh}(cx^2)}{3d} (dx)^{\frac{3}{2}} + \frac{db\sqrt{2}}{6c} \ln \left( \left( dx - \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx + \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right) \frac{1}{\sqrt[4]{\frac{d^2}{c}}} + \frac{db}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x)`

[Out]  $\frac{2}{3} \frac{a}{d} (d*x)^{\frac{3}{2}} + \frac{2}{3} \frac{b}{d} (d*x)^{\frac{3}{2}} \operatorname{arctanh}(c*x^2) + \frac{1}{6} \frac{d*b}{d^2/c} (d^2/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \ln \left( \frac{(d*x - (d^2/c)^{\frac{1}{4}}) * (d*x)^{\frac{1}{2}} * 2^{\frac{1}{2}} + (d^2/c)^{\frac{1}{4}}}{(d*x + (d^2/c)^{\frac{1}{4}}) * (d*x)^{\frac{1}{2}} * 2^{\frac{1}{2}} + (d^2/c)^{\frac{1}{4}}} \right) + \frac{1}{3} \frac{d*b}{d^2/c} (d^2/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left( \frac{2^{\frac{1}{2}}}{(d^2/c)^{\frac{1}{4}} * (d*x)^{\frac{1}{2}} + 1} \right) + \frac{1}{3} \frac{d*b}{d^2/c} (d^2/c)^{\frac{1}{4}} * 2^{\frac{1}{2}} * \arctan \left( \frac{2^{\frac{1}{2}}}{(d^2/c)^{\frac{1}{4}} * (d*x)^{\frac{1}{2}} - 1} \right) + \frac{2}{3} \frac{d*b}{d^2/c} (d^2/c)^{\frac{1}{4}} * \arctan \left( \frac{(d*x)^{\frac{1}{2}}}{(d^2/c)^{\frac{1}{4}}} \right) - \frac{1}{3} \frac{d*b}{d^2/c} (d^2/c)^{\frac{1}{4}} * \ln \left( \frac{(d*x)^{\frac{1}{2}} + (d^2/c)^{\frac{1}{4}}}{(d*x)^{\frac{1}{2}} - (d^2/c)^{\frac{1}{4}}} \right)$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 2.15992, size = 80, normalized size = 0.27

$$\frac{1}{3} \left( bx \log \left( -\frac{cx^2 + 1}{cx^2 - 1} \right) + 2ax \right) \sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")`

[Out]  $\frac{1}{3}(b*x*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*x)*\sqrt{d*x}$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**(1/2)*(a+b*atanh(c*x**2)),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b \operatorname{artanh}(cx^2) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")`

[Out] `integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a), x)`



$$3.85 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=285

$$\frac{2\sqrt{dx}(a+b \tanh^{-1}(cx^2))}{d} - \frac{b \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} + \frac{b \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} - \frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}$$

[Out]  $(-2*b*ArcTan[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) - (Sqrt[2]*b*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) + (Sqrt[2]*b*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) + (2*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*c^{(1/4)}*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*c^{(1/4)}*Sqrt[d])$

**Rubi [A]** time = 0.24214, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {6097, 16, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{2\sqrt{dx}(a+b \tanh^{-1}(cx^2))}{d} - \frac{b \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} + \frac{b \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} - \frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{d}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/Sqrt[d\*x], x]

[Out]  $(-2*b*ArcTan[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) - (Sqrt[2]*b*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) + (Sqrt[2]*b*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) + (2*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - (2*b*ArcTanh[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/(c^{(1/4)}*Sqrt[d]) - (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*c^{(1/4)}*Sqrt[d]) + (b*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*c^{(1/4)}*Sqrt[d])$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c^n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /;

FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 16

Int[(u\_)\*(v\_)^(m\_)\*((b\_)\*(v\_))^(n\_), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 329

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 301

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[x^(m - n/2)/(r + s\*x^(n/2)), x], x] - Dist[s/(2\*b), Int[x^(m - n/2)/(r - s\*x^(n/2)), x], x] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ[m, n] && !GtQ[a/b, 0]

### Rule 211

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 212

```
Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b),
2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

### Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{\sqrt{dx}} dx &= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - \frac{(4bc) \int \frac{x\sqrt{dx}}{1-c^2x^4} dx}{d} \\
&= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - \frac{(4bc) \int \frac{(dx)^{3/2}}{1-c^2x^4} dx}{d^2} \\
&= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - \frac{(8bc) \operatorname{Subst}\left(\int \frac{x^4}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx}\right)}{d^3} \\
&= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - (4bd) \operatorname{Subst}\left(\int \frac{1}{d^2 - cx^4} dx, x, \sqrt{dx}\right) + (4bd) \operatorname{Subst}\left(\int \frac{1}{d^2 + cx^4} dx, x, \sqrt{dx}\right) \\
&= \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - (2b) \operatorname{Subst}\left(\int \frac{1}{d - \sqrt{cx}^2} dx, x, \sqrt{dx}\right) - (2b) \operatorname{Subst}\left(\int \frac{1}{d + \sqrt{cx}^2} dx, x, \sqrt{dx}\right) \\
&= -\frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - \frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\frac{d}{\sqrt{c}} - \frac{\sqrt{2}\sqrt{dx}}{\sqrt[4]{c}}} dx, x, \sqrt{dx}\right)}{\sqrt{d}} \\
&= -\frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d} - \frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{b \log(\sqrt{d} + \sqrt{c}\sqrt{dx})}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} \\
&= -\frac{2b \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{\sqrt{2}b \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{\sqrt{2}b \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}(a + b \tanh^{-1}(cx^2))}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.0586972, size = 227, normalized size = 0.8

$$\frac{\sqrt{x}(4a\sqrt[4]{c}\sqrt{x} + 4b\sqrt[4]{c}\sqrt{x}\tanh^{-1}(cx^2) + 2b\log(1 - \sqrt[4]{c}\sqrt{x}) - 2b\log(\sqrt[4]{c}\sqrt{x} + 1) - \sqrt{2}b\log(\sqrt{cx} - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1) + \sqrt{2}b\log(\sqrt{cx} + \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1))}{2\sqrt[4]{c}\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/Sqrt[d\*x], x]

[Out] (Sqrt[x]\*(4\*a\*c^(1/4)\*Sqrt[x] - 2\*Sqrt[2]\*b\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 2\*Sqrt[2]\*b\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 4\*b\*ArcTan[c^(1/4)\*Sqrt[x]] + 4\*b\*c^(1/4)\*Sqrt[x]\*ArcTanh[c\*x^2] + 2\*b\*Log[1 - c^(1/4)\*Sqrt[x]] - 2\*b\*Log[1 + c^(1/4)\*Sqrt[x]] - Sqrt[2]\*b\*Log[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] + Sqrt[2]\*b\*Log[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] + Sqrt[c]\*x)) / (2\*sqrt[4]{c}\*sqrt{dx})

$(2*c^{(1/4)}*\text{Sqrt}[d*x])$

**Maple [A]** time = 0.013, size = 273, normalized size = 1.

$$2 \frac{a\sqrt{dx}}{d} + 2 \frac{b\sqrt{dx} \text{Artanh}(cx^2)}{d} + \frac{b\sqrt{2} \sqrt[4]{d^2}}{2d} \ln \left( \left( dx + \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx - \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right) + \frac{b\sqrt{2} \sqrt[4]{d^2}}{d} \sqrt[4]{\frac{d^2}{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))/(d*x)^(1/2),x)`

[Out]  $2/d*a*(d*x)^{(1/2)} + 2/d*b*(d*x)^{(1/2)}*\text{arctanh}(c*x^2) + 1/2/d*b*(d^2/c)^{(1/4)}*2^{(1/2)}*\ln((d*x+(d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)}+(d^2/c)^{(1/2)))/(d*x-(d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)}+(d^2/c)^{(1/2))}) + 1/d*b*(d^2/c)^{(1/4)}*2^{(1/2)}*\text{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)}+1) + 1/d*b*(d^2/c)^{(1/4)}*2^{(1/2)}*\text{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)}-1) - 1/d*b*(d^2/c)^{(1/4)}*\ln(((d*x)^{(1/2)}+(d^2/c)^{(1/4)))/((d*x)^{(1/2)}-(d^2/c)^{(1/4))}) - 2/d*b*(d^2/c)^{(1/4)}*\text{arctan}((d*x)^{(1/2)}/(d^2/c)^{(1/4)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.22864, size = 72, normalized size = 0.25

$$\frac{\sqrt{dx} \left( b \log \left( -\frac{cx^2+1}{cx^2-1} \right) + 2a \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(1/2),x, algorithm="fricas")

[Out] sqrt(d\*x)\*(b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 2\*a)/d

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx^2)}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/(d\*x)\*\*(1/2),x)

[Out] Integral((a + b\*atanh(c\*x\*\*2))/sqrt(d\*x), x)

**Giac [B]** time = 1.24348, size = 666, normalized size = 2.34

$$\left( \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} + \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} - \frac{2\sqrt{2}(-c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} - \frac{2\sqrt{2}(-c^3d^2)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(-\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(1/2),x, algorithm="giac")

[Out] 1/2\*((c\*d^2\*(2\*sqrt(2)\*(c^3\*d^2)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) + 2\*sqrt(d\*x))/(d^2/c)^(1/4)))/(c^2\*d^2) + 2\*sqrt(2)\*(c^3\*d^2)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) - 2\*sqrt(d\*x))/(d^2/c)^(1/4)))/(c^2\*d^2) - 2\*sqrt(2)\*(-c^3\*d^2)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) + 2\*sqrt(d\*x))/(-d^2/c)^(1/4)))/(c^2\*d^2) - 2\*sqrt(2)\*(-c^3\*d^2)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) - 2\*sqrt(d\*x))/(-d^2/c)^(1/4)))/(c^2\*d^2) + sqrt(2)\*(c^3\*d^2)^(1/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4)

$$\begin{aligned}
& ) + \sqrt{d^2/c})/(c^2*d^2) - \sqrt{2}*(c^3*d^2)^{(1/4)}*\log(d*x - \sqrt{2}*\sqrt{d*x}*(d^2/c)^{(1/4)} + \sqrt{d^2/c})/(c^2*d^2) - \sqrt{2}*(-c^3*d^2)^{(1/4)}*\log \\
& (d*x + \sqrt{2}*\sqrt{d*x}*(-d^2/c)^{(1/4)} + \sqrt{-d^2/c})/(c^2*d^2) + \sqrt{2} \\
& *(-c^3*d^2)^{(1/4)}*\log(d*x - \sqrt{2}*\sqrt{d*x}*(-d^2/c)^{(1/4)} + \sqrt{-d^2/c} \\
& )/(c^2*d^2)) + 2*\sqrt{d*x}*\log(-(c*x^2 + 1)/(c*x^2 - 1))*b + 4*\sqrt{d*x}*a \\
& )/d
\end{aligned}$$

$$3.86 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=285

$$\frac{2(a+b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{b\sqrt[4]{c} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}d^{3/2}} - \frac{b\sqrt[4]{c} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}d^{3/2}} - \frac{2b\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d}}{\sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d}}\right)}{d^{3/2}}$$

[Out]  $(-2*b*c^{(1/4)}*ArcTan[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} - (Sqrt[2]*b*c^{(1/4)}*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (Sqrt[2]*b*c^{(1/4)}*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} - (2*(a + b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^{(1/4)}*ArcTanh[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (b*c^{(1/4)}*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*d^{(3/2)}) - (b*c^{(1/4)}*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*d^{(3/2)})$

**Rubi [A]** time = 0.248436, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {6097, 16, 329, 300, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(a+b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{b\sqrt[4]{c} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}d^{3/2}} - \frac{b\sqrt[4]{c} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{\sqrt{2}d^{3/2}} - \frac{2b\sqrt[4]{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d}}{\sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^2])/(d\*x)^(3/2), x]

[Out]  $(-2*b*c^{(1/4)}*ArcTan[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} - (Sqrt[2]*b*c^{(1/4)}*ArcTan[1 - (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (Sqrt[2]*b*c^{(1/4)}*ArcTan[1 + (Sqrt[2]*c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} - (2*(a + b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + (2*b*c^{(1/4)}*ArcTanh[(c^{(1/4)}*Sqrt[d*x])/Sqrt[d]])/d^{(3/2)} + (b*c^{(1/4)}*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*d^{(3/2)}) - (b*c^{(1/4)}*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^{(1/4)}*Sqrt[d*x]])/(Sqrt[2]*d^{(3/2)})$

**Rule 6097**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol]  
 := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n])/(d\*(m + 1)), x] - Dist[(b\*c^n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /;



FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] := Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m+1)-1)\*(a+(b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 300

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[x^m/(r+s\*x^(n/2)), x], x] + Dist[r/(2\*a), Int[x^m/(r-s\*x^(n/2)), x], x] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] && !GtQ[a/b, 0]

### Rule 297

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*s), Int[(r+s\*x^2)/(a+b\*x^4), x], x] - Dist[1/(2\*s), Int[(r-s\*x^2)/(a+b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q-x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{(dx)^{3/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{(4bc) \int \frac{x}{\sqrt{dx}(1-c^2x^4)} dx}{d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{(4bc) \int \frac{\sqrt{dx}}{1-c^2x^4} dx}{d^2} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{(8bc) \text{Subst} \left( \int \frac{x^2}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx} \right)}{d^3} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{(4bc) \text{Subst} \left( \int \frac{x^2}{d^2-cx^4} dx, x, \sqrt{dx} \right)}{d} + \frac{(4bc) \text{Subst} \left( \int \frac{x^2}{d^2+cx^4} dx, x, \sqrt{dx} \right)}{d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{(2b\sqrt{c}) \text{Subst} \left( \int \frac{1}{d-\sqrt{c}x^2} dx, x, \sqrt{dx} \right)}{d} - \frac{(2b\sqrt{c}) \text{Subst} \left( \int \frac{1}{d+\sqrt{c}x^2} dx, x, \sqrt{dx} \right)}{d} \\
&= -\frac{2b\sqrt[4]{c} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{2b\sqrt[4]{c} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{(b\sqrt[4]{c}) \text{Subst} \left( \int \frac{1}{d-\sqrt{c}x^2} dx, x, \sqrt{dx} \right)}{d} \\
&= -\frac{2b\sqrt[4]{c} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}} + \frac{2b\sqrt[4]{c} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{b\sqrt[4]{c} \log(\sqrt{d} + \sqrt{c}\sqrt{x})}{d} \\
&= -\frac{2b\sqrt[4]{c} \tan^{-1} \left( \frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} - \frac{\sqrt{2}b\sqrt[4]{c} \tan^{-1} \left( 1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{\sqrt{2}b\sqrt[4]{c} \tan^{-1} \left( 1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}} \right)}{d^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{d\sqrt{dx}}
\end{aligned}$$

**Mathematica [A]** time = 0.102945, size = 268, normalized size = 0.94

$$\frac{x(4a + 4b \tanh^{-1}(cx^2)) + 2b\sqrt[4]{c}\sqrt{x} \log(1 - \sqrt[4]{c}\sqrt{x}) - 2b\sqrt[4]{c}\sqrt{x} \log(\sqrt[4]{c}\sqrt{x} + 1) - \sqrt{2}b\sqrt[4]{c}\sqrt{x} \log(\sqrt{c}x - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/(d\*x)^(3/2), x]

[Out] -(x\*(4\*a + 2\*Sqrt[2]\*b\*c^(1/4)\*Sqrt[x]\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 2\*Sqrt[2]\*b\*c^(1/4)\*Sqrt[x]\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 4\*b\*c^(1/4)\*Sqrt[x]\*ArcTan[c^(1/4)\*Sqrt[x]] + 4\*b\*ArcTanh[c\*x^2] + 2\*b\*c^(1/4)\*Sqrt[x]\*Log[1 - c^(1/4)\*Sqrt[x]] - 2\*b\*c^(1/4)\*Sqrt[x]\*Log[1 + c^(1/4)\*Sqrt[x]] - Sqrt[2]\*b\*c^(1/4)\*Sqrt[x]\*Log[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x)

+ Sqrt[2]\*b\*c^(1/4)\*Sqrt[x]\*Log[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x))/  
(2\*(d\*x)^(3/2))

**Maple [A]** time = 0.013, size = 272, normalized size = 1.

$$-2 \frac{a}{d\sqrt{dx}} - 2 \frac{b \operatorname{Arctanh}(cx^2)}{d\sqrt{dx}} + \frac{b\sqrt{2}}{2d} \ln \left( \left( dx - \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx + \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right) \frac{1}{\sqrt[4]{\frac{d^2}{c}}} + \frac{b\sqrt{2}}{d} \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/(d\*x)^(3/2),x)

[Out]  $-2/d*a/(d*x)^{(1/2)} - 2/d*b/(d*x)^{(1/2)}*\operatorname{arctanh}(c*x^2) + 1/2/d*b/(d^2/c)^{(1/4)}*2^{(1/2)}*\ln((d*x - (d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)} + (d^2/c)^{(1/2)})/(d*x + (d^2/c)^{(1/4)}*(d*x)^{(1/2)}*2^{(1/2)} + (d^2/c)^{(1/2)})) + 1/d*b/(d^2/c)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)} + 1) + 1/d*b/(d^2/c)^{(1/4)}*2^{(1/2)}*\operatorname{arctan}(2^{(1/2)}/(d^2/c)^{(1/4)}*(d*x)^{(1/2)} - 1) - 2/d*b/(d^2/c)^{(1/4)}*\operatorname{arctan}((d*x)^{(1/2)}/(d^2/c)^{(1/4)}) + 1/d*b/(d^2/c)^{(1/4)}*\ln(((d*x)^{(1/2)} + (d^2/c)^{(1/4)})/((d*x)^{(1/2)} - (d^2/c)^{(1/4)}))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.2664, size = 81, normalized size = 0.28

$$-\frac{\sqrt{dx} \left( b \log \left( -\frac{cx^2+1}{cx^2-1} \right) + 2a \right)}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(3/2),x, algorithm="fricas")

[Out] -sqrt(d\*x)\*(b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 2\*a)/(d^2\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/(d\*x)\*\*(3/2),x)

[Out] Timed out

**Giac [B]** time = 1.90645, size = 682, normalized size = 2.39

$$\frac{1}{2} bcd^2 \left( \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^3d^5} + \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^3d^5} + \frac{2\sqrt{2}(-c^3d^2)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^3d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(3/2),x, algorithm="giac")

[Out] 1/2\*b\*c\*d^2\*(2\*sqrt(2)\*(c^3\*d^2)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) + 2\*sqrt(d\*x))/(d^2/c)^(1/4))/(c^3\*d^5) + 2\*sqrt(2)\*(c^3\*d^2)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) - 2\*sqrt(d\*x))/(d^2/c)^(1/4))/(c^3\*d^5) + 2\*sqrt(2)\*(-c^3\*d^2)^(3/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) + 2\*sqrt(d\*x))/(-d^2/c)^(1/4))/(c^3\*d^5) + 2\*sqrt(2)\*(-c^3\*d^2)^(3/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) - 2\*sqrt(d\*x))/(-d^2/c)^(1/4))/(c^3\*d^5) - sqrt(2)\*(c^3\*d^2)^(3/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^3\*d^5) + sqrt(2)\*(c^3\*d^2)^(3/4)\*log(d\*x - sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^3\*d^5) - sqrt(2)\*(-c^3\*d^2)^(3/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) - sqrt(d^2/c))/(c^3\*d^5) + sqrt(2)\*(-c^3\*d^2)^(3/4)\*log(d\*x - sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) - sqrt(d^2/c))/(c^3\*d^5)

$$\begin{aligned} & (d*x + \sqrt{2}*\sqrt{d*x})*(-d^2/c)^{(1/4)} + \sqrt{-d^2/c})/(c^3*d^5) + \sqrt{2} \\ & *(-c^3*d^2)^{(3/4)}*\log(d*x - \sqrt{2}*\sqrt{d*x})*(-d^2/c)^{(1/4)} + \sqrt{-d^2/c} \\ & )/(c^3*d^5) - (b*\log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/\sqrt{d*x} + 2*a \\ & / \sqrt{d*x})/d \end{aligned}$$

$$3.87 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=301

$$\frac{2(a+b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} - \frac{bc^{3/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{3\sqrt{2}d^{5/2}} + \frac{bc^{3/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{3\sqrt{2}d^{5/2}} + \frac{2bc^{3/4} \tan^{-1}(\dots)}{3d^5}$$

```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (Sqrt[2]*b*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) + (Sqrt[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)))
```

**Rubi [A]** time = 0.249511, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {6097, 16, 329, 214, 212, 208, 205, 211, 1165, 628, 1162, 617, 204}

$$\frac{2(a+b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} - \frac{bc^{3/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{3\sqrt{2}d^{5/2}} + \frac{bc^{3/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{3\sqrt{2}d^{5/2}} + \frac{2bc^{3/4} \tan^{-1}(\dots)}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2), x]
```

```
[Out] (2*b*c^(3/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (Sqrt[2]*b*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) + (Sqrt[2]*b*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (2*b*c^(3/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(3*d^(5/2)) - (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)) + (b*c^(3/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(3*Sqrt[2]*d^(5/2)))
```

### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_)*(v_)^{(m_)}*((b_)*(v_))^{(n_)}, x\_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}\{b, n\}, x\} \&\& \text{IntegerQ}[m]$

### Rule 329

$\text{Int}[((c_)*(x_))^{(m_)}*((a_)+(b_)*(x_))^{(n_)}^{(p_)}, x\_Symbol] := \text{With}\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)}))/c^{(n)}^{(p)}, x], x, (c*x)^{(1/k)}], x]] /;$   
 $\text{FreeQ}\{a, b, c, p\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{FractionQ}[m] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 214

$\text{Int}[((a_)+(b_)*(x_))^{(n_)}^{(-1)}, x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^{(n/2)}), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{IGtQ}[n/4, 1] \&\& \text{!GtQ}[a/b, 0]$

### Rule 212

$\text{Int}[((a_)+(b_)*(x_))^{(4)}^{(-1)}, x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[r/(2*a), \text{Int}[1/(r - s*x^2), x], x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{!GtQ}[a/b, 0]$

### Rule 208

$\text{Int}[((a_)+(b_)*(x_))^{(2)}^{(-1)}, x\_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

### Rule 205

$\text{Int}[((a_)+(b_)*(x_))^{(2)}^{(-1)}, x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

### Rule 211

$\text{Int}[((a_)+(b_)*(x_))^{(4)}^{(-1)}, x\_Symbol] := \text{With}\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /;$   
 $\text{FreeQ}\{a, b\}, x\} \&\& (\text{GtQ}[a/b, 0] \mid\mid (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\&$



AtomQ[SplitProduct[SumBaseQ, b]])

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{(dx)^{5/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{(4bc) \int \frac{x}{(dx)^{3/2}(1-c^2x^4)} dx}{3d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{(4bc) \int \frac{1}{\sqrt{dx}(1-c^2x^4)} dx}{3d^2} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{(8bc) \operatorname{Subst}\left(\int \frac{1}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx}\right)}{3d^3} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{(4bc) \operatorname{Subst}\left(\int \frac{1}{d^2-cx^4} dx, x, \sqrt{dx}\right)}{3d} + \frac{(4bc) \operatorname{Subst}\left(\int \frac{1}{d^2+cx^4} dx, x, \sqrt{dx}\right)}{3d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{(2bc) \operatorname{Subst}\left(\int \frac{1}{d-\sqrt{cx^2}} dx, x, \sqrt{dx}\right)}{3d^2} + \frac{(2bc) \operatorname{Subst}\left(\int \frac{1}{d+\sqrt{cx^2}} dx, x, \sqrt{dx}\right)}{3d^2} \\
&= \frac{2bc^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{2bc^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{(bc^{3/4}) \operatorname{Subst}\left(\int \frac{1}{d^2-cx^4} dx, x, \sqrt{dx}\right)}{3d^3} \\
&= \frac{2bc^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}} + \frac{2bc^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{bc^{3/4} \log(\sqrt{d} + \sqrt{cx})}{3\sqrt{d}} \\
&= \frac{2bc^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{\sqrt{2}bc^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{\sqrt{2}bc^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{3d(dx)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.105879, size = 268, normalized size = 0.89

$$\frac{x(4a + 2bc^{3/4}x^{3/2} \log(1 - \sqrt[4]{c}\sqrt{x}) - 2bc^{3/4}x^{3/2} \log(\sqrt[4]{c}\sqrt{x} + 1) + \sqrt{2}bc^{3/4}x^{3/2} \log(\sqrt{cx} - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1) - \sqrt{2}bc^{3/4}x^{3/2} \log(\sqrt{cx} + \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1))}{3d^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/(d\*x)^(5/2), x]

[Out] -(x\*(4\*a + 2\*Sqrt[2]\*b\*c^(3/4)\*x^(3/2)\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 2\*Sqrt[2]\*b\*c^(3/4)\*x^(3/2)\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 4\*b\*c^(3/4)\*x^(3/2)\*ArcTan[c^(1/4)\*Sqrt[x]] + 4\*b\*ArcTanh[c\*x^2] + 2\*b\*c^(3/4)\*x^(3/2)\*Log[1 - c^(1/4)\*Sqrt[x]] - 2\*b\*c^(3/4)\*x^(3/2)\*Log[1 + c^(1/4)\*Sqrt[x]] + Sqrt[2]\*b\*c^(3/4)\*x^(3/2)\*Log[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x] + Sqrt[c]\*x)

$$-\frac{\sqrt{2} b c^{3/4} x^{3/2} \operatorname{Log}\left[1 + \sqrt{2} c^{1/4} \sqrt{x} + \sqrt{c x}\right]}{6 (d x)^{5/2}}$$

**Maple [A]** time = 0.01, size = 280, normalized size = 0.9

$$-\frac{2a}{3d} (dx)^{-\frac{3}{2}} - \frac{2b \operatorname{Artanh}(cx^2)}{3d} (dx)^{-\frac{3}{2}} + \frac{bc\sqrt{2}}{6d^3} \sqrt{\frac{d^2}{c}} \ln \left( \left( dx + \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx - \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^2))/(d*x)^(5/2),x)`

[Out] 
$$-\frac{2}{3} \frac{a}{d} (d x)^{-3/2} - \frac{2}{3} \frac{b}{d} (d x)^{-3/2} \operatorname{arctanh}(c x^2) + \frac{1}{6} \frac{b c}{d^3} (d^2/c)^{1/4} 2^{1/2} \ln \left( \frac{(d x + (d^2/c)^{1/4} (d x)^{1/2} 2^{1/2} + (d^2/c)^{1/2})}{(d x - (d^2/c)^{1/4} (d x)^{1/2} 2^{1/2} + (d^2/c)^{1/2})} \right) + \frac{1}{3} \frac{b c}{d^3} (d^2/c)^{1/4} 2^{1/2} \operatorname{arctan} \left( \frac{2^{1/2}}{(d^2/c)^{1/4} (d x)^{1/2} + 1} \right) + \frac{1}{3} \frac{b c}{d^3} (d^2/c)^{1/4} 2^{1/2} \operatorname{arctan} \left( \frac{2^{1/2}}{(d^2/c)^{1/4} (d x)^{1/2} - 1} \right) + \frac{1}{3} \frac{b c}{d^3} (d^2/c)^{1/4} \ln \left( \frac{(d x)^{1/2} + (d^2/c)^{1/4}}{(d x)^{1/2} - (d^2/c)^{1/4}} \right) + \frac{2}{3} \frac{b c}{d^3} (d^2/c)^{1/4} \operatorname{arctan} \left( \frac{(d x)^{1/2}}{(d^2/c)^{1/4}} \right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.26762, size = 89, normalized size = 0.3

$$-\frac{\sqrt{dx} \left( b \log \left( -\frac{cx^2+1}{cx^2-1} \right) + 2a \right)}{3d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(5/2),x, algorithm="fricas")

[Out] -1/3\*sqrt(d\*x)\*(b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 2\*a)/(d^3\*x^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/(d\*x)\*\*(5/2),x)

[Out] Timed out

**Giac [B]** time = 3.18579, size = 698, normalized size = 2.32

$$\frac{1}{6}bcd^2 \left( \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^5} + \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^5} + \frac{2\sqrt{2}(-c^3d^2)^{\frac{1}{4}} \arctan\left(\dots\right)}{cd^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(5/2),x, algorithm="giac")

[Out] 1/6\*b\*c\*d^2\*(2\*sqrt(2)\*(c^3\*d^2)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) + 2\*sqrt(d\*x))/(d^2/c)^(1/4))/(c\*d^5) + 2\*sqrt(2)\*(c^3\*d^2)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(d^2/c)^(1/4) - 2\*sqrt(d\*x))/(d^2/c)^(1/4))/(c\*d^5) + 2\*sqrt(2)\*(-c^3\*d^2)^(1/4)\*arctan(1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) + 2\*sqrt(d\*x))/(-d^2/c)^(1/4))/(c\*d^5) + 2\*sqrt(2)\*(-c^3\*d^2)^(1/4)\*arctan(-1/2\*sqrt(2)\*(sqrt(2)\*(-d^2/c)^(1/4) - 2\*sqrt(d\*x))/(-d^2/c)^(1/4))/(c\*d^5) + sqrt(2)\*(c^3\*d^2)^(1/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) + sqrt(d^2/c))/(c\*d^5) - sqrt(2)\*(c^3\*d^2)^(1/4)\*log(d\*x - sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) + sqrt(d^2/c))/(c\*d^5) + sqrt(2)\*(-c^3\*d^2)^(1/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) + sqrt(d^2/c))/(c\*d^5) + sqrt(2)\*(-c^3\*d^2)^(1/4)\*log(d\*x + sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) - sqrt(d^2/c))/(c\*d^5) + sqrt(2)\*(-c^3\*d^2)^(1/4)\*log(d\*x - sqrt(2)\*sqrt(d\*x)\*(d^2/c)^(1/4) - sqrt(d^2/c))/(c\*d^5)

$$\begin{aligned}
& 2) * \sqrt{d*x} * (-d^2/c)^{(1/4)} + \sqrt{-d^2/c}) / (c*d^5) - \sqrt{2} * (-c^3*d^2)^{(1/4)} \\
& / 4 * \log(d*x - \sqrt{2} * \sqrt{d*x} * (-d^2/c)^{(1/4)} + \sqrt{-d^2/c}) / (c*d^5)) - 1 \\
& / 3 * (b * \log(-(c*d^2*x^2 + d^2) / (c*d^2*x^2 - d^2)) / (\sqrt{d*x} * d*x) + 2*a / (\sqrt{d*x} * d*x)) / d
\end{aligned}$$

$$3.88 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{7/2}} dx$$

**Optimal.** Leaf size=317

$$\frac{2(a+b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} - \frac{bc^{5/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{5\sqrt{2}d^{7/2}} + \frac{bc^{5/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{5\sqrt{2}d^{7/2}} - \frac{2bc^{5/4} \tan^{-1}}{5d^{7/2}}$$

[Out]  $(-8*b*c)/(5*d^3*\text{Sqrt}[d*x]) - (2*b*c^{(5/4)}*\text{ArcTan}[(c^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(5*d^{(7/2)}) + (\text{Sqrt}[2]*b*c^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(5*d^{(7/2)}) - (\text{Sqrt}[2]*b*c^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(5*d^{(7/2)}) - (2*(a + b*\text{ArcTanh}[c*x^2]))/(5*d*(d*x)^{(5/2)}) + (2*b*c^{(5/4)}*\text{ArcTanh}[(c^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(5*d^{(7/2)}) - (b*c^{(5/4)}*\text{Log}[\text{Sqrt}[d] + \text{Sqrt}[c]*\text{Sqrt}[d]*x - \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x]])/(5*\text{Sqrt}[2]*d^{(7/2)}) + (b*c^{(5/4)}*\text{Log}[\text{Sqrt}[d] + \text{Sqrt}[c]*\text{Sqrt}[d]*x + \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x]])/(5*\text{Sqrt}[2]*d^{(7/2)})$

**Rubi [A]** time = 0.277762, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6097, 16, 325, 329, 301, 297, 1162, 617, 204, 1165, 628, 298, 205, 208}

$$\frac{2(a+b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} - \frac{bc^{5/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{5\sqrt{2}d^{7/2}} + \frac{bc^{5/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{5\sqrt{2}d^{7/2}} - \frac{2bc^{5/4} \tan^{-1}}{5d^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c*x^2])/(d*x)^{(7/2)}, x]$

[Out]  $(-8*b*c)/(5*d^3*\text{Sqrt}[d*x]) - (2*b*c^{(5/4)}*\text{ArcTan}[(c^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(5*d^{(7/2)}) + (\text{Sqrt}[2]*b*c^{(5/4)}*\text{ArcTan}[1 - (\text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(5*d^{(7/2)}) - (\text{Sqrt}[2]*b*c^{(5/4)}*\text{ArcTan}[1 + (\text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x])/\text{Sqrt}[d]])/(5*d^{(7/2)}) - (2*(a + b*\text{ArcTanh}[c*x^2]))/(5*d*(d*x)^{(5/2)}) + (2*b*c^{(5/4)}*\text{ArcTanh}[(c^{(1/4)}*\text{Sqrt}[d*x])/ \text{Sqrt}[d]])/(5*d^{(7/2)}) - (b*c^{(5/4)}*\text{Log}[\text{Sqrt}[d] + \text{Sqrt}[c]*\text{Sqrt}[d]*x - \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x]])/(5*\text{Sqrt}[2]*d^{(7/2)}) + (b*c^{(5/4)}*\text{Log}[\text{Sqrt}[d] + \text{Sqrt}[c]*\text{Sqrt}[d]*x + \text{Sqrt}[2]*c^{(1/4)}*\text{Sqrt}[d*x]])/(5*\text{Sqrt}[2]*d^{(7/2)})$

**Rule 6097**

$\text{Int}[(a + \text{ArcTanh}[c*x^n])*(d*x)^m, x]$   
 $\text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c$

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_.)*(v_)^{(m_.)}*((b_.)*(v_))^{(n_.)}, x\_Symbol] \text{:>} \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

### Rule 325

$\text{Int}[((c_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \text{:>} \text{Simp}[(c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)}/(a*c*(m + 1)), x] - \text{Dist}[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), \text{Int}[(c*x)^{(m + n)}*(a + b*x^n)^p, x], x] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 329

$\text{Int}[((c_.)*(x_))^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \text{:>} \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)})/c^n)^p, x], x, (c*x)^{(1/k)}], x]] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 301

$\text{Int}[(x_)^{(m_.)}/((a_.) + (b_.)*(x_)^{(n_.)}), x\_Symbol] \text{:>} \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)}/(r + s*x^{(n/2)}), x], x] - \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)}/(r - s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n/4, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{LeQ}[n/2, m] \ \&\& \ \text{LtQ}[m, n] \ \&\& \ \text{!GtQ}[a/b, 0]$

### Rule 297

$\text{Int}[(x_)^2/((a_.) + (b_.)*(x_)^4), x\_Symbol] \text{:>} \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*s), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x] - \text{Dist}[1/(2*s), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 1162

$\text{Int}[((d_.) + (e_.)*(x_)^2)/((a_.) + (c_.)*(x_)^4), x\_Symbol] \text{:>} \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /;$   
 $\text{FreeQ}[\{a, c, d, e\}, x] \ \&$

& EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]



Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{(dx)^{7/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(4bc) \int \frac{x}{(dx)^{5/2}(1-c^2x^4)} dx}{5d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(4bc) \int \frac{1}{(dx)^{3/2}(1-c^2x^4)} dx}{5d^2} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(4bc^3) \int \frac{(dx)^{5/2}}{1-c^2x^4} dx}{5d^6} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(8bc^3) \text{Subst}\left(\int \frac{x^6}{1-\frac{c^2x^8}{d^4}} dx, x, \sqrt{dx}\right)}{5d^7} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(4bc^2) \text{Subst}\left(\int \frac{x^2}{d^2-cx^4} dx, x, \sqrt{dx}\right)}{5d^3} - \frac{(4bc^2) \text{Subst}\left(\int \frac{1}{d-\sqrt{cx^2}} dx, x, \sqrt{dx}\right)}{5d^3} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{(2bc^{3/2}) \text{Subst}\left(\int \frac{1}{d-\sqrt{cx^2}} dx, x, \sqrt{dx}\right)}{5d^3} - \frac{(2bc^{3/2}) \text{Subst}\left(\int \frac{1}{d-\sqrt{cx^2}} dx, x, \sqrt{dx}\right)}{5d^3} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{2bc^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{bc^5}{5d^3\sqrt{dx}} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{5d(dx)^{5/2}} + \frac{2bc^{5/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{bc^5}{5d^3\sqrt{dx}} \\
&= -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{\sqrt{2}bc^{5/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{\sqrt{2}bc^{5/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0857637, size = 275, normalized size = 0.87

$$x \left( -4a - 2bc^{5/4}x^{5/2} \log\left(1 - \sqrt[4]{c}\sqrt{x}\right) + 2bc^{5/4}x^{5/2} \log\left(\sqrt[4]{c}\sqrt{x} + 1\right) - \sqrt{2}bc^{5/4}x^{5/2} \log\left(\sqrt{cx} - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1\right) + \sqrt{2}bc^{5/4}x^{5/2} \log\left(\sqrt{cx} + \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])/(d\*x)^(7/2), x]

```
[Out] (x*(-4*a - 16*b*c*x^2 + 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*c^(5/4)*x^(5/2)*ArcTan[c^(1/4)*Sqrt[x]] - 4*b*ArcTanh[c*x^2] - 2*b*c^(5/4)*x^(5/2)*Log[1 - c^(1/4)*Sqrt[x]] + 2*b*c^(5/4)*x^(5/2)*Log[1 + c^(1/4)*Sqrt[x]] - Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(10*(d*x)^(7/2))
```

**Maple [A]** time = 0.016, size = 292, normalized size = 0.9

$$-\frac{2a}{5d}(dx)^{-\frac{5}{2}} - \frac{2b \operatorname{Artanh}(cx^2)}{5d}(dx)^{-\frac{5}{2}} - \frac{bc\sqrt{2}}{10d^3} \ln \left( \left( dx - \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx + \sqrt[4]{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right) \frac{1}{\sqrt[4]{\frac{d^2}{c}}} - \frac{b}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^2))/(d*x)^(7/2),x)
```

```
[Out] -2/5/d*a/(d*x)^(5/2)-2/5/d*b/(d*x)^(5/2)*arctanh(c*x^2)-1/10/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))-1/5/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)-1/5/d^3*b*c/(d^2/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)-8/5*b*c/d^3/(d*x)^(1/2)-2/5/d^3*b*c/(d^2/c)^(1/4)*arctan((d*x)^(1/2)/(d^2/c)^(1/4))+1/5/d^3*b*c/(d^2/c)^(1/4)*ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4))))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.23702, size = 105, normalized size = 0.33

$$\frac{\left(8bcx^2 + b \log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2a\right)\sqrt{dx}}{5d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(7/2),x, algorithm="fricas")

[Out] -1/5\*(8\*b\*c\*x^2 + b\*log(-(c\*x^2 + 1)/(c\*x^2 - 1)) + 2\*a)\*sqrt(d\*x)/(d^4\*x^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/(d\*x)\*\*(7/2),x)

[Out] Timed out

**Giac [B]** time = 9.77957, size = 717, normalized size = 2.26

$$-\frac{1}{10}bc^3 \left( \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^4d^5} + \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^4d^5} - \frac{2\sqrt{2}(-c^3d^2)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^4d^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(7/2),x, algorithm="giac")

```
[Out] -1/10*b*c^3*(2*sqrt(2)*(c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4)))/(c^4*d^5) + 2*sqrt(2)*(c^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4)))/(c^4*d^5) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4)))/(c^4*d^5) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4)))/(c^4*d^5) - sqrt(2)*(c^3*d^2)^(3/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^4*d^5) + sqrt(2)*(c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^4*d^5) + sqrt(2)*(-c^3*d^2)^(3/4)*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^4*d^5) - sqrt(2)*(-c^3*d^2)^(3/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^4*d^5) - 1/5*(b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^2*x^2) + 2*(4*b*c*d^2*x^2 + a*d^2)/(sqrt(d*x)*d^4*x^2))/d
```

$$3.89 \quad \int \frac{a+b \tanh^{-1}(cx^2)}{(dx)^{9/2}} dx$$

**Optimal.** Leaf size=317

$$\frac{2(a+b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{bc^{7/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}d^{9/2}} - \frac{bc^{7/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}d^{9/2}} + \frac{2bc^{7/4} \tan^{-1}(\dots)}{7d^{9/2}}$$

```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```

**Rubi [A]** time = 0.286934, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6097, 16, 325, 329, 301, 211, 1165, 628, 1162, 617, 204, 212, 208, 205}

$$\frac{2(a+b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{bc^{7/4} \log(\sqrt{c}\sqrt{dx} - \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}d^{9/2}} - \frac{bc^{7/4} \log(\sqrt{c}\sqrt{dx} + \sqrt{2}\sqrt[4]{c}\sqrt{dx} + \sqrt{d})}{7\sqrt{2}d^{9/2}} + \frac{2bc^{7/4} \tan^{-1}(\dots)}{7d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2), x]
```

```
[Out] (-8*b*c)/(21*d^3*(d*x)^(3/2)) + (2*b*c^(7/4)*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (Sqrt[2]*b*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (Sqrt[2]*b*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) - (2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7/2)) + (2*b*c^(7/4)*ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]])/(7*d^(9/2)) + (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x - Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2)) - (b*c^(7/4)*Log[Sqrt[d] + Sqrt[c]*Sqrt[d]*x + Sqrt[2]*c^(1/4)*Sqrt[d*x]])/(7*Sqrt[2]*d^(9/2))
```

**Rule 6097**

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
```

$n)/(d*(m + 1)), \text{Int}[(x^{(n - 1)}*(d*x)^{(m + 1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}[\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 16

$\text{Int}[(u_)*(v_)^{(m_)}*((b_)*(v_))^{(n_)}, x\_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m + n)}, x], x] /;$   
 $\text{FreeQ}[\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

### Rule 325

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_))^{(n_)}^{(p_)}, x\_Symbol] := \text{Simp}[(c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)}/(a*c*(m + 1)), x] - \text{Dist}[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), \text{Int}[(c*x)^{(m + n)}*(a + b*x^n)^p, x], x] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 329

$\text{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_))^{(n_)}^{(p_)}, x\_Symbol] := \text{With}[\{k = \text{Denominator}[m]\}, \text{Dist}[k/c, \text{Subst}[\text{Int}[x^{(k*(m + 1) - 1)}*(a + (b*x^{(k*n)}))/c^n]^p, x], x, (c*x)^{(1/k)}, x]] /;$   
 $\text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{FractionQ}[m] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 301

$\text{Int}[(x_)^{(m_)} / ((a_) + (b_)*(x_))^{(n_)}, x\_Symbol] := \text{With}[\{r = \text{Numerator}[\text{Rt}[-(a/b), 2]], s = \text{Denominator}[\text{Rt}[-(a/b), 2]]\}, \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)} / (r + s*x^{(n/2)}), x], x] - \text{Dist}[s/(2*b), \text{Int}[x^{(m - n/2)} / (r - s*x^{(n/2)}), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{IGtQ}[n/4, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{LeQ}[n/2, m] \ \&\& \ \text{LtQ}[m, n] \ \&\& \ \text{!GtQ}[a/b, 0]$

### Rule 211

$\text{Int}[(a_ + (b_)*(x_)^4)^{-1}, x\_Symbol] := \text{With}[\{r = \text{Numerator}[\text{Rt}[a/b, 2]], s = \text{Denominator}[\text{Rt}[a/b, 2]]\}, \text{Dist}[1/(2*r), \text{Int}[(r - s*x^2)/(a + b*x^4), x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /;$   
 $\text{FreeQ}[\{a, b\}, x] \ \&\& \ (\text{GtQ}[a/b, 0] \ || \ (\text{PosQ}[a/b] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \ \&\& \ \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

### Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2) / ((a_ + (c_)*(x_)^4), x\_Symbol] := \text{With}[\{q = \text{Rt}[(-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /;$   
 $\text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{GtQ}[d, 0] \ \&\& \ \text{GtQ}[e, 0] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[q, 0] \ \&\& \ \text{GtQ}[a/b, 0]$

$eQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& NegQ[d*e]$

### Rule 628

$Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x\_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[\{a, b, c, d, e\}, x] \&\& EqQ[2*c*d - b*e, 0]$

### Rule 1162

$Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x\_Symbol] := With[\{q = Rt[(2*d)/e, 2]\}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[\{a, c, d, e\}, x] \&\& EqQ[c*d^2 - a*e^2, 0] \&\& PosQ[d*e]$

### Rule 617

$Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^{-1}, x\_Symbol] := With[\{q = 1 - 4*Simplify[(a*c)/b^2]\}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] \&\& (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; FreeQ[\{a, b, c\}, x] \&\& NeQ[b^2 - 4*a*c, 0]$

### Rule 204

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b] \&\& (LtQ[a, 0] || LtQ[b, 0])$

### Rule 212

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := With[\{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]\}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[\{a, b\}, x] \&\& !GtQ[a/b, 0]$

### Rule 208

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[\{a, b\}, x] \&\& NegQ[a/b]$

### Rule 205

$Int[((a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[\{a, b\}, x] \&\& PosQ[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^2)}{(dx)^{9/2}} dx &= -\frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(4bc) \int \frac{x}{(dx)^{7/2}(1-c^2x^4)} dx}{7d} \\
&= -\frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(4bc) \int \frac{1}{(dx)^{5/2}(1-c^2x^4)} dx}{7d^2} \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(4bc^3) \int \frac{(dx)^{3/2}}{1-c^2x^4} dx}{7d^6} \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(8bc^3) \text{Subst}\left(\int \frac{x^4}{1-c^2x^8} dx, x, \sqrt{dx}\right)}{7d^7} \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(4bc^2) \text{Subst}\left(\int \frac{1}{d^2-cx^4} dx, x, \sqrt{dx}\right)}{7d^3} - \frac{(4bc^2) \text{Subst}}{7d^3} \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{(2bc^2) \text{Subst}\left(\int \frac{1}{d-\sqrt{cx}^2} dx, x, \sqrt{dx}\right)}{7d^4} + \frac{(2bc^2) \text{Subst}}{7d^4} \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{2bc^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \dots \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{2(a + b \tanh^{-1}(cx^2))}{7d(dx)^{7/2}} + \frac{2bc^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \dots \\
&= -\frac{8bc}{21d^3(dx)^{3/2}} + \frac{2bc^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \frac{\sqrt{2}bc^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} - \frac{\sqrt{2}bc^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.0976679, size = 281, normalized size = 0.89

$$\sqrt{dx} \left( -12a - 6bc^{7/4}x^{7/2} \log\left(1 - \sqrt[4]{c}\sqrt{x}\right) + 6bc^{7/4}x^{7/2} \log\left(\sqrt[4]{c}\sqrt{x} + 1\right) + 3\sqrt{2}bc^{7/4}x^{7/2} \log\left(\sqrt{cx} - \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1\right) - 3\sqrt{2}bc^{7/4}x^{7/2} \log\left(\sqrt{cx} + \sqrt{2}\sqrt[4]{c}\sqrt{x} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTan[c\*x^2])/(d\*x)^(9/2), x]

[Out] (Sqrt[d\*x]\*(-12\*a - 16\*b\*c\*x^2 + 6\*Sqrt[2]\*b\*c^(7/4)\*x^(7/2)\*ArcTan[1 - Sqrt[2]\*c^(1/4)\*Sqrt[x]] - 6\*Sqrt[2]\*b\*c^(7/4)\*x^(7/2)\*ArcTan[1 + Sqrt[2]\*c^(1/4)\*Sqrt[x]] + 3\*sqrt(2)\*b\*c^(7/4)\*x^(7/2)\*log[1 - sqrt(2)\*sqrt(c)\*sqrt(x)] - 3\*sqrt(2)\*b\*c^(7/4)\*x^(7/2)\*log[1 + sqrt(2)\*sqrt(c)\*sqrt(x)] + 3\*sqrt(2)\*b\*c^(7/4)\*x^(7/2)\*log[1 - sqrt(2)\*sqrt(c)\*sqrt(x)] - 3\*sqrt(2)\*b\*c^(7/4)\*x^(7/2)\*log[1 + sqrt(2)\*sqrt(c)\*sqrt(x)])



$$\begin{aligned} & /4) * \text{Sqrt}[x]] + 12 * b * c^{(7/4)} * x^{(7/2)} * \text{ArcTan}[c^{(1/4)} * \text{Sqrt}[x]] - 12 * b * \text{ArcTanh}[ \\ & c * x^2] - 6 * b * c^{(7/4)} * x^{(7/2)} * \text{Log}[1 - c^{(1/4)} * \text{Sqrt}[x]] + 6 * b * c^{(7/4)} * x^{(7/2)} \\ & * \text{Log}[1 + c^{(1/4)} * \text{Sqrt}[x]] + 3 * \text{Sqrt}[2] * b * c^{(7/4)} * x^{(7/2)} * \text{Log}[1 - \text{Sqrt}[2] * c^{(1/4)} \\ & * \text{Sqrt}[x] + \text{Sqrt}[c] * x] - 3 * \text{Sqrt}[2] * b * c^{(7/4)} * x^{(7/2)} * \text{Log}[1 + \text{Sqrt}[2] * c^{(1/4)} \\ & * \text{Sqrt}[x] + \text{Sqrt}[c] * x]) / (42 * d^5 * x^4) \end{aligned}$$

**Maple [A]** time = 0.014, size = 302, normalized size = 1.

$$-\frac{2a}{7d}(dx)^{-\frac{7}{2}} - \frac{2b \text{Artanh}(cx^2)}{7d}(dx)^{-\frac{7}{2}} - \frac{bc^2\sqrt{2}}{14d^5} \sqrt{\frac{d^2}{c}} \ln \left( \left( dx + \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right) \left( dx - \sqrt{\frac{d^2}{c}} \sqrt{dx} \sqrt{2} + \sqrt{\frac{d^2}{c}} \right)^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))/(d\*x)^(9/2), x)

[Out] 
$$\begin{aligned} & -2/7/d*a/(d*x)^{(7/2)} - 2/7/d*b/(d*x)^{(7/2)} * \text{arctanh}(c*x^2) - 1/14/d^5*b*c^2*(d^2 \\ & /c)^{(1/4)} * 2^{(1/2)} * \ln((d*x+(d^2/c)^{(1/4)}*(d*x)^{(1/2)} * 2^{(1/2)} + (d^2/c)^{(1/2)}) / \\ & (d*x - (d^2/c)^{(1/4)}*(d*x)^{(1/2)} * 2^{(1/2)} + (d^2/c)^{(1/2)})) - 1/7/d^5*b*c^2*(d^2/c \\ & )^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (d^2/c)^{(1/4)} * (d*x)^{(1/2)} + 1) - 1/7/d^5*b*c^2*( \\ & d^2/c)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (d^2/c)^{(1/4)} * (d*x)^{(1/2)} - 1) + 1/7/d^5*b* \\ & c^2*(d^2/c)^{(1/4)} * \ln(((d*x)^{(1/2)} + (d^2/c)^{(1/4)}) / ((d*x)^{(1/2)} - (d^2/c)^{(1/4)} \\ & )) + 2/7/d^5*b*c^2*(d^2/c)^{(1/4)} * \arctan((d*x)^{(1/2)} / (d^2/c)^{(1/4)}) - 8/21*b*c/d \\ & ^3/(d*x)^{(3/2)} \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(9/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.23562, size = 109, normalized size = 0.34

$$\frac{\left( 8bcx^2 + 3b \log\left( -\frac{cx^2+1}{cx^2-1} \right) + 6a \right) \sqrt{dx}}{21d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(9/2),x, algorithm="fricas")

[Out]  $-1/21*(8*b*c*x^2 + 3*b*\log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)*\sqrt{d*x}/(d^5*x^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))/(d\*x)\*\*(9/2),x)

[Out] Timed out

**Giac [B]** time = 25.291, size = 720, normalized size = 2.27

$$-\frac{1}{14}bc^3 \left[ \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} + 2\sqrt{dx}}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^5} + \frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^5} - \frac{2\sqrt{2}(-c^3d^2)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}} - 2\sqrt{dx}}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))/(d\*x)^(9/2),x, algorithm="giac")

[Out]  $-1/14*b*c^3*(2*\sqrt{2}*(c^3*d^2)^{(1/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(d^2/c)^{(1/4)} + 2*\sqrt{d*x})/(d^2/c)^{(1/4)})/(c^2*d^5) + 2*\sqrt{2}*(c^3*d^2)^{(1/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(d^2/c)^{(1/4)} - 2*\sqrt{d*x})/(d^2/c)^{(1/4)})/(c^2*d^5) - 2*\sqrt{2}*(-c^3*d^2)^{(1/4)}*\arctan(1/2*\sqrt{2}*(\sqrt{2}*(-d^2/c)^{(1/4)} + 2*\sqrt{d*x})/(-d^2/c)^{(1/4)})/(c^2*d^5) - 2*\sqrt{2}*(-c^3*d^2)^{(1/4)}*\arctan(-1/2*\sqrt{2}*(\sqrt{2}*(-d^2/c)^{(1/4)} - 2*\sqrt{d*x})/(-d^2/c)^{(1/4)})/(c^2*d^5)$

$$\begin{aligned}
& c^2 d^5 + \sqrt{2} (c^3 d^2)^{1/4} \log(dx + \sqrt{2} \sqrt{dx} (d^2/c)^{1/4} \\
& + \sqrt{d^2/c}) / (c^2 d^5) - \sqrt{2} (c^3 d^2)^{1/4} \log(dx - \sqrt{2} \sqrt{dx} \\
& (d^2/c)^{1/4} + \sqrt{d^2/c}) / (c^2 d^5) - \sqrt{2} (-c^3 d^2)^{1/4} \log \\
& (dx + \sqrt{2} \sqrt{dx} (-d^2/c)^{1/4} + \sqrt{-d^2/c}) / (c^2 d^5) + \sqrt{2} \\
& * (-c^3 d^2)^{1/4} \log(dx - \sqrt{2} \sqrt{dx} (-d^2/c)^{1/4} + \sqrt{-d^2/c}) \\
& / (c^2 d^5) - 1/21 * (3b \log(-c d^2 x^2 + d^2) / (c d^2 x^2 - d^2)) / (\sqrt{dx} \\
& * d^3 x^3) + 2 * (4b c d^2 x^2 + 3a d^2) / (\sqrt{dx} * d^5 x^3) / d
\end{aligned}$$

### 3.90 $\int \sqrt{dx} \left( a + b \tanh^{-1} (cx^2) \right)^2 dx$

**Optimal.** Leaf size=6327

result too large to display

```
[Out] (-8*a*b*x*Sqrt[d*x])/9 - (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 - Sqrt[2]*c^(1/4)
)*Sqrt[x]]/(3*c^(3/4)*Sqrt[x]) + (2*Sqrt[2]*a*b*Sqrt[d*x]*ArcTan[1 + Sqrt[
2]*c^(1/4)*Sqrt[x]]/(3*c^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcTan[
(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(3/4)*Sqrt[x]) - (((2*I)/3)*b^2*Sqrt[d*x]*ArcT
an[c^(1/4)*Sqrt[x]]^2)/(c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1
/4)*Sqrt[x]]^2)/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[c^(1/4)*S
qrt[x]]^2)/(3*c^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x
]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*
x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/
4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))]/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-
c)^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(
1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))]/((I*Sqrt[-Sq
rt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sqrt[x]) +
(2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*S
qrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (4*b^2*Sqrt[
d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(
3/4)*Sqrt[x]) - (4*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-
c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(
1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))]/((Sqrt[-Sq
rt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sqrt[x]) -
(2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-S
qrt[-c]]*Sqrt[x]))]/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]
)))/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*L
og[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))]/((Sqrt[-Sqrt[c]] - (-c)^(1/
4))*(1 + (-c)^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*A
rcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))]/
((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sq
rt[x]) + (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)
^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)*Sqrt[x]) + (4*b
^2*Sqrt[d*x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(3*c^(3
/4)*Sqrt[x]) - (2*b^2*Sqrt[d*x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4
)*(1 - c^(1/4)*Sqrt[x]))]/(((c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x
])))/(3*(-c)^(3/4)*Sqrt[x]) + (2*b^2*Sqrt[d*x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]
*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))]/(((c)^(1/4) - c^(1/4))*(1 + (-c)
^(1/4)*Sqrt[x])))/(3*(-c)^(3/4)*Sqrt[x]) + (4*b^2*Sqrt[d*x]*ArcTan[c^(1/4)
*Sqrt[x]]*Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(3*c^(3/4)*Sqrt[x]) - (2*b^2*Sqrt
[d*x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x])])
```

$$\begin{aligned}
& ) / ((I \sqrt{-\sqrt{-c}} - c^{1/4}) * (1 - I * c^{1/4} * \sqrt{x})) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[(2 * c^{1/4} * (1 + \sqrt{-\sqrt{-c}}) * \sqrt{x})] / ((I * \sqrt{-\sqrt{-c}} + c^{1/4}) * (1 - I * c^{1/4} * \sqrt{x}))]) \\
& ) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[(-2 * c^{1/4} * (1 - (-c)^{1/4} * \sqrt{x})) / ((I * (-c)^{1/4} - c^{1/4}) * (1 - I * c^{1/4} * \sqrt{x}))]) \\
& ) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[(2 * c^{1/4} * (1 + (-c)^{1/4} * \sqrt{x})) / ((I * (-c)^{1/4} + c^{1/4}) * (1 - I * c^{1/4} * \sqrt{x}))]) \\
& ) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[(1 + I) * (1 - c^{1/4} * \sqrt{x})] / (1 - I * c^{1/4} * \sqrt{x})) / (3 * c^{3/4} * \sqrt{x}) - (4 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[2 / (1 + I * c^{1/4} * \sqrt{x})]) \\
& ) / (3 * c^{3/4} * \sqrt{x}) - (4 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[2 / (1 + c^{1/4} * \sqrt{x})]) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(-2 * c^{1/4} * (1 - \sqrt{-\sqrt{-c}}) * \sqrt{x})] / ((\sqrt{-\sqrt{-c}} - c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))) \\
& ) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(2 * c^{1/4} * (1 + \sqrt{-\sqrt{-c}}) * \sqrt{x})] / ((\sqrt{-\sqrt{-c}} + c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(-2 * c^{1/4} * (1 - \sqrt{-\sqrt{c}}) * \sqrt{x})] / ((\sqrt{-\sqrt{c}} - c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))) \\
& ) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(2 * c^{1/4} * (1 + \sqrt{-\sqrt{c}}) * \sqrt{x})] / ((\sqrt{-\sqrt{c}} + c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(-2 * c^{1/4} * (1 - (-c)^{1/4} * \sqrt{x})) / (((-c)^{1/4} - c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))]) \\
& ) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[(2 * c^{1/4} * (1 + (-c)^{1/4} * \sqrt{x})) / (((-c)^{1/4} + c^{1/4}) * (1 + c^{1/4} * \sqrt{x}))]) / (3 * c^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[(2 * (-c)^{1/4} * (1 + c^{1/4} * \sqrt{x})) / (((-c)^{1/4} + I * c^{1/4}) * (1 - I * (-c)^{1/4} * \sqrt{x}))]) \\
& ) / (3 * (-c)^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[(2 * (-c)^{1/4} * (1 + c^{1/4} * \sqrt{x})) / (((-c)^{1/4} + c^{1/4}) * (1 + (-c)^{1/4} * \sqrt{x}))]) / (3 * (-c)^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[(1 - I) * (1 + c^{1/4} * \sqrt{x})] / (1 - I * c^{1/4} * \sqrt{x})) \\
& ) / (3 * c^{3/4} * \sqrt{x}) + (\sqrt{2} * a * b * \sqrt{d * x} * \text{Log}[1 - \sqrt{2} * c^{1/4} * \sqrt{x} + \sqrt{c} * x]) / (3 * c^{3/4} * \sqrt{x}) - (\sqrt{2} * a * b * \sqrt{d * x} * \text{Log}[1 + \sqrt{2} * c^{1/4} * \sqrt{x} + \sqrt{c} * x]) / (3 * c^{3/4} * \sqrt{x}) \\
& + (4 * b^2 * x * \sqrt{d * x} * \text{Log}[1 - c * x^2]) / 9 + (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[1 - c * x^2]) / (3 * (-c)^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[1 - c * x^2]) / (3 * (-c)^{3/4} * \sqrt{x}) + (4 * b * x * \sqrt{d * x} * (2 * a - b * \text{Log}[1 - c * x^2])) / 9 + (2 * b * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * (2 * a - b * \text{Log}[1 - c * x^2])) / (3 * c^{3/4} * \sqrt{x}) - (2 * b * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * (2 * a - b * \text{Log}[1 - c * x^2])) / (3 * c^{3/4} * \sqrt{x}) + (x * \sqrt{d * x} * (2 * a - b * \text{Log}[1 - c * x^2])^2) / 6 + (2 * a * b * x * \sqrt{d * x} * \text{Log}[1 + c * x^2]) / 3 - (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[1 + c * x^2]) / (3 * (-c)^{3/4} * \sqrt{x}) \\
& + (2 * b^2 * \sqrt{d * x} * \text{ArcTan}[c^{1/4} * \sqrt{x}] * \text{Log}[1 + c * x^2]) / (3 * c^{3/4} * \sqrt{x}) + (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[(-c)^{1/4} * \sqrt{x}] * \text{Log}[1 + c * x^2]) / (3 * (-c)^{3/4} * \sqrt{x}) - (2 * b^2 * \sqrt{d * x} * \text{ArcTanh}[c^{1/4} * \sqrt{x}] * \text{Log}[1 + c * x^2]) / (3 * c^{3/4} * \sqrt{x}) - (b^2 * x * \sqrt{d * x} * \text{Log}[1 - c * x^2] * \text{Log}[1 + c * x^2]) / 3 +
\end{aligned}$$

$$\begin{aligned}
& (b^2*x*\text{Sqrt}[d*x]*\text{Log}[1 + c*x^2]^2)/6 + (2*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 \\
& - (-c)^{(1/4)}*\text{Sqrt}[x]])/(3*(-c)^{(3/4)}*\text{Sqrt}[x]) - (((2*I)/3)*b^2*\text{Sqrt}[d*x]* \\
& \text{PolyLog}[2, 1 - 2/(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x]])/((-c)^{(3/4)}*\text{Sqrt}[x]) + ((I/3) \\
& *b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/( \\
& (I*\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)}*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])))/((-c)^{(3/4)}*\text{S} \\
& \text{qrt}[x]) + ((I/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt} \\
& [c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[-\text{Sqrt}[c]] + (-c)^{(1/4)}*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])) \\
& )/((-c)^{(3/4)}*\text{Sqrt}[x]) - ((I/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - \\
& (-c)^{(1/4)}*\text{Sqrt}[x]))/(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x]))/((-c)^{(3/4)}*\text{Sqrt}[x]) - ( \\
& ((2*I)/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 + I*(-c)^{(1/4)}*\text{Sqrt}[x]))/((-c) \\
& ^{(3/4)}*\text{Sqrt}[x]) + (2*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 + (-c)^{(1/4)}*\text{Sqrt}[x] \\
& )]/(3*(-c)^{(3/4)}*\text{Sqrt}[x]) + (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*(-c)^{(1/4)}*(1 \\
& - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] - (-c)^{(1/4)}*(1 + (-c)^{(1/4) \\
& }*\text{Sqrt}[x])))/((3*(-c)^{(3/4)}*\text{Sqrt}[x]) + (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*(-c) \\
& )^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] + (-c)^{(1/4)}*(1 + \\
& (-c)^{(1/4)}*\text{Sqrt}[x])))/((3*(-c)^{(3/4)}*\text{Sqrt}[x]) - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, \\
& 1 + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/ \\
& 4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])))/((3*(-c)^{(3/4)}*\text{Sqrt}[x]) - (b^2*\text{Sqrt}[d*x]*\text{Pol} \\
& \text{yLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] + \\
& (-c)^{(1/4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])))/((3*(-c)^{(3/4)}*\text{Sqrt}[x]) - ((I/3)*b^2 \\
& * \text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - ((1 - I)*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))/(1 - I*(-c)^{(1 \\
& /4)}*\text{Sqrt}[x]))/((-c)^{(3/4)}*\text{Sqrt}[x]) + (2*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 \\
& - c^{(1/4)}*\text{Sqrt}[x]))/(3*c^{(3/4)}*\text{Sqrt}[x]) + ((I/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, \\
& 1 - (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x]))/(((c)^{(1/4)} - I*c^{(1/4)}*(1 - I*( \\
& -c)^{(1/4)}*\text{Sqrt}[x])))/((-c)^{(3/4)}*\text{Sqrt}[x]) - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - \\
& (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x]))/(((c)^{(1/4)} - c^{(1/4)}*(1 + (-c)^{(1/4) \\
& }*\text{Sqrt}[x])))/((3*(-c)^{(3/4)}*\text{Sqrt}[x]) - (((2*I)/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, \\
& 1 - 2/(1 - I*c^{(1/4)}*\text{Sqrt}[x]))/(c^{(3/4)}*\text{Sqrt}[x]) + ((I/3)*b^2*\text{Sqrt}[d*x]*\text{Pol} \\
& \text{yLog}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[-\text{Sqrt}[-c]] \\
& - c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))/((c)^{(3/4)}*\text{Sqrt}[x]) + ((I/3)*b^2*\text{Sqrt}[d \\
& *x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[-\text{Sqrt} \\
& [-c]] + c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))/((c)^{(3/4)}*\text{Sqrt}[x]) + ((I/3)*b^2* \\
& \text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - (-c)^{(1/4)}*\text{Sqrt}[x]))/((I*(-c)^{(1/4) \\
& } - c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))/((c)^{(3/4)}*\text{Sqrt}[x]) + ((I/3)*b^2*\text{Sqrt} \\
& [d*x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))/((I*(-c)^{(1/4)} + \\
& c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))/((c)^{(3/4)}*\text{Sqrt}[x]) - ((I/3)*b^2*\text{Sqrt}[d*x] \\
& ]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - c^{(1/4)}*\text{Sqrt}[x]))/(1 - I*c^{(1/4)}*\text{Sqrt}[x])))/ \\
& (c^{(3/4)}*\text{Sqrt}[x]) - (((2*I)/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 + I*c^{(1/4) \\
& }*\text{Sqrt}[x]))/((c)^{(3/4)}*\text{Sqrt}[x]) + (2*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - 2/(1 + c^{( \\
& 1/4)}*\text{Sqrt}[x]))/(3*c^{(3/4)}*\text{Sqrt}[x]) - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*c^{(1 \\
& /4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)}*(1 + c^{(1/4) \\
& }*\text{Sqrt}[x])))/((3*c^{(3/4)}*\text{Sqrt}[x]) - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*c^{(1/4) \\
& }*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)}*(1 + c^{(1/4)* \\
& }*\text{Sqrt}[x])))/((3*c^{(3/4)}*\text{Sqrt}[x]) + (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)* \\
& }*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] - c^{(1/4)}*(1 + c^{(1/4)*\text{Sqrt}
\end{aligned}$$

$$\frac{[x])]}{(3*c^{(3/4)*\text{Sqrt}[x])} + (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))]/((\text{Sqrt}[-\text{Sqrt}[c]] + c^{(1/4)})*(1 + c^{(1/4)*\text{Sqrt}[x]})])]/(3*c^{(3/4)*\text{Sqrt}[x])} - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - (-c)^{(1/4)*\text{Sqrt}[x]})]/(((-c)^{(1/4)} - c^{(1/4)})*(1 + c^{(1/4)*\text{Sqrt}[x]})])]/(3*c^{(3/4)*\text{Sqrt}[x])} - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + (-c)^{(1/4)*\text{Sqrt}[x]})]/(((-c)^{(1/4)} + c^{(1/4)})*(1 + c^{(1/4)*\text{Sqrt}[x]})])]/(3*c^{(3/4)*\text{Sqrt}[x])} + ((I/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + c^{(1/4)*\text{Sqrt}[x]})]/(((-c)^{(1/4)} + I*c^{(1/4)})*(1 - I*(-c)^{(1/4)*\text{Sqrt}[x]})])]/((-c)^{(3/4)*\text{Sqrt}[x]}) - (b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + c^{(1/4)*\text{Sqrt}[x]})]/(((-c)^{(1/4)} + c^{(1/4)})*(1 + (-c)^{(1/4)*\text{Sqrt}[x]})])]/(3*(-c)^{(3/4)*\text{Sqrt}[x]}) - ((I/3)*b^2*\text{Sqrt}[d*x]*\text{PolyLog}[2, 1 - ((1 - I)*(1 + c^{(1/4)*\text{Sqrt}[x]})]/(1 - I*c^{(1/4)*\text{Sqrt}[x]})])]/(c^{(3/4)*\text{Sqrt}[x]})$$

**Rubi [F]** time = 0.0256037, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \sqrt{dx} (a + b \tanh^{-1}(cx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x^2])^2, x]

[Out] Defer[Int][Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x^2])^2, x]

Rubi steps

$$\int \sqrt{dx} (a + b \tanh^{-1}(cx^2))^2 dx = \int \sqrt{dx} (a + b \tanh^{-1}(cx^2))^2 dx$$

**Mathematica [F]** time = 61.9463, size = 0, normalized size = 0.

$$\int \sqrt{dx} (a + b \tanh^{-1}(cx^2))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x^2])^2, x]

[Out] Integrate[Sqrt[d\*x]\*(a + b\*ArcTanh[c\*x^2])^2, x]

**Maple [F]** time = 0.313, size = 0, normalized size = 0.

$$\int \sqrt{dx} (a + b \operatorname{Artanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(1/2)\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] int((d\*x)^(1/2)\*(a+b\*arctanh(c\*x^2))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2\right)\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)\*sqrt(d\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(1/2)\*(a+b\*atanh(c\*x\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}(b \operatorname{artanh}(cx^2) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(1/2)\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="giac")

[Out] integrate(sqrt(d\*x)\*(b\*arctanh(c\*x^2) + a)^2, x)

$$3.91 \quad \int \frac{\left(a+b \tanh^{-1}(cx^2)\right)^2}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=6177

result too large to display

```
[Out] (2*a^2*x)/Sqrt[d*x] - (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) + (2*Sqrt[2]*a*b*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(1/4)*Sqrt[d*x]) - (4*a*b*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) + ((2*I)*b^2*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]^2)/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/((-c)^(1/4)*Sqrt[d*x]) - (4*a*b*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]])/(c^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(c^(1/4)*Sqrt[d*x]) + (4*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) + (4*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (4*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x])])/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x])])/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x])])/((Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))/((-c)^(1/4)*Sqrt[d*x]) - (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x])])/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))/((-c)^(1/4)*Sqrt[d*x]) + (4*b^2*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/((c^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x])])/((-c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))/((-c)^(1/4)*Sqrt[d*x]) + (2*b^2*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt[x])])/((-c)^(1/4) - c^(1/4))*(1 + (-c
```

$$\begin{aligned}
& )^{1/4} \sqrt{x} \Big) \Big) / \Big( (-c)^{1/4} \sqrt{dx} \Big) - (4b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] \sqrt{x}] * \operatorname{Log}[2/(1 - I * c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(-2c^{1/4} * (1 - \sqrt{-\sqrt{-c}}) \sqrt{x})] / (I * \sqrt{-\sqrt{-c}} - c^{1/4}) * (1 - I * c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) \\
& ) + (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(2c^{1/4} * (1 + \sqrt{-\sqrt{-c}}) \sqrt{x})] / ((I * \sqrt{-\sqrt{-c}} + c^{1/4}) * (1 - I * c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(-2c^{1/4} * (1 - (-c)^{1/4} \sqrt{x})] / ((I * (-c)^{1/4} - c^{1/4}) * (1 - I * c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(2c^{1/4} * (1 + (-c)^{1/4} \sqrt{x})] / ((I * (-c)^{1/4} + c^{1/4}) * (1 - I * c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(1 + I) * (1 - c^{1/4} \sqrt{x})] / (1 - I * c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) \\
& + (4b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[2/(1 + I * c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) - (4b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[2/(1 + c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(-2c^{1/4} * (1 - \sqrt{-\sqrt{-c}}) \sqrt{x})] / ((\sqrt{-\sqrt{-c}} - c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(2c^{1/4} * (1 + \sqrt{-\sqrt{-c}}) \sqrt{x})] / ((\sqrt{-\sqrt{-c}} + c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(-2c^{1/4} * (1 - \sqrt{-\sqrt{c}}) \sqrt{x})] / ((\sqrt{-\sqrt{c}} - c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(2c^{1/4} * (1 + \sqrt{-\sqrt{c}}) \sqrt{x})] / ((\sqrt{-\sqrt{c}} + c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(-2c^{1/4} * (1 - (-c)^{1/4} \sqrt{x})] / (((-c)^{1/4} - c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(2c^{1/4} * (1 + (-c)^{1/4} \sqrt{x})] / (((-c)^{1/4} + c^{1/4}) * (1 + c^{1/4} \sqrt{x}))]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[(2 * (-c)^{1/4} * (1 + c^{1/4} \sqrt{x})] / (((-c)^{1/4} + I * c^{1/4}) * (1 - I * (-c)^{1/4} \sqrt{x}))]) / (((-c)^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[(2 * (-c)^{1/4} * (1 + c^{1/4} \sqrt{x})] / (((-c)^{1/4} + c^{1/4}) * (1 + (-c)^{1/4} \sqrt{x}))]) / (((-c)^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[(1 - I) * (1 + c^{1/4} \sqrt{x})] / (1 - I * c^{1/4} \sqrt{x})]) / (c^{1/4} \sqrt{dx}) - (\sqrt{2} * a * b * \sqrt{x} * \operatorname{Log}[1 - \sqrt{2} * c^{1/4} \sqrt{x} + \sqrt{c} * x]) / (c^{1/4} \sqrt{dx}) + (\sqrt{2} * a * b * \sqrt{x} * \operatorname{Log}[1 + \sqrt{2} * c^{1/4} \sqrt{x} + \sqrt{c} * x]) / (c^{1/4} \sqrt{dx}) - (2 * a * b * x * \operatorname{Log}[1 - c * x^2]) / \sqrt{dx} - (2b^2 \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[1 - c * x^2]) / ((-c)^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[1 - c * x^2]) / (c^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[1 - c * x^2]) / ((-c)^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[1 - c * x^2]) / (c^{1/4} \sqrt{dx}) + (b^2 * x * \operatorname{Log}[1 - c * x^2]^2) / (2 * \sqrt{dx}) + (2 * a * b * x * \operatorname{Log}[1 + c * x^2]) / \sqrt{dx} + (2b^2 \sqrt{x} \operatorname{ArcTan}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[1 + c * x^2]) / ((-c)^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTan}[c^{1/4} \sqrt{x}] * \operatorname{Log}[1 + c * x^2]) / (c^{1/4} \sqrt{dx}) + (2b^2 \sqrt{x} \operatorname{ArcTanh}[(-c)^{1/4} \sqrt{x}] * \operatorname{Log}[1 + c * x^2]) / ((-c)^{1/4} \sqrt{dx}) - (2b^2 \sqrt{x} \operatorname{ArcTanh}[c^{1/4} \sqrt{x}] * \operatorname{Log}[1 + c * x^2]) / (c^{1/4} \sqrt{dx})
\end{aligned}$$

$$\begin{aligned}
& ) * \text{Sqrt}[x]] * \text{Log}[1 + c*x^2]) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (b^2*x * \text{Log}[1 - c*x^2] * \text{Log}[ \\
& 1 + c*x^2]) / \text{Sqrt}[d*x] + (b^2*x * \text{Log}[1 + c*x^2]^2) / (2 * \text{Sqrt}[d*x]) + (2*b^2 * \text{Sqr} \\
& \text{t}[x] * \text{PolyLog}[2, 1 - 2/(1 - (-c)^{(1/4)} * \text{Sqrt}[x])]) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + ( \\
& (2*I)*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - 2/(1 - I*(-c)^{(1/4)} * \text{Sqrt}[x])]) / ((-c)^{(1/4)} \\
& * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 + (2*(-c)^{(1/4)} * (1 - \text{Sqrt}[-\text{Sqrt}[c] \\
& ] * \text{Sqrt}[x])]) / ((I * \text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)}) * (1 - I*(-c)^{(1/4)} * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*(-c)^{(1/4)} * (1 + \\
& \text{Sqrt}[-\text{Sqrt}[c]] * \text{Sqrt}[x])]) / ((I * \text{Sqrt}[-\text{Sqrt}[c]] + (-c)^{(1/4)}) * (1 - I*(-c)^{(1/4)} \\
& * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - ((1 + I) \\
& ) * (1 - (-c)^{(1/4)} * \text{Sqrt}[x])]) / (1 - I*(-c)^{(1/4)} * \text{Sqrt}[x]) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + ((2*I)*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - 2/(1 + I*(-c)^{(1/4)} * \text{Sqrt}[x] \\
& )]) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (2*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - 2/(1 + (-c)^{(1/4)} * \text{Sqrt}[x] \\
& )]) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 + (2*(-c)^{(1/4)} * (1 \\
& - \text{Sqrt}[-\text{Sqrt}[-c]] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[-c]] - (-c)^{(1/4)}) * (1 + (-c)^{(1/4)} \\
& ) * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*(-c)^{(1/4)} * (1 + \text{Sqrt}[-\text{Sqrt}[-c] \\
& ] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[-c]] + (-c)^{(1/4)}) * (1 + (-c)^{(1/4)} * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) - (b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 + (2*(-c)^{(1/4)} * (1 - \text{Sqrt}[-\text{Sqrt}[c] \\
& ] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)}) * (1 + (-c)^{(1/4)} * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - ((1 - I) * (1 + (-c)^{(1/4)} * \text{Sqrt}[x] \\
& )]) / (1 - I*(-c)^{(1/4)} * \text{Sqrt}[x]) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + (2*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - 2/(1 - c^{(1/4)} * \text{Sqr} \\
& \text{t}[x])]) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*(-c)^{(1/4)} * \\
& (1 - c^{(1/4)} * \text{Sqrt}[x])]) / (((-c)^{(1/4)} - I*c^{(1/4)}) * (1 - I*(-c)^{(1/4)} * \text{Sqrt}[x]) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) - (b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*(-c)^{(1/4)} * (1 - \\
& c^{(1/4)} * \text{Sqrt}[x])]) / (((-c)^{(1/4)} - c^{(1/4)}) * (1 + (-c)^{(1/4)} * \text{Sqrt}[x])) \\
& ) / ((-c)^{(1/4)} * \text{Sqrt}[d*x]) + ((2*I)*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - 2/(1 - I*c^{(1/4)} * \text{Sqr} \\
& \text{t}[x])]) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 + (2*c^{(1/4)} * (1 - \\
& \text{Sqrt}[-\text{Sqrt}[-c]] * \text{Sqrt}[x])]) / ((I * \text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)}) * (1 - I*c^{(1/4)} * \text{Sqr} \\
& \text{t}[x])) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*c^{(1/4)} * (1 \\
& + \text{Sqrt}[-\text{Sqrt}[-c]] * \text{Sqrt}[x])]) / ((I * \text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)}) * (1 - I*c^{(1/4)} * \\
& \text{Sqrt}[x])) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 + (2*c^{(1/4)} * \\
& (1 - (-c)^{(1/4)} * \text{Sqrt}[x])]) / ((I*(-c)^{(1/4)} - c^{(1/4)}) * (1 - I*c^{(1/4)} * \text{Sqrt}[x]) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - (2*c^{(1/4)} * (1 + (-c) \\
& )^{(1/4)} * \text{Sqrt}[x])]) / ((I*(-c)^{(1/4)} + c^{(1/4)}) * (1 - I*c^{(1/4)} * \text{Sqrt}[x])) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) + (I*b^2 * \text{Sqrt}[x] * \text{PolyLog}[2, 1 - ((1 + I) * (1 - c^{(1/4)} * \text{Sqrt}[x] \\
& )]) / (1 - I*c^{(1/4)} * \text{Sqrt}[x]) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) + ((2*I)*b^2 * \text{Sqrt}[x] * \text{Poly} \\
& \text{Log}[2, 1 - 2/(1 + I*c^{(1/4)} * \text{Sqrt}[x])]) / (c^{(1/4)} * \text{Sqrt}[d*x]) + (2*b^2 * \text{Sqrt}[x] \\
& * \text{PolyLog}[2, 1 - 2/(1 + c^{(1/4)} * \text{Sqrt}[x])]) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (b^2 * \text{Sqrt}[x] \\
& * \text{PolyLog}[2, 1 + (2*c^{(1/4)} * (1 - \text{Sqrt}[-\text{Sqrt}[-c]] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[-c] \\
& ] - c^{(1/4)}) * (1 + c^{(1/4)} * \text{Sqrt}[x])) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) - (b^2 * \text{Sqrt}[x] * \text{P} \\
& \text{olyLog}[2, 1 - (2*c^{(1/4)} * (1 + \text{Sqrt}[-\text{Sqrt}[-c]] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[-c]] + \\
& c^{(1/4)}) * (1 + c^{(1/4)} * \text{Sqrt}[x])) \\
& ) / (c^{(1/4)} * \text{Sqrt}[d*x]) + (b^2 * \text{Sqrt}[x] * \text{PolyL} \\
& \text{og}[2, 1 + (2*c^{(1/4)} * (1 - \text{Sqrt}[-\text{Sqrt}[c]] * \text{Sqrt}[x])]) / ((\text{Sqrt}[-\text{Sqrt}[c]] - c^{(1/4)}
\end{aligned}$$

4))\*(1 + c^(1/4)\*Sqrt[x])))/(c^(1/4)\*Sqrt[d\*x]) + (b^2\*Sqrt[x]\*PolyLog[2, 1 - (2\*c^(1/4)\*(1 + Sqrt[-Sqrt[c]]\*Sqrt[x]))]/((Sqrt[-Sqrt[c]] + c^(1/4))\*(1 + c^(1/4)\*Sqrt[x])))]/(c^(1/4)\*Sqrt[d\*x]) - (b^2\*Sqrt[x]\*PolyLog[2, 1 + (2\*c^(1/4)\*(1 - (-c)^(1/4)\*Sqrt[x]))]/(((-c)^(1/4) - c^(1/4))\*(1 + c^(1/4)\*Sqrt[x])))]/(c^(1/4)\*Sqrt[d\*x]) - (b^2\*Sqrt[x]\*PolyLog[2, 1 - (2\*c^(1/4)\*(1 + (-c)^(1/4)\*Sqrt[x]))]/(((-c)^(1/4) + c^(1/4))\*(1 + c^(1/4)\*Sqrt[x])))]/(c^(1/4)\*Sqrt[d\*x]) - (I\*b^2\*Sqrt[x]\*PolyLog[2, 1 - (2\*(-c)^(1/4)\*(1 + c^(1/4)\*Sqrt[x]))]/(((-c)^(1/4) + I\*c^(1/4))\*(1 - I\*(-c)^(1/4)\*Sqrt[x])))]/((-c)^(1/4)\*Sqrt[d\*x]) - (b^2\*Sqrt[x]\*PolyLog[2, 1 - (2\*(-c)^(1/4)\*(1 + c^(1/4)\*Sqrt[x]))]/(((-c)^(1/4) + c^(1/4))\*(1 + (-c)^(1/4)\*Sqrt[x])))]/((-c)^(1/4)\*Sqrt[d\*x]) + (I\*b^2\*Sqrt[x]\*PolyLog[2, 1 - ((1 - I)\*(1 + c^(1/4)\*Sqrt[x]))]/(1 - I\*c^(1/4)\*Sqrt[x]))]/(c^(1/4)\*Sqrt[d\*x]))

**Rubi [F]** time = 0.025645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{\sqrt{dx}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/Sqrt[d\*x], x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^2])^2/Sqrt[d\*x], x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{\sqrt{dx}} dx = \int \frac{(a + b \tanh^{-1}(cx^2))^2}{\sqrt{dx}} dx$$

**Mathematica [F]** time = 59.3506, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{\sqrt{dx}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/Sqrt[d\*x],x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^2])^2/Sqrt[d\*x], x]

**Maple [F]** time = 0.213, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Artanh}(cx^2))^2 \frac{1}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(1/2),x)

[Out] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(1/2),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\left( b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2 \right) \sqrt{dx}}{dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(1/2),x, algorithm="fricas")

[Out] `integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^2))^2}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**2))**2/(d*x)**(1/2), x)`

[Out] `Integral((a + b*atanh(c*x**2))**2/sqrt(d*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{\sqrt{dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2), x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^2) + a)^2/sqrt(d*x), x)`

$$3.92 \quad \int \frac{\left(a+b \tanh^{-1}(cx^2)\right)^2}{(dx)^{3/2}} dx$$

**Optimal.** Leaf size=6334

result too large to display

```
[Out] (-2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]/(d*Sqr
t[d*x]) + (2*Sqrt[2]*a*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]
]/(d*Sqrt[d*x]) + ((2*I)*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]
^2)/(d*Sqrt[d*x]) + ((2*I)*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]^2]/(
d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/(d*
Sqrt[d*x]) + (2*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]^2)/(d*Sqrt[d*x
]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^(
1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4
)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[
c]])*Sqrt[x])]/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))
)/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[
(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x])]/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4)
)*(1 - I*(-c)^(1/4)*Sqrt[x]))]/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*A
rcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c
)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)^(1/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (4*b^2*(-c)
^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])
/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[
(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]])*Sqrt[x])]/((Sqrt[-Sqrt[-c]] - (-c)^(1/4
))*(1 + (-c)^(1/4)*Sqrt[x]))]/(d*Sqrt[d*x]) + (2*b^2*(-c)^(1/4)*Sqrt[x]*Ar
cTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]])*Sqrt[x])]/
((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]/(d*Sqrt[d*x]) -
(2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(
1 - Sqrt[-Sqrt[c]])*Sqrt[x])]/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)
)*Sqrt[x]))]/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]])*Sqrt[x])]/((Sqrt[-Sqrt[c]] +
(-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]/(d*Sqrt[d*x]) - (2*b^2*(-c)^(1/4)*S
qrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x]))/(1
- I*(-c)^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTanh[c
^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/(d*Sqrt[d*x]) + (2*b^2*(-c)^(
1/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)*Sqrt
[x])]/(((c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))]/(d*Sqrt[d*x])
- (2*b^2*(-c)^(1/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*
(1 - c^(1/4)*Sqrt[x])]/(((c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]/(
d*Sqrt[d*x]) - (4*b^2*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2/(1 - I
```



$$\begin{aligned}
& *c^{(1/4)*\text{Sqrt}[x]})/(d*\text{Sqrt}[d*x]) + (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] \\
& * \text{Log}[(-2*c^{(1/4)*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])})]/((I*\text{Sqrt}[-\text{Sqrt}[-c]] \\
& - c^{(1/4)*(1 - I*c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) + (2*b^2*c^{(1/4)*\text{Sqrt}[x]} \\
& * \text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(2*c^{(1/4)*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])})]/((I \\
& * \text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)*(1 - I*c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) + (2*b \\
& ^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(-2*c^{(1/4)*(1 - (-c)^{(1/4)*\text{Sqrt}[x]})}) \\
& ])/((I*(-c)^{(1/4)} - c^{(1/4)*(1 - I*c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x] \\
& ) + (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(2*c^{(1/4)*(1 + (-c) \\
& ^{(1/4)*\text{Sqrt}[x]})})]/((I*(-c)^{(1/4)} + c^{(1/4)*(1 - I*c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sq} \\
& \text{rt}[d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(1 + I)*(1 - \\
& c^{(1/4)*\text{Sqrt}[x]})]/(1 - I*c^{(1/4)*\text{Sqrt}[x]})/(d*\text{Sqrt}[d*x]) + (4*b^2*c^{(1/4) \\
& * \text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[2/(1 + I*c^{(1/4)*\text{Sqrt}[x]})])/(d*\text{Sqrt}[d* \\
& x]) + (4*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[2/(1 + c^{(1/4)*\text{S} \\
& \text{qrt}[x]})])/(d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Lo} \\
& \text{g}[(-2*c^{(1/4)*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])})]/((\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)*(1 + \\
& c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1 \\
& /4)*\text{Sqrt}[x]}] * \text{Log}[(2*c^{(1/4)*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])})]/((\text{Sqrt}[-\text{Sqrt}[-c] \\
& ] + c^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) + (2*b^2*c^{(1/4)*\text{Sqrt}[x]} \\
& * \text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(-2*c^{(1/4)*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x])})]/(( \\
& \text{Sqrt}[-\text{Sqrt}[c]] - c^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) + (2*b^2*c \\
& ^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(2*c^{(1/4)*(1 + \text{Sqrt}[-\text{Sqrt}[c]]* \\
& \text{Sqrt}[x])})]/((\text{Sqrt}[-\text{Sqrt}[c]] + c^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x] \\
& ) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(-2*c^{(1/4)*(1 - (- \\
& c)^{(1/4)*\text{Sqrt}[x]})})]/(((c)^{(1/4)} - c^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt} \\
& [d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(2*c^{(1/4)*(1 \\
& + (-c)^{(1/4)*\text{Sqrt}[x]})})]/(((c)^{(1/4)} + c^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})/(d* \\
& \text{Sqrt}[d*x]) + (2*b^2*(-c)^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[(-c)^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[(2*(- \\
& c)^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})]/(((c)^{(1/4)} + I*c^{(1/4)*(1 - I*(-c)^{(1/4) \\
& * \text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) - (2*b^2*(-c)^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[(-c)^{(1/4)*\text{S} \\
& \text{qrt}[x]}] * \text{Log}[(2*(-c)^{(1/4)*(1 + c^{(1/4)*\text{Sqrt}[x]})})]/(((c)^{(1/4)} + c^{(1/4)*(1 \\
& + (-c)^{(1/4)*\text{Sqrt}[x]})})/(d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{( \\
& 1/4)*\text{Sqrt}[x]}] * \text{Log}[(1 - I)*(1 + c^{(1/4)*\text{Sqrt}[x]})]/(1 - I*c^{(1/4)*\text{Sqrt}[x]}) \\
& ]/(d*\text{Sqrt}[d*x]) + (\text{Sqrt}[2]*a*b*c^{(1/4)*\text{Sqrt}[x]} * \text{Log}[1 - \text{Sqrt}[2]*c^{(1/4)*\text{Sqrt}[ \\
& x] + \text{Sqrt}[c]*x])/(d*\text{Sqrt}[d*x]) - (\text{Sqrt}[2]*a*b*c^{(1/4)*\text{Sqrt}[x]} * \text{Log}[1 + \text{Sqrt}[ \\
& 2]*c^{(1/4)*\text{Sqrt}[x] + \text{Sqrt}[c]*x])/(d*\text{Sqrt}[d*x]) - (2*b^2*(-c)^{(1/4)*\text{Sqrt}[x]} * \\
& \text{ArcTan}[(-c)^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[1 - c*x^2])/(d*\text{Sqrt}[d*x]) + (2*b^2*(-c)^{(1/4) \\
& )*\text{Sqrt}[x]*\text{ArcTanh}[(-c)^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[1 - c*x^2])/(d*\text{Sqrt}[d*x]) - (2*b* \\
& c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[c^{(1/4)*\text{Sqrt}[x]}] * (2*a - b*\text{Log}[1 - c*x^2]))/(d*\text{Sqrt}[d \\
& *x]) + (2*b*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * (2*a - b*\text{Log}[1 - c*x^2 \\
& ])))/(d*\text{Sqrt}[d*x]) - (2*a - b*\text{Log}[1 - c*x^2])^2/(2*d*\text{Sqrt}[d*x]) - (2*a*b*\text{Log} \\
& [1 + c*x^2])/(d*\text{Sqrt}[d*x]) + (2*b^2*(-c)^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan}[(-c)^{(1/4)*\text{S} \\
& \text{qrt}[x]}] * \text{Log}[1 + c*x^2])/(d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1/4)*\text{Sqrt}[x]}*\text{ArcTan} \\
& h[(-c)^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[1 + c*x^2])/(d*\text{Sqrt}[d*x]) + (2*b^2*c^{(1/4)*\text{Sqrt}[x]} \\
& * \text{ArcTanh}[c^{(1/4)*\text{Sqrt}[x]}] * \text{Log}[1 + c*x^2])/(d*\text{Sqrt}[d*x]) + (b^2*\text{Log}[1 - c*x
\end{aligned}$$

$$\begin{aligned}
& ^2] * \text{Log}[1 + c*x^2]) / (d*\text{Sqrt}[d*x]) - (b^2 * \text{Log}[1 + c*x^2]^2) / (2*d*\text{Sqrt}[d*x]) \\
& - (2*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - (-c)^{(1/4)}*\text{Sqrt}[x])]) / (d* \\
& \text{Sqrt}[d*x]) + ((2*I)*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - I*(-c)^{(1/4)} \\
& /4)*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (I*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*( \\
& -c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x])) / ((I*\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)})*(1 \\
& - I*(-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) - (I*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyL} \\
& \text{og}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x])) / ((I*\text{Sqrt}[-\text{Sqrt}[c]] + \\
& (-c)^{(1/4)})*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) + (I*b^2*(-c)^{(1/4)} \\
& *\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - (-c)^{(1/4)}*\text{Sqrt}[x])) / (1 - I*(-c)^{(1/4)} \\
& )*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) + ((2*I)*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2 \\
& / (1 + I*(-c)^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (2*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{Pol} \\
& \text{yLog}[2, 1 - 2/(1 + (-c)^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (b^2*(-c)^{(1/4)}*\text{Sq} \\
& \text{rt}[x]*\text{PolyLog}[2, 1 + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])) / ((\text{Sqrt}[-\text{S} \\
& \text{qrt}[-c]] - (-c)^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) - (b^2*(-c) \\
& )^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])) \\
& / ((\text{Sqrt}[-\text{Sqrt}[-c]] + (-c)^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) \\
& + (b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]* \\
& \text{Sqrt}[x])) / ((\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqr} \\
& \text{t}[d*x]) + (b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{S} \\
& \text{qrt}[c]]*\text{Sqrt}[x])) / ((\text{Sqrt}[-\text{Sqrt}[c]] + (-c)^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))]) \\
& ) / (d*\text{Sqrt}[d*x]) + (I*b^2*(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 - I)*(1 + (- \\
& c)^{(1/4)}*\text{Sqrt}[x])) / (1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1 \\
& /4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - c^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (I*b^2 \\
& *(-c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x])) / ((( \\
& -c)^{(1/4)} - I*c^{(1/4)})*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) + (b^2*( \\
& -c)^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x])) / (((-c) \\
& )^{(1/4)} - c^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) + ((2*I)*b^2*c \\
& ^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - I*c^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - ( \\
& I*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x \\
& ])) / ((I*\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)})*(1 - I*c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) \\
& - (I*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqr} \\
& \text{t}[x])) / ((I*\text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)})*(1 - I*c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d \\
& *x]) - (I*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - (-c)^{(1/4)}*\text{Sqr} \\
& \text{t}[x])) / ((I*(-c)^{(1/4)} - c^{(1/4)})*(1 - I*c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) - \\
& (I*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])) \\
& / ((I*(-c)^{(1/4)} + c^{(1/4)})*(1 - I*c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) + (I*b^ \\
& 2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - c^{(1/4)}*\text{Sqrt}[x])) / (1 - I*c^{( \\
& 1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) + ((2*I)*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2 \\
& / (1 + I*c^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) - (2*b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2 \\
& , 1 - 2/(1 + c^{(1/4)}*\text{Sqrt}[x])]) / (d*\text{Sqrt}[d*x]) + (b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLo} \\
& \text{g}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])) / ((\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1 \\
& /4)})*(1 + c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) + (b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[ \\
& 2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x])) / ((\text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4) \\
& ))*(1 + c^{(1/4)}*\text{Sqrt}[x]))]) / (d*\text{Sqrt}[d*x]) - (b^2*c^{(1/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, \\
& 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x])) / ((\text{Sqrt}[-\text{Sqrt}[c]] - c^{(1/4)})*(
\end{aligned}$$

$$\begin{aligned} & (1 + c^{1/4} \sqrt{x})) / (d \sqrt{dx}) - (b^2 c^{1/4} \sqrt{x} \text{PolyLog}[2, 1 - \\ & (2c^{1/4}(1 + \sqrt{-\sqrt{c}}) \sqrt{x})) / ((\sqrt{-\sqrt{c}} + c^{1/4})(1 + \\ & c^{1/4} \sqrt{x}))]) / (d \sqrt{dx}) + (b^2 c^{1/4} \sqrt{x} \text{PolyLog}[2, 1 + (2 \\ & c^{1/4}(1 - (-c)^{1/4} \sqrt{x})) / (((-c)^{1/4} - c^{1/4})(1 + c^{1/4} \sqrt{x}))]) / (d \sqrt{dx}) + (b^2 c^{1/4} \sqrt{x} \text{PolyLog}[2, 1 - (2c^{1/4}(1 + \\ & (-c)^{1/4} \sqrt{x})) / (((-c)^{1/4} + c^{1/4})(1 + c^{1/4} \sqrt{x}))]) / (d \sqrt{dx}) - (I b^2 (-c)^{1/4} \sqrt{x} \text{PolyLog}[2, 1 - (2(-c)^{1/4}(1 + c^{1/4} \\ & \sqrt{x})) / (((-c)^{1/4} + I c^{1/4})(1 - I(-c)^{1/4} \sqrt{x}))]) / (d \sqrt{dx}) + (b^2 (-c)^{1/4} \sqrt{x} \text{PolyLog}[2, 1 - (2(-c)^{1/4}(1 + c^{1/4} \\ & \sqrt{x})) / (((-c)^{1/4} + c^{1/4})(1 + (-c)^{1/4} \sqrt{x}))]) / (d \sqrt{dx}) + (I b^2 c^{1/4} \sqrt{x} \text{PolyLog}[2, 1 - ((1 - I)(1 + c^{1/4} \sqrt{x})) / \\ & (1 - I c^{1/4} \sqrt{x})]) / (d \sqrt{dx}) \end{aligned}$$

**Rubi [F]** time = 0.0292299, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(3/2), x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(3/2), x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{3/2}} dx = \int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{3/2}} dx$$

**Mathematica [F]** time = 94.0147, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(3/2), x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(3/2), x]

**Maple [F]** time = 0.272, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Artanh}(cx^2))^2 (dx)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(3/2), x)

[Out] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(3/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\left( b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2 \right) \sqrt{dx}}{d^2 x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(3/2), x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)\*sqrt(d\*x)/(d^2\*x^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/(d\*x)\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2/(d\*x)^(3/2), x)

$$3.93 \quad \int \frac{\left(a+b \tanh^{-1}(cx^2)\right)^2}{(dx)^{5/2}} dx$$

**Optimal.** Leaf size=6520

result too large to display

```
[Out] (-2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]/(3*d^2
*Sqrt[d*x]) + (2*Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqr
t[x]]/(3*d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1
/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) - (((2*I)/3)*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(
1/4)*Sqrt[x]]^2)/(d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(
1/4)*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)
*Sqrt[x]]^2)/(3*d^2*Sqrt[d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/
4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) + (4*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x]
)])/((3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]
]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))]/((I*Sqrt[-Sqrt[c]] - (-c
)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x])))/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)
*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*S
qrt[x]))]/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x])))/(3*
d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(
(1 + I)*(1 - (-c)^(1/4)*Sqrt[x])]/(1 - I*(-c)^(1/4)*Sqrt[x])])/((3*d^2*Sqrt[
d*x]) - (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(
-c)^(1/4)*Sqrt[x])])/((3*d^2*Sqrt[d*x]) + (4*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[
(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/((3*d^2*Sqrt[d*x]) + (2
*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 -
Sqrt[-Sqrt[-c]]*Sqrt[x]))]/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*
Sqrt[x])))/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)
)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))]/((Sqrt[-Sqrt[-c
]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])))/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c
)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-S
qrt[c]]*Sqrt[x]))]/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))]
)/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]
]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))]/((Sqrt[-Sqrt[c]] + (-c)^(1
/4))*(1 + (-c)^(1/4)*Sqrt[x])))/(3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^(3/4)*Sqrt
[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + (-c)^(1/4)*Sqrt[x])]/(1 -
I*(-c)^(1/4)*Sqrt[x])))/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTanh[
c^(1/4)*Sqrt[x]]*Log[2/(1 - c^(1/4)*Sqrt[x])])/((3*d^2*Sqrt[d*x]) - (2*b^2*(
-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 - c^(1/4)
)*Sqrt[x]))]/(((c)^(1/4) - I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x])))/(3*d^2*S
qrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-
c)^(1/4)*(1 - c^(1/4)*Sqrt[x]))]/(((c)^(1/4) - c^(1/4))*(1 + (-c)^(1/4)*Sqr
```

$$\begin{aligned}
& t[x])))/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]] \\
& *Log[2/(1 - I*c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x] \\
& *ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))]/((I \\
& *Sqrt[-Sqrt[-c]] - c^(1/4))*(1 - I*c^(1/4)*Sqrt[x])))/(3*d^2*Sqrt[d*x]) - \\
& (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sq \\
& rt[-c]]*Sqrt[x]))]/((I*Sqrt[-Sqrt[-c]] + c^(1/4))*(1 - I*c^(1/4)*Sqrt[x])))) \\
& /((3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[(-2 \\
& *c^(1/4)*(1 - (-c)^(1/4)*Sqrt[x]))]/((I*(-c)^(1/4) - c^(1/4))*(1 - I*c^(1/4) \\
& *Sqrt[x])))))/(3*d^2*Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt \\
& [x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/4)*Sqrt[x]))]/((I*(-c)^(1/4) + c^(1/4))*(1 \\
& - I*c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c \\
& ^{(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x]) \\
& ])/(3*d^2*Sqrt[d*x]) - (4*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[2 \\
& /((1 + I*c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) + (4*b^2*c^(3/4)*Sqrt[x]*ArcTa \\
& nh[c^(1/4)*Sqrt[x]]*Log[2/(1 + c^(1/4)*Sqrt[x])])/(3*d^2*Sqrt[d*x]) - (2*b^ \\
& 2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[ \\
& -c]]*Sqrt[x]))]/((Sqrt[-Sqrt[-c]] - c^(1/4))*(1 + c^(1/4)*Sqrt[x])))]/(3*d^2 \\
& *Sqrt[d*x]) - (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4) \\
& )*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))]/((Sqrt[-Sqrt[-c]] + c^(1/4))*(1 + c^(1/4)* \\
& Sqrt[x])))]/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt \\
& [x]]*Log[(-2*c^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))]/((Sqrt[-Sqrt[c]] - c^(1/ \\
& 4))*(1 + c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*Arc \\
& Tanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))]/((Sqrt[- \\
& Sqrt[c]] + c^(1/4))*(1 + c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) - (2*b^2*c^( \\
& 3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(-2*c^(1/4)*(1 - (-c)^(1/4)*Sqrt[ \\
& x]))]/(((c)^(1/4) - c^(1/4))*(1 + c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) - ( \\
& 2*b^2*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[x]]*Log[(2*c^(1/4)*(1 + (-c)^(1/ \\
& 4)*Sqrt[x]))]/(((c)^(1/4) + c^(1/4))*(1 + c^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d \\
& *x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/ \\
& 4)*(1 + c^(1/4)*Sqrt[x]))]/(((c)^(1/4) + I*c^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[ \\
& x])))]/(3*d^2*Sqrt[d*x]) - (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqr \\
& t[x]]*Log[(2*(-c)^(1/4)*(1 + c^(1/4)*Sqrt[x]))]/(((c)^(1/4) + c^(1/4))*(1 + \\
& (-c)^(1/4)*Sqrt[x])))]/(3*d^2*Sqrt[d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c \\
& ^{(1/4)*Sqrt[x]]*Log[((1 - I)*(1 + c^(1/4)*Sqrt[x]))/(1 - I*c^(1/4)*Sqrt[x]) \\
& ])/(3*d^2*Sqrt[d*x]) - (Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4) \\
& *Sqrt[x] + Sqrt[c]*x])/((3*d^2*Sqrt[d*x]) + (Sqrt[2]*a*b*c^(3/4)*Sqrt[x]*Log \\
& [1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/((3*d^2*Sqrt[d*x]) + (2*b^2*(-c)^( \\
& 3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/((3*d^2*Sqrt[d*x]) \\
& + (2*b^2*(-c)^(3/4)*Sqrt[x]*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[1 - c*x^2])/((3* \\
& d^2*Sqrt[d*x]) + (2*b*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*(2*a - b*Log[ \\
& 1 - c*x^2])))/(3*d^2*Sqrt[d*x]) + (2*b*c^(3/4)*Sqrt[x]*ArcTanh[c^(1/4)*Sqrt[ \\
& x]]*(2*a - b*Log[1 - c*x^2])))/(3*d^2*Sqrt[d*x]) - (2*a - b*Log[1 - c*x^2])^ \\
& 2/((6*d^2*x*Sqrt[d*x]) - (2*a*b*Log[1 + c*x^2])/((3*d^2*x*Sqrt[d*x]) - (2*b^2 \\
& *(-c)^(3/4)*Sqrt[x]*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/((3*d^2*Sqrt[ \\
& d*x]) + (2*b^2*c^(3/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]]*Log[1 + c*x^2])/((3*d
\end{aligned}$$

$$\begin{aligned}
& ^2\text{Sqrt}[d*x]) - (2*b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{ArcTanh}[(-c)^{(1/4)}*\text{Sqrt}[x]]*\text{Log}[1 \\
& + c*x^2])/(3*d^2*\text{Sqrt}[d*x]) + (2*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{ArcTanh}[c^{(1/4)}*\text{Sqrt}[ \\
& x]]*\text{Log}[1 + c*x^2])/(3*d^2*\text{Sqrt}[d*x]) + (b^2*\text{Log}[1 - c*x^2]*\text{Log}[1 + c*x^2]) \\
& /((3*d^2*x*\text{Sqrt}[d*x]) - (b^2*\text{Log}[1 + c*x^2]^2)/(6*d^2*x*\text{Sqrt}[d*x]) - (2*b^2* \\
& (-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - (-c)^{(1/4)}*\text{Sqrt}[x])])/(3*d^2*\text{Sqrt}[ \\
& d*x]) - (((2*I)/3)*b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - I*(-c)^{(1/4)} \\
& )*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) + ((I/3)*b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + \\
& (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)} \\
& ))*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) + ((I/3)*b^2*(-c)^{(3/4)}*\text{Sqr \\
& t}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[- \\
& \text{Sqrt}[c]] + (-c)^{(1/4)}*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - ((I/ \\
& 3)*b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - (-c)^{(1/4)}*\text{Sqrt}[x])) \\
& /((1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - (((2*I)/3)*b^2*(-c)^{(3/4)}*S \\
& qrt}[x]*\text{PolyLog}[2, 1 - 2/(1 + I*(-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - (2*b \\
& ^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 + (-c)^{(1/4)}*\text{Sqrt}[x])])/(3*d^2*Sq \\
& rt[d*x]) - (b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[- \\
& \text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] - (-c)^{(1/4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x] \\
& )))]/(3*d^2*\text{Sqrt}[d*x]) - (b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/ \\
& 4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] + (-c)^{(1/4)}*(1 + (-c) \\
& ^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) + (b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 \\
& + (2*(-c)^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] - (-c)^{(1/4)} \\
& ))*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])])/(3*d^2*\text{Sqrt}[d*x]) + (b^2*(-c)^{(3/4)}*\text{Sqrt}[x]* \\
& \text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] \\
& + (-c)^{(1/4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) - ((I/3)*b^2*( \\
& -c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 - I)*(1 + (-c)^{(1/4)}*\text{Sqrt}[x]))/(1 - I* \\
& (-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - (2*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 \\
& - 2/(1 - c^{(1/4)}*\text{Sqrt}[x])])/(3*d^2*\text{Sqrt}[d*x]) + ((I/3)*b^2*(-c)^{(3/4)}*\text{Sqrt} \\
& [x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x]))/((( -c)^{(1/4)} - I*c^{(1/4)} \\
& )*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) + (b^2*(-c)^{(3/4)}*\text{Sqrt} \\
& [x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 - c^{(1/4)}*\text{Sqrt}[x]))/((( -c)^{(1/4)} - c^{(1/ \\
& 4)}*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) - (((2*I)/3)*b^2*c^{(3/4)} \\
& * \text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 - I*c^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) + ((I/3) \\
& )*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]]*\text{Sqrt}[x] \\
& ))/((I*\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))]/(d^2*\text{Sqrt}[d*x \\
& ]) + ((I/3)*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[- \\
& c]]*\text{Sqrt}[x]))/((I*\text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))]/(d^ \\
& 2*\text{Sqrt}[d*x]) + ((I/3)*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - (- \\
& c)^{(1/4)}*\text{Sqrt}[x]))/((I*(-c)^{(1/4)} - c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))]/(d^ \\
& 2*\text{Sqrt}[d*x]) + ((I/3)*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + (- \\
& c)^{(1/4)}*\text{Sqrt}[x]))/((I*(-c)^{(1/4)} + c^{(1/4)}*(1 - I*c^{(1/4)}*\text{Sqrt}[x])))]/(d^ \\
& 2*\text{Sqrt}[d*x]) - ((I/3)*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 + I)*(1 - c^{(1/ \\
& 4)}*\text{Sqrt}[x]))/(1 - I*c^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - (((2*I)/3)*b^2*c^ \\
& (3/4)*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 + I*c^{(1/4)}*\text{Sqrt}[x])])/(d^2*\text{Sqrt}[d*x]) - \\
& (2*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - 2/(1 + c^{(1/4)}*\text{Sqrt}[x])])/(3*d^2*\text{Sqrt} \\
& [d*x]) + (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[-c]
\end{aligned}$$



$$\begin{aligned} & ]*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] - c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))/(3*d^2*\text{Sqrt}[d*x]) \\ & + (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[-c]])*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[-c]] + c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & - (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - \text{Sqrt}[-\text{Sqrt}[c]])*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] - c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & - (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + \text{Sqrt}[-\text{Sqrt}[c]])*\text{Sqrt}[x]))/((\text{Sqrt}[-\text{Sqrt}[c]] + c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & + (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 + (2*c^{(1/4)}*(1 - (-c)^{(1/4)})*\text{Sqrt}[x]))/((( -c)^{(1/4)} - c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & + (b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*c^{(1/4)}*(1 + (-c)^{(1/4)})*\text{Sqrt}[x]))/((( -c)^{(1/4)} + c^{(1/4)})*(1 + c^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & + ((I/3)*b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + c^{(1/4)}*\text{Sqrt}[x]))/((( -c)^{(1/4)} + I*c^{(1/4)})*(1 - I*(-c)^{(1/4)}*\text{Sqrt}[x])))]/(d^2*\text{Sqrt}[d*x]) \\ & + (b^2*(-c)^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - (2*(-c)^{(1/4)}*(1 + c^{(1/4)}*\text{Sqrt}[x]))/((( -c)^{(1/4)} + c^{(1/4)})*(1 + (-c)^{(1/4)}*\text{Sqrt}[x])))]/(3*d^2*\text{Sqrt}[d*x]) \\ & - ((I/3)*b^2*c^{(3/4)}*\text{Sqrt}[x]*\text{PolyLog}[2, 1 - ((1 - I)*(1 + c^{(1/4)}*\text{Sqrt}[x]))/(1 - I*c^{(1/4)}*\text{Sqrt}[x])))]/(d^2*\text{Sqrt}[d*x]) \end{aligned}$$

**Rubi [F]** time = 0.030478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(5/2), x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(5/2), x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{5/2}} dx = \int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{5/2}} dx$$

**Mathematica [F]** time = 50.7384, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^2))^2}{(dx)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(5/2), x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^2])^2/(d\*x)^(5/2), x]

**Maple [F]** time = 0.275, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Arctanh}(cx^2))^2 (dx)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(5/2), x)

[Out] int((a+b\*arctanh(c\*x^2))^2/(d\*x)^(5/2), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(5/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{\left( b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2 \right) \sqrt{dx}}{d^3 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(5/2), x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)\*sqrt(d\*x)/(d^3\*x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*2))\*\*2/(d\*x)\*\*(5/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^2))^2/(d\*x)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^2) + a)^2/(d\*x)^(5/2), x)

$$3.94 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^3, x]

**Rubi [A]** time = 0.0239834, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^3,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$$

**Mathematica [A]** time = 1.75129, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^3,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^3, x]

**Maple [A]** time = 0.122, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^2))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^2))^3,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^2))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{artanh}(cx^2)^3 + 3ab^2 \operatorname{artanh}(cx^2)^2 + 3a^2b \operatorname{artanh}(cx^2) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2))^3,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x^2)^3 + 3\*a\*b^2\*arctanh(c\*x^2)^2 + 3\*a^2\*b\*arctanh(c\*x^2) + a^3)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**3,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^3*(d*x)^m, x)
```

$$3.95 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left((dx)^m (a + b \tanh^{-1}(cx^2))^2, x\right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^2, x]

**Rubi [A]** time = 0.0237911, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^2 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$$

**Mathematica [A]** time = 1.16718, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2])^2, x]

**Maple [A]** time = 0.119, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^2))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^2))^2,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^2 \operatorname{artanh}(cx^2)^2 + 2ab \operatorname{artanh}(cx^2) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^2)^2 + 2\*a\*b\*arctanh(c\*x^2) + a^2)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((d*x)**m*(a+b*atanh(c*x**2))**2,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)^2*(d*x)^m, x)
```

### 3.96 $\int (dx)^m \left( a + b \tanh^{-1}(cx^2) \right) dx$

**Optimal.** Leaf size=74

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx^2) \right)}{d(m+1)} - \frac{2bc(dx)^{m+3} \text{Hypergeometric2F1} \left( 1, \frac{m+3}{4}, \frac{m+7}{4}, c^2x^4 \right)}{d^3(m+1)(m+3)}$$

[Out]  $((d*x)^{(1+m)}*(a + b*ArcTanh[c*x^2]))/(d*(1+m)) - (2*b*c*(d*x)^{(3+m)}*Hypergeometric2F1[1, (3+m)/4, (7+m)/4, c^2*x^4])/(d^3*(1+m)*(3+m))$

**Rubi [A]** time = 0.0397164, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx^2) \right)}{d(m+1)} - \frac{2bc(dx)^{m+3} {}_2F_1 \left( 1, \frac{m+3}{4}; \frac{m+7}{4}; c^2x^4 \right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2]),x]

[Out]  $((d*x)^{(1+m)}*(a + b*ArcTanh[c*x^2]))/(d*(1+m)) - (2*b*c*(d*x)^{(3+m)}*Hypergeometric2F1[1, (3+m)/4, (7+m)/4, c^2*x^4])/(d^3*(1+m)*(3+m))$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a

)]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \tanh^{-1}(cx^2)) dx &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^2))}{d(1+m)} - \frac{(2bc) \int \frac{x(dx)^{1+m}}{1-c^2x^4} dx}{d(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^2))}{d(1+m)} - \frac{(2bc) \int \frac{(dx)^{2+m}}{1-c^2x^4} dx}{d^2(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^2))}{d(1+m)} - \frac{2bc(dx)^{3+m} {}_2F_1\left(1, \frac{3+m}{4}; \frac{7+m}{4}; c^2x^4\right)}{d^3(1+m)(3+m)} \end{aligned}$$

**Mathematica [A]** time = 0.0659015, size = 64, normalized size = 0.86

$$\frac{x(dx)^m \left( 2bcx^2 \text{Hypergeometric2F1}\left(1, \frac{m+3}{4}, \frac{m+7}{4}, c^2x^4\right) - (m+3)(a + b \tanh^{-1}(cx^2)) \right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^2]), x]

[Out] -((x\*(d\*x)^m\*(-((3 + m)\*(a + b\*ArcTanh[c\*x^2])) + 2\*b\*c\*x^2\*Hypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2\*x^4]))/((1 + m)\*(3 + m)))

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \text{Artanh}(cx^2)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^2)), x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^2)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \operatorname{artanh}\left(cx^2\right) + a\right) (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^2)),x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^2) + a)\*(d\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*atanh(c\*x\*\*2)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^2) + a) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

$$3.97 \quad \int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b \tanh^{-1}(cx^2)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^2]), x]

**Rubi [A]** time = 0.0287267, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^2]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^2]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx = \int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$$

**Mathematica [A]** time = 0.348211, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^2)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^2]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^2]), x]

---

**Maple [A]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{Artanh}(cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^2)),x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^2)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^2)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x^2) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^2)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arctanh(c\*x^2) + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*atanh(c\*x\*\*2)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^2) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^2)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x^2) + a), x)



$$3.98 \quad \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^2])^2, x]

**Rubi [A]** time = 0.0271365, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^2])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^2])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2} dx$$

**Mathematica [A]** time = 0.413779, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^2)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^2])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^2])^2, x]

**Maple [A]** time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{Artanh}(cx^2))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^2))^2,x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(c^2 d^m x^4 - d^m) x^m}{b^2 c x \log(cx^2 + 1) - b^2 c x \log(-cx^2 + 1) + 2 abc x} + \int -\frac{(c^2 d^m (m + 3) x^4 - d^m (m - 1)) x^m}{b^2 c x^2 \log(cx^2 + 1) - b^2 c x^2 \log(-cx^2 + 1) + 2 abc x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^2))^2,x, algorithm="maxima")

[Out] (c^2\*d^m\*x^4 - d^m)\*x^m/(b^2\*c\*x\*log(c\*x^2 + 1) - b^2\*c\*x\*log(-c\*x^2 + 1) + 2\*a\*b\*c\*x) + integrate(-(c^2\*d^m\*(m + 3)\*x^4 - d^m\*(m - 1))\*x^m/(b^2\*c\*x^2\*log(c\*x^2 + 1) - b^2\*c\*x^2\*log(-c\*x^2 + 1) + 2\*a\*b\*c\*x^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}(cx^2)^2 + 2 ab \operatorname{artanh}(cx^2) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^2))^2,x, algorithm="fricas")

[Out] `integral((d*x)^m/(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m/(a+b*atanh(c*x**2))**2,x)`

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{artanh}(cx^2) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="giac")`

[Out] `integrate((d*x)^m/(b*arctanh(c*x^2) + a)^2, x)`

### 3.99 $\int x^{11} (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=54

$$\frac{1}{12}x^{12}(a + b \tanh^{-1}(cx^3)) + \frac{bx^3}{12c^3} - \frac{b \tanh^{-1}(cx^3)}{12c^4} + \frac{bx^9}{36c}$$

[Out] (b\*x^3)/(12\*c^3) + (b\*x^9)/(36\*c) - (b\*ArcTanh[c\*x^3])/(12\*c^4) + (x^12\*(a + b\*ArcTanh[c\*x^3]))/12

**Rubi [A]** time = 0.0382175, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 302, 206}

$$\frac{1}{12}x^{12}(a + b \tanh^{-1}(cx^3)) + \frac{bx^3}{12c^3} - \frac{b \tanh^{-1}(cx^3)}{12c^4} + \frac{bx^9}{36c}$$

Antiderivative was successfully verified.

[In] Int[x^11\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (b\*x^3)/(12\*c^3) + (b\*x^9)/(36\*c) - (b\*ArcTanh[c\*x^3])/(12\*c^4) + (x^12\*(a + b\*ArcTanh[c\*x^3]))/12

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

#### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

$Q[m, 2*n - 1]$

### Rule 206

$\text{Int}[\left((a_) + (b_)*(x_)^2\right)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\left(1*\text{ArcTanh}[\left(\text{Rt}[-b, 2]*x\right)/\text{Rt}[a, 2]]\right)/\left(\text{Rt}[a, 2]*\text{Rt}[-b, 2]\right), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int x^{11} (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{12} x^{12} (a + b \tanh^{-1}(cx^3)) - \frac{1}{4} (bc) \int \frac{x^{14}}{1 - c^2 x^6} dx \\ &= \frac{1}{12} x^{12} (a + b \tanh^{-1}(cx^3)) - \frac{1}{12} (bc) \text{Subst} \left( \int \frac{x^4}{1 - c^2 x^2} dx, x, x^3 \right) \\ &= \frac{1}{12} x^{12} (a + b \tanh^{-1}(cx^3)) - \frac{1}{12} (bc) \text{Subst} \left( \int \left( -\frac{1}{c^4} - \frac{x^2}{c^2} + \frac{1}{c^4 (1 - c^2 x^2)} \right) dx, x, x^3 \right) \\ &= \frac{bx^3}{12c^3} + \frac{bx^9}{36c} + \frac{1}{12} x^{12} (a + b \tanh^{-1}(cx^3)) - \frac{b \text{Subst} \left( \int \frac{1}{1 - c^2 x^2} dx, x, x^3 \right)}{12c^3} \\ &= \frac{bx^3}{12c^3} + \frac{bx^9}{36c} - \frac{b \tanh^{-1}(cx^3)}{12c^4} + \frac{1}{12} x^{12} (a + b \tanh^{-1}(cx^3)) \end{aligned}$$

**Mathematica [A]** time = 0.0200856, size = 78, normalized size = 1.44

$$\frac{ax^{12}}{12} + \frac{bx^3}{12c^3} + \frac{b \log(1 - cx^3)}{24c^4} - \frac{b \log(cx^3 + 1)}{24c^4} + \frac{bx^9}{36c} + \frac{1}{12} bx^{12} \tanh^{-1}(cx^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^11\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (b\*x^3)/(12\*c^3) + (b\*x^9)/(36\*c) + (a\*x^12)/12 + (b\*x^12\*ArcTanh[c\*x^3])/12 + (b\*Log[1 - c\*x^3])/(24\*c^4) - (b\*Log[1 + c\*x^3])/(24\*c^4)

**Maple [A]** time = 0.009, size = 66, normalized size = 1.2

$$\frac{x^{12}a}{12} + \frac{bx^{12}\text{Artanh}(cx^3)}{12} + \frac{bx^9}{36c} + \frac{bx^3}{12c^3} + \frac{b \ln(cx^3 - 1)}{24c^4} - \frac{b \ln(cx^3 + 1)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(a+b*arctanh(c*x^3)),x)`

[Out]  $\frac{1}{12}x^{12}a + \frac{1}{12}bx^{12}\operatorname{arctanh}(cx^3) + \frac{1}{36}bx^9/c + \frac{1}{12}bx^3/c^3 + \frac{1}{24}b/c^4 \ln(cx^3-1) - \frac{1}{24}b/c^4 \ln(cx^3+1)$

**Maxima [A]** time = 0.953359, size = 93, normalized size = 1.72

$$\frac{1}{12}ax^{12} + \frac{1}{72}\left(6x^{12}\operatorname{artanh}(cx^3) + c\left(\frac{2(c^2x^9 + 3x^3)}{c^4} - \frac{3\log(cx^3 + 1)}{c^5} + \frac{3\log(cx^3 - 1)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out]  $\frac{1}{12}ax^{12} + \frac{1}{72}(6x^{12}\operatorname{arctanh}(cx^3) + c(2(c^2x^9 + 3x^3)/c^4 - 3\log(cx^3 + 1)/c^5 + 3\log(cx^3 - 1)/c^5))b$

**Fricas [A]** time = 2.04169, size = 138, normalized size = 2.56

$$\frac{6ac^4x^{12} + 2bc^3x^9 + 6bcx^3 + 3(bc^4x^{12} - b)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{72c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]  $\frac{1}{72}(6ac^4x^{12} + 2bc^3x^9 + 6bcx^3 + 3(bc^4x^{12} - b)\log(-(cx^3 + 1)/(cx^3 - 1)))/c^4$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: `KeyError`

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

**Giac [A]** time = 1.20153, size = 105, normalized size = 1.94

$$\frac{1}{24}bx^{12}\log\left(-\frac{cx^3+1}{cx^3-1}\right) + \frac{1}{12}ax^{12} + \frac{bx^9}{36c} + \frac{bx^3}{12c^3} - \frac{b\log(cx^3+1)}{24c^4} + \frac{b\log(cx^3-1)}{24c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/24*b*x^12*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/12*a*x^12 + 1/36*b*x^9/c + 1/12*b*x^3/c^3 - 1/24*b*log(c*x^3 + 1)/c^4 + 1/24*b*log(c*x^3 - 1)/c^4
```

### 3.100 $\int x^8 (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=48

$$\frac{1}{9}x^9(a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2x^6)}{18c^3} + \frac{bx^6}{18c}$$

[Out] (b\*x^6)/(18\*c) + (x^9\*(a + b\*ArcTanh[c\*x^3]))/9 + (b\*Log[1 - c^2\*x^6])/(18\*c^3)

**Rubi [A]** time = 0.0359128, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6097, 266, 43}

$$\frac{1}{9}x^9(a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2x^6)}{18c^3} + \frac{bx^6}{18c}$$

Antiderivative was successfully verified.

[In] Int[x^8\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (b\*x^6)/(18\*c) + (x^9\*(a + b\*ArcTanh[c\*x^3]))/9 + (b\*Log[1 - c^2\*x^6])/(18\*c^3)

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```



Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned}
 \int x^8 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{9} x^9 (a + b \tanh^{-1}(cx^3)) - \frac{1}{3} (bc) \int \frac{x^{11}}{1 - c^2 x^6} dx \\
 &= \frac{1}{9} x^9 (a + b \tanh^{-1}(cx^3)) - \frac{1}{18} (bc) \text{Subst} \left( \int \frac{x}{1 - c^2 x} dx, x, x^6 \right) \\
 &= \frac{1}{9} x^9 (a + b \tanh^{-1}(cx^3)) - \frac{1}{18} (bc) \text{Subst} \left( \int \left( -\frac{1}{c^2} - \frac{1}{c^2(-1 + c^2 x)} \right) dx, x, x^6 \right) \\
 &= \frac{bx^6}{18c} + \frac{1}{9} x^9 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2 x^6)}{18c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0158537, size = 53, normalized size = 1.1

$$\frac{ax^9}{9} + \frac{b \log(1 - c^2 x^6)}{18c^3} + \frac{bx^6}{18c} + \frac{1}{9} bx^9 \tanh^{-1}(cx^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^8\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (b\*x^6)/(18\*c) + (a\*x^9)/9 + (b\*x^9\*ArcTanh[c\*x^3])/9 + (b\*Log[1 - c^2\*x^6])/(18\*c^3)

**Maple [A]** time = 0.008, size = 45, normalized size = 0.9

$$\frac{x^9 a}{9} + \frac{bx^9 \text{Artanh}(cx^3)}{9} + \frac{bx^6}{18c} + \frac{b \ln(c^2 x^6 - 1)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(a+b\*arctanh(c\*x^3)),x)

[Out] 1/9\*x^9\*a+1/9\*b\*x^9\*arctanh(c\*x^3)+1/18\*b\*x^6/c+1/18\*b/c^3\*ln(c^2\*x^6-1)

**Maxima [A]** time = 1.02131, size = 62, normalized size = 1.29

$$\frac{1}{9}ax^9 + \frac{1}{18}\left(2x^9 \operatorname{artanh}(cx^3) + \left(\frac{x^6}{c^2} + \frac{\log(c^2x^6 - 1)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(a+b\*arctanh(c\*x^3)),x, algorithm="maxima")

[Out] 1/9\*a\*x^9 + 1/18\*(2\*x^9\*arctanh(c\*x^3) + (x^6/c^2 + log(c^2\*x^6 - 1)/c^4)\*c)\*b

**Fricas [A]** time = 1.91646, size = 134, normalized size = 2.79

$$\frac{bc^3x^9 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2ac^3x^9 + bc^2x^6 + b \log(c^2x^6 - 1)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(a+b\*arctanh(c\*x^3)),x, algorithm="fricas")

[Out] 1/18\*(b\*c^3\*x^9\*log(-(c\*x^3 + 1)/(c\*x^3 - 1)) + 2\*a\*c^3\*x^9 + b\*c^2\*x^6 + b\*log(c^2\*x^6 - 1))/c^3

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*8\*(a+b\*atanh(c\*x\*\*3)),x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.19606, size = 77, normalized size = 1.6

$$\frac{1}{18}bx^9 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + \frac{1}{9}ax^9 + \frac{bx^6}{18c} + \frac{b \log(c^2x^6 - 1)}{18c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*b*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/9*a*x^9 + 1/18*b*x^6/c + 1/18*  
b*log(c^2*x^6 - 1)/c^3
```

### 3.101 $\int x^5 (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=43

$$\frac{1}{6}x^6(a + b \tanh^{-1}(cx^3)) - \frac{b \tanh^{-1}(cx^3)}{6c^2} + \frac{bx^3}{6c}$$

[Out] (b\*x^3)/(6\*c) - (b\*ArcTanh[c\*x^3])/(6\*c^2) + (x^6\*(a + b\*ArcTanh[c\*x^3]))/6

**Rubi [A]** time = 0.030026, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 321, 206}

$$\frac{1}{6}x^6(a + b \tanh^{-1}(cx^3)) - \frac{b \tanh^{-1}(cx^3)}{6c^2} + \frac{bx^3}{6c}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (b\*x^3)/(6\*c) - (b\*ArcTanh[c\*x^3])/(6\*c^2) + (x^6\*(a + b\*ArcTanh[c\*x^3]))/6

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

#### Rule 321

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p]
```

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned} \int x^5 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^3)) - \frac{1}{2} (bc) \int \frac{x^8}{1 - c^2 x^6} dx \\ &= \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^3)) - \frac{1}{6} (bc) \text{Subst} \left( \int \frac{x^2}{1 - c^2 x^2} dx, x, x^3 \right) \\ &= \frac{bx^3}{6c} + \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^3)) - \frac{b \text{Subst} \left( \int \frac{1}{1 - c^2 x^2} dx, x, x^3 \right)}{6c} \\ &= \frac{bx^3}{6c} - \frac{b \tanh^{-1}(cx^3)}{6c^2} + \frac{1}{6} x^6 (a + b \tanh^{-1}(cx^3)) \end{aligned}$$

**Mathematica [A]** time = 0.0156988, size = 67, normalized size = 1.56

$$\frac{ax^6}{6} + \frac{b \log(1 - cx^3)}{12c^2} - \frac{b \log(cx^3 + 1)}{12c^2} + \frac{bx^3}{6c} + \frac{1}{6} bx^6 \tanh^{-1}(cx^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] (b\*x^3)/(6\*c) + (a\*x^6)/6 + (b\*x^6\*ArcTanh[c\*x^3])/6 + (b\*Log[1 - c\*x^3])/(12\*c^2) - (b\*Log[1 + c\*x^3])/(12\*c^2)

**Maple [A]** time = 0.012, size = 57, normalized size = 1.3

$$\frac{x^6 a}{6} + \frac{bx^6 \text{Artanh}(cx^3)}{6} + \frac{bx^3}{6c} + \frac{b \ln(cx^3 - 1)}{12c^2} - \frac{b \ln(cx^3 + 1)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arctanh(c*x^3)),x)`

[Out]  $\frac{1}{6}x^6a + \frac{1}{6}bx^6\operatorname{arctanh}(cx^3) + \frac{1}{6}bx^3/c + \frac{1}{12}b/c^2\ln(cx^3-1) - \frac{1}{12}b/c^2\ln(cx^3+1)$

**Maxima [A]** time = 0.979988, size = 78, normalized size = 1.81

$$\frac{1}{6}ax^6 + \frac{1}{12}\left(2x^6\operatorname{artanh}(cx^3) + c\left(\frac{2x^3}{c^2} - \frac{\log(cx^3+1)}{c^3} + \frac{\log(cx^3-1)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out]  $\frac{1}{6}ax^6 + \frac{1}{12}(2x^6\operatorname{arctanh}(cx^3) + c(2x^3/c^2 - \log(cx^3+1)/c^3 + \log(cx^3-1)/c^3))b$

**Fricas [A]** time = 1.79863, size = 113, normalized size = 2.63

$$\frac{2ac^2x^6 + 2bcx^3 + (bc^2x^6 - b)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]  $\frac{1}{12}(2ac^2x^6 + 2bcx^3 + (bc^2x^6 - b)\log(-(cx^3+1)/(cx^3-1)))/c^2$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: `KeyError`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*atanh(c*x**3)),x)`

[Out] Exception raised: KeyError

**Giac [A]** time = 1.14918, size = 93, normalized size = 2.16

$$\frac{1}{12} b x^6 \log\left(-\frac{c x^3 + 1}{c x^3 - 1}\right) + \frac{1}{6} a x^6 + \frac{b x^3}{6 c} - \frac{b \log(c x^3 + 1)}{12 c^2} + \frac{b \log(c x^3 - 1)}{12 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^3)),x, algorithm="giac")

[Out] 1/12\*b\*x^6\*log(-(c\*x^3 + 1)/(c\*x^3 - 1)) + 1/6\*a\*x^6 + 1/6\*b\*x^3/c - 1/12\*b\*log(c\*x^3 + 1)/c^2 + 1/12\*b\*log(c\*x^3 - 1)/c^2

### 3.102 $\int x^2 (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=37

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2x^6)}{6c}$$

[Out] (x^3\*(a + b\*ArcTanh[c\*x^3]))/3 + (b\*Log[1 - c^2\*x^6])/(6\*c)

**Rubi [A]** time = 0.020476, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 260}

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2x^6)}{6c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] (x^3\*(a + b\*ArcTanh[c\*x^3]))/3 + (b\*Log[1 - c^2\*x^6])/(6\*c)

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rubi steps

$$\begin{aligned} \int x^2 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^3)) - (bc) \int \frac{x^5}{1 - c^2x^6} dx \\ &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^2x^6)}{6c} \end{aligned}$$



**Mathematica [A]** time = 0.008322, size = 42, normalized size = 1.14

$$\frac{ax^3}{3} + \frac{b \log(1 - c^2x^6)}{6c} + \frac{1}{3}bx^3 \tanh^{-1}(cx^3)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (a\*x^3)/3 + (b\*x^3\*ArcTanh[c\*x^3])/3 + (b\*Log[1 - c^2\*x^6])/(6\*c)

**Maple [A]** time = 0.003, size = 37, normalized size = 1.

$$\frac{x^3a}{3} + \frac{bx^3 \operatorname{Artanh}(cx^3)}{3} + \frac{b \ln(-c^2x^6 + 1)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^3)),x)

[Out] 1/3\*x^3\*a+1/3\*b\*x^3\*arctanh(c\*x^3)+1/6\*b\*ln(-c^2\*x^6+1)/c

**Maxima [A]** time = 1.01133, size = 50, normalized size = 1.35

$$\frac{1}{3}ax^3 + \frac{(2cx^3 \operatorname{artanh}(cx^3) + \log(-c^2x^6 + 1))b}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^3)),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/6\*(2\*c\*x^3\*arctanh(c\*x^3) + log(-c^2\*x^6 + 1))\*b/c

**Fricas [A]** time = 1.9613, size = 108, normalized size = 2.92

$$\frac{bcx^3 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2acx^3 + b \log(c^2x^6 - 1)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(b*c*x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3 + b*log(c^2*x^6 - 1)
)/c
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c*x**3)),x)
```

```
[Out] Exception raised: KeyError
```

**Giac [A]** time = 1.1694, size = 66, normalized size = 1.78

$$\frac{1}{3}ax^3 + \frac{1}{6}\left(x^3 \log\left(-\frac{cx^3 + 1}{cx^3 - 1}\right) + \frac{\log(|c^2x^6 - 1|)}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/3*a*x^3 + 1/6*(x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + log(abs(c^2*x^6 - 1))/
c)*b
```

$$3.103 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x} dx$$

**Optimal.** Leaf size=30

$$-\frac{1}{6}b\text{PolyLog}(2, -cx^3) + \frac{1}{6}b\text{PolyLog}(2, cx^3) + a \log(x)$$

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^3)])/6 + (b\*PolyLog[2, c\*x^3])/6

**Rubi [A]** time = 0.0337, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6095, 5912}

$$-\frac{1}{6}b\text{PolyLog}(2, -cx^3) + \frac{1}{6}b\text{PolyLog}(2, cx^3) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x, x]

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^3)])/6 + (b\*PolyLog[2, c\*x^3])/6

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}(cx^3)}{x} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, x^3 \right) \\ &= a \log(x) - \frac{1}{6}b\text{Li}_2(-cx^3) + \frac{1}{6}b\text{Li}_2(cx^3) \end{aligned}$$

**Mathematica [A]** time = 0.0136819, size = 28, normalized size = 0.93

$$\frac{1}{6}b \left( \text{PolyLog} \left( 2, cx^3 \right) - \text{PolyLog} \left( 2, -cx^3 \right) \right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x,x]

[Out] a\*Log[x] + (b\*(-PolyLog[2, -(c\*x^3)] + PolyLog[2, c\*x^3]))/6

**Maple [C]** time = 0.029, size = 92, normalized size = 3.1

$$a \ln(x) + b \ln(x) \text{Artanh}(cx^3) + \frac{b}{2} \sum_{_R1=\text{RootOf}(c_Z^3-1)} \ln(x) \ln\left(\frac{-_R1-x}{_R1}\right) + \text{dilog}\left(\frac{-_R1-x}{_R1}\right) - \frac{b}{2} \sum_{_R1=\text{RootOf}(c_Z^3+1)} \ln(x) \ln\left(\frac{-_R1-x}{_R1}\right) + \text{dilog}\left(\frac{-_R1-x}{_R1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))/x,x)

[Out] a\*ln(x)+b\*ln(x)\*arctanh(c\*x^3)+1/2\*b\*sum(ln(x)\*ln((\_R1-x)/\_R1)+dilog((\_R1-x)/\_R1),\_R1=RootOf(\_Z^3\*c-1))-1/2\*b\*sum(ln(x)\*ln((\_R1-x)/\_R1)+dilog((\_R1-x)/\_R1),\_R1=RootOf(\_Z^3\*c+1))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b \int \frac{\log(cx^3 + 1) - \log(-cx^3 + 1)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x,x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(c\*x^3 + 1) - log(-c\*x^3 + 1))/x, x) + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx^3) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^3) + a)/x, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^3) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^3) + a)/x, x)

$$3.104 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^4} dx$$

**Optimal.** Leaf size=40

$$-\frac{a+b \tanh^{-1}(cx^3)}{3x^3} - \frac{1}{6}bc \log(1-c^2x^6) + bc \log(x)$$

[Out]  $-(a + b*\text{ArcTanh}[c*x^3])/(3*x^3) + b*c*\text{Log}[x] - (b*c*\text{Log}[1 - c^2*x^6])/6$

**Rubi [A]** time = 0.0270264, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 266, 36, 29, 31}

$$-\frac{a+b \tanh^{-1}(cx^3)}{3x^3} - \frac{1}{6}bc \log(1-c^2x^6) + bc \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c*x^3])/x^4, x]$

[Out]  $-(a + b*\text{ArcTanh}[c*x^3])/(3*x^3) + b*c*\text{Log}[x] - (b*c*\text{Log}[1 - c^2*x^6])/6$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n])/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^3)}{x^4} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{3x^3} + (bc) \int \frac{1}{x(1 - c^2x^6)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{3x^3} + \frac{1}{6}(bc) \operatorname{Subst}\left(\int \frac{1}{x(1 - c^2x)} dx, x, x^6\right) \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{3x^3} + \frac{1}{6}(bc) \operatorname{Subst}\left(\int \frac{1}{x} dx, x, x^6\right) + \frac{1}{6}(bc^3) \operatorname{Subst}\left(\int \frac{1}{1 - c^2x} dx, x, x^6\right) \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{3x^3} + bc \log(x) - \frac{1}{6}bc \log(1 - c^2x^6)
 \end{aligned}$$

**Mathematica [A]** time = 0.0122482, size = 45, normalized size = 1.12

$$-\frac{a}{3x^3} - \frac{1}{6}bc \log(1 - c^2x^6) - \frac{b \tanh^{-1}(cx^3)}{3x^3} + bc \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^4, x]

[Out] -a/(3\*x^3) - (b\*ArcTanh[c\*x^3])/(3\*x^3) + b\*c\*Log[x] - (b\*c\*Log[1 - c^2\*x^6])/6

**Maple [A]** time = 0.013, size = 49, normalized size = 1.2

$$-\frac{a}{3x^3} - \frac{b \operatorname{Artanh}(cx^3)}{3x^3} + bc \ln(x) - \frac{bc \ln(cx^3 - 1)}{6} - \frac{bc \ln(cx^3 + 1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^3))/x^4,x)`

[Out]  $-1/3*a/x^3-1/3*b/x^3*arctanh(c*x^3)+b*c*\ln(x)-1/6*b*c*\ln(c*x^3-1)-1/6*b*c*\ln(c*x^3+1)$

**Maxima [A]** time = 1.02135, size = 55, normalized size = 1.38

$$-\frac{1}{6} \left( c \left( \log(c^2 x^6 - 1) - \log(x^6) \right) + \frac{2 \operatorname{artanh}(cx^3)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="maxima")`

[Out]  $-1/6*(c*(\log(c^2*x^6 - 1) - \log(x^6)) + 2*arctanh(c*x^3)/x^3)*b - 1/3*a/x^3$

**Fricas [A]** time = 1.98002, size = 130, normalized size = 3.25

$$\frac{bcx^3 \log(c^2 x^6 - 1) - 6bcx^3 \log(x) + b \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="fricas")`

[Out]  $-1/6*(b*c*x^3*\log(c^2*x^6 - 1) - 6*b*c*x^3*\log(x) + b*\log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^3$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: `KeyError`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**3))/x**4,x)`



[Out] Exception raised: KeyError

**Giac [A]** time = 1.13407, size = 69, normalized size = 1.72

$$-\frac{1}{6}bc \log(c^2x^6 - 1) + bc \log(x) - \frac{b \log\left(\frac{-cx^3+1}{cx^3-1}\right)}{6x^3} - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^4,x, algorithm="giac")

[Out]  $-\frac{1}{6}bc \log(c^2x^6 - 1) + bc \log(x) - \frac{1}{6}b \log\left(\frac{-cx^3 + 1}{cx^3 - 1}\right) / x^3 - \frac{1}{3}a/x^3$

$$3.105 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^7} dx$$

**Optimal.** Leaf size=41

$$-\frac{a+b \tanh^{-1}(cx^3)}{6x^6} + \frac{1}{6}bc^2 \tanh^{-1}(cx^3) - \frac{bc}{6x^3}$$

[Out]  $-(b*c)/(6*x^3) + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)$

**Rubi [A]** time = 0.0270646, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 275, 325, 206}

$$-\frac{a+b \tanh^{-1}(cx^3)}{6x^6} + \frac{1}{6}bc^2 \tanh^{-1}(cx^3) - \frac{bc}{6x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^7, x]

[Out]  $-(b*c)/(6*x^3) + (b*c^2*ArcTanh[c*x^3])/6 - (a + b*ArcTanh[c*x^3])/(6*x^6)$

### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 325

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^3)}{x^7} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{6x^6} + \frac{1}{2}(bc) \int \frac{1}{x^4(1 - c^2x^6)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{6x^6} + \frac{1}{6}(bc) \operatorname{Subst}\left(\int \frac{1}{x^2(1 - c^2x^2)} dx, x, x^3\right) \\
 &= -\frac{bc}{6x^3} - \frac{a + b \tanh^{-1}(cx^3)}{6x^6} + \frac{1}{6}(bc^3) \operatorname{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, x^3\right) \\
 &= -\frac{bc}{6x^3} + \frac{1}{6}bc^2 \tanh^{-1}(cx^3) - \frac{a + b \tanh^{-1}(cx^3)}{6x^6}
 \end{aligned}$$

**Mathematica [A]** time = 0.0182664, size = 65, normalized size = 1.59

$$-\frac{a}{6x^6} - \frac{1}{12}bc^2 \log(1 - cx^3) + \frac{1}{12}bc^2 \log(cx^3 + 1) - \frac{bc}{6x^3} - \frac{b \tanh^{-1}(cx^3)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^7, x]

[Out] -a/(6\*x^6) - (b\*c)/(6\*x^3) - (b\*ArcTanh[c\*x^3])/(6\*x^6) - (b\*c^2\*Log[1 - c\*x^3])/12 + (b\*c^2\*Log[1 + c\*x^3])/12

**Maple [A]** time = 0.013, size = 55, normalized size = 1.3

$$-\frac{a}{6x^6} - \frac{b \operatorname{Arctanh}(cx^3)}{6x^6} - \frac{bc^2 \ln(cx^3 - 1)}{12} + \frac{bc^2 \ln(cx^3 + 1)}{12} - \frac{bc}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^3))/x^7,x)`

[Out]  $-1/6*a/x^6 - 1/6*b/x^6*arctanh(c*x^3) - 1/12*b*c^2*\ln(c*x^3-1) + 1/12*b*c^2*\ln(c*x^3+1) - 1/6*b*c/x^3$

**Maxima [A]** time = 1.01283, size = 69, normalized size = 1.68

$$\frac{1}{12} \left( \left( c \log(cx^3 + 1) - c \log(cx^3 - 1) - \frac{2}{x^3} \right) c - \frac{2 \operatorname{artanh}(cx^3)}{x^6} \right) b - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="maxima")`

[Out]  $1/12*((c*\log(c*x^3 + 1) - c*\log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6)*b - 1/6*a/x^6$

**Fricas [A]** time = 1.93639, size = 104, normalized size = 2.54

$$\frac{2bcx^3 - (bc^2x^6 - b)\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2a}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="fricas")`

[Out]  $-1/12*(2*b*c*x^3 - (b*c^2*x^6 - b)*\log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^6$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**3))/x**7,x)`

[Out] Timed out

**Giac [A]** time = 1.16228, size = 90, normalized size = 2.2

$$\frac{1}{12}bc^2 \log(cx^3 + 1) - \frac{1}{12}bc^2 \log(cx^3 - 1) - \frac{b \log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12x^6} - \frac{bcx^3 + a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^7,x, algorithm="giac")

[Out] 1/12\*b\*c^2\*log(c\*x^3 + 1) - 1/12\*b\*c^2\*log(c\*x^3 - 1) - 1/12\*b\*log(-(c\*x^3 + 1)/(c\*x^3 - 1))/x^6 - 1/6\*(b\*c\*x^3 + a)/x^6

$$3.106 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^{10}} dx$$

**Optimal.** Leaf size=56

$$-\frac{a+b \tanh^{-1}(cx^3)}{9x^9} - \frac{1}{18}bc^3 \log(1-c^2x^6) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{18x^6}$$

[Out]  $-(b*c)/(18*x^6) - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^6])/18$

**Rubi [A]** time = 0.0363813, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6097, 266, 44}

$$-\frac{a+b \tanh^{-1}(cx^3)}{9x^9} - \frac{1}{18}bc^3 \log(1-c^2x^6) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{18x^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^10, x]

[Out]  $-(b*c)/(18*x^6) - (a + b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/3 - (b*c^3*Log[1 - c^2*x^6])/18$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```

& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^3)}{x^{10}} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{9x^9} + \frac{1}{3}(bc) \int \frac{1}{x^7(1 - c^2x^6)} dx \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{9x^9} + \frac{1}{18}(bc) \text{Subst} \left( \int \frac{1}{x^2(1 - c^2x)} dx, x, x^6 \right) \\
 &= -\frac{a + b \tanh^{-1}(cx^3)}{9x^9} + \frac{1}{18}(bc) \text{Subst} \left( \int \left( \frac{1}{x^2} + \frac{c^2}{x} - \frac{c^4}{-1 + c^2x} \right) dx, x, x^6 \right) \\
 &= -\frac{bc}{18x^6} - \frac{a + b \tanh^{-1}(cx^3)}{9x^9} + \frac{1}{3}bc^3 \log(x) - \frac{1}{18}bc^3 \log(1 - c^2x^6)
 \end{aligned}$$

**Mathematica [A]** time = 0.0117102, size = 61, normalized size = 1.09

$$-\frac{a}{9x^9} - \frac{1}{18}bc^3 \log(1 - c^2x^6) + \frac{1}{3}bc^3 \log(x) - \frac{bc}{18x^6} - \frac{b \tanh^{-1}(cx^3)}{9x^9}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^10,x]

[Out] -a/(9\*x^9) - (b\*c)/(18\*x^6) - (b\*ArcTanh[c\*x^3])/(9\*x^9) + (b\*c^3\*Log[x])/3 - (b\*c^3\*Log[1 - c^2\*x^6])/18

**Maple [A]** time = 0.014, size = 63, normalized size = 1.1

$$-\frac{a}{9x^9} - \frac{b \text{Artanh}(cx^3)}{9x^9} - \frac{bc}{18x^6} + \frac{bc^3 \ln(x)}{3} - \frac{bc^3 \ln(cx^3 - 1)}{18} - \frac{bc^3 \ln(cx^3 + 1)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))/x^10,x)

[Out]  $-1/9*a/x^9 - 1/9*b/x^9*\operatorname{arctanh}(c*x^3) - 1/18*b*c/x^6 + 1/3*b*c^3*\ln(x) - 1/18*b*c^3*\ln(c*x^3-1) - 1/18*b*c^3*\ln(c*x^3+1)$

---

**Maxima [A]** time = 0.964576, size = 69, normalized size = 1.23

$$-\frac{1}{18} \left( \left( c^2 \log(c^2 x^6 - 1) - c^2 \log(x^6) + \frac{1}{x^6} \right) c + \frac{2 \operatorname{artanh}(cx^3)}{x^9} \right) b - \frac{a}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="maxima")`

[Out]  $-1/18*((c^2*\log(c^2*x^6 - 1) - c^2*\log(x^6) + 1/x^6)*c + 2*\operatorname{arctanh}(c*x^3)/x^9)*b - 1/9*a/x^9$

---

**Fricas [A]** time = 2.15736, size = 150, normalized size = 2.68

$$\frac{bc^3x^9 \log(c^2x^6 - 1) - 6bc^3x^9 \log(x) + bcx^3 + b \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2a}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="fricas")`

[Out]  $-1/18*(b*c^3*x^9*\log(c^2*x^6 - 1) - 6*b*c^3*x^9*\log(x) + b*c*x^3 + b*\log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^9$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**3))/x**10,x)`

[Out] Timed out



---

**Giac [A]** time = 1.17147, size = 88, normalized size = 1.57

$$-\frac{1}{18}bc^3 \log(c^2x^6 - 1) + \frac{1}{3}bc^3 \log(x) - \frac{b \log\left(-\frac{cx^3+1}{cx^3-1}\right)}{18x^9} - \frac{bcx^3 + 2a}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^10,x, algorithm="giac")

[Out] -1/18\*b\*c^3\*log(c^2\*x^6 - 1) + 1/3\*b\*c^3\*log(x) - 1/18\*b\*log(-(c\*x^3 + 1)/(c\*x^3 - 1))/x^9 - 1/18\*(b\*c\*x^3 + 2\*a)/x^9

### 3.107 $\int x^3 (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=174

$$\frac{1}{4}x^4(a + b \tanh^{-1}(cx^3)) + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{16c^{4/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{16c^{4/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}}$$

[Out] (3\*b\*x)/(4\*c) + (Sqrt[3]\*b\*ArcTan[1/Sqrt[3] - (2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) - (Sqrt[3]\*b\*ArcTan[1/Sqrt[3] + (2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) - (b\*ArcTanh[c^(1/3)\*x]/(4\*c^(4/3)) + (x^4\*(a + b\*ArcTanh[c\*x^3]))/4 + (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3))

**Rubi [A]** time = 0.219734, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6097, 321, 210, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4(a + b \tanh^{-1}(cx^3)) + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{16c^{4/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{16c^{4/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (3\*b\*x)/(4\*c) + (Sqrt[3]\*b\*ArcTan[1/Sqrt[3] - (2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) - (Sqrt[3]\*b\*ArcTan[1/Sqrt[3] + (2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) - (b\*ArcTanh[c^(1/3)\*x]/(4\*c^(4/3)) + (x^4\*(a + b\*ArcTanh[c\*x^3]))/4 + (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3))

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c^n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[

$(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x],$   
 $x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n*p$   
 $+ 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 210

$\text{Int}[(a_.) + (b_.)*(x_)^{(n_)}]^{-1}, x\_Symbol] \text{ :> Module}\{r = \text{Numerator}[\text{Rt}[-(a/b), n]], s = \text{Denominator}[\text{Rt}[-(a/b), n]], k, u\}, \text{Simp}[u = \text{Int}[(r - s*\text{Cos}[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*\text{Cos}[(2*k*Pi)/n]*x + s^2*x^2), x] + \text{Int}[(r + s*\text{Cos}[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*\text{Cos}[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*\text{Int}[1/(r^2 - s^2*x^2), x])/(a*n) + \text{Dist}[(2*r)/(a*n), \text{Sum}[u, \{k, 1, (n - 2)/4\}], x], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{IGtQ}[(n - 2)/4, 0] \ \&\& \ \text{NegQ}[a/b]$

### Rule 634

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \text{ :> Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rule 618

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x\_Symbol] \text{ :> Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \text{ :> -Simp}[\text{ArcTan}[\text{Rt}[-b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 628

$\text{Int}[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol] \text{ :> Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 206

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \text{ :> Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3)) - \frac{1}{4}(3bc) \int \frac{x^6}{1 - c^2x^6} dx \\
&= \frac{3bx}{4c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3)) - \frac{(3b) \int \frac{1}{1 - c^2x^6} dx}{4c} \\
&= \frac{3bx}{4c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3)) - \frac{b \int \frac{1}{1 - c^{2/3}x^2} dx}{4c} - \frac{b \int \frac{1 - \frac{\sqrt[3]{cx}}{2}}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{4c} - \frac{b \int \frac{1 + \frac{\sqrt[3]{cx}}{2}}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{4c} \\
&= \frac{3bx}{4c} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{4c^{4/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3)) + \frac{b \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{16c^{4/3}} - \frac{b \int \frac{\sqrt[3]{c} + 2c^{2/3}x}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{16c^{4/3}} \\
&= \frac{3bx}{4c} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{4c^{4/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - \sqrt[3]{cx} + c^{2/3}x^2)}{16c^{4/3}} - \frac{b \log(1 + \sqrt[3]{cx} + c^{2/3}x^2)}{16c^{4/3}} \\
&= \frac{3bx}{4c} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{4c^{4/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^3))
\end{aligned}$$

**Mathematica [A]** time = 0.0424553, size = 196, normalized size = 1.13

$$\frac{ax^4}{4} + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{16c^{4/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{16c^{4/3}} + \frac{b \log(1 - \sqrt[3]{cx})}{8c^{4/3}} - \frac{b \log(\sqrt[3]{cx} + 1)}{8c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{cx} - 1}{\sqrt{3}}\right)}{8c^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (3\*b\*x)/(4\*c) + (a\*x^4)/4 - (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) - (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(8\*c^(4/3)) + (b\*x^4\*ArcTanh[c\*x^3])/4 + (b\*Log[1 - c^(1/3)\*x])/(8\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*x])/(8\*c^(4/3)) + (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(16\*c^(4/3))

**Maple [A]** time = 0.015, size = 184, normalized size = 1.1

$$\frac{x^4 a}{4} + \frac{bx^4 \operatorname{Artanh}(cx^3)}{4} + \frac{3bx}{4c} + \frac{b}{8c^2} \ln\left(x - \sqrt[3]{c^{-1}}\right) (c^{-1})^{-\frac{2}{3}} - \frac{b}{16c^2} \ln\left(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}}\right) (c^{-1})^{-\frac{2}{3}} - \frac{b\sqrt{3}}{8c^2} \arctan\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctanh(c*x^3)),x)
```

```
[Out] 1/4*x^4*a+1/4*b*x^4*arctanh(c*x^3)+3/4*b*x/c+1/8*b/c^2/(1/c)^(2/3)*ln(x-(1/c)^(1/3))-1/16*b/c^2/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3))+1/16*b/c^2/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.07704, size = 2611, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqrt((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))]/c^2, 1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*sqrt(3)*b*c*sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(1/3))*sqrt(-(-c)^(1/3)/c)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sq
```

```

rt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(
c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)
) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (
-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/16*(2*b*c^2*x^4*log(-c
*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)^(1/3)/c)*log((
2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqrt((-c)^(1/3)/c)
+ 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)
*(2*c^(2/3)*x + c^(1/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c
)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*
(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/1
6*(2*b*c^2*x^4*log(-c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 - 2*sqrt(3)*b*c*
sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x + (-c)^(1/3))*sqrt(-
(-c)^(1/3)/c)) - 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1
/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/
3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x +
(-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2]

```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*3)),x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.31662, size = 279, normalized size = 1.6

$$\frac{1}{16}bc^7 \left( \frac{2 \left(-\frac{1}{c}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{c^8} - \frac{2\sqrt{3}|c|^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x + \frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^9} - \frac{2\sqrt{3}(-c^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^9} - \frac{|c|^{\frac{2}{3}} \log\left(\frac{2c^{\frac{1}{3}}x + 1}{c^{\frac{1}{3}}}\right)}{c^9} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^3)),x, algorithm="giac")

[Out]  $\frac{1}{16}bc^7(2(-1/c)^{1/3}\log(\text{abs}(x - (-1/c)^{1/3}))) / c^8 - 2\sqrt{3}\text{abs}(c)^{2/3}\arctan(1/3\sqrt{3}c^{1/3}(2x + 1/c^{1/3})) / c^9 - 2\sqrt{3}(-c^2)^{1/3}\arctan(1/3\sqrt{3}(2x + (-1/c)^{1/3}) / (-1/c)^{1/3}) / c^9 - \text{abs}(c)^{2/3}\log(x^2 + x/c^{1/3} + 1/c^{2/3}) / c^9 + 2\log(\text{abs}(x - 1/c^{1/3})) / c^{25/3} - (-c^2)^{1/3}\log(x^2 + x(-1/c)^{1/3} + (-1/c)^{2/3}) / c^9 + 1/8bx^4\log(-(cx^3 + 1)/(cx^3 - 1)) + 1/4ax^4 + 3/4bx/c$

### 3.108 $\int (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=101

$$ax + \frac{b \log(1 - c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \log(c^{4/3}x^4 + c^{2/3}x^2 + 1)}{4\sqrt[3]{c}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx \tanh^{-1}(cx^3)$$

[Out] a\*x + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/(2\*c^(1/3)) + b\*x\*ArcTanh[c\*x^3] + (b\*Log[1 - c^(2/3)\*x^2])/(2\*c^(1/3)) - (b\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/(4\*c^(1/3))

**Rubi [A]** time = 0.0966456, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$ , Rules used = {6091, 275, 292, 31, 634, 617, 204, 628}

$$ax + \frac{b \log(1 - c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \log(c^{4/3}x^4 + c^{2/3}x^2 + 1)}{4\sqrt[3]{c}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx \tanh^{-1}(cx^3)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*x^3], x]

[Out] a\*x + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/(2\*c^(1/3)) + b\*x\*ArcTanh[c\*x^3] + (b\*Log[1 - c^(2/3)\*x^2])/(2\*c^(1/3)) - (b\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/(4\*c^(1/3))

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 292

Int[(x\_)/((a\_) + (b\_.)\*(x\_)^3), x\_Symbol] := -Dist[(3\*Rt[a, 3]\*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]\*Rt[b, 3]), I



```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(cx^3)) dx &= ax + b \int \tanh^{-1}(cx^3) dx \\
&= ax + bx \tanh^{-1}(cx^3) - (3bc) \int \frac{x^3}{1 - c^2x^6} dx \\
&= ax + bx \tanh^{-1}(cx^3) - \frac{1}{2}(3bc) \operatorname{Subst}\left(\int \frac{x}{1 - c^2x^3} dx, x, x^2\right) \\
&= ax + bx \tanh^{-1}(cx^3) - \frac{1}{2}(b\sqrt[3]{c}) \operatorname{Subst}\left(\int \frac{1}{1 - c^{2/3}x} dx, x, x^2\right) + \frac{1}{2}(b\sqrt[3]{c}) \operatorname{Subst}\left(\int \frac{1}{1 + c^{2/3}x} dx, x, x^2\right) \\
&= ax + bx \tanh^{-1}(cx^3) + \frac{b \log(1 - c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \operatorname{Subst}\left(\int \frac{c^{2/3} + 2c^{4/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2\right)}{4\sqrt[3]{c}} + \frac{1}{4}(3b\sqrt[3]{c}) \operatorname{Subst}\left(\int \frac{1}{-3 - x} dx, x, x^2\right) \\
&= ax + bx \tanh^{-1}(cx^3) + \frac{b \log(1 - c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \log(1 + c^{2/3}x^2 + c^{4/3}x^4)}{4\sqrt[3]{c}} - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{-3 - x} dx, x, x^2\right)}{2} \\
&= ax + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx \tanh^{-1}(cx^3) + \frac{b \log(1 - c^{2/3}x^2)}{2\sqrt[3]{c}} - \frac{b \log(1 + c^{2/3}x^2 + c^{4/3}x^4)}{4\sqrt[3]{c}}
\end{aligned}$$

**Mathematica [A]** time = 0.042819, size = 136, normalized size = 1.35

$$ax - \frac{b \left( \log(c^{2/3}x^2 - \sqrt[3]{c}x + 1) + \log(c^{2/3}x^2 + \sqrt[3]{c}x + 1) - 2 \log(1 - \sqrt[3]{c}x) - 2 \log(\sqrt[3]{c}x + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{c}x - 1}{\sqrt{3}}\right) + 2\sqrt{3} \right)}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*x^3], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x^3] - (b\*(-2\*Sqrt[3]\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]] + 2\*Sqrt[3]\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]] - 2\*Log[1 - c^(1/3)\*x] - 2\*Log[1 + c^(1/3)\*x] + Log[1 - c^(1/3)\*x + c^(2/3)\*x^2] + Log[1 + c^(1/3)\*x + c^(2/3)\*x^2]))/(4\*c^(1/3))

**Maple [A]** time = 0.006, size = 99, normalized size = 1.

$$ax + bx \operatorname{Artanh}(cx^3) + \frac{b}{2c} \ln\left(x^2 - \sqrt[3]{c^{-2}}\right) \frac{1}{\sqrt[3]{c^{-2}}} - \frac{b}{4c} \ln\left(x^4 + \sqrt[3]{c^{-2}}x^2 + (c^{-2})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-2}}} + \frac{b\sqrt{3}}{2c} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \frac{x^2}{\sqrt[3]{c^{-2}}} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctanh(c\*x^3),x)

[Out] a\*x+b\*x\*arctanh(c\*x^3)+1/2\*b/c/(1/c^2)^(1/3)\*ln(x^2-(1/c^2)^(1/3))-1/4\*b/c/(1/c^2)^(1/3)\*ln(x^4+(1/c^2)^(1/3)\*x^2+(1/c^2)^(2/3))+1/2\*b\*3^(1/2)/c/(1/c^2)^(1/3)\*arctan(1/3\*3^(1/2)\*(2/(1/c^2)^(1/3)\*x^2+1))

**Maxima [A]** time = 1.4484, size = 143, normalized size = 1.42

$$\frac{1}{4} \left( \frac{2\sqrt{3}(c^2)^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}(c^2)^{\frac{1}{3}}\left(2x^2 + \frac{1}{c^2}\right)\right)}{c^2} - \frac{(c^2)^{\frac{1}{3}} \log\left(x^4 + \frac{1}{c^2}x^2 + \frac{1}{c^2}\right)}{c^2} + \frac{2(c^2)^{\frac{1}{3}} \log\left(x^2 - \frac{1}{c^2}\right)}{c^2} \right) + 4x \arctan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^3),x, algorithm="maxima")

[Out] 1/4\*(c\*(2\*sqrt(3)\*(c^2)^(1/3)\*arctan(1/3\*sqrt(3)\*(c^2)^(1/3)\*(2\*x^2 + (c^(-2))^(1/3)))/c^2 - (c^2)^(1/3)\*log(x^4 + (c^(-2))^(1/3)\*x^2 + (c^(-2))^(2/3))/c^2 + 2\*(c^2)^(1/3)\*log(x^2 - (c^(-2))^(1/3))/c^2) + 4\*x\*arctanh(c\*x^3)\*b + a\*x

**Fricas [A]** time = 1.86935, size = 674, normalized size = 6.67

$$\frac{\sqrt{3}bc \sqrt{-\frac{1}{2}} \log\left(\frac{2c^2x^6 - 3c^{\frac{2}{3}}x^2 + \sqrt{3}\left(2c^{\frac{5}{3}}x^4 - cx^2 - c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{2}+1}}{c^2x^6-1}\right) + 2bcx \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 4acx - bc^{\frac{2}{3}} \log\left(c^2x^4 + c^{\frac{4}{3}}x^2 + c^{\frac{2}{3}}\right) + 2bc}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^3),x, algorithm="fricas")

[Out] [1/4\*(sqrt(3)\*b\*c\*sqrt(-1/c^(2/3))\*log((2\*c^2\*x^6 - 3\*c^(2/3)\*x^2 + sqrt(3))\*(2\*c^(5/3)\*x^4 - c\*x^2 - c^(1/3))\*sqrt(-1/c^(2/3)) + 1)/(c^2\*x^6 - 1)) + 2

```
*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c, 1/4*(2*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(2*c*x^2 + c^(1/3))/c^(1/3)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c\*x\*\*3),x)

[Out] Timed out

**Giac [A]** time = 1.18351, size = 147, normalized size = 1.46

$$\frac{1}{4} \left[ c \left( \frac{2\sqrt{3}|c|^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2 + \frac{1}{|c|^{\frac{2}{3}}}\right)\right)}{c^2} - \frac{|c|^{\frac{2}{3}} \log\left(x^4 + \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2} + \frac{2 \log\left(\left|x^2 - \frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right) + 2x \log\left(-\frac{cx^3 + 1}{cx^3 - 1}\right) \right] b +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^3),x, algorithm="giac")

[Out] 1/4\*(c\*(2\*sqrt(3)\*abs(c)^(2/3)\*arctan(1/3\*sqrt(3)\*(2\*x^2 + 1/abs(c)^(2/3))\*abs(c)^(2/3))/c^2 - abs(c)^(2/3)\*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/c^2 + 2\*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) + 2\*x\*log(-(c\*x^3 + 1)/(c\*x^3 - 1)))\*b + a\*x

$$3.109 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^3} dx$$

**Optimal.** Leaf size=165

$$-\frac{a+b \tanh^{-1}(cx^3)}{2x^2} - \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) - \frac{1}{4}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right) +$$

[Out]  $-(\text{Sqrt}[3]*b*c^{(2/3)*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)*x}/\text{Sqrt}[3])]/4 + (\text{Sqrt}[3]*b*c^{(2/3)*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)*x}/\text{Sqrt}[3])]/4 + (b*c^{(2/3)*\text{ArcTan}}[c^{(1/3)*x}])/2 - (a + b*\text{ArcTanh}[c*x^3])/(2*x^2) - (b*c^{(2/3)*\text{Log}[1 - c^{(1/3)*x} + c^{(2/3)*x^2}])/8 + (b*c^{(2/3)*\text{Log}[1 + c^{(1/3)*x} + c^{(2/3)*x^2}])/8$

**Rubi [A]** time = 0.197406, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6097, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b \tanh^{-1}(cx^3)}{2x^2} - \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) - \frac{1}{4}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right) +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^3,x]

[Out]  $-(\text{Sqrt}[3]*b*c^{(2/3)*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)*x}/\text{Sqrt}[3])]/4 + (\text{Sqrt}[3]*b*c^{(2/3)*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)*x}/\text{Sqrt}[3])]/4 + (b*c^{(2/3)*\text{ArcTan}}[c^{(1/3)*x}])/2 - (a + b*\text{ArcTanh}[c*x^3])/(2*x^2) - (b*c^{(2/3)*\text{Log}[1 - c^{(1/3)*x} + c^{(2/3)*x^2}])/8 + (b*c^{(2/3)*\text{Log}[1 + c^{(1/3)*x} + c^{(2/3)*x^2}])/8$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(m\_), x\_Symbol] :> Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s\*Cos[(2\*k\*Pi)/n]\*x)/(r^2 - 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x] + Int[(r + s\*Co

```
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^3)}{x^3} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{2x^2} + \frac{1}{2}(3bc) \int \frac{1}{1 - c^2x^6} dx \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{2x^2} + \frac{1}{2}(bc) \int \frac{1}{1 - c^{2/3}x^2} dx + \frac{1}{2}(bc) \int \frac{1 - \frac{\sqrt[3]{cx}}{2}}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx + \frac{1}{2}(bc) \int \frac{1 + \frac{\sqrt[3]{cx}}{2}}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx \\
&= \frac{1}{2}bc^{2/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{2x^2} - \frac{1}{8}(bc^{2/3}) \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx + \frac{1}{8}(bc^{2/3}) \int \frac{\sqrt[3]{c} + 2c^{2/3}x}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx \\
&= \frac{1}{2}bc^{2/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{2x^2} - \frac{1}{8}bc^{2/3} \log(1 - \sqrt[3]{cx} + c^{2/3}x^2) + \frac{1}{8}bc^{2/3} \log(1 + \sqrt[3]{cx} + c^{2/3}x^2) \\
&= -\frac{1}{4}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right) + \frac{1}{4}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1 + 2\sqrt[3]{cx}}{\sqrt{3}}\right) + \frac{1}{2}bc^{2/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0523718, size = 187, normalized size = 1.13

$$-\frac{a}{2x^2} - \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{8}bc^{2/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) - \frac{1}{4}bc^{2/3} \log(1 - \sqrt[3]{cx}) + \frac{1}{4}bc^{2/3} \log(\sqrt[3]{cx} + 1) + \frac{1}{2}bc^{2/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^3,x]

[Out] -a/(2\*x^2) + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/4 + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/4 - (b\*ArcTanh[c\*x^3])/(2\*x^2) - (b\*c^(2/3)\*Log[1 - c^(1/3)\*x])/4 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*x])/4 - (b\*c^(2/3)\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/8 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/8

**Maple [A]** time = 0.01, size = 159, normalized size = 1.

$$-\frac{a}{2x^2} - \frac{b \operatorname{Artanh}(cx^3)}{2x^2} - \frac{b}{4} \ln\left(x - \sqrt[3]{c^{-1}}\right) (c^{-1})^{-\frac{2}{3}} + \frac{b}{8} \ln\left(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}}\right) (c^{-1})^{-\frac{2}{3}} + \frac{b\sqrt{3}}{4} \arctan\left(\frac{\sqrt{3}}{3} \left(2\frac{x}{\sqrt[3]{c^{-1}}} + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))/x^3,x)

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctanh(c*x^3)-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3))+1/8
*b/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/3))
-1/8*b/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 1.9275, size = 598, normalized size = 3.62

$$2\sqrt{3}(-c^2)^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}(-c^2)^{\frac{2}{3}}x+\sqrt{3}c}{3c}\right)-2\sqrt{3}b(c^2)^{\frac{1}{3}}x^2\arctan\left(\frac{2\sqrt{3}(c^2)^{\frac{2}{3}}x-\sqrt{3}c}{3c}\right)+(-c^2)^{\frac{1}{3}}bx^2\log\left(c^2x^2-(-c^2)^{\frac{1}{3}}cx+\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(3)*(-c^2)^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*x + s
qrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x^2*arctan(1/3*(2*sqrt(3)*(c^2)^(2/3
)*x - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x^2*log(c^2*x^2 - (-c^2)^(1/3)*c*x + (
-c^2)^(2/3)) + b*(c^2)^(1/3)*x^2*log(c^2*x^2 - (c^2)^(1/3)*c*x + (c^2)^(2/3
)) - 2*(-c^2)^(1/3)*b*x^2*log(c*x + (-c^2)^(1/3)) - 2*b*(c^2)^(1/3)*x^2*log
(c*x + (c^2)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^2
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))/x\*\*3,x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.29789, size = 223, normalized size = 1.35

$$\frac{1}{8} \left( \frac{2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x + \frac{1}{|c|^{1/3}}\right)|c|^{1/3}\right)}{|c|^{1/3}} + \frac{2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x - \frac{1}{|c|^{1/3}}\right)|c|^{1/3}\right)}{|c|^{1/3}} + \frac{\log\left(x^2 + \frac{x}{|c|^{1/3}} + \frac{1}{|c|^{2/3}}\right)}{|c|^{1/3}} - \frac{\log\left(x^2 - \frac{x}{|c|^{1/3}} + \frac{1}{|c|^{2/3}}\right)}{|c|^{1/3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^3,x, algorithm="giac")

[Out] 1/8\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x + 1/abs(c)^(1/3))\*abs(c)^(1/3))/abs(c)^(1/3) + 2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - 1/abs(c)^(1/3))\*abs(c)^(1/3))/abs(c)^(1/3) + log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) - log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2\*log(abs(x + 1/abs(c)^(1/3)))/abs(c)^(1/3) - 2\*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(1/3))\*b\*c - 1/4\*b\*log(-(c\*x^3 + 1)/(c\*x^3 - 1))/x^2 - 1/2\*a/x^2

$$3.110 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^6} dx$$

**Optimal.** Leaf size=115

$$-\frac{a+b \tanh^{-1}(cx^3)}{5x^5} - \frac{1}{10}bc^{5/3} \log(1-c^{2/3}x^2) + \frac{1}{20}bc^{5/3} \log(c^{4/3}x^4 + c^{2/3}x^2 + 1) - \frac{1}{10}\sqrt{3}bc^{5/3} \tan^{-1}\left(\frac{2c^{2/3}x^2 + 1}{\sqrt{3}}\right) - \frac{3b}{10x}$$

[Out]  $(-3*b*c)/(10*x^2) - (\text{Sqrt}[3]*b*c^{(5/3)}*\text{ArcTan}[(1 + 2*c^{(2/3)}*x^2)/\text{Sqrt}[3]])/10 - (a + b*\text{ArcTanh}[c*x^3])/(5*x^5) - (b*c^{(5/3)}*\text{Log}[1 - c^{(2/3)}*x^2])/10 + (b*c^{(5/3)}*\text{Log}[1 + c^{(2/3)}*x^2 + c^{(4/3)}*x^4])/20$

**Rubi [A]** time = 0.0945154, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {6097, 275, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{a+b \tanh^{-1}(cx^3)}{5x^5} - \frac{1}{10}bc^{5/3} \log(1-c^{2/3}x^2) + \frac{1}{20}bc^{5/3} \log(c^{4/3}x^4 + c^{2/3}x^2 + 1) - \frac{1}{10}\sqrt{3}bc^{5/3} \tan^{-1}\left(\frac{2c^{2/3}x^2 + 1}{\sqrt{3}}\right) - \frac{3b}{10x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^6, x]

[Out]  $(-3*b*c)/(10*x^2) - (\text{Sqrt}[3]*b*c^{(5/3)}*\text{ArcTan}[(1 + 2*c^{(2/3)}*x^2)/\text{Sqrt}[3]])/10 - (a + b*\text{ArcTanh}[c*x^3])/(5*x^5) - (b*c^{(5/3)}*\text{Log}[1 - c^{(2/3)}*x^2])/10 + (b*c^{(5/3)}*\text{Log}[1 + c^{(2/3)}*x^2 + c^{(4/3)}*x^4])/20$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n])/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1 /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^3)}{x^6} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{5x^5} + \frac{1}{5}(3bc) \int \frac{1}{x^3(1 - c^2x^6)} dx \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{5x^5} + \frac{1}{10}(3bc) \text{Subst} \left( \int \frac{1}{x^2(1 - c^2x^3)} dx, x, x^2 \right) \\
&= -\frac{3bc}{10x^2} - \frac{a + b \tanh^{-1}(cx^3)}{5x^5} + \frac{1}{10}(3bc^3) \text{Subst} \left( \int \frac{x}{1 - c^2x^3} dx, x, x^2 \right) \\
&= -\frac{3bc}{10x^2} - \frac{a + b \tanh^{-1}(cx^3)}{5x^5} + \frac{1}{10}(bc^{7/3}) \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x} dx, x, x^2 \right) - \frac{1}{10}(bc^{7/3}) \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x} dx, x, x^2 \right) \\
&= -\frac{3bc}{10x^2} - \frac{a + b \tanh^{-1}(cx^3)}{5x^5} - \frac{1}{10}bc^{5/3} \log(1 - c^{2/3}x^2) + \frac{1}{20}(bc^{5/3}) \text{Subst} \left( \int \frac{c^{2/3} + 2c^{4/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2 \right) \\
&= -\frac{3bc}{10x^2} - \frac{a + b \tanh^{-1}(cx^3)}{5x^5} - \frac{1}{10}bc^{5/3} \log(1 - c^{2/3}x^2) + \frac{1}{20}bc^{5/3} \log(1 + c^{2/3}x^2 + c^{4/3}x^4) + \frac{1}{20}bc^{5/3} \log(1 + c^{2/3}x^2 + c^{4/3}x^4) \\
&= -\frac{3bc}{10x^2} - \frac{1}{10}\sqrt{3}bc^{5/3} \tan^{-1} \left( \frac{1 + 2c^{2/3}x^2}{\sqrt{3}} \right) - \frac{a + b \tanh^{-1}(cx^3)}{5x^5} - \frac{1}{10}bc^{5/3} \log(1 - c^{2/3}x^2) + \frac{1}{20}bc^{5/3} \log(1 + c^{2/3}x^2 + c^{4/3}x^4)
\end{aligned}$$

**Mathematica [A]** time = 0.0496929, size = 196, normalized size = 1.7

$$-\frac{a}{5x^5} + \frac{1}{20}bc^{5/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{20}bc^{5/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) - \frac{1}{10}bc^{5/3} \log(1 - \sqrt[3]{cx}) - \frac{1}{10}bc^{5/3} \log(\sqrt[3]{cx} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^6, x]

[Out] -a/(5\*x^5) - (3\*b\*c)/(10\*x^2) - (Sqrt[3]\*b\*c^(5/3)\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/10 + (Sqrt[3]\*b\*c^(5/3)\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/10 - (b\*ArcTanh[c\*x^3])/(5\*x^5) - (b\*c^(5/3)\*Log[1 - c^(1/3)\*x])/10 - (b\*c^(5/3)\*Log[1 + c^(1/3)\*x])/10 + (b\*c^(5/3)\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/20 + (b\*c^(5/3)\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/20

**Maple [A]** time = 0.014, size = 172, normalized size = 1.5

$$-\frac{a}{5x^5} - \frac{b \text{Artanh}(cx^3)}{5x^5} - \frac{3bc}{10x^2} - \frac{bc}{10} \ln(x - \sqrt[3]{c^{-1}})(c^{-1})^{-\frac{2}{3}} + \frac{bc}{20} \ln(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}})(c^{-1})^{-\frac{2}{3}} + \frac{bc\sqrt{3}}{10} \arctan\left(\frac{\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^3))/x^6,x)`

[Out] 
$$-1/5*a/x^5 - 1/5*b/x^5*arctanh(c*x^3) - 3/10*b*c/x^2 - 1/10*b*c/(1/c)^{(2/3)}*\ln(x - (1/c)^{(1/3)}) + 1/20*b*c/(1/c)^{(2/3)}*\ln(x^2 + (1/c)^{(1/3)}*x + (1/c)^{(2/3)}) + 1/10*b*c/(1/c)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x + 1)) - 1/10*b*c/(1/c)^{(2/3)}*\ln(x + (1/c)^{(1/3)}) + 1/20*b*c/(1/c)^{(2/3)}*\ln(x^2 - (1/c)^{(1/3)}*x + (1/c)^{(2/3)}) - 1/10*b*c/(1/c)^{(2/3)}*3^{(1/2)}*arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x - 1))$$

**Maxima [A]** time = 1.44875, size = 144, normalized size = 1.25

$$-\frac{1}{20} \left( \left( 2\sqrt{3}(c^2)^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}(c^2)^{\frac{1}{3}}\left(2x^2 + \frac{1}{c^2}\right)\right) - (c^2)^{\frac{1}{3}} \log\left(x^4 + \frac{1}{c^2}x^2 + \frac{1}{c^2}\right) + 2(c^2)^{\frac{1}{3}} \log\left(x^2 - \frac{1}{c^2}\right) + \frac{6}{x^2} \right) c + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="maxima")`

[Out] 
$$-1/20*((2*\sqrt{3}*(c^2)^{(1/3)}*arctan(1/3*\sqrt{3}*(c^2)^{(1/3)}*(2*x^2 + (c^(-2))^{(1/3)}))) - (c^2)^{(1/3)}*\log(x^4 + (c^(-2))^{(1/3)}*x^2 + (c^(-2))^{(2/3)}) + 2*(c^2)^{(1/3)}*\log(x^2 - (c^(-2))^{(1/3)}) + 6/x^2)*c + 4*arctanh(c*x^3)/x^5)*b - 1/5*a/x^5$$

**Fricas [A]** time = 1.76675, size = 367, normalized size = 3.19

$$\frac{2\sqrt{3}(-c^2)^{\frac{1}{3}}bcx^5 \arctan\left(\frac{2}{3}\sqrt{3}(-c^2)^{\frac{1}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) + (-c^2)^{\frac{1}{3}}bcx^5 \log\left(c^2x^4 + (-c^2)^{\frac{2}{3}}x^2 - (-c^2)^{\frac{1}{3}}\right) - 2(-c^2)^{\frac{1}{3}}bcx^5 \log\left(\dots\right)}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="fricas")`

[Out] 
$$-1/20*(2*\sqrt{3}*(-c^2)^{(1/3)}*b*c*x^5*arctan(2/3*\sqrt{3}*(-c^2)^{(1/3)}*x^2 - 1/3*\sqrt{3}) + (-c^2)^{(1/3)}*b*c*x^5*\log(c^2*x^4 + (-c^2)^{(2/3)}*x^2 - (-c^2)^{(1/3)}) - 2*(-c^2)^{(1/3)}*b*c*x^5*\log(c^2*x^2 - (-c^2)^{(2/3)}) + 6*b*c*x^3 + 2*b*\log(-c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^5$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))/x\*\*6,x)

[Out] Timed out

---

**Giac [A]** time = 1.21764, size = 169, normalized size = 1.47

$$-\frac{1}{20}bc^3 \left( \frac{2\sqrt{3}|c|^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2 + \frac{1}{2}\right)|c|^{\frac{2}{3}}\right)}{c^2} - \frac{|c|^{\frac{2}{3}} \log\left(x^4 + \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{4|c|^{\frac{2}{3}}}\right)}{c^2} + \frac{2 \log\left(\left|x^2 - \frac{1}{2|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}} \right) - \frac{b \log\left(\frac{-cx^3+1}{cx^3-1}\right)}{10x^5} - \frac{3b}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^6,x, algorithm="giac")

[Out] -1/20\*b\*c^3\*(2\*sqrt(3)\*abs(c)^(2/3)\*arctan(1/3\*sqrt(3)\*(2\*x^2 + 1/abs(c)^(2/3))\*abs(c)^(2/3))/c^2 - abs(c)^(2/3)\*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/c^2 + 2\*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) - 1/10\*b\*log(-(c\*x^3 + 1)/(c\*x^3 - 1))/x^5 - 1/10\*(3\*b\*c\*x^3 + 2\*a)/x^5

### 3.111 $\int x^7 (a + b \tanh^{-1}(cx^3)) dx$

**Optimal.** Leaf size=176

$$\frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{32c^{8/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{32c^{8/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}}$$

[Out]  $(3bx^5)/(40c) - (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(16*c^{(8/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(16*c^{(8/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*x])/(8*c^{(8/3)}) + (x^8*(a + b*\text{ArcTanh}[c*x^3]))/8 + (b*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/(32*c^{(8/3)}) - (b*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/(32*c^{(8/3)})$

**Rubi [A]** time = 0.283719, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6097, 321, 296, 634, 618, 204, 628, 206}

$$\frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{32c^{8/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{32c^{8/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^7*(a + b*\text{ArcTanh}[c*x^3]), x]$

[Out]  $(3bx^5)/(40c) - (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(16*c^{(8/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(16*c^{(8/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*x])/(8*c^{(8/3)}) + (x^8*(a + b*\text{ArcTanh}[c*x^3]))/8 + (b*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/(32*c^{(8/3)}) - (b*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/(32*c^{(8/3)})$

#### Rule 6097

$\text{Int}[(a + \text{ArcTanh}[c*x^n])*(b*x^m), x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{n-1}*(d*x)^{(m+1)})/(1 - c^2*x^{2*n}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 321

$\text{Int}[(c*x)^m*(a + b*x^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{n-1}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{n-1}*(d*x)^{(m+1)})/(1 - c^2*x^{2*n}), x], x] /;$

$(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x],$   
 $x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p  
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 296

$\text{Int}[(x_)^{(m_.)}/((a_) + (b_)*(x_)^{(n_)}), x\_Symbol] :=$  Module[{r = Numerator  
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r\*Cos  
[(2\*k\*m\*Pi)/n] - s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/(r^2 - 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*  
x + s^2\*x^2), x] + Int[(r\*Cos[(2\*k\*m\*Pi)/n] + s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/  
(r^2 + 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x]; (2\*r^(m + 2)\*Int[1/(r^2 - s^  
2\*x^2), x])/(a\*n\*s^m) + Dist[(2\*r^(m + 1))/(a\*n\*s^m), Sum[u, {k, 1, (n - 2)  
/4}], x], x] /;

FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt  
Q[m, n - 1] && NegQ[a/b]

### Rule 634

$\text{Int}(((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol) :=$  D  
ist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In  
t[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /;

FreeQ[{a, b, c, d, e}, x] && NeQ  
[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 618

$\text{Int}(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^{-1}, x\_Symbol) :=$  Dist[-2, Subst[  
Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /;

FreeQ[{a, b, c},  
x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

$\text{Int}(((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol) :=$  -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[  
-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /;

FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[  
a, 0] || LtQ[b, 0])

### Rule 628

$\text{Int}(((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x\_Symbol) :=$  S  
imp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /;

FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 206

$\text{Int}(((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol) :=$  Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/  
Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /;

FreeQ[{a, b}, x] && NegQ[a/b] && (Gt



Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^7 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) - \frac{1}{8}(3bc) \int \frac{x^{10}}{1 - c^2x^6} dx \\
 &= \frac{3bx^5}{40c} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) - \frac{(3b) \int \frac{x^4}{1 - c^2x^6} dx}{8c} \\
 &= \frac{3bx^5}{40c} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) - \frac{b \int \frac{1}{1 - c^{2/3}x^2} dx}{8c^{7/3}} - \frac{b \int \frac{-\frac{1}{2} - \frac{\sqrt[3]{cx}}{2}}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{8c^{7/3}} - \frac{b \int \frac{-\frac{1}{2} + \frac{\sqrt[3]{cx}}{2}}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{8c^{7/3}} \\
 &= \frac{3bx^5}{40c} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{8c^{8/3}} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) + \frac{b \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{32c^{8/3}} - \frac{b \int \frac{\sqrt[3]{c} + 2c^{2/3}x}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{32c^{8/3}} \\
 &= \frac{3bx^5}{40c} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{8c^{8/3}} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - \sqrt[3]{cx} + c^{2/3}x^2)}{32c^{8/3}} - \frac{b \log(1 + \sqrt[3]{cx} + c^{2/3}x^2)}{32c^{8/3}} \\
 &= \frac{3bx^5}{40c} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{16c^{8/3}} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{8c^{8/3}} + \frac{1}{8}x^8 (a + b \tanh^{-1}(cx^3))
 \end{aligned}$$

**Mathematica [A]** time = 0.0302007, size = 198, normalized size = 1.12

$$\frac{ax^8}{8} + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{32c^{8/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{32c^{8/3}} + \frac{b \log(1 - \sqrt[3]{cx})}{16c^{8/3}} - \frac{b \log(\sqrt[3]{cx} + 1)}{16c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{cx} - 1}{\sqrt{3}}\right)}{16c^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^7\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] (3\*b\*x^5)/(40\*c) + (a\*x^8)/8 + (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(16\*c^(8/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(16\*c^(8/3)) + (b\*x^8\*ArcTanh[c\*x^3])/8 + (b\*Log[1 - c^(1/3)\*x])/(16\*c^(8/3)) - (b\*Log[1 + c^(1/3)\*x])/(16\*c^(8/3)) + (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(32\*c^(8/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(32\*c^(8/3))

**Maple [A]** time = 0.013, size = 186, normalized size = 1.1

$$\frac{x^8 a}{8} + \frac{x^8 b \operatorname{Artanh}(cx^3)}{8} + \frac{3bx^5}{40c} + \frac{b}{16c^3} \ln\left(x - \sqrt[3]{c^{-1}}\right) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{b}{32c^3} \ln\left(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{b\sqrt{3}}{16c^3} \arctan\left(\frac{2\sqrt[3]{cx} - 1}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a+b*arctanh(c*x^3)),x)`

[Out]  $\frac{1}{8}x^8a + \frac{1}{8}x^8b \operatorname{arctanh}(cx^3) + \frac{3}{40}bx^5/c + \frac{1}{16}b/c^3/(1/c)^{1/3} \ln(x - (1/c)^{1/3}) - \frac{1}{32}b/c^3/(1/c)^{1/3} \ln(x^2 + (1/c)^{1/3}x + (1/c)^{2/3}) + \frac{1}{16}b/c^3 \cdot 3^{1/2}/(1/c)^{1/3} \operatorname{arctan}(1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}x + 1)) - \frac{1}{16}b/c^3/(1/c)^{1/3} \ln(x + (1/c)^{1/3}) + \frac{1}{32}b/c^3/(1/c)^{1/3} \ln(x^2 - (1/c)^{1/3}x + (1/c)^{2/3}) + \frac{1}{16}b/c^3 \cdot 3^{1/2}/(1/c)^{1/3} \operatorname{arctan}(1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}x - 1))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.88365, size = 659, normalized size = 3.74

$$10bc^4x^8 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 20ac^4x^8 + 12bc^3x^5 + 10\sqrt{3}bc\sqrt{-(-c^2)^{\frac{1}{3}}}\operatorname{arctan}\left(\frac{\sqrt{3}\left(2cx+(-c^2)^{\frac{1}{3}}\right)\sqrt{-(-c^2)^{\frac{1}{3}}}}{3c}\right) + 10\sqrt{3}b(c^2)^{\frac{1}{6}}c \operatorname{arctan}\left(\frac{\sqrt{3}\left(2cx+(-c^2)^{\frac{1}{3}}\right)\sqrt{-(-c^2)^{\frac{1}{3}}}}{3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]  $\frac{1}{160}(10bc^4x^8 \log(-cx^3 + 1)/(cx^3 - 1)) + 20ac^4x^8 + 12bc^3x^5 + 10\sqrt{3}bc\sqrt{-(-c^2)^{1/3}} \operatorname{arctan}(1/3\sqrt{3}(2cx + (-c^2)^{1/3})\sqrt{-(-c^2)^{1/3}}/c) + 10\sqrt{3}b(c^2)^{1/6}c \operatorname{arctan}(1/3\sqrt{3}(c^2)^{1/6}(2cx + (-c^2)^{1/3})/c) + 5(-c^2)^{2/3}b \log(c^2x^2 + (-c^2)^{1/3}cx + (-c^2)^{2/3}) - 5b(c^2)^{2/3} \log(c^2x^2 + (-c^2)^{1/3}cx + (-c^2)^{2/3}) - 10(-c^2)^{2/3}b \log(cx - (-c^2)^{1/3}) + 10b(c^2)^{1/6}c \operatorname{arctan}(1/3\sqrt{3}(c^2)^{1/6}(2cx + (-c^2)^{1/3})/c)$

$$2)^{(2/3)} * \log(c*x - (c^2)^{(1/3)}) / c^4$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(a+b\*atanh(c\*x\*\*3)),x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.49266, size = 281, normalized size = 1.6

$$-\frac{1}{32}bc^{15} \left( \frac{2 \left(-\frac{1}{c}\right)^{\frac{2}{3}} \log\left(x - \left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{c^{17}} - \frac{2\sqrt{3}|c|^{\frac{4}{3}} \arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x + \frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^{19}} + \frac{|c|^{\frac{4}{3}} \log\left(x^2 + \frac{x}{c^{\frac{1}{3}}} + \frac{1}{c^{\frac{2}{3}}}\right)}{c^{19}} - \frac{2 \log\left(x - \frac{1}{c^{\frac{1}{3}}}\right)}{c^{\frac{53}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c\*x^3)),x, algorithm="giac")

[Out]  $-1/32*b*c^{15}*(2*(-1/c)^{(2/3)}*\log(\text{abs}(x - (-1/c)^{(1/3)}))/c^{17} - 2*\text{sqrt}(3)*\text{abs}(c)^{(4/3)}*\arctan(1/3*\text{sqrt}(3)*c^{(1/3)}*(2*x + 1/c^{(1/3)}))/c^{19} + \text{abs}(c)^{(4/3)}*\log(x^2 + x/c^{(1/3)} + 1/c^{(2/3)})/c^{19} - 2*\log(\text{abs}(x - 1/c^{(1/3)}))/c^{(53/3)}) + 2*\text{sqrt}(3)*(-c^2)^{(2/3)}*\arctan(1/3*\text{sqrt}(3)*(2*x + (-1/c)^{(1/3)})/(-1/c)^{(1/3)})/c^{19} - (-c^2)^{(2/3)}*\log(x^2 + x*(-1/c)^{(1/3)} + (-1/c)^{(2/3)})/c^{19} + 1/16*b*x^8*\log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/8*a*x^8 + 3/40*b*x^5/c$

### 3.112 $\int x^4 \left( a + b \tanh^{-1} (cx^3) \right) dx$

**Optimal.** Leaf size=117

$$\frac{1}{5}x^5 \left( a + b \tanh^{-1} (cx^3) \right) + \frac{b \log(1 - c^{2/3}x^2)}{10c^{5/3}} - \frac{b \log(c^{4/3}x^4 + c^{2/3}x^2 + 1)}{20c^{5/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{2c^{2/3}x^2 + 1}{\sqrt{3}} \right)}{10c^{5/3}} + \frac{3bx^2}{10c}$$

[Out] (3\*b\*x^2)/(10\*c) - (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/(10\*c^(5/3)) + (x^5\*(a + b\*ArcTanh[c\*x^3]))/5 + (b\*Log[1 - c^(2/3)\*x^2])/(10\*c^(5/3)) - (b\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/(20\*c^(5/3))

**Rubi [A]** time = 0.0972647, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {6097, 275, 321, 200, 31, 634, 617, 204, 628}

$$\frac{1}{5}x^5 \left( a + b \tanh^{-1} (cx^3) \right) + \frac{b \log(1 - c^{2/3}x^2)}{10c^{5/3}} - \frac{b \log(c^{4/3}x^4 + c^{2/3}x^2 + 1)}{20c^{5/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{2c^{2/3}x^2 + 1}{\sqrt{3}} \right)}{10c^{5/3}} + \frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] (3\*b\*x^2)/(10\*c) - (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/(10\*c^(5/3)) + (x^5\*(a + b\*ArcTanh[c\*x^3]))/5 + (b\*Log[1 - c^(2/3)\*x^2])/(10\*c^(5/3)) - (b\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/(20\*c^(5/3))

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c^n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int x^4 (a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) - \frac{1}{5}(3bc) \int \frac{x^7}{1 - c^2x^6} dx \\
&= \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) - \frac{1}{10}(3bc) \operatorname{Subst}\left(\int \frac{x^3}{1 - c^2x^3} dx, x, x^2\right) \\
&= \frac{3bx^2}{10c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{1 - c^2x^3} dx, x, x^2\right)}{10c} \\
&= \frac{3bx^2}{10c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{1 - c^{2/3}x} dx, x, x^2\right)}{10c} - \frac{b \operatorname{Subst}\left(\int \frac{2 + c^{2/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2\right)}{10c} \\
&= \frac{3bx^2}{10c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^{2/3}x^2)}{10c^{5/3}} - \frac{b \operatorname{Subst}\left(\int \frac{c^{2/3} + 2c^{4/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2\right)}{20c^{5/3}} \\
&= \frac{3bx^2}{10c} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^{2/3}x^2)}{10c^{5/3}} - \frac{b \log(1 + c^{2/3}x^2 + c^{4/3}x^4)}{20c^{5/3}} + \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2\right)}{10c} \\
&= \frac{3bx^2}{10c} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}} + \frac{1}{5}x^5 (a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - c^{2/3}x^2)}{10c^{5/3}} - \frac{b \log(1 + c^{2/3}x^2 + c^{4/3}x^4)}{20c^{5/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.0282765, size = 198, normalized size = 1.69

$$\frac{ax^5}{5} - \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{20c^{5/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{20c^{5/3}} + \frac{b \log(1 - \sqrt[3]{cx})}{10c^{5/3}} + \frac{b \log(\sqrt[3]{cx} + 1)}{10c^{5/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{cx} - 1}{\sqrt{3}}\right)}{10c^{5/3}} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] (3\*b\*x^2)/(10\*c) + (a\*x^5)/5 - (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(10\*c^(5/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(10\*c^(5/3)) + (b\*x^5\*ArcTanh[c\*x^3])/5 + (b\*Log[1 - c^(1/3)\*x])/(10\*c^(5/3)) + (b\*Log[1 + c^(1/3)\*x])/(10\*c^(5/3)) - (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(20\*c^(5/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(20\*c^(5/3))

**Maple [A]** time = 0.007, size = 114, normalized size = 1.

$$\frac{ax^5}{5} + \frac{x^5 b \operatorname{Artanh}(cx^3)}{5} + \frac{3bx^2}{10c} + \frac{b}{10c^3} \ln\left(x^2 - \sqrt[3]{c^{-2}}\right) (c^{-2})^{-\frac{2}{3}} - \frac{b}{20c^3} \ln\left(x^4 + \sqrt[3]{c^{-2}x^2} + (c^{-2})^{\frac{2}{3}}\right) (c^{-2})^{-\frac{2}{3}} - \frac{b\sqrt{3}}{10c^3} \operatorname{arctan}\left(\frac{1 + 2c^{2/3}x^2}{\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*arctanh(c*x^3)),x)`

[Out]  $\frac{1}{5}ax^5 + \frac{1}{5}x^5b\operatorname{arctanh}(cx^3) + \frac{3}{10}bx^2/c + \frac{1}{10}b/c^3/(1/c^2)^{(2/3)} * \ln(x^2 - (1/c^2)^{(1/3)}) - \frac{1}{20}b/c^3/(1/c^2)^{(2/3)} * \ln(x^4 + (1/c^2)^{(1/3)}x^2 + (1/c^2)^{(2/3)}) - \frac{1}{10}b/c^3/(1/c^2)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(1/c^2)^{(1/3)}x^2 + 1))$

**Maxima [A]** time = 1.50079, size = 161, normalized size = 1.38

$$\frac{1}{5}ax^5 + \frac{1}{20} \left( 4x^5 \operatorname{arctanh}(cx^3) + c \left( \frac{6x^2}{c^2} - \frac{2\sqrt{3}(c^2)^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}(c^2)^{\frac{1}{3}}\left(2x^2 + \frac{1}{c^2}\right)\right)}{c^4} - \frac{(c^2)^{\frac{2}{3}} \log\left(x^4 + \frac{1}{c^2}x^2 + \frac{1}{c^2}\right)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out]  $\frac{1}{5}ax^5 + \frac{1}{20} * (4x^5 \operatorname{arctanh}(cx^3) + c * (6x^2/c^2 - 2\sqrt{3}(c^2)^{(2/3)} * (c^2)^{(1/3)} * \arctan(1/3 * \sqrt{3}(c^2)^{(1/3)} * (2x^2 + (c^2)^{(1/3)}))) / c^4 - (c^2)^{(2/3)} * \log(x^4 + (c^2)^{(1/3)}x^2 + (c^2)^{(2/3)}) / c^4 + 2 * (c^2)^{(2/3)} * \log(x^2 - (c^2)^{(1/3)}) / c^4) * b$

**Fricas [A]** time = 1.79069, size = 414, normalized size = 3.54

$$\frac{2bc^3x^5 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 4ac^3x^5 + 6bc^2x^2 - 2\sqrt{3}b(c^2)^{\frac{1}{6}}c \arctan\left(-\frac{\sqrt{3}\left(4c^2x^4 - 2(c^2)^{\frac{2}{3}}x^2 + (c^2)^{\frac{1}{3}}\right)(c^2)^{\frac{1}{6}}}{8c^3x^6 + c}\right) - b(c^2)^{\frac{2}{3}} \log\left(c^2x^4 + (c^2)\right)}{20c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]  $\frac{1}{20} * (2 * b * c^3 * x^5 * \log(-(c * x^3 + 1) / (c * x^3 - 1)) + 4 * a * c^3 * x^5 + 6 * b * c^2 * x^2 - 2 * \sqrt{3} * b * (c^2)^{(1/6)} * c * \arctan(-\sqrt{3} * (4 * c^2 * x^4 - 2 * (c^2)^{(2/3)} * x^2)) / (8 * c^3 * x^6 + c)) - b * (c^2)^{(2/3)} * \log(c^2 * x^4 + c^2)$

$$+ (c^2)^{(1/3)} * (c^2)^{(1/6)} / (8 * c^3 * x^6 + c) - b * (c^2)^{(2/3)} * \log(c^2 * x^4 + (c^2)^{(2/3)} * x^2 + (c^2)^{(1/3)}) + 2 * b * (c^2)^{(2/3)} * \log(c^2 * x^2 - (c^2)^{(2/3)}) / c^3$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*atanh(c\*x\*\*3)),x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.18607, size = 170, normalized size = 1.45

$$-\frac{1}{20} b c^9 \left( \frac{2 \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} \left(2x^2 + \frac{1}{2}\right) |c|^{\frac{2}{3}}\right)}{c^{10} |c|^{\frac{2}{3}}} + \frac{\log\left(x^4 + \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10} |c|^{\frac{2}{3}}} - \frac{2 \log\left(\left|x^2 - \frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{c^{10} |c|^{\frac{2}{3}}} \right) + \frac{1}{10} b x^5 \log\left(-\frac{c x^3 + 1}{c x^3 - 1}\right) + \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c\*x^3)),x, algorithm="giac")

[Out] -1/20\*b\*c^9\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x^2 + 1/abs(c)^(2/3))\*abs(c)^(2/3))/(c^10\*abs(c)^(2/3)) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c^10\*abs(c)^(2/3)) - 2\*log(abs(x^2 - 1/abs(c)^(2/3)))/(c^10\*abs(c)^(2/3))) + 1/10\*b\*x^5\*log(-(c\*x^3 + 1)/(c\*x^3 - 1)) + 1/5\*a\*x^5 + 3/10\*b\*x^2/c



### 3.113 $\int x \left( a + b \tanh^{-1} \left( cx^3 \right) \right) dx$

**Optimal.** Leaf size=165

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( cx^3 \right) \right) + \frac{b \log \left( c^{2/3}x^2 - \sqrt[3]{cx} + 1 \right)}{8c^{2/3}} - \frac{b \log \left( c^{2/3}x^2 + \sqrt[3]{cx} + 1 \right)}{8c^{2/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}} \right)}{4c^{2/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}} \right)}{4c^{2/3}}$$

[Out]  $-(\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(4*c^{(2/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(4*c^{(2/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*x])/(2*c^{(2/3)}) + (x^2*(a + b*\text{ArcTanh}[c*x^3]))/2 + (b*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)}) - (b*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)})$

**Rubi [A]** time = 0.248606, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {6097, 296, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( cx^3 \right) \right) + \frac{b \log \left( c^{2/3}x^2 - \sqrt[3]{cx} + 1 \right)}{8c^{2/3}} - \frac{b \log \left( c^{2/3}x^2 + \sqrt[3]{cx} + 1 \right)}{8c^{2/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}} \right)}{4c^{2/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{cx}}{\sqrt{3}} \right)}{4c^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{ArcTanh}[c*x^3]), x]$

[Out]  $-(\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(4*c^{(2/3)}) + (\text{Sqrt}[3]*b*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/(4*c^{(2/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*x])/(2*c^{(2/3)}) + (x^2*(a + b*\text{ArcTanh}[c*x^3]))/2 + (b*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)}) - (b*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/(8*c^{(2/3)})$

#### Rule 6097

$\text{Int}[\left((a_{.}) + \text{ArcTanh}[(c_{.})*(x_{.})^{(n_{.})}]*b_{.}\right)*\left((d_{.})*(x_{.})\right)^{(m_{.})}, x\_Symbol]$   
 $:= \text{Simp}[\left((d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])\right)/(d*(m+1)), x] - \text{Dist}[\left(b*c^n/(d*(m+1))\right), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 296

$\text{Int}[(x_{.})^{(m_{.})}/\left((a_{.}) + (b_{.})*(x_{.})^{(n_{.})}\right), x\_Symbol] := \text{Module}[\{r = \text{Numerator}[\text{Rt}[-(a/b), n]], s = \text{Denominator}[\text{Rt}[-(a/b), n]], k, u\}, \text{Simp}[u = \text{Int}[(r*\text{Cos}$

```

[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]

```

### Rule 634

```

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

```

### Rule 618

```

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]

```

### Rule 204

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])

```

### Rule 628

```

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]

```

### Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

### Rubi steps

$$\begin{aligned}
\int x(a + b \tanh^{-1}(cx^3)) dx &= \frac{1}{2}x^2(a + b \tanh^{-1}(cx^3)) - \frac{1}{2}(3bc) \int \frac{x^4}{1 - c^2x^6} dx \\
&= \frac{1}{2}x^2(a + b \tanh^{-1}(cx^3)) - \frac{b \int \frac{1}{1 - c^{2/3}x^2} dx}{2\sqrt[3]{c}} - \frac{b \int \frac{-\frac{1}{2} - \frac{\sqrt[3]{cx}}{2}}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{2\sqrt[3]{c}} - \frac{b \int \frac{-\frac{1}{2} + \frac{\sqrt[3]{cx}}{2}}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{2\sqrt[3]{c}} \\
&= -\frac{b \tanh^{-1}(\sqrt[3]{cx})}{2c^{2/3}} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx^3)) + \frac{b \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx}{8c^{2/3}} - \frac{b \int \frac{\sqrt[3]{c} + 2c^{2/3}x}{1 + \sqrt[3]{cx} + c^{2/3}x^2} dx}{8c^{2/3}} \\
&= -\frac{b \tanh^{-1}(\sqrt[3]{cx})}{2c^{2/3}} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx^3)) + \frac{b \log(1 - \sqrt[3]{cx} + c^{2/3}x^2)}{8c^{2/3}} - \frac{b \log(1 + \sqrt[3]{cx} + c^{2/3}x^2)}{8c^{2/3}} \\
&= -\frac{\sqrt{3}b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{4c^{2/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{cx}}{\sqrt{3}}\right)}{4c^{2/3}} - \frac{b \tanh^{-1}(\sqrt[3]{cx})}{2c^{2/3}} + \frac{1}{2}x^2(a + b \tanh^{-1}(cx^3))
\end{aligned}$$

**Mathematica [A]** time = 0.0255684, size = 187, normalized size = 1.13

$$\frac{ax^2}{2} + \frac{b \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1)}{8c^{2/3}} - \frac{b \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1)}{8c^{2/3}} + \frac{b \log(1 - \sqrt[3]{cx})}{4c^{2/3}} - \frac{b \log(\sqrt[3]{cx} + 1)}{4c^{2/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{cx} - 1}{\sqrt{3}}\right)}{4c^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^3]),x]

[Out] (a\*x^2)/2 + (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(4\*c^(2/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/(4\*c^(2/3)) + (b\*x^2\*ArcTanh[c\*x^3])/2 + (b\*Log[1 - c^(1/3)\*x])/(4\*c^(2/3)) - (b\*Log[1 + c^(1/3)\*x])/(4\*c^(2/3)) + (b\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/(8\*c^(2/3)) - (b\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/(8\*c^(2/3))

**Maple [A]** time = 0.01, size = 177, normalized size = 1.1

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{Artanh}(cx^3)}{2} + \frac{b}{4c} \ln\left(x - \sqrt[3]{c^{-1}}\right) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{b}{8c} \ln\left(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{b\sqrt{3}}{4c} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \frac{x}{\sqrt[3]{c^{-1}}} + \dots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c\*x^3)),x)

[Out]  $\frac{1}{2}ax^2 + \frac{1}{2}bx^2 \operatorname{arctanh}(cx^3) + \frac{1}{4}b/c/(1/c)^{1/3} \ln(x - (1/c)^{1/3}) - \frac{1}{8}b/c/(1/c)^{1/3} \ln(x^2 + (1/c)^{1/3}x + (1/c)^{2/3}) + \frac{1}{4}b \cdot 3^{1/2}/c/(1/c)^{1/3} \operatorname{arctan}(1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}x + 1)) - \frac{1}{4}b/c/(1/c)^{1/3} \ln(x + (1/c)^{1/3}) + \frac{1}{8}b/c/(1/c)^{1/3} \ln(x^2 - (1/c)^{1/3}x + (1/c)^{2/3}) + \frac{1}{4}b \cdot 3^{1/2}/c/(1/c)^{1/3} \operatorname{arctan}(1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}x - 1))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.85752, size = 622, normalized size = 3.77

$$2bc^2x^2 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 4ac^2x^2 + 2\sqrt{3}bc\sqrt{-(-c^2)^{\frac{1}{3}}}\operatorname{arctan}\left(\frac{\sqrt{3}\left(2cx+(-c^2)^{\frac{1}{3}}\right)\sqrt{-(-c^2)^{\frac{1}{3}}}}{3c}\right) + 2\sqrt{3}b(c^2)^{\frac{1}{6}}c\operatorname{arctan}\left(\frac{\sqrt{3}(c^2)^{\frac{1}{6}}\left(2cx+(-c^2)^{\frac{1}{3}}\right)}{3c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="fricas")`

[Out]  $\frac{1}{8} \cdot (2bc^2x^2 \log(-cx^3 + 1)/(cx^3 - 1) + 4ac^2x^2 + 2\sqrt{3}bc\sqrt{-(-c^2)^{1/3}} \operatorname{arctan}(1/3\sqrt{3}(2cx + (-c^2)^{1/3})\sqrt{-(-c^2)^{1/3}}/c) + 2\sqrt{3}b(c^2)^{1/6}c \operatorname{arctan}(1/3\sqrt{3}(c^2)^{1/6}(2cx + (-c^2)^{1/3})/c) + (-c^2)^{2/3}b \log(c^2x^2 + (-c^2)^{1/3}cx + (-c^2)^{2/3}) - b(c^2)^{2/3} \log(c^2x^2 + (c^2)^{1/3}cx + (c^2)^{2/3}) - 2(-c^2)^{2/3}b \log(cx - (-c^2)^{1/3}) + 2b(c^2)^{2/3} \log(cx - (c^2)^{1/3}))/c^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c*x**3)),x)`

[Out] Timed out

**Giac [A]** time = 1.31032, size = 269, normalized size = 1.63

$$-\frac{1}{8}bc^5 \left( \frac{2 \left(-\frac{1}{c}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^5} - \frac{2\sqrt{3}|c|^{\frac{4}{3}} \arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2x + \frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^7} + \frac{|c|^{\frac{4}{3}} \log\left(x^2 + \frac{x}{c^{\frac{1}{3}}} + \frac{1}{c^{\frac{2}{3}}}\right)}{c^7} - \frac{2 \log\left(\left|x - \frac{1}{c^{\frac{1}{3}}}\right|\right)}{c^{\frac{17}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="giac")`

[Out] `-1/8*b*c^5*(2*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^5 - 2*sqrt(3)*abs(c)^(4/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^7 + abs(c)^(4/3)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^7 - 2*log(abs(x - 1/c^(1/3)))/c^(17/3) + 2*sqrt(3)*(-c^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/c^7 - (-c^2)^(2/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^7) + 1/4*b*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2`

$$3.114 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^2} dx$$

**Optimal.** Leaf size=104

$$-\frac{a+b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1-c^{2/3}x^2) + \frac{1}{4}b\sqrt[3]{c} \log(c^{4/3}x^4 + c^{2/3}x^2 + 1) + \frac{1}{2}\sqrt{3}b\sqrt[3]{c} \tan^{-1}\left(\frac{2c^{2/3}x^2 + 1}{\sqrt{3}}\right)$$

[Out] (Sqrt[3]\*b\*c^(1/3)\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/2 - (a + b\*ArcTanh[c\*x^3])/x - (b\*c^(1/3)\*Log[1 - c^(2/3)\*x^2])/2 + (b\*c^(1/3)\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/4

**Rubi [A]** time = 0.0827732, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6097, 275, 200, 31, 634, 617, 204, 628}

$$-\frac{a+b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1-c^{2/3}x^2) + \frac{1}{4}b\sqrt[3]{c} \log(c^{4/3}x^4 + c^{2/3}x^2 + 1) + \frac{1}{2}\sqrt{3}b\sqrt[3]{c} \tan^{-1}\left(\frac{2c^{2/3}x^2 + 1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^2, x]

[Out] (Sqrt[3]\*b\*c^(1/3)\*ArcTan[(1 + 2\*c^(2/3)\*x^2)/Sqrt[3]])/2 - (a + b\*ArcTanh[c\*x^3])/x - (b\*c^(1/3)\*Log[1 - c^(2/3)\*x^2])/2 + (b\*c^(1/3)\*Log[1 + c^(2/3)\*x^2 + c^(4/3)\*x^4])/4

### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

### Rule 31

```
Int[((a_) + (b_)*(x_)^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^3)}{x^2} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{x} + (3bc) \int \frac{x}{1 - c^2x^6} dx \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{x} + \frac{1}{2}(3bc) \text{Subst} \left( \int \frac{1}{1 - c^2x^3} dx, x, x^2 \right) \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{x} + \frac{1}{2}(bc) \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x} dx, x, x^2 \right) + \frac{1}{2}(bc) \text{Subst} \left( \int \frac{2 + c^{2/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2 \right) \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1 - c^{2/3}x^2) + \frac{1}{4}(b\sqrt[3]{c}) \text{Subst} \left( \int \frac{c^{2/3} + 2c^{4/3}x}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2 \right) \\
&= -\frac{a + b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1 - c^{2/3}x^2) + \frac{1}{4}b\sqrt[3]{c} \log(1 + c^{2/3}x^2 + c^{4/3}x^4) - \frac{1}{2}(3b\sqrt[3]{c}) \text{Subst} \left( \int \frac{1}{1 + c^{2/3}x + c^{4/3}x^2} dx, x, x^2 \right) \\
&= \frac{1}{2}\sqrt{3}b\sqrt[3]{c} \tan^{-1} \left( \frac{1 + 2c^{2/3}x^2}{\sqrt{3}} \right) - \frac{a + b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1 - c^{2/3}x^2) + \frac{1}{4}b\sqrt[3]{c} \log(1 + c^{2/3}x^2 + c^{4/3}x^4)
\end{aligned}$$

**Mathematica [A]** time = 0.0290422, size = 183, normalized size = 1.76

$$-\frac{a}{x} + \frac{1}{4}b\sqrt[3]{c} \log(c^{2/3}x^2 - \sqrt[3]{c}x + 1) + \frac{1}{4}b\sqrt[3]{c} \log(c^{2/3}x^2 + \sqrt[3]{c}x + 1) - \frac{b \tanh^{-1}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c} \log(1 - \sqrt[3]{c}x) - \frac{1}{2}b\sqrt[3]{c} \log(1 + \sqrt[3]{c}x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^2, x]

[Out] -(a/x) + (Sqrt[3]\*b\*c^(1/3)\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/2 - (Sqrt[3]\*b\*c^(1/3)\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/2 - (b\*ArcTanh[c\*x^3])/x - (b\*c^(1/3)\*Log[1 - c^(1/3)\*x])/2 - (b\*c^(1/3)\*Log[1 + c^(1/3)\*x])/2 + (b\*c^(1/3)\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/4 + (b\*c^(1/3)\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/4

**Maple [A]** time = 0.005, size = 105, normalized size = 1.

$$-\frac{a}{x} - \frac{b \text{Arctanh}(cx^3)}{x} - \frac{b}{2c} \ln(x^2 - \sqrt[3]{c^{-2}})(c^{-2})^{-\frac{2}{3}} + \frac{b}{4c} \ln(x^4 + \sqrt[3]{c^{-2}}x^2 + (c^{-2})^{\frac{2}{3}})(c^{-2})^{-\frac{2}{3}} + \frac{b\sqrt{3}}{2c} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \frac{x^2}{\sqrt[3]{c^{-2}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))/x^2, x)



[Out]  $-a/x - b/x \cdot \operatorname{arctanh}(c \cdot x^3) - 1/2 \cdot b/c / (1/c^2)^{2/3} \cdot \ln(x^2 - (1/c^2)^{1/3}) + 1/4 \cdot b/c / (1/c^2)^{2/3} \cdot \ln(x^4 + (1/c^2)^{1/3} \cdot x^2 + (1/c^2)^{2/3}) + 1/2 \cdot b/c / (1/c^2)^{2/3} \cdot 3^{1/2} \cdot \arctan(1/3 \cdot 3^{1/2} \cdot (2/(1/c^2)^{1/3} \cdot x^2 + 1))$

**Maxima [A]** time = 1.52792, size = 149, normalized size = 1.43

$$\frac{1}{4} \left( c \left( \frac{2\sqrt{3}(c^2)^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}(c^2)^{\frac{1}{3}}\left(2x^2 + \frac{1}{c^2}\right)\right)}{c^2} + \frac{(c^2)^{\frac{2}{3}} \log\left(x^4 + \frac{1}{c^2}x^2 + \frac{1}{c^2}\right)}{c^2} - \frac{2(c^2)^{\frac{2}{3}} \log\left(x^2 - \frac{1}{c^2}\right)}{c^2} \right) - \frac{4 \operatorname{arctanh}(c \cdot x^3)}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="maxima")`

[Out]  $1/4 \cdot (c \cdot (2 \cdot \sqrt{3}) \cdot (c^2)^{2/3} \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (c^2)^{1/3} \cdot (2 \cdot x^2 + (c^{-2})^{1/3}))) / c^2 + (c^2)^{2/3} \cdot \log(x^4 + (c^{-2})^{1/3} \cdot x^2 + (c^{-2})^{2/3}) / c^2 - 2 \cdot (c^2)^{2/3} \cdot \log(x^2 - (c^{-2})^{1/3}) / c^2 - 4 \cdot \operatorname{arctanh}(c \cdot x^3) / x) \cdot b - a/x$

**Fricas [A]** time = 1.82913, size = 312, normalized size = 3.

$$\frac{2\sqrt{3}b(-c)^{\frac{1}{3}}x \arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{2}{3}}x^2 + \frac{1}{3}\sqrt{3}\right) + b(-c)^{\frac{1}{3}}x \log\left(c^2x^4 - (-c)^{\frac{1}{3}}cx^2 + (-c)^{\frac{2}{3}}\right) - 2b(-c)^{\frac{1}{3}}x \log\left(cx^2 + (-c)^{\frac{1}{3}}\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="fricas")`

[Out]  $-1/4 \cdot (2 \cdot \sqrt{3}) \cdot b \cdot (-c)^{1/3} \cdot x \cdot \arctan(2/3 \cdot \sqrt{3} \cdot (-c)^{2/3} \cdot x^2 + 1/3 \cdot \sqrt{3}) + b \cdot (-c)^{1/3} \cdot x \cdot \log(c^2 \cdot x^4 - (-c)^{1/3} \cdot c \cdot x^2 + (-c)^{2/3}) - 2 \cdot b \cdot (-c)^{1/3} \cdot x \cdot \log(c \cdot x^2 + (-c)^{1/3}) + 2 \cdot b \cdot \log(-(c \cdot x^3 + 1)/(c \cdot x^3 - 1)) + 4 \cdot a) / x$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))/x\*\*2,x)

[Out] Exception raised: KeyError

**Giac [A]** time = 1.18253, size = 143, normalized size = 1.38

$$\frac{1}{4}bc \left( \frac{2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2 + \frac{1}{2}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}} + \frac{\log\left(x^4 + \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{4|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}} - \frac{2 \log\left(\left|x^2 - \frac{1}{2|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{2}{3}}} \right) - \frac{b \log\left(-\frac{cx^3+1}{cx^3-1}\right)}{2x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^2,x, algorithm="giac")

[Out] 1/4\*b\*c\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x^2 + 1/abs(c)^(2/3))\*abs(c)^(2/3))/abs(c)^(2/3) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3) - 2\*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(2/3) - 1/2\*b\*log(-(c\*x^3 + 1)/(c\*x^3 - 1))/x - a/x

$$3.115 \quad \int \frac{a+b \tanh^{-1}(cx^3)}{x^5} dx$$

**Optimal.** Leaf size=174

$$-\frac{a+b \tanh^{-1}(cx^3)}{4x^4} - \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) + \frac{1}{8}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)$$

[Out]  $(-3*b*c)/(4*x) + (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/8 - (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/8 + (b*c^{(4/3)}*\text{ArcTanh}[c^{(1/3)}*x])/4 - (a + b*\text{ArcTanh}[c*x^3])/(4*x^4) - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/16 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/16$

**Rubi [A]** time = 0.267174, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6097, 325, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b \tanh^{-1}(cx^3)}{4x^4} - \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) + \frac{1}{8}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{cx}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])/x^5, x]

[Out]  $(-3*b*c)/(4*x) + (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] - (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/8 - (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[1/\text{Sqrt}[3] + (2*c^{(1/3)}*x)/\text{Sqrt}[3]])/8 + (b*c^{(4/3)}*\text{ArcTanh}[c^{(1/3)}*x])/4 - (a + b*\text{ArcTanh}[c*x^3])/(4*x^4) - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)}*x + c^{(2/3)}*x^2])/16 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*x + c^{(2/3)}*x^2])/16$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1))

+ 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 296

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r\*Cos[(2\*k\*m\*Pi)/n] - s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/(r^2 - 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x] + Int[(r\*Cos[(2\*k\*m\*Pi)/n] + s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/(r^2 + 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x]; (2\*r^(m + 2)\*Int[1/(r^2 - s^2\*x^2), x])/(a\*n\*s^m) + Dist[(2\*r^(m + 1))/(a\*n\*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^3)}{x^5} dx &= -\frac{a + b \tanh^{-1}(cx^3)}{4x^4} + \frac{1}{4}(3bc) \int \frac{1}{x^2(1 - c^2x^6)} dx \\
 &= -\frac{3bc}{4x} - \frac{a + b \tanh^{-1}(cx^3)}{4x^4} + \frac{1}{4}(3bc^3) \int \frac{x^4}{1 - c^2x^6} dx \\
 &= -\frac{3bc}{4x} - \frac{a + b \tanh^{-1}(cx^3)}{4x^4} + \frac{1}{4}(bc^{5/3}) \int \frac{1}{1 - c^{2/3}x^2} dx + \frac{1}{4}(bc^{5/3}) \int \frac{-\frac{1}{2} - \frac{\sqrt[3]{cx}}{2}}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx + \frac{1}{16} \\
 &= -\frac{3bc}{4x} + \frac{1}{4}bc^{4/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{4x^4} - \frac{1}{16}(bc^{4/3}) \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{cx} + c^{2/3}x^2} dx + \frac{1}{16}bc \\
 &= -\frac{3bc}{4x} + \frac{1}{4}bc^{4/3} \tanh^{-1}(\sqrt[3]{cx}) - \frac{a + b \tanh^{-1}(cx^3)}{4x^4} - \frac{1}{16}bc^{4/3} \log(1 - \sqrt[3]{cx} + c^{2/3}x^2) + \frac{1}{16}bc \\
 &= -\frac{3bc}{4x} + \frac{1}{8}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{cx}}{\sqrt{3}}\right) - \frac{1}{8}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1 + 2\sqrt[3]{cx}}{\sqrt{3}}\right) + \frac{1}{4}bc^{4/3} \tanh^{-1}(\sqrt[3]{cx})
 \end{aligned}$$

**Mathematica [A]** time = 0.0516986, size = 196, normalized size = 1.13

$$-\frac{a}{4x^4} - \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 - \sqrt[3]{cx} + 1) + \frac{1}{16}bc^{4/3} \log(c^{2/3}x^2 + \sqrt[3]{cx} + 1) - \frac{1}{8}bc^{4/3} \log(1 - \sqrt[3]{cx}) + \frac{1}{8}bc^{4/3} \log(\sqrt[3]{cx} + 1) -$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])/x^5,x]

[Out] -a/(4\*x^4) - (3\*b\*c)/(4\*x) - (Sqrt[3]\*b\*c^(4/3)\*ArcTan[(-1 + 2\*c^(1/3)\*x)/Sqrt[3]])/8 - (Sqrt[3]\*b\*c^(4/3)\*ArcTan[(1 + 2\*c^(1/3)\*x)/Sqrt[3]])/8 - (b\*ArcTanh[c\*x^3])/(4\*x^4) - (b\*c^(4/3)\*Log[1 - c^(1/3)\*x])/8 + (b\*c^(4/3)\*Log[1 + c^(1/3)\*x])/8 - (b\*c^(4/3)\*Log[1 - c^(1/3)\*x + c^(2/3)\*x^2])/16 + (b\*c^(4/3)\*Log[1 + c^(1/3)\*x + c^(2/3)\*x^2])/16

**Maple [A]** time = 0.013, size = 172, normalized size = 1.

$$-\frac{a}{4x^4} - \frac{b \operatorname{Artanh}(cx^3)}{4x^4} - \frac{3bc}{4x} - \frac{bc}{8} \ln\left(x - \sqrt[3]{c^{-1}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{bc}{16} \ln\left(x^2 + \sqrt[3]{c^{-1}}x + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{bc\sqrt{3}}{8} \arctan\left(\frac{\sqrt{3}}{3}\left(2 - \sqrt[3]{c^{-1}}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^3))/x^5,x)`

[Out] 
$$-1/4*a/x^4 - 1/4*b/x^4*arctanh(c*x^3) - 3/4*b*c/x - 1/8*b*c/(1/c)^{(1/3)}*\ln(x - (1/c)^{(1/3)}) + 1/16*b*c/(1/c)^{(1/3)}*\ln(x^2 + (1/c)^{(1/3)}*x + (1/c)^{(2/3)}) - 1/8*b*c*3^{(1/2)}/(1/c)^{(1/3)}*arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x + 1)) + 1/8*b*c/(1/c)^{(1/3)}*\ln(x + (1/c)^{(1/3)}) - 1/16*b*c/(1/c)^{(1/3)}*\ln(x^2 - (1/c)^{(1/3)}*x + (1/c)^{(2/3)}) - 1/8*b*c*3^{(1/2)}/(1/c)^{(1/3)}*arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x - 1))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.67314, size = 547, normalized size = 3.14

$$2\sqrt{3}b(-c)^{\frac{1}{3}}cx^4\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{1}{3}}x - \frac{1}{3}\sqrt{3}\right) + 2\sqrt{3}bc^{\frac{4}{3}}x^4\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}x - \frac{1}{3}\sqrt{3}\right) + b(-c)^{\frac{1}{3}}cx^4\log\left(cx^2 + (-c)^{\frac{2}{3}}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="fricas")`

[Out] 
$$-1/16*(2*\sqrt{3}*b*(-c)^{(1/3)}*c*x^4*\arctan(2/3*\sqrt{3}*(-c)^{(1/3)}*x - 1/3*\sqrt{3}) + 2*\sqrt{3}*b*c^{(4/3)}*x^4*\arctan(2/3*\sqrt{3}*c^{(1/3)}*x - 1/3*\sqrt{3}) + b*(-c)^{(1/3)}*c*x^4*\log(c*x^2 + (-c)^{(2/3)}*x - (-c)^{(1/3)}) + b*c^{(4/3)}*x^4*\log(c*x^2 - c^{(2/3)}*x + c^{(1/3)}) - 2*b*(-c)^{(1/3)}*c*x^4*\log(c*x - (-c)^{(2/3)}) - 2*b*c^{(4/3)}*x^4*\log(c*x + c^{(2/3)}) + 12*b*c*x^3 + 2*b*\log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^4$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))/x\*\*5,x)

[Out] Exception raised: KeyError

**Giac [A]** time = 2.08691, size = 261, normalized size = 1.5

$$-\frac{1}{16}bc^3 \left( \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x + \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2} + \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x - \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2} - \frac{|c|^{\frac{1}{3}} \log\left(x^2 + \frac{x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))/x^5,x, algorithm="giac")

[Out]  $-1/16*b*c^3*(2*\sqrt{3}*abs(c)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + 1/abs(c)^{(1/3}))*abs(c)^{(1/3}))/c^2 + 2*\sqrt{3}*abs(c)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x - 1/abs(c)^{(1/3}))*abs(c)^{(1/3}))/c^2 - abs(c)^{(1/3)}*\log(x^2 + x/abs(c)^{(1/3)} + 1/abs(c)^{(2/3}))/c^2 + abs(c)^{(1/3)}*\log(x^2 - x/abs(c)^{(1/3)} + 1/abs(c)^{(2/3}))/c^2 - 2*abs(c)^{(1/3)}*\log(abs(x + 1/abs(c)^{(1/3}))/c^2 + 2*abs(c)^{(1/3)}*\log(abs(x - 1/abs(c)^{(1/3}))/c^2 - 1/8*b*\log(-(c*x^3 + 1)/(c*x^3 - 1))/x^4 - 1/4*(3*b*c*x^3 + a)/x^4$

### 3.116 $\int x^{11} (a + b \tanh^{-1}(cx^3))^2 dx$

**Optimal.** Leaf size=125

$$\frac{abx^3}{6c^3} - \frac{(a + b \tanh^{-1}(cx^3))^2}{12c^4} + \frac{1}{12}x^{12}(a + b \tanh^{-1}(cx^3))^2 + \frac{bx^9(a + b \tanh^{-1}(cx^3))}{18c} + \frac{b^2x^6}{36c^2} + \frac{b^2 \log(1 - c^2x^6)}{9c^4} + \frac{b^2 \log(1 + c^2x^6)}{9c^4}$$

[Out] (a\*b\*x^3)/(6\*c^3) + (b^2\*x^6)/(36\*c^2) + (b^2\*x^3\*ArcTanh[c\*x^3])/(6\*c^3) + (b\*x^9\*(a + b\*ArcTanh[c\*x^3]))/(18\*c) - (a + b\*ArcTanh[c\*x^3])^2/(12\*c^4) + (x^12\*(a + b\*ArcTanh[c\*x^3])^2)/12 + (b^2\*Log[1 - c^2\*x^6])/(9\*c^4) + (b^2\*Log[1 + c^2\*x^6])/(9\*c^4)

**Rubi [C]** time = 1.54585, antiderivative size = 636, normalized size of antiderivative = 5.09, number of steps used = 62, number of rules used = 19, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 1.187, Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right)}{24c^4} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)}{24c^4} + \frac{abx^3}{12c^3} - \frac{bx^6(2a - b \log(1 - cx^3))}{48c^2} - \frac{1}{288}b \left( -\frac{3(1 - cx^3)^4}{c^4} + \frac{3(1 + cx^3)^4}{c^4} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^11\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (a\*b\*x^3)/(12\*c^3) + (23\*b^2\*x^3)/(288\*c^3) + (b^2\*x^6)/(192\*c^2) - (7\*b^2\*x^9)/(864\*c) - (b^2\*x^12)/384 + (b^2\*(1 - c\*x^3)^2)/(16\*c^4) - (b^2\*(1 - c\*x^3)^3)/(54\*c^4) + (b^2\*(1 - c\*x^3)^4)/(384\*c^4) - (5\*b^2\*Log[1 - c\*x^3])/(288\*c^4) + (b^2\*(1 - c\*x^3)\*Log[1 - c\*x^3])/(24\*c^4) + (b^2\*Log[1 - c\*x^3]^2)/(48\*c^4) - (b\*x^6\*(2\*a - b\*Log[1 - c\*x^3]))/(48\*c^2) + (b\*x^9\*(2\*a - b\*Log[1 - c\*x^3]))/(72\*c) - (b\*x^12\*(2\*a - b\*Log[1 - c\*x^3]))/96 + (x^12\*(2\*a - b\*Log[1 - c\*x^3])^2)/48 - (b\*(2\*a - b\*Log[1 - c\*x^3])\*((48\*(1 - c\*x^3))/c^4 - (36\*(1 - c\*x^3)^2)/c^4 + (16\*(1 - c\*x^3)^3)/c^4 - (3\*(1 - c\*x^3)^4)/c^4 - (12\*Log[1 - c\*x^3])/c^4))/288 - (b\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(24\*c^4) + (b^2\*Log[1 + c\*x^3])/(36\*c^4) + (b^2\*x^9\*Log[1 + c\*x^3])/(36\*c) + (b^2\*(1 + c\*x^3)\*Log[1 + c\*x^3])/(12\*c^4) + (b^2\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/(24\*c^4) + (b\*x^12\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/24 - (b^2\*Log[1 + c\*x^3]^2)/(48\*c^4) + (b^2\*x^12\*Log[1 + c\*x^3]^2)/48 + (b^2\*PolyLog[2, (1 - c\*x^3)/2])/(24\*c^4) + (b^2\*PolyLog[2, (1 + c\*x^3)/2])/(24\*c^4)

**Rule 6099**



```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x]
/; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

#### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol]
:> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol]
:> Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x]
/; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x]
/; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2301

Int[((a\_.) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2395

Int[((a\_.) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 2439

Int[((a\_.) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_.)^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.)\*(x\_)^(r\_.)), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m])/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m])/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

Rule 2416

Int[((a\_.) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_.)^(p\_.)\*((h\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2389

Int[((a\_.) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_.)^(p\_.), x\_Symbol] :

```
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :=> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol] :=> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p)/((f_) + (g_.)*(x_))^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^{11} (a + b \tanh^{-1}(cx^3))^2 dx &= \int \left( \frac{1}{4} x^{11} (2a - b \log(1 - cx^3))^2 - \frac{1}{2} b x^{11} (-2a + b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{4} b^2 x^{11} \log^2(1 + cx^3) \right) dx \\
&= \frac{1}{4} \int x^{11} (2a - b \log(1 - cx^3))^2 dx - \frac{1}{2} b \int x^{11} (-2a + b \log(1 - cx^3)) \log(1 + cx^3) dx \\
&= \frac{1}{12} \text{Subst} \left( \int x^3 (2a - b \log(1 - cx))^2 dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int x^3 (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^3 \right) \\
&= \frac{1}{48} x^{12} (2a - b \log(1 - cx^3))^2 + \frac{1}{24} b x^{12} (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{48} b^2 x^{12} \log^2(1 + cx^3) \\
&= \frac{1}{48} x^{12} (2a - b \log(1 - cx^3))^2 + \frac{1}{24} b x^{12} (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{48} b^2 x^{12} \log^2(1 + cx^3) \\
&= \frac{1}{48} x^{12} (2a - b \log(1 - cx^3))^2 - \frac{1}{288} b (2a - b \log(1 - cx^3)) \left( \frac{48(1 - cx^3)}{c^4} - \frac{36(1 - cx^3)}{c^4} \right) \\
&= \frac{abx^3}{12c^3} - \frac{bx^6(2a - b \log(1 - cx^3))}{48c^2} + \frac{bx^9(2a - b \log(1 - cx^3))}{72c} - \frac{1}{96} bx^{12} (2a - b \log(1 - cx^3)) \\
&= \frac{abx^3}{12c^3} - \frac{bx^6(2a - b \log(1 - cx^3))}{48c^2} + \frac{bx^9(2a - b \log(1 - cx^3))}{72c} - \frac{1}{96} bx^{12} (2a - b \log(1 - cx^3)) \\
&= \frac{abx^3}{12c^3} + \frac{55b^2x^3}{288c^3} - \frac{5b^2x^6}{576c^2} + \frac{b^2x^9}{864c} - \frac{b^2x^{12}}{384} + \frac{b^2(1 - cx^3)^2}{16c^4} - \frac{b^2(1 - cx^3)^3}{54c^4} + \frac{b^2(1 - cx^3)^4}{384c^4} \\
&= \frac{abx^3}{12c^3} + \frac{55b^2x^3}{288c^3} - \frac{5b^2x^6}{576c^2} + \frac{b^2x^9}{864c} - \frac{b^2x^{12}}{384} + \frac{b^2(1 - cx^3)^2}{16c^4} - \frac{b^2(1 - cx^3)^3}{54c^4} + \frac{b^2(1 - cx^3)^4}{384c^4}
\end{aligned}$$

**Mathematica [A]** time = 0.0761196, size = 146, normalized size = 1.17

$$\frac{3a^2c^4x^{12} + 2abc^3x^9 + 2bcx^3 \tanh^{-1}(cx^3)(3ac^3x^9 + b(c^2x^6 + 3)) + 6abcx^3 + b(3a + 4b) \log(1 - cx^3) - 3ab \log(cx^3 + 1)}{36c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^11\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (6\*a\*b\*c\*x^3 + b^2\*c^2\*x^6 + 2\*a\*b\*c^3\*x^9 + 3\*a^2\*c^4\*x^12 + 2\*b\*c\*x^3\*(3\*a\*c^3\*x^9 + b\*(3 + c^2\*x^6))\*ArcTanh[c\*x^3] + 3\*b^2\*(-1 + c^4\*x^12)\*ArcTanh

$$\frac{[c*x^3]^2 + b*(3*a + 4*b)*\text{Log}[1 - c*x^3] - 3*a*b*\text{Log}[1 + c*x^3] + 4*b^2*\text{Log}[1 + c*x^3]}{(36*c^4)}$$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x^{11} (a + b \operatorname{Artanh}(cx^3))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11\*(a+b\*arctanh(c\*x^3))^2,x)

[Out] int(x^11\*(a+b\*arctanh(c\*x^3))^2,x)

**Maxima [A]** time = 0.985566, size = 293, normalized size = 2.34

$$\frac{1}{12} b^2 x^{12} \operatorname{artanh}(cx^3)^2 + \frac{1}{12} a^2 x^{12} + \frac{1}{36} \left( 6 x^{12} \operatorname{artanh}(cx^3) + c \left( \frac{2(c^2 x^9 + 3x^3)}{c^4} - \frac{3 \log(cx^3 + 1)}{c^5} + \frac{3 \log(cx^3 - 1)}{c^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11\*(a+b\*arctanh(c\*x^3))^2,x, algorithm="maxima")

[Out] 1/12\*b^2\*x^12\*arctanh(c\*x^3)^2 + 1/12\*a^2\*x^12 + 1/36\*(6\*x^12\*arctanh(c\*x^3) + c\*(2\*(c^2\*x^9 + 3\*x^3)/c^4 - 3\*log(c\*x^3 + 1)/c^5 + 3\*log(c\*x^3 - 1)/c^5))\*a\*b + 1/144\*(4\*c\*(2\*(c^2\*x^9 + 3\*x^3)/c^4 - 3\*log(c\*x^3 + 1)/c^5 + 3\*log(c\*x^3 - 1)/c^5)\*arctanh(c\*x^3) + (4\*c^2\*x^6 - 2\*(3\*log(c\*x^3 - 1) - 8)\*log(c\*x^3 + 1) + 3\*log(c\*x^3 + 1)^2 + 3\*log(c\*x^3 - 1)^2 + 16\*log(c\*x^3 - 1))/c^4)\*b^2

**Fricas [A]** time = 1.85454, size = 381, normalized size = 3.05

$$\frac{12 a^2 c^4 x^{12} + 8 a b c^3 x^9 + 4 b^2 c^2 x^6 + 24 a b c x^3 + 3 (b^2 c^4 x^{12} - b^2) \log\left(-\frac{c x^3 + 1}{c x^3 - 1}\right)^2 - 4 (3 a b - 4 b^2) \log(c x^3 + 1) + 4 (3 a b + 4 b^2) \log(c x^3 - 1)}{144 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/144*(12*a^2*c^4*x^12 + 8*a*b*c^3*x^9 + 4*b^2*c^2*x^6 + 24*a*b*c*x^3 + 3*(
b^2*c^4*x^12 - b^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(3*a*b - 4*b^2)*log
(c*x^3 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^3 - 1) + 4*(3*a*b*c^4*x^12 + b^2*c^
3*x^9 + 3*b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^4
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.28881, size = 236, normalized size = 1.89

$$\frac{1}{12}a^2x^{12} + \frac{abx^9}{18c} + \frac{b^2x^6}{36c^2} + \frac{1}{48}\left(b^2x^{12} - \frac{b^2}{c^4}\right)\log\left(\frac{-cx^3+1}{cx^3-1}\right)^2 + \frac{abx^3}{6c^3} + \frac{1}{36}\left(3abx^{12} + \frac{b^2x^9}{c} + \frac{3b^2x^3}{c^3}\right)\log\left(\frac{-cx^3+1}{cx^3-1}\right) - \left(\frac{1}{12}a^2x^{12} + \frac{abx^9}{18c} + \frac{b^2x^6}{36c^2} + \frac{1}{48}\left(b^2x^{12} - \frac{b^2}{c^4}\right)\log\left(\frac{-cx^3+1}{cx^3-1}\right)^2 + \frac{abx^3}{6c^3} + \frac{1}{36}\left(3abx^{12} + \frac{b^2x^9}{c} + \frac{3b^2x^3}{c^3}\right)\log\left(\frac{-cx^3+1}{cx^3-1}\right) - \frac{1}{36}\left(3ab - 4b^2\right)\log\left(\frac{cx^3+1}{cx^3-1}\right) + \frac{1}{36}\left(3ab + 4b^2\right)\log\left(\frac{cx^3-1}{cx^3+1}\right)\right)/c^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/12*a^2*x^12 + 1/18*a*b*x^9/c + 1/36*b^2*x^6/c^2 + 1/48*(b^2*x^12 - b^2/c^
4)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*a*b*x^3/c^3 + 1/36*(3*a*b*x^12 + b
^2*x^9/c + 3*b^2*x^3/c^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/36*(3*a*b - 4*b
^2)*log(c*x^3 + 1)/c^4 + 1/36*(3*a*b + 4*b^2)*log(c*x^3 - 1)/c^4
```

### 3.117 $\int x^8 (a + b \tanh^{-1}(cx^3))^2 dx$

**Optimal.** Leaf size=146

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right)}{9c^3} + \frac{(a + b \tanh^{-1}(cx^3))^2}{9c^3} - \frac{2b \log\left(\frac{2}{1-cx^3}\right)(a + b \tanh^{-1}(cx^3))}{9c^3} + \frac{1}{9}x^9 (a + b \tanh^{-1}(cx^3))^2$$

[Out] (b^2\*x^3)/(9\*c^2) - (b^2\*ArcTanh[c\*x^3])/(9\*c^3) + (b\*x^6\*(a + b\*ArcTanh[c\*x^3]))/(9\*c) + (a + b\*ArcTanh[c\*x^3])^2/(9\*c^3) + (x^9\*(a + b\*ArcTanh[c\*x^3])^2)/9 - (2\*b\*(a + b\*ArcTanh[c\*x^3])\*Log[2/(1 - c\*x^3)])/(9\*c^3) - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x^3)])/(9\*c^3)

**Rubi [B]** time = 1.30934, antiderivative size = 536, normalized size of antiderivative = 3.67, number of steps used = 53, number of rules used = 19, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.187$ , Rules used = {6099, 2454, 2398, 2411, 43, 2334, 12, 14, 2301, 2395, 2439, 2416, 2389, 2295, 2394, 2393, 2391, 2410, 2390}

$$-\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right)}{18c^3} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)}{18c^3} - \frac{abx^3}{9c^2} - \frac{1}{108}b \left( \frac{2(1 - cx^3)^3}{c^3} - \frac{9(1 - cx^3)^2}{c^3} + \frac{18(1 - cx^3)}{c^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^8\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] -(a\*b\*x^3)/(9\*c^2) + (19\*b^2\*x^3)/(108\*c^2) - (5\*b^2\*x^6)/(216\*c) - (b^2\*x^9)/162 + (b^2\*(1 - c\*x^3)^2)/(24\*c^3) - (b^2\*(1 - c\*x^3)^3)/(162\*c^3) + (b^2\*Log[1 - c\*x^3])/(108\*c^3) - (b^2\*(1 - c\*x^3)\*Log[1 - c\*x^3])/(18\*c^3) + (b^2\*Log[1 - c\*x^3]^2)/(36\*c^3) + (b\*x^6\*(2\*a - b\*Log[1 - c\*x^3]))/(36\*c) - (b\*x^9\*(2\*a - b\*Log[1 - c\*x^3]))/54 + (x^9\*(2\*a - b\*Log[1 - c\*x^3])^2)/36 - (b\*(2\*a - b\*Log[1 - c\*x^3])\*((18\*(1 - c\*x^3))/c^3 - (9\*(1 - c\*x^3)^2)/c^3 + (2\*(1 - c\*x^3)^3)/c^3 - (6\*Log[1 - c\*x^3])/c^3))/108 + (b\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(18\*c^3) - (b^2\*Log[1 + c\*x^3])/(18\*c^3) + (b^2\*x^6\*Log[1 + c\*x^3])/(18\*c) + (b^2\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/(18\*c^3) + (b\*x^9\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/18 + (b^2\*Log[1 + c\*x^3]^2)/(36\*c^3) + (b^2\*x^9\*Log[1 + c\*x^3]^2)/36 - (b^2\*PolyLog[2, (1 - c\*x^3)/2])/(18\*c^3) + (b^2\*PolyLog[2, (1 + c\*x^3)/2])/(18\*c^3)

**Rule 6099**

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Int[ExpandIntegrand[(d*x)^(m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

#### Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

#### Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol]
:= Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol]
:= Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol]
:= Int[ExpandIntegrand[(a + b*x)^(m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol]
:= With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```



Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^(2/(2\*b\*n)), x] /; FreeQ[{a, b, c, n}, x]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e^n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*(x\_)^(r\_.), x\_Symbol] := Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m])/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m])/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_)^(m\_.))\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :

```
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

### Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rubi steps

$$\begin{aligned}
\int x^8 (a + b \tanh^{-1}(cx^3))^2 dx &= \int \left( \frac{1}{4} x^8 (2a - b \log(1 - cx^3))^2 - \frac{1}{2} b x^8 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{4} b^2 x^8 \log^2(1 + cx^3) \right) dx \\
&= \frac{1}{4} \int x^8 (2a - b \log(1 - cx^3))^2 dx - \frac{1}{2} b \int x^8 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) dx + \frac{1}{4} \int b^2 x^8 \log^2(1 + cx^3) dx \\
&= \frac{1}{12} \text{Subst} \left( \int x^2 (2a - b \log(1 - cx))^2 dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int x^2 (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^3 \right) + \frac{1}{4} \int b^2 x^8 \log^2(1 + cx^3) dx \\
&= \frac{1}{36} x^9 (2a - b \log(1 - cx^3))^2 + \frac{1}{18} b x^9 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{36} b^2 x^9 \log^2(1 + cx^3) \\
&= \frac{1}{36} x^9 (2a - b \log(1 - cx^3))^2 + \frac{1}{18} b x^9 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{36} b^2 x^9 \log^2(1 + cx^3) \\
&= \frac{1}{36} x^9 (2a - b \log(1 - cx^3))^2 - \frac{1}{108} b (2a - b \log(1 - cx^3)) \left( \frac{18(1 - cx^3)}{c^3} - \frac{9(1 - cx^3)^2}{c^3} \right) \\
&= -\frac{abx^3}{9c^2} + \frac{bx^6(2a - b \log(1 - cx^3))}{36c} - \frac{1}{54} b x^9 (2a - b \log(1 - cx^3)) + \frac{1}{36} x^9 (2a - b \log(1 - cx^3))^2 \\
&= -\frac{abx^3}{9c^2} + \frac{bx^6(2a - b \log(1 - cx^3))}{36c} - \frac{1}{54} b x^9 (2a - b \log(1 - cx^3)) + \frac{1}{36} x^9 (2a - b \log(1 - cx^3))^2 \\
&= -\frac{abx^3}{9c^2} + \frac{13b^2x^3}{108c^2} + \frac{b^2x^6}{216c} - \frac{b^2x^9}{162} + \frac{b^2(1 - cx^3)^2}{24c^3} - \frac{b^2(1 - cx^3)^3}{162c^3} + \frac{b^2 \log(1 - cx^3)}{108c^3} \\
&= -\frac{abx^3}{9c^2} + \frac{13b^2x^3}{108c^2} + \frac{b^2x^6}{216c} - \frac{b^2x^9}{162} + \frac{b^2(1 - cx^3)^2}{24c^3} - \frac{b^2(1 - cx^3)^3}{162c^3} + \frac{b^2 \log(1 - cx^3)}{108c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.271671, size = 132, normalized size = 0.9

$$\frac{b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^3)}\right) + a^2 c^3 x^9 + abc^2 x^6 + ab \log(c^2 x^6 - 1) + b \tanh^{-1}(cx^3) \left(2ac^3 x^9 + bc^2 x^6 - 2b \log\left(e^{-2 \tanh^{-1}(cx^3)}\right)\right)}{9c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^8\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (b^2\*c\*x^3 + a\*b\*c^2\*x^6 + a^2\*c^3\*x^9 + b^2\*(-1 + c^3\*x^9)\*ArcTanh[c\*x^3]^2 + b\*ArcTanh[c\*x^3]\*(-b + b\*c^2\*x^6 + 2\*a\*c^3\*x^9 - 2\*b\*Log[1 + E^(-2\*ArcT

$\text{anh}[c*x^3])) + a*b*\text{Log}[-1 + c^2*x^6] + b^2*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x^3])})]/(9*c^3)$

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x^8 (a + b \text{Artanh}(cx^3))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(a+b*arctanh(c*x^3))^2,x)`

[Out] `int(x^8*(a+b*arctanh(c*x^3))^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{9} a^2 x^9 + \frac{1}{9} \left( 2x^9 \text{artanh}(cx^3) + \left( \frac{x^6}{c^2} + \frac{\log(c^2 x^6 - 1)}{c^4} \right) c \right) ab + \frac{1}{648} \left( 18x^9 \log(-cx^3 + 1)^2 - 2c^4 \left( \frac{2(c^2 x^9 + 3x^3)}{c^6} - \frac{3 \log(-cx^3 + 1)}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")`

[Out] `1/9*a^2*x^9 + 1/9*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*c)*a*b + 1/648*(18*x^9*log(-c*x^3 + 1)^2 - 2*c^4*(2*(c^2*x^9 + 3*x^3)/c^6 - 3*log(c*x^3 + 1)/c^7 + 3*log(c*x^3 - 1)/c^7) + 3*(x^6/c^4 + log(c^2*x^6 - 1)/c^6)*c^3 + 1944*c^3*integrate(1/9*x^11*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) - 9*c^2*(2*x^3/c^4 - log(c*x^3 + 1)/c^5 + log(c*x^3 - 1)/c^5) - 6*c*((2*c^2*x^9 + 3*c*x^6 + 6*x^3)/c^3 + 6*log(c*x^3 - 1)/c^4)*log(-c*x^3 + 1) + 972*c*integrate(1/9*x^5*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) + 6*(3*c^3*x^9*log(c*x^3 + 1)^2 + (2*c^3*x^9 - 3*c^2*x^6 + 6*c*x^3 - 6*(c^3*x^9 + 1))*log(c*x^3 + 1))*log(-c*x^3 + 1)/c^3 + (4*c^3*x^9 + 15*c^2*x^6 + 66*c*x^3 + 18*log(c*x^3 - 1)^2 + 66*log(c*x^3 - 1))/c^3 - 18*log(9*c^4*x^6 - 9*c^2)/c^3 + 972*integrate(1/9*x^2*log(c*x^3 + 1)/(c^4*x^6 - c^2), x))*b^2`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 x^8 \text{artanh}(cx^3)^2 + 2 ab x^8 \text{artanh}(cx^3) + a^2 x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^8*arctanh(c*x^3)^2 + 2*a*b*x^8*arctanh(c*x^3) + a^2*x^8, x)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: `KeyError`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*(a+b*atanh(c*x**3))**2,x)`

[Out] Exception raised: `KeyError`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^2 x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^3) + a)^2*x^8, x)`

### 3.118 $\int x^5 \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$

**Optimal.** Leaf size=91

$$-\frac{(a + b \tanh^{-1}(cx^3))^2}{6c^2} + \frac{abx^3}{3c} + \frac{1}{6}x^6(a + b \tanh^{-1}(cx^3))^2 + \frac{b^2 \log(1 - c^2x^6)}{6c^2} + \frac{b^2x^3 \tanh^{-1}(cx^3)}{3c}$$

[Out] (a\*b\*x^3)/(3\*c) + (b^2\*x^3\*ArcTanh[c\*x^3])/(3\*c) - (a + b\*ArcTanh[c\*x^3])^2/(6\*c^2) + (x^6\*(a + b\*ArcTanh[c\*x^3])^2)/6 + (b^2\*Log[1 - c^2\*x^6])/(6\*c^2)

**Rubi [C]** time = 0.985499, antiderivative size = 524, normalized size of antiderivative = 5.76, number of steps used = 44, number of rules used = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 2439, 2416, 2394, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right)}{12c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)}{12c^2} + \frac{(1 - cx^3)^2 (2a - b \log(1 - cx^3))^2}{24c^2} - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))}{12c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (a\*b\*x^3)/(2\*c) - (b^2\*x^6)/24 + (b^2\*(1 - c\*x^3)^2)/(48\*c^2) + (b^2\*(1 + c\*x^3)^2)/(48\*c^2) - (b^2\*Log[1 - c\*x^3])/(24\*c^2) + (b^2\*(1 - c\*x^3)\*Log[1 - c\*x^3])/(4\*c^2) - (b\*x^6\*(2\*a - b\*Log[1 - c\*x^3]))/24 + (b\*(1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3]))/(24\*c^2) - ((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(12\*c^2) + ((1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3])^2)/(24\*c^2) - (b\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(12\*c^2) - (b^2\*Log[1 + c\*x^3])/(24\*c^2) + (b^2\*x^6\*Log[1 + c\*x^3])/24 + (b^2\*(1 + c\*x^3)\*Log[1 + c\*x^3])/(4\*c^2) - (b^2\*(1 + c\*x^3)^2\*Log[1 + c\*x^3])/(24\*c^2) + (b^2\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/(12\*c^2) + (b\*x^6\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/12 - (b^2\*(1 + c\*x^3)\*Log[1 + c\*x^3]^2)/(12\*c^2) + (b^2\*(1 + c\*x^3)^2\*Log[1 + c\*x^3]^2)/(24\*c^2) + (b^2\*PolyLog[2, (1 - c\*x^3)/2])/(12\*c^2) + (b^2\*PolyLog[2, (1 + c\*x^3)/2])/(12\*c^2)

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte

gerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n

\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :=  
Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] :=  
Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2439

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((x\_)^(r\_.)), x\_Symbol] :=  
Simp[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]))/(r + 1), x] + (-Dist[(g\*j\*m)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[(b\*e\*n\*p)/(r + 1), Int[(x^(r + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m]))/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((h\_.)\*(x\_)^(m\_.))\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] :=  
Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2394



```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \tanh^{-1}(cx^3))^2 dx &= \int \left( \frac{1}{4} x^5 (2a - b \log(1 - cx^3))^2 - \frac{1}{2} b x^5 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{4} b^2 x^5 \log^2(1 + cx^3) \right) dx \\
&= \frac{1}{4} \int x^5 (2a - b \log(1 - cx^3))^2 dx - \frac{1}{2} b \int x^5 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) dx + \frac{1}{4} b^2 \int x^5 \log^2(1 + cx^3) dx \\
&= \frac{1}{12} \text{Subst} \left( \int x (2a - b \log(1 - cx))^2 dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int x (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^3 \right) + \frac{1}{4} b^2 \int x^5 \log^2(1 + cx^3) dx \\
&= \frac{1}{12} b x^6 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{12} \text{Subst} \left( \int \left( \frac{(2a - b \log(1 - cx))^2}{c} - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c^2} \right) dx, x, x^3 \right) \\
&= \frac{1}{12} b x^6 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{\text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, x^3 \right)}{12c} - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c^2} + \frac{(1 - cx^3)^2}{12c^2} \\
&= \frac{1}{12} b x^6 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{12} b \text{Subst} \left( \int x (-2a + b \log(1 - cx)) dx, x, x^3 \right) + \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c^2} + \frac{(1 - cx^3)^2}{12c^2} \\
&= \frac{abx^3}{6c} - \frac{1}{24} b x^6 (2a - b \log(1 - cx^3)) - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c^2} + \frac{(1 - cx^3)^2}{12c^2} \\
&= \frac{abx^3}{2c} - \frac{b^2 x^3}{4c} + \frac{b^2 (1 - cx^3)^2}{48c^2} + \frac{b^2 (1 + cx^3)^2}{48c^2} - \frac{1}{24} b x^6 (2a - b \log(1 - cx^3)) + \frac{b (1 - cx^3)^2}{12c^2} \\
&= \frac{abx^3}{2c} - \frac{b^2 x^6}{24} + \frac{b^2 (1 - cx^3)^2}{48c^2} + \frac{b^2 (1 + cx^3)^2}{48c^2} - \frac{b^2 \log(1 - cx^3)}{24c^2} + \frac{b^2 (1 - cx^3) \log(1 - cx^3)}{4c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0555193, size = 106, normalized size = 1.16

$$\frac{a^2c^2x^6 + 2abcx^3 + b(a+b)\log(1-cx^3) - ab\log(cx^3+1) + 2bcx^3 \tanh^{-1}(cx^3)(acx^3+b) + b^2(c^2x^6-1)\tanh^{-1}(cx^3)^2}{6c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (2\*a\*b\*c\*x^3 + a^2\*c^2\*x^6 + 2\*b\*c\*x^3\*(b + a\*c\*x^3)\*ArcTanh[c\*x^3] + b^2\*(-1 + c^2\*x^6)\*ArcTanh[c\*x^3]^2 + b\*(a + b)\*Log[1 - c\*x^3] - a\*b\*Log[1 + c\*x^3] + b^2\*Log[1 + c\*x^3])/(6\*c^2)

**Maple [B]** time = 0.18, size = 247, normalized size = 2.7

$$\frac{b^2(c^2x^6-1)(\ln(cx^3+1))^2}{24c^2} + \frac{b(-x^6b\ln(-cx^3+1)c^2 + 2ac^2x^6 + 2bcx^3 + b\ln(-cx^3+1))\ln(cx^3+1)}{12c^2} + \frac{b^2x^6(\ln(-cx^3+1))^2}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arctanh(c\*x^3))^2,x)

[Out] 1/24\*b^2\*(c^2\*x^6-1)/c^2\*ln(c\*x^3+1)^2+1/12\*b\*(-x^6\*b\*ln(-c\*x^3+1)\*c^2+2\*a\*c^2\*x^6+2\*b\*c\*x^3+b\*ln(-c\*x^3+1))/c^2\*ln(c\*x^3+1)+1/24\*b^2\*x^6\*ln(-c\*x^3+1)^2-1/6\*a\*b\*x^6\*ln(-c\*x^3+1)+1/6\*x^6\*a^2-1/6/c\*b^2\*x^3\*ln(-c\*x^3+1)+1/3\*a\*b\*x^3/c-1/24/c^2\*b^2\*ln(-c\*x^3+1)^2-1/6/c^2\*b\*ln(-c\*x^3-1)\*a+1/6/c^2\*b^2\*ln(-c\*x^3-1)+1/6/c^2\*b\*ln(-c\*x^3+1)\*a+1/6/c^2\*b^2\*ln(-c\*x^3+1)

**Maxima [B]** time = 1.12297, size = 251, normalized size = 2.76

$$\frac{1}{6}b^2x^6 \operatorname{artanh}(cx^3)^2 + \frac{1}{6}a^2x^6 + \frac{1}{6}\left(2x^6 \operatorname{artanh}(cx^3) + c\left(\frac{2x^3}{c^2} - \frac{\log(cx^3+1)}{c^3} + \frac{\log(cx^3-1)}{c^3}\right)\right)ab + \frac{1}{24}\left(4c\left(\frac{2x^3}{c^2} - \frac{\log(cx^3+1)}{c^3}\right) + \frac{\log(cx^3-1)}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^3))^2,x, algorithm="maxima")

[Out]  $\frac{1}{6}b^2x^6\operatorname{arctanh}(cx^3)^2 + \frac{1}{6}a^2x^6 + \frac{1}{6}(2x^6\operatorname{arctanh}(cx^3) + c(2x^3/c^2 - \log(cx^3 + 1)/c^3 + \log(cx^3 - 1)/c^3))ab + \frac{1}{24}(4c(2x^3/c^2 - \log(cx^3 + 1)/c^3 + \log(cx^3 - 1)/c^3)\operatorname{arctanh}(cx^3) - (2(\log(cx^3 - 1) - 2)\log(cx^3 + 1) - \log(cx^3 + 1)^2 - \log(cx^3 - 1)^2 - 4\log(cx^3 - 1))/c^2)b^2$

**Fricas [A]** time = 1.6891, size = 292, normalized size = 3.21

$$\frac{4a^2c^2x^6 + 8abcx^3 + (b^2c^2x^6 - b^2)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2 - 4(ab - b^2)\log(cx^3 + 1) + 4(ab + b^2)\log(cx^3 - 1) + 4(abc^2x^6 + b^2c^2x^3)}{24c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{24}(4a^2c^2x^6 + 8a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*\log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(a*b - b^2)*\log(c*x^3 + 1) + 4*(a*b + b^2)*\log(c*x^3 - 1) + 4*(a*b*c^2*x^6 + b^2*c*x^3)*\log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^2$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*atanh(c*x**3))**2,x)`

[Out] Exception raised: KeyError

**Giac [A]** time = 1.19832, size = 186, normalized size = 2.04

$$\frac{1}{6}a^2x^6 + \frac{abx^3}{3c} + \frac{1}{24}\left(b^2x^6 - \frac{b^2}{c^2}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2 + \frac{1}{6}\left(abx^6 + \frac{b^2x^3}{c}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right) - \frac{(ab - b^2)\log(cx^3 + 1)}{6c^2} + \frac{(ab + b^2)\log(cx^3 - 1)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*a^2*x^6 + 1/3*a*b*x^3/c + 1/24*(b^2*x^6 - b^2/c^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*(a*b*x^6 + b^2*x^3/c)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/6*(a*b - b^2)*log(c*x^3 + 1)/c^2 + 1/6*(a*b + b^2)*log(c*x^3 - 1)/c^2
```

### 3.119 $\int x^2 \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$

**Optimal.** Leaf size=96

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right)}{3c} + \frac{1}{3}x^3 \left( a + b \tanh^{-1}(cx^3) \right)^2 + \frac{\left( a + b \tanh^{-1}(cx^3) \right)^2}{3c} - \frac{2b \log\left(\frac{2}{1-cx^3}\right) \left( a + b \tanh^{-1}(cx^3) \right)}{3c}$$

[Out] (a + b\*ArcTanh[c\*x^3])^2/(3\*c) + (x^3\*(a + b\*ArcTanh[c\*x^3])^2)/3 - (2\*b\*(a + b\*ArcTanh[c\*x^3])\*Log[2/(1 - c\*x^3)])/(3\*c) - (b^2\*PolyLog[2, 1 - 2/(1 - c\*x^3)])/(3\*c)

**Rubi [B]** time = 0.591888, antiderivative size = 207, normalized size of antiderivative = 2.16, number of steps used = 28, number of rules used = 12, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$-\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right)}{6c} + \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)}{6c} + \frac{1}{6}bx^3 \log(cx^3 + 1) (2a - b \log(1 - cx^3)) - \frac{(1 - cx^3)(2a}{6c}$$

Warning: Unable to verify antiderivative.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] -((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(12\*c) + (b\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(6\*c) + (b^2\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/(6\*c) + (b\*x^3\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/6 + (b^2\*(1 + c\*x^3)\*Log[1 + c\*x^3]^2)/(12\*c) - (b^2\*PolyLog[2, (1 - c\*x^3)/2])/(6\*c) + (b^2\*PolyLog[2, (1 + c\*x^3)/2])/(6\*c)

#### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)])\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x^p)])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},

x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

### Rule 2295

Int[Log[(c\_.)\*(x\_.)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 6715

Int[(u\_.)\*(x\_.)^(m\_.), x\_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_.))^(m\_.)]\*(g\_.)), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m))]/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx^3))^2 dx &= \int \left( \frac{1}{4} x^2 (2a - b \log(1 - cx^3))^2 - \frac{1}{2} bx^2 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) + \frac{1}{4} b^2 x^2 \log^2(1 + cx^3) \right) dx \\
&= \frac{1}{4} \int x^2 (2a - b \log(1 - cx^3))^2 dx - \frac{1}{2} b \int x^2 (-2a + b \log(1 - cx^3)) \log(1 + cx^3) dx + \frac{1}{4} \int b^2 x^2 \log^2(1 + cx^3) dx \\
&= \frac{1}{12} \text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int (-2a + b \log(1 - cx)) \log(1 + cx) dx, x, x^3 \right) + \frac{1}{4} \int b^2 x^2 \log^2(1 + cx^3) dx \\
&= \frac{1}{6} bx^3 (2a - b \log(1 - cx^3)) \log(1 + cx^3) - \frac{\text{Subst} \left( \int (2a - b \log(x))^2 dx, x, 1 - cx^3 \right)}{12c} + \frac{b^2}{4} \int x^2 \log^2(1 + cx^3) dx \\
&= -\frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c} + \frac{1}{6} bx^3 (2a - b \log(1 - cx^3)) \log(1 + cx^3) + \frac{b^2}{4} \int x^2 \log^2(1 + cx^3) dx \\
&= \frac{1}{3} abx^3 + \frac{b^2 x^3}{6} - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c} - \frac{b^2 (1 + cx^3) \log(1 + cx^3)}{6c} + \frac{1}{6} bx^3 (2a - b \log(1 - cx^3)) \log(1 + cx^3) \\
&= \frac{b^2 x^3}{3} + \frac{b^2 (1 - cx^3) \log(1 - cx^3)}{6c} - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c} + \frac{b (2a - b \log(1 - cx^3)) \log(1 + cx^3)}{6c} \\
&= \frac{b^2 x^3}{6} + \frac{b^2 (1 - cx^3) \log(1 - cx^3)}{6c} - \frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c} + \frac{b (2a - b \log(1 - cx^3)) \log(1 + cx^3)}{6c} \\
&= -\frac{(1 - cx^3) (2a - b \log(1 - cx^3))^2}{12c} + \frac{b (2a - b \log(1 - cx^3)) \log\left(\frac{1}{2}(1 + cx^3)\right)}{6c} + \frac{b^2 \log^2(1 + cx^3)}{4c}
\end{aligned}$$

**Mathematica [A]** time = 0.152375, size = 99, normalized size = 1.03

$$\frac{b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^3)}\right) + a(acx^3 + b \log(1 - c^2 x^6)) + 2b \tanh^{-1}(cx^3) \left(acx^3 - b \log\left(e^{-2 \tanh^{-1}(cx^3)} + 1\right)\right) + b^2 (cx^3 \log(1 + cx^3) - \log(1 - cx^3))}{3c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^3])^2,x]

[Out] (b^2\*(-1 + c\*x^3)\*ArcTanh[c\*x^3]^2 + 2\*b\*ArcTanh[c\*x^3]\*(a\*c\*x^3 - b\*Log[1 + E^(-2\*ArcTanh[c\*x^3])]) + a\*(a\*c\*x^3 + b\*Log[1 - c^2\*x^6]) + b^2\*PolyLog[2, -E^(-2\*ArcTanh[c\*x^3])])/(3\*c)



**Maple [A]** time = 0.004, size = 145, normalized size = 1.5

$$\frac{(\operatorname{Artanh}(cx^3))^2 x^3 b^2}{3} + \frac{2 abx^3 \operatorname{Artanh}(cx^3)}{3} + \frac{x^3 a^2}{3} - \frac{2 \operatorname{Artanh}(cx^3) b^2}{3c} \ln\left(\frac{(cx^3+1)^2}{-c^2 x^6+1} + 1\right) + \frac{b^2 (\operatorname{Artanh}(cx^3))^2}{3c} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^3))^2,x)`

[Out] `1/3*arctanh(c*x^3)^2*x^3*b^2+2/3*a*b*x^3*arctanh(c*x^3)+1/3*x^3*a^2-2/3/c*a  
rctanh(c*x^3)*ln((c*x^3+1)^2/(-c^2*x^6+1)+1)*b^2+1/3/c*b^2*arctanh(c*x^3)^2  
+1/3/c*a*b*ln(-c^2*x^6+1)-1/3/c*polylog(2,-(c*x^3+1)^2/(-c^2*x^6+1))*b^2`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^2 x^3 + \frac{1}{12} \left( x^3 \log(-cx^3 + 1)^2 - c^2 \left( \frac{2x^3}{c^2} - \frac{\log(cx^3 + 1)}{c^3} + \frac{\log(cx^3 - 1)}{c^3} \right) \right) - 2 \left( \frac{x^3}{c} + \frac{\log(cx^3 - 1)}{c^2} \right) c \log(-cx^3 + 1) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")`

[Out] `1/3*a^2*x^3 + 1/12*(x^3*log(-c*x^3 + 1)^2 - c^2*(2*x^3/c^2 - log(c*x^3 + 1)  
/c^3 + log(c*x^3 - 1)/c^3) - 2*(x^3/c + log(c*x^3 - 1)/c^2)*c*log(-c*x^3 +  
1) + 18*c*integrate(x^5*log(c*x^3 + 1)/(c^2*x^6 - 1), x) + (c*x^3*log(c*x^3  
+ 1)^2 + 2*(c*x^3 - (c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c + (2*c*  
x^3 + log(c*x^3 - 1)^2 + 2*log(c*x^3 - 1))/c - log(c^2*x^6 - 1)/c + 6*integ  
rate(x^2*log(c*x^3 + 1)/(c^2*x^6 - 1), x))*b^2 + 1/3*(2*c*x^3*arctanh(c*x^3  
) + log(-c^2*x^6 + 1))*a*b/c`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2 x^2 \operatorname{artanh}(cx^3)^2 + 2 abx^2 \operatorname{artanh}(cx^3) + a^2 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^2*arctanh(c*x^3)^2 + 2*a*b*x^2*arctanh(c*x^3) + a^2*x^2, x)`

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: `KeyError`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c*x**3))**2,x)`

[Out] Exception raised: `KeyError`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^3) + a)^2*x^2, x)`

$$3.120 \quad \int \frac{(a + b \tanh^{-1}(cx^3))^2}{x} dx$$

**Optimal.** Leaf size=140

$$-\frac{1}{3}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^3}\right)(a + b \tanh^{-1}(cx^3)) + \frac{1}{3}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^3} - 1\right)(a + b \tanh^{-1}(cx^3)) + \frac{1}{6}b^2 \operatorname{PolyLog}$$

[Out] (2\*(a + b\*ArcTanh[c\*x^3])^2\*ArcTanh[1 - 2/(1 - c\*x^3)]/3 - (b\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, 1 - 2/(1 - c\*x^3)]/3 + (b\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, -1 + 2/(1 - c\*x^3)]/3 + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^3)]/6 - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^3)]/6

**Rubi [A]** time = 0.334869, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{1}{3}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^3}\right)(a + b \tanh^{-1}(cx^3)) + \frac{1}{3}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^3} - 1\right)(a + b \tanh^{-1}(cx^3)) + \frac{1}{6}b^2 \operatorname{PolyLog}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])^2/x, x]

[Out] (2\*(a + b\*ArcTanh[c\*x^3])^2\*ArcTanh[1 - 2/(1 - c\*x^3)]/3 - (b\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, 1 - 2/(1 - c\*x^3)]/3 + (b\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, -1 + 2/(1 - c\*x^3)]/3 + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^3)]/6 - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^3)]/6

### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

### Rule 5914

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcTanh[c\*x])^p\*ArcTanh[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[(a + b\*ArcTanh[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 - c\*x)]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^2}{x} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{3} (4bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \tanh^{-1}(cx)}{1 - c^2 x^2} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) + \frac{1}{3} (2bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \log \left( \frac{1 + cx}{1 - cx} \right)}{1 - c^2 x^2} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{3} b (a + b \tanh^{-1}(cx^3)) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^3} \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{3} b (a + b \tanh^{-1}(cx^3)) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^3} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.111432, size = 143, normalized size = 1.02

$$\frac{1}{6} \left( b \left( 2 \text{PolyLog} \left( 2, \frac{cx^3 + 1}{1 - cx^3} \right) (a + b \tanh^{-1}(cx^3)) - 2 \text{PolyLog} \left( 2, \frac{cx^3 + 1}{cx^3 - 1} \right) (a + b \tanh^{-1}(cx^3)) + b \left( \text{PolyLog} \left( 3, \frac{cx^3}{cx^3} \right) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^2/x,x]

[Out] (4\*(a + b\*ArcTanh[c\*x^3])^2\*ArcTanh[1 + 2/(-1 + c\*x^3)] + b\*(2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, (1 + c\*x^3)/(1 - c\*x^3)] - 2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, (1 + c\*x^3)/(-1 + c\*x^3)] + b\*(-PolyLog[3, (1 + c\*x^3)/(1 - c\*x^3)] + PolyLog[3, (1 + c\*x^3)/(-1 + c\*x^3)]))/6

**Maple [F]** time = 0.144, size = 0, normalized size = 0.

$$\int \frac{(a + b \text{Artanh}(cx^3))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))^2/x,x)

[Out] `int((a+b*arctanh(c*x^3))^2/x,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2(\log(cx^3 + 1) - \log(-cx^3 + 1))^2}{4x} + \frac{ab(\log(cx^3 + 1) - \log(-cx^3 + 1))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="maxima")`

[Out] `a^2*log(x) + integrate(1/4*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + a*b*(log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx^3)^2 + 2ab \operatorname{artanh}(cx^3) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c*x^3))^2 + 2*a*b*arctanh(c*x^3) + a^2)/x, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**3))**2/x,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x, x)
```

$$3.121 \quad \int \frac{(a + b \tanh^{-1}(cx^3))^2}{x^4} dx$$

**Optimal.** Leaf size=90

$$-\frac{1}{3}b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx^3+1} - 1\right) + \frac{1}{3}c(a + b \tanh^{-1}(cx^3))^2 - \frac{(a + b \tanh^{-1}(cx^3))^2}{3x^3} + \frac{2}{3}bc \log\left(2 - \frac{2}{cx^3+1}\right)(a + b \tanh^{-1}(cx^3))$$

[Out] (c\*(a + b\*ArcTanh[c\*x^3])^2)/3 - (a + b\*ArcTanh[c\*x^3])^2/(3\*x^3) + (2\*b\*c\*(a + b\*ArcTanh[c\*x^3])\*Log[2 - 2/(1 + c\*x^3)])/3 - (b^2\*c\*PolyLog[2, -1 + 2/(1 + c\*x^3)])/3

**Rubi [B]** time = 0.619188, antiderivative size = 237, normalized size of antiderivative = 2.63, number of steps used = 24, number of rules used = 13, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.812$ , Rules used = {6099, 2454, 2397, 2392, 2391, 2395, 36, 29, 31, 2439, 2416, 2394, 2393}

$$-\frac{1}{3}b^2c \operatorname{PolyLog}(2, -cx^3) + \frac{1}{3}b^2c \operatorname{PolyLog}(2, cx^3) + \frac{1}{6}b^2c \operatorname{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right) - \frac{1}{6}b^2c \operatorname{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right) - \frac{1}{6}b^2c \operatorname{PolyLog}(2, -cx^3)$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c\*x^3])^2/x^4, x]

[Out] 2\*a\*b\*c\*Log[x] - ((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(12\*x^3) - (b\*c\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/6 - (b^2\*c\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/6 - (b\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/(6\*x^3) - (b^2\*(1 + c\*x^3)\*Log[1 + c\*x^3]^2)/(12\*x^3) - (b^2\*c\*PolyLog[2, -(c\*x^3)])/3 + (b^2\*c\*PolyLog[2, c\*x^3])/3 + (b^2\*c\*PolyLog[2, (1 - c\*x^3)/2])/6 - (b^2\*c\*PolyLog[2, (1 + c\*x^3)/2])/6

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Lo



```
g[c*(d + e*x)^p]^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

### Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_))^2, x_Symbol] := Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f
- d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d +
e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] &&
NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

### Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] := Simp[
(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_)
^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^2}{x^4} dx &= \int \left( \frac{(2a - b \log(1 - cx^3))^2}{4x^4} - \frac{b(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{2x^4} + \frac{b^2 \log^2(1 + cx^3)}{4x^4} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - cx^3))^2}{x^4} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{x^4} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + cx^3)}{x^4} dx \\
&= \frac{1}{12} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^2} dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log(1 + cx)}{x^2} dx, x, x^3 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x^2} dx, x, x^3 \right) \\
&= -\frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{6x^3} - \frac{b^2(1 + cx^3)}{12x^3} \\
&= abc \log(x) - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{6x^3} - \frac{b^2(1 + cx^3)}{12x^3} \\
&= abc \log(x) - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{6x^3} - \frac{b^2(1 + cx^3)}{12x^3} \\
&= 2abc \log(x) - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{1}{6} bc(2a - b \log(1 - cx^3)) \log\left(\frac{1}{2}(1 + cx^3)\right) - \frac{b^2(1 + cx^3)}{12x^3} \\
&= 2abc \log(x) - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{1}{6} bc(2a - b \log(1 - cx^3)) \log\left(\frac{1}{2}(1 + cx^3)\right) - \frac{b^2(1 + cx^3)}{12x^3} \\
&= 2abc \log(x) - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12x^3} - \frac{1}{6} bc(2a - b \log(1 - cx^3)) \log\left(\frac{1}{2}(1 + cx^3)\right) - \frac{b^2(1 + cx^3)}{12x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.158228, size = 117, normalized size = 1.3

$$\frac{-b^2 cx^3 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx^3)}\right) - a(a + bcx^3 \log(1 - c^2 x^6) - 2bcx^3 \log(cx^3)) + 2b \tanh^{-1}(cx^3) \left( bcx^3 \log\left(1 - e^{-2 \tanh^{-1}(cx^3)}\right) \right)}{3x^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^2/x^4, x]

[Out] (b^2\*(-1 + c\*x^3)\*ArcTanh[c\*x^3]^2 + 2\*b\*ArcTanh[c\*x^3]\*(-a + b\*c\*x^3\*Log[1 - E^(-2\*ArcTanh[c\*x^3])]) - a\*(a - 2\*b\*c\*x^3\*Log[c\*x^3] + b\*c\*x^3\*Log[1 - c^2\*x^6]) - b^2\*c\*x^3\*PolyLog[2, E^(-2\*ArcTanh[c\*x^3])])/(3\*x^3)

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^3))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))^2/x^4,x)

[Out] int((a+b\*arctanh(c\*x^3))^2/x^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3} \left( c(\log(c^2x^6 - 1) - \log(x^6)) + \frac{2 \operatorname{artanh}(cx^3)}{x^3} \right) ab - \frac{1}{12} b^2 \left( \frac{\log(-cx^3 + 1)^2}{x^3} + 3 \int -\frac{(cx^3 - 1) \log(cx^3 + 1)^2 + 2(cx^3 - 1) \log(cx^3 + 1) \log(-cx^3 + 1)}{(cx^7 - x^4)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^2/x^4,x, algorithm="maxima")

[Out] -1/3\*(c\*(log(c^2\*x^6 - 1) - log(x^6)) + 2\*arctanh(c\*x^3)/x^3)\*a\*b - 1/12\*b^2\*(log(-c\*x^3 + 1)^2/x^3 + 3\*integrate(-((c\*x^3 - 1)\*log(c\*x^3 + 1)^2 + 2\*(c\*x^3 - (c\*x^3 - 1)\*log(c\*x^3 + 1))\*log(-c\*x^3 + 1))/(c\*x^7 - x^4), x)) - 1/3\*a^2/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^2 \operatorname{artanh}(cx^3)^2 + 2ab \operatorname{artanh}(cx^3) + a^2}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^2/x^4,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^3)^2 + 2\*a\*b\*arctanh(c\*x^3) + a^2)/x^4, x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))\*\*2/x\*\*4,x)

[Out] Exception raised: KeyError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^2/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^3) + a)^2/x^4, x)

$$3.122 \quad \int \frac{(a+b \tanh^{-1}(cx^3))^2}{x^7} dx$$

**Optimal.** Leaf size=88

$$\frac{1}{6}c^2(a+b \tanh^{-1}(cx^3))^2 - \frac{bc(a+b \tanh^{-1}(cx^3))}{3x^3} - \frac{(a+b \tanh^{-1}(cx^3))^2}{6x^6} - \frac{1}{6}b^2c^2 \log(1-c^2x^6) + b^2c^2 \log(x)$$

[Out]  $-(b*c*(a + b*ArcTanh[c*x^3]))/(3*x^3) + (c^2*(a + b*ArcTanh[c*x^3])^2)/6 - (a + b*ArcTanh[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^6])/6$

**Rubi [C]** time = 1.06043, antiderivative size = 360, normalized size of antiderivative = 4.09, number of steps used = 46, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2395, 44, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{12}b^2c^2 \text{PolyLog}\left(2, \frac{1}{2}(1-cx^3)\right) - \frac{1}{12}b^2c^2 \text{PolyLog}\left(2, \frac{1}{2}(cx^3+1)\right) + \frac{1}{12}bc^2 \log\left(\frac{1}{2}(cx^3+1)\right)(2a-b \log(1-cx^3)) + \dots$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c\*x^3])^2/x^7, x]

[Out]  $b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c*x^3])/12 - (b*c*(2*a - b*Log[1 - c*x^3]))/(12*x^3) - (b*c*(1 - c*x^3)*(2*a - b*Log[1 - c*x^3]))/(12*x^3) + (c^2*(2*a - b*Log[1 - c*x^3])^2)/24 - (2*a - b*Log[1 - c*x^3])^2/(24*x^6) + (b*c^2*(2*a - b*Log[1 - c*x^3])*Log[(1 + c*x^3)/2])/12 - (b^2*c^2*Log[1 + c*x^3])/6 - (b^2*c*Log[1 + c*x^3])/(6*x^3) - (b^2*c^2*Log[(1 - c*x^3)/2])*Log[1 + c*x^3]/12 - (b*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/(12*x^6) + (b^2*c^2*Log[1 + c*x^3]^2)/24 - (b^2*Log[1 + c*x^3]^2)/(24*x^6) - (b^2*c^2*PolyLog[2, (1 - c*x^3)/2])/12 - (b^2*c^2*PolyLog[2, (1 + c*x^3)/2])/12$

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol) :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.)^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2398

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(g*(q + 1)), x] - Dist[(b*e*n*p)/(g*(q + 1)), Int[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2*p, 2*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2347

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.))*((d_) + (e_.)*(x_)^(q_))/(x_), x_Symbol] := Dist[1/d, Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

Rule 2344

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Dist[1/d, Int[(a + b*Log[c*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b*Log[c*x^n])^p/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] := Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)*(x_)^(r_.), x_Symbol] := Simp[(x^
(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
```



[p, 1] || GtQ[r, 0] && NeQ[r, -1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 2392

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*d])\*Log[x], x] + Dist[b, Int[Log[1 + (e\*x)/d]/x, x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[c\*d, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^2}{x^7} dx &= \int \left( \frac{(2a - b \log(1 - cx^3))^2}{4x^7} - \frac{b(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{2x^7} + \frac{b^2 \log^2(1 + cx^3)}{4x^7} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - cx^3))^2}{x^7} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{x^7} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + cx^3)}{x^7} dx \\
&= \frac{1}{12} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^3} dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log(1 + cx)}{x^3} dx, x, x^3 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x^3} dx, x, x^3 \right) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{24x^6} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{12x^6} - \frac{b^2 \log^2(1 + cx^3)}{24x^6} + \frac{1}{12} b^2 \log(x) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{24x^6} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{12x^6} - \frac{b^2 \log^2(1 + cx^3)}{24x^6} - \frac{1}{12} b^2 \log(x) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{24x^6} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{12x^6} - \frac{b^2 \log^2(1 + cx^3)}{24x^6} - \frac{1}{12} b^2 \log(x) \\
&= -\frac{1}{2} abc^2 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{12x^3} - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))}{12x^3} - \frac{(2a - b \log(1 - cx^3))^2}{12x^3} \\
&= \frac{1}{4} b^2 c^2 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{12x^3} - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))}{12x^3} + \frac{1}{24} c^2 (2a - b \log(1 - cx^3))^2 \\
&= \frac{1}{2} b^2 c^2 \log(x) - \frac{1}{12} b^2 c^2 \log(1 - cx^3) - \frac{bc(2a - b \log(1 - cx^3))}{12x^3} - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))}{12x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0831599, size = 111, normalized size = 1.26

$$\frac{1}{6} \left( -\frac{a^2}{x^6} - bc^2(a+b) \log(1-cx^3) + bc^2(a-b) \log(cx^3+1) - \frac{2abc}{x^3} - \frac{2b \tanh^{-1}(cx^3)(a+bcx^3)}{x^6} + \frac{b^2(c^2x^6-1) \tanh^{-1}(cx^3)}{x^6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^2/x^7, x]

[Out]  $(-a^2/x^6) - (2*a*b*c)/x^3 - (2*b*(a + b*c*x^3)*ArcTanh[c*x^3])/x^6 + (b^2*(-1 + c^2*x^6)*ArcTanh[c*x^3]^2)/x^6 + 6*b^2*c^2*Log[x] - b*(a + b)*c^2*Log[1 - c*x^3] + (a - b)*b*c^2*Log[1 + c*x^3])/6$

**Maple [B]** time = 0.201, size = 257, normalized size = 2.9

$$\frac{b^2(c^2x^6-1)(\ln(cx^3+1))^2}{24x^6} - \frac{b(x^6b \ln(-cx^3+1)c^2 + 2bcx^3 - b \ln(-cx^3+1) + 2a) \ln(cx^3+1)}{12x^6} - \frac{-b^2c^2x^6(\ln(-cx^3+1))}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))^2/x^7, x)

[Out]  $1/24*b^2*(c^2*x^6-1)/x^6*\ln(cx^3+1)^2 - 1/12*b*(x^6*b*\ln(-cx^3+1)*c^2 + 2*b*c*x^3 - b*\ln(-cx^3+1) + 2*a)/x^6*\ln(cx^3+1) - 1/24*(-b^2*c^2*x^6*\ln(-cx^3+1)^2 + 4*b*c^2*\ln(cx^3-1)*x^6*a + 4*b^2*c^2*\ln(cx^3-1)*x^6 - 4*b*c^2*\ln(cx^3+1)*x^6*a + 4*b^2*c^2*\ln(cx^3+1)*x^6 - 24*b^2*c^2*\ln(x)*x^6 - 4*b^2*c*x^3*\ln(-cx^3+1) + 8*a*b*c*x^3 + b^2*\ln(-cx^3+1)^2 - 4*b*\ln(-cx^3+1)*a + 4*a^2)/x^6$

**Maxima [B]** time = 0.99658, size = 236, normalized size = 2.68

$$\frac{1}{6} \left( \left( c \log(cx^3+1) - c \log(cx^3-1) - \frac{2}{x^3} \right) c - \frac{2 \operatorname{artanh}(cx^3)}{x^6} \right) ab + \frac{1}{24} \left( \left( 2(\log(cx^3-1) - 2) \log(cx^3+1) - \log(cx^3+1) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^2/x^7, x, algorithm="maxima")

```
[Out] 1/6*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a*b + 1/24*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - 1
og(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1) -
c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*b^2 - 1/6*b^2*arctanh(c*x^3)^2
/x^6 - 1/6*a^2/x^6
```

**Fricas [A]** time = 1.77071, size = 324, normalized size = 3.68

$$\frac{24b^2c^2x^6 \log(x) + 4(ab - b^2)c^2x^6 \log(cx^3 + 1) - 4(ab + b^2)c^2x^6 \log(cx^3 - 1) - 8abcx^3 + (b^2c^2x^6 - b^2) \log\left(-\frac{cx^3+1}{cx^3-1}\right)^2}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^2*c^2*x^6*log(x) + 4*(a*b - b^2)*c^2*x^6*log(c*x^3 + 1) - 4*(a*b
+ b^2)*c^2*x^6*log(c*x^3 - 1) - 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*
x^3 + 1)/(c*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^3 + a*b)*log(-(c*x^3 + 1)/(c*x
^3 - 1)))/x^6
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**7,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^2}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^7, x)
```

$$3.123 \quad \int \frac{(a + b \tanh^{-1}(cx^3))^2}{x^{10}} dx$$

**Optimal.** Leaf size=144

$$-\frac{1}{9}b^2c^3\text{PolyLog}\left(2, \frac{2}{cx^3+1} - 1\right) + \frac{1}{9}c^3(a + b \tanh^{-1}(cx^3))^2 + \frac{2}{9}bc^3 \log\left(2 - \frac{2}{cx^3+1}\right)(a + b \tanh^{-1}(cx^3)) - \frac{bc(a + b \tanh^{-1}(cx^3))}{cx^3+1}$$

[Out]  $-(b^2c^2)/(9x^3) + (b^2c^3\text{ArcTanh}[cx^3])/9 - (bc(a + b\text{ArcTanh}[cx^3]))/(9x^6) + (c^3(a + b\text{ArcTanh}[cx^3])^2)/9 - (a + b\text{ArcTanh}[cx^3])^2/(9x^9) + (2bc^3(a + b\text{ArcTanh}[cx^3])\text{Log}[2 - 2/(1 + cx^3)])/9 - (b^2c^3\text{PolyLog}[2, -1 + 2/(1 + cx^3)])/9$

**Rubi [B]** time = 1.31132, antiderivative size = 420, normalized size of antiderivative = 2.92, number of steps used = 59, number of rules used = 24, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.5$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2395, 2439, 2416, 36, 29, 2392, 2391, 2394, 2393, 2410, 2390}

$$-\frac{1}{9}b^2c^3\text{PolyLog}(2, -cx^3) + \frac{1}{9}b^2c^3\text{PolyLog}(2, cx^3) + \frac{1}{18}b^2c^3\text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right) - \frac{1}{18}b^2c^3\text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c\*x^3])^2/x^10, x]

[Out]  $-(b^2c^2)/(9x^3) + (2ab^2c^3\text{Log}[x])/3 - (bc(2a - b\text{Log}[1 - cx^3]))/(18x^6) + (bc^2(2a - b\text{Log}[1 - cx^3]))/(18x^3) - (bc^2(1 - cx^3)(2a - b\text{Log}[1 - cx^3]))/(18x^3) + (c^3(2a - b\text{Log}[1 - cx^3])^2)/36 - (2a - b\text{Log}[1 - cx^3])^2/(36x^9) - (bc^3(2a - b\text{Log}[1 - cx^3])\text{Log}[(1 + cx^3)/2])/18 + (b^2c^3\text{Log}[1 + cx^3])/18 - (b^2c\text{Log}[1 + cx^3])/(18x^6) - (b^2c^3\text{Log}[(1 - cx^3)/2]\text{Log}[1 + cx^3])/18 - (b(2a - b\text{Log}[1 - cx^3])\text{Log}[1 + cx^3])/(18x^9) - (b^2c^3\text{Log}[1 + cx^3]^2)/36 - (b^2\text{Log}[1 + cx^3]^2)/(36x^9) - (b^2c^3\text{PolyLog}[2, -(cx^3)])/9 + (b^2c^3\text{PolyLog}[2, cx^3])/9 + (b^2c^3\text{PolyLog}[2, (1 - cx^3)/2])/18 - (b^2c^3\text{PolyLog}[2, (1 + cx^3)/2])/18$

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 -

$c*x^n)/2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

#### Rule 2454

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{GtQ}[(m+1)/n, 0] \parallel \text{IGtQ}[q, 0]) \&\& !(\text{EqQ}[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

#### Rule 2398

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_.)^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])^p/(g*(q+1)), x] - \text{Dist}[(b*e*n*p)/(g*(q+1)), \text{Int}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])^{(p-1)}]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[q, -1] \&\& \text{IntegerQ}[2*p, 2*q] \&\& (!\text{IGtQ}[q, 0] \parallel (\text{EqQ}[p, 2] \&\& \text{NeQ}[q, 1]))$

#### Rule 2411

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_.)^{(q_.)}*((h_.) + (i_.)*(x_.)^{(r_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x\} \&\& \text{EqQ}[e*f - d*g, 0] \&\& (\text{IGtQ}[p, 0] \parallel \text{IGtQ}[r, 0]) \&\& \text{IntegerQ}[2*r]$

#### Rule 2347

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.) + (e_.)*(x_.)^{(q_.)})/(x_.), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(d + e*x)^{(q+1)}*(a + b*\text{Log}[c*x^n])^p]/x, x], x] - \text{Dist}[e/d, \text{Int}[(d + e*x)^q*(a + b*\text{Log}[c*x^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1] \&\& \text{IntegerQ}[2*q]$

#### Rule 2344

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}/((x_.)*((d_.) + (e_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/x, x], x] - \text{Dist}[e/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[p, 0]$

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)\*(b\_.)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[ ((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_), x\_Symbol] := Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r\*(q + 1) + 1, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2319

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/(e\*(q + 1)), x] - Dist[(b\*n\*p)/(e\*(q + 1)), Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2395



```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/(g*(q + 1)), x] - Dist[(b*e^n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && NeQ[q, -1]
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*x^(r_.), x_Symbol] := Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p]/(i + j*x), x], x] - Dist[(b*e^n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

### Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol] := Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^2}{x^{10}} dx &= \int \left( \frac{(2a - b \log(1 - cx^3))^2}{4x^{10}} - \frac{b(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{2x^{10}} + \frac{b^2 \log^2(1 + cx^3)}{4x^{10}} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - cx^3))^2}{x^{10}} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - cx^3)) \log(1 + cx^3)}{x^{10}} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + cx^3)}{x^{10}} dx \\
&= \frac{1}{12} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^4} dx, x, x^3 \right) - \frac{1}{6} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx)) \log(1 + cx)}{x^4} dx, x, x^3 \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x^4} dx, x, x^3 \right) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{36x^9} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{18x^9} - \frac{b^2 \log^2(1 + cx^3)}{36x^9} + \frac{1}{18} \log(x) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{36x^9} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{18x^9} - \frac{b^2 \log^2(1 + cx^3)}{36x^9} - \frac{1}{18} \log(x) \\
&= -\frac{(2a - b \log(1 - cx^3))^2}{36x^9} - \frac{b(2a - b \log(1 - cx^3)) \log(1 + cx^3)}{18x^9} - \frac{b^2 \log^2(1 + cx^3)}{36x^9} - \frac{1}{18} \log(x) \\
&= \frac{1}{3} abc^3 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{18x^6} + \frac{bc^2(2a - b \log(1 - cx^3))}{18x^3} - \frac{(2a - b \log(1 - cx^3))^2}{36x^9} \\
&= \frac{1}{3} abc^3 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{18x^6} + \frac{bc^2(2a - b \log(1 - cx^3))}{18x^3} - \frac{bc^2(1 - cx^3)(2a - b \log(1 - cx^3))}{18x^9} \\
&= -\frac{b^2c^2}{18x^3} + \frac{2}{3} abc^3 \log(x) + \frac{1}{6} b^2c^3 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{18x^6} + \frac{bc^2(2a - b \log(1 - cx^3))}{18x^3} \\
&= -\frac{b^2c^2}{18x^3} + \frac{2}{3} abc^3 \log(x) + \frac{1}{6} b^2c^3 \log(x) - \frac{bc(2a - b \log(1 - cx^3))}{18x^6} + \frac{bc^2(2a - b \log(1 - cx^3))}{18x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.362807, size = 159, normalized size = 1.1

$$\frac{b^2c^3x^9 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx^3)}\right) + a^2 - 2abc^3x^9 \log(cx^3) + abc^3x^9 \log(1 - c^2x^6) + b \tanh^{-1}(cx^3) \left(2a - bc^3x^9 - 2bc^2x^6\right)}{9x^9}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^2/x^10, x]

[Out]  $-(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - c^3*x^9)*\text{ArcTanh}[c*x^3]^2 + b*\text{ArcTanh}[c*x^3]*(2*a + b*c*x^3 - b*c^3*x^9 - 2*b*c^3*x^9*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c*x^3])}]) - 2*a*b*c^3*x^9*\text{Log}[c*x^3] + a*b*c^3*x^9*\text{Log}[1 - c^2*x^6] + b^2*c^3*x^9*\text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c*x^3])}]))/(9*x^9)$

**Maple [F]** time = 0.181, size = 0, normalized size = 0.

$$\int \frac{(a + b \text{Artanh}(cx^3))^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^3))^2/x^10,x)`

[Out] `int((a+b*arctanh(c*x^3))^2/x^10,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{9} \left( \left( c^2 \log(c^2 x^6 - 1) - c^2 \log(x^6) + \frac{1}{x^6} \right) c + \frac{2 \operatorname{artanh}(cx^3)}{x^9} \right) ab - \frac{1}{36} b^2 \left( \frac{\log(-cx^3 + 1)^2}{x^9} + 9 \int -\frac{3(cx^3 - 1) \log(cx^3 + 1)}{x^9} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="maxima")`

[Out]  $-1/9*((c^2*\log(c^2*x^6 - 1) - c^2*\log(x^6) + 1/x^6)*c + 2*\text{arctanh}(c*x^3)/x^9)*a*b - 1/36*b^2*(\log(-c*x^3 + 1)^2/x^9 + 9*\text{integrate}(-1/3*(3*(c*x^3 - 1)*\log(c*x^3 + 1)^2 + 2*(c*x^3 - 3*(c*x^3 - 1)*\log(c*x^3 + 1))*\log(-c*x^3 + 1))/(c*x^13 - x^10), x) - 1/9*a^2/x^9$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 \operatorname{artanh}(cx^3)^2 + 2ab \operatorname{artanh}(cx^3) + a^2}{x^{10}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^10, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**2/x**10,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^2}{x^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2/x^10, x)
```

### 3.124 $\int x^8 \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$

**Optimal.** Leaf size=231

$$-\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right) (a + b \tanh^{-1}(cx^3))}{3c^3} + \frac{b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)}{6c^3} + \frac{ab^2 x^3}{3c^2} + \frac{(a + b \tanh^{-1}(cx^3))^3}{9c^3} - \frac{b(a + b \tanh^{-1}(cx^3))}{9c^3}$$

[Out] (a\*b^2\*x^3)/(3\*c^2) + (b^3\*x^3\*ArcTanh[c\*x^3])/(3\*c^2) - (b\*(a + b\*ArcTanh[c\*x^3])^2)/(6\*c^3) + (b\*x^6\*(a + b\*ArcTanh[c\*x^3])^2)/(6\*c) + (a + b\*ArcTanh[c\*x^3])^3/(9\*c^3) + (x^9\*(a + b\*ArcTanh[c\*x^3])^3)/9 - (b\*(a + b\*ArcTanh[c\*x^3])^2\*Log[2/(1 - c\*x^3)])/(3\*c^3) + (b^3\*Log[1 - c^2\*x^6])/(6\*c^3) - (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, 1 - 2/(1 - c\*x^3)])/(3\*c^3) + (b^3\*PolyLog[3, 1 - 2/(1 - c\*x^3)])/(6\*c^3)

**Rubi [B]** time = 6.35017, antiderivative size = 1421, normalized size of antiderivative = 6.15, number of steps used = 239, number of rules used = 32, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2398, 2411, 43, 2334, 12, 14, 2301, 2439, 2416, 2396, 2433, 2374, 6589, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2410, 2425}

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[x^8\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] (2\*a\*b^2\*x^3)/(3\*c^2) - (7\*b^3\*x^3)/(216\*c^2) - (23\*b^3\*x^6)/(432\*c) + (b^3\*x^9)/324 + (b^3\*(1 - c\*x^3)^2)/(48\*c^3) + (b^3\*(1 + c\*x^3)^2)/(24\*c^3) - (b^3\*(1 + c\*x^3)^3)/(324\*c^3) - (b^3\*Log[1 - c\*x^3])/(24\*c^3) + (b^3\*(1 - c\*x^3)\*Log[1 - c\*x^3])/(3\*c^3) - (b^3\*Log[1 - c\*x^3]^2)/(72\*c^3) - (b^2\*x^6\*(2\*a - b\*Log[1 - c\*x^3]))/(24\*c) + (b^2\*(1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3]))/(12\*c^3) - (b^2\*(1 - c\*x^3)^3\*(2\*a - b\*Log[1 - c\*x^3]))/(108\*c^3) - (b\*x^9\*(2\*a - b\*Log[1 - c\*x^3])^2)/72 - (b\*(1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(8\*c^3) + (b\*(1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3])^2)/(12\*c^3) - (b\*(1 - c\*x^3)^3\*(2\*a - b\*Log[1 - c\*x^3])^2)/(72\*c^3) - ((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^3)/(24\*c^3) + ((1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3])^3)/(24\*c^3) - ((1 - c\*x^3)^3\*(2\*a - b\*Log[1 - c\*x^3])^3)/(72\*c^3) + (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*(18\*(1 - c\*x^3))/c^3 - (9\*(1 - c\*x^3)^2)/c^3 + (2\*(1 - c\*x^3)^3)/c^3 - (6\*Log[1 - c\*x^3])/c^3)/216 - (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(12\*c^3) + (b\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[(1 + c\*x^3)/2])/(12\*c^3) - (7\*b^3\*Log[1 + c\*x^3])/(108\*c^3) + (b^3\*x^6\*Log[1 + c\*x^3])

```

)/(18*c) - (b^3*x^9*Log[1 + c*x^3])/108 + (11*b^3*(1 + c*x^3)*Log[1 + c*x^3
])/ (36*c^3) - (b^3*(1 + c*x^3)^2*Log[1 + c*x^3])/ (12*c^3) + (b^3*(1 + c*x^3
)^3*Log[1 + c*x^3])/ (108*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3])/ (12
*c^3) + (b^2*x^6*(2*a - b*Log[1 - c*x^3])*Log[1 + c*x^3])/ (12*c) - (b*(2*a
- b*Log[1 - c*x^3])^2*Log[1 + c*x^3])/ (24*c^3) + (b*x^9*(2*a - b*Log[1 - c*
x^3])^2*Log[1 + c*x^3])/24 + (b^3*Log[1 + c*x^3]^2)/(72*c^3) + (b^3*x^9*Log
[1 + c*x^3]^2)/72 - (b^3*(1 + c*x^3)*Log[1 + c*x^3]^2)/(8*c^3) + (b^3*(1 +
c*x^3)^2*Log[1 + c*x^3]^2)/(12*c^3) - (b^3*(1 + c*x^3)^3*Log[1 + c*x^3]^2)/
(72*c^3) + (b^3*Log[(1 - c*x^3)/2]*Log[1 + c*x^3]^2)/(12*c^3) + (b^2*(2*a -
b*Log[1 - c*x^3])*Log[1 + c*x^3]^2)/(24*c^3) + (b^2*x^9*(2*a - b*Log[1 - c
*x^3])*Log[1 + c*x^3]^2)/24 + (b^3*(1 + c*x^3)*Log[1 + c*x^3]^3)/(24*c^3) -
(b^3*(1 + c*x^3)^2*Log[1 + c*x^3]^3)/(24*c^3) + (b^3*(1 + c*x^3)^3*Log[1 +
c*x^3]^3)/(72*c^3) + (b^3*PolyLog[2, (1 - c*x^3)/2])/ (12*c^3) - (b^2*(2*a
- b*Log[1 - c*x^3])*PolyLog[2, (1 - c*x^3)/2])/ (6*c^3) + (b^3*PolyLog[2, (1
+ c*x^3)/2])/ (12*c^3) + (b^3*Log[1 + c*x^3]*PolyLog[2, (1 + c*x^3)/2])/ (6*
c^3) - (b^3*PolyLog[3, (1 - c*x^3)/2])/ (6*c^3) - (b^3*PolyLog[3, (1 + c*x^3
)/2])/ (6*c^3)

```

### Rule 6099

```

Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Sy
mbol] :> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 -
c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte
gerQ[m] && IntegerQ[n]

```

### Rule 2454

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
!(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

```

### Rule 2401

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.)*(f_.) + (g_.
)*(x_)^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d
+ e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f -
d*g, 0] && IGtQ[q, 0]

```

### Rule 2389

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a

```

, b, c, d, e, n, p}, x]

### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

### Rule 2411



```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)]^(n_.)]*(b_.)*(x_)]^(m_.)*((d_) + (e_.)*(x_)]^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_)]^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)]^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*(x_)]^(r_.), x_Symbol] := Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p/(i + j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
```

, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ[p, 1] || GtQ[r, 0]) && NeQ[r, -1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)), x_Symbol] :> Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m))]/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.)]^(r_.))*((a_.) + Log[(c_.)*(x_))^(n_
.))*((b_.))^(p_.)/(x_), x_Symbol] :> Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
:= Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
  Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b,
  c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2410

```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symbol]
:= Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ[
  {c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

### Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.)))/
(x_), x_Symbol] := Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n])/(2*m), x] -
  Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int x^8 (a + b \tanh^{-1}(cx^3))^3 dx &= \int \left( \frac{1}{8} x^8 (2a - b \log(1 - cx^3))^3 + \frac{3}{8} b x^8 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) - \frac{3}{8} b^2 x^8 \right. \\
&= \frac{1}{8} \int x^8 (2a - b \log(1 - cx^3))^3 dx + \frac{1}{8} (3b) \int x^8 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) \\
&= \frac{1}{24} \text{Subst} \left( \int x^2 (2a - b \log(1 - cx))^3 dx, x, x^3 \right) + \frac{1}{8} b \text{Subst} \left( \int x^2 (-2a + b \log(1 - cx)) \right. \\
&= \frac{1}{24} b x^9 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{24} b^2 x^9 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= \frac{1}{24} b x^9 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{24} b^2 x^9 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= \frac{1}{24} b x^9 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{24} b^2 x^9 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= -\frac{1}{72} b x^9 (2a - b \log(1 - cx^3))^2 - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c^3} + \frac{(1 - cx^3)^2 (2a - b \log(1 - cx^3))}{24c^3} \\
&= -\frac{1}{72} b x^9 (2a - b \log(1 - cx^3))^2 - \frac{b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{12c^3} + \frac{b(1 - cx^3)^2 (2a - b \log(1 - cx^3))}{12c^3} \\
&= \frac{ab^2 x^3}{3c^2} - \frac{b^3 x^3}{6c^2} + \frac{b^3 (1 - cx^3)^2}{32c^3} - \frac{b^3 (1 - cx^3)^3}{324c^3} + \frac{b^3 (1 + cx^3)^2}{32c^3} - \frac{b^3 (1 + cx^3)^3}{324c^3} + \frac{b^2 (1 - cx^3) \log(1 + cx^3)}{6c^3} \\
&= \frac{ab^2 x^3}{3c^2} + \frac{b^3 (1 - cx^3)^2}{32c^3} - \frac{b^3 (1 - cx^3)^3}{324c^3} + \frac{b^3 (1 + cx^3)^2}{32c^3} - \frac{b^3 (1 + cx^3)^3}{324c^3} + \frac{b^3 (1 - cx^3) \log(1 + cx^3)}{6c^3} \\
&= \frac{ab^2 x^3}{2c^2} - \frac{b^3 x^3}{18c^2} + \frac{b^3 (1 - cx^3)^2}{24c^3} - \frac{b^3 (1 - cx^3)^3}{324c^3} + \frac{b^3 (1 + cx^3)^2}{24c^3} - \frac{b^3 (1 + cx^3)^3}{324c^3} + \frac{b^3 (1 - cx^3) \log(1 + cx^3)}{6c^3} \\
&= \frac{ab^2 x^3}{2c^2} - \frac{7b^3 x^3}{216c^2} - \frac{5b^3 x^6}{432c} + \frac{b^3 x^9}{324} + \frac{b^3 (1 - cx^3)^2}{48c^3} + \frac{b^3 (1 + cx^3)^2}{24c^3} - \frac{b^3 (1 + cx^3)^3}{324c^3} + \frac{b^3 (1 - cx^3) \log(1 + cx^3)}{6c^3} \\
&= \frac{ab^2 x^3}{2c^2} - \frac{7b^3 x^3}{216c^2} - \frac{5b^3 x^6}{432c} + \frac{b^3 x^9}{324} + \frac{b^3 (1 - cx^3)^2}{48c^3} + \frac{b^3 (1 + cx^3)^2}{24c^3} - \frac{b^3 (1 + cx^3)^3}{324c^3} + \frac{b^3 (1 - cx^3) \log(1 + cx^3)}{6c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.470237, size = 334, normalized size = 1.45

$$6b^2 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^3)}\right) (a + b \tanh^{-1}(cx^3)) + 3b^3 \text{PolyLog}\left(3, -e^{-2 \tanh^{-1}(cx^3)}\right) + 3a^2 bc^2 x^6 + 3a^2 b \log(1 - c^2 x^6)$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[x^8\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] (6\*a\*b^2\*c\*x^3 + 3\*a^2\*b\*c^2\*x^6 + 2\*a^3\*c^3\*x^9 - 6\*a\*b^2\*ArcTanh[c\*x^3] + 6\*b^3\*c\*x^3\*ArcTanh[c\*x^3] + 6\*a\*b^2\*c^2\*x^6\*ArcTanh[c\*x^3] + 6\*a^2\*b\*c^3\*x^9\*ArcTanh[c\*x^3] - 6\*a\*b^2\*ArcTanh[c\*x^3]^2 - 3\*b^3\*ArcTanh[c\*x^3]^2 + 3\*b^3\*c^2\*x^6\*ArcTanh[c\*x^3]^2 + 6\*a\*b^2\*c^3\*x^9\*ArcTanh[c\*x^3]^2 - 2\*b^3\*ArcTanh[c\*x^3]^3 + 2\*b^3\*c^3\*x^9\*ArcTanh[c\*x^3]^3 - 12\*a\*b^2\*ArcTanh[c\*x^3]\*Log[1 + E^(-2\*ArcTanh[c\*x^3])] - 6\*b^3\*ArcTanh[c\*x^3]^2\*Log[1 + E^(-2\*ArcTanh[c\*x^3])] + 3\*a^2\*b\*Log[1 - c^2\*x^6] + 3\*b^3\*Log[1 - c^2\*x^6] + 6\*b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, -E^(-2\*ArcTanh[c\*x^3])] + 3\*b^3\*PolyLog[3, -E^(-2\*ArcTanh[c\*x^3])])/(18\*c^3)

---

**Maple [F]** time = 0.217, size = 0, normalized size = 0.

$$\int x^8 (a + b \text{Artanh}(cx^3))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8\*(a+b\*arctanh(c\*x^3))^3,x)

[Out] int(x^8\*(a+b\*arctanh(c\*x^3))^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{9} a^3 x^9 + \frac{1}{6} \left( 2x^9 \text{artanh}(cx^3) + \left( \frac{x^6}{c^2} + \frac{\log(c^2 x^6 - 1)}{c^4} \right) c \right) a^2 b - \frac{(b^3 c^3 x^9 - b^3) \log(-cx^3 + 1)^3 - 3(2ab^2 c^3 x^9 + b^3 c^2 x^6 + b^3 c^3 x^9)}{72 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="maxima")

```
[Out] 1/9*a^3*x^9 + 1/6*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*a^2*b - 1/72*((b^3*c^3*x^9 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c^3*x^9
+ b^3*c^2*x^6 + (b^3*c^3*x^9 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c^3
- integrate(-1/8*((b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^3 + 6*(a*b^2
*c^3*x^11 - a*b^2*c^2*x^8)*log(c*x^3 + 1)^2 - (4*a*b^2*c^3*x^11 + 2*b^3*c^2
*x^8 + 3*(b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^2 - 2*(6*a*b^2*c^2*x^8
- (6*a*b^2*c^3 + b^3*c^3)*x^11 - b^3*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))
/(c^3*x^3 - c^2), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^8 \operatorname{artanh}(cx^3)^3 + 3ab^2x^8 \operatorname{artanh}(cx^3)^2 + 3a^2bx^8 \operatorname{artanh}(cx^3) + a^3x^8, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctanh(c*x^3)^3 + 3*a*b^2*x^8*arctanh(c*x^3)^2 + 3*a^2*b*
x^8*arctanh(c*x^3) + a^3*x^8, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^3 x^8 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^8, x)
```



### 3.125 $\int x^5 \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$

**Optimal.** Leaf size=139

$$\frac{b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right)}{2c^2} - \frac{b^2 \log\left(\frac{2}{1-cx^3}\right) \left(a + b \tanh^{-1}(cx^3)\right)}{c^2} - \frac{\left(a + b \tanh^{-1}(cx^3)\right)^3}{6c^2} + \frac{b \left(a + b \tanh^{-1}(cx^3)\right)^2}{2c^2} + \dots$$

[Out] (b\*(a + b\*ArcTanh[c\*x^3])^2)/(2\*c^2) + (b\*x^3\*(a + b\*ArcTanh[c\*x^3])^2)/(2\*c) - (a + b\*ArcTanh[c\*x^3])^3/(6\*c^2) + (x^6\*(a + b\*ArcTanh[c\*x^3])^3)/6 - (b^2\*(a + b\*ArcTanh[c\*x^3])\*Log[2/(1 - c\*x^3)])/c^2 - (b^3\*PolyLog[2, 1 - 2/(1 - c\*x^3)])/(2\*c^2)

**Rubi [B]** time = 4.20449, antiderivative size = 479, normalized size of antiderivative = 3.45, number of steps used = 155, number of rules used = 30, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.875$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2439, 2416, 2396, 2433, 2374, 6589, 2411, 43, 2334, 12, 14, 2301, 6742, 2430, 2394, 2393, 2391, 2395, 2375, 2317, 2425}

$$\frac{b^3 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right)}{4c^2} + \frac{b^3 \text{PolyLog}\left(2, \frac{1}{2}(cx^3 + 1)\right)}{4c^2} - \frac{b^2 \log^2(cx^3 + 1) (2a - b \log(1 - cx^3))}{16c^2} + \frac{b^2 \log\left(\frac{1}{2}(cx^3 + 1)\right) (2a - b \log(1 - cx^3))}{16c^2} + \dots$$

Warning: Unable to verify antiderivative.

[In] Int[x^5\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] -(b\*(1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(8\*c^2) - ((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^3)/(24\*c^2) + ((1 - c\*x^3)^2\*(2\*a - b\*Log[1 - c\*x^3])^3)/(48\*c^2) + (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*Log[(1 + c\*x^3)/2])/(4\*c^2) + (b^3\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3])/(4\*c^2) + (b^2\*x^3\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3])/(4\*c) - (b\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[1 + c\*x^3])/(16\*c^2) + (b\*x^6\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[1 + c\*x^3])/16 + (b^3\*(1 + c\*x^3)\*Log[1 + c\*x^3]^2)/(8\*c^2) - (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3]^2)/(16\*c^2) + (b^2\*x^6\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3]^2)/16 - (b^3\*(1 + c\*x^3)\*Log[1 + c\*x^3]^3)/(24\*c^2) + (b^3\*(1 + c\*x^3)^2\*Log[1 + c\*x^3]^3)/(48\*c^2) - (b^3\*PolyLog[2, (1 - c\*x^3)/2])/(4\*c^2) + (b^3\*PolyLog[2, (1 + c\*x^3)/2])/(4\*c^2)

#### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 -

$c*x^n)/2)^p, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 2439

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*x^(r_.), x_Symbol]
:> Simp[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]))/(r + 1), x
] + (-Dist[(g*j*m)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d + e*x)^n])^p)/(i
+ j*x), x], x] - Dist[(b*e*n*p)/(r + 1), Int[(x^(r + 1)*(a + b*Log[c*(d +
e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m]))/(d + e*x), x], x]) /; FreeQ[{a
, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0] && IntegerQ[r] && (EqQ
[p, 1] || GtQ[r, 0]) && NeQ[r, -1]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((h_.)*(x_)
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.
)*(x_)), x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.), x_Sym
bol]
:> Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_.) + (g_.)*(x_)^(q_.))*((h_.) + (i_.)*(x_)^(r_.), x_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_.))^(q_.), x_Symbol] := With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q, 1] && EqQ[m, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
```

```
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.)), x_Symbol] := Simp[x*(a + b*Log[c
*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a +
b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*L
og[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m)))/(d + e*x), x], x]) /
; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^q, x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
```

$(g*(q + 1)), x] - \text{Dist}[(b*e*n)/(g*(q + 1)), \text{Int}[(f + g*x)^(q + 1)/(d + e*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 2375

$\text{Int}[(\text{Log}[(d_.) * ((e_.) + (f_.) * (x_)^(m_.))^(r_.)] * ((a_.) + \text{Log}[(c_.) * (x_)^(n_.)] * (b_.))^(p_.)) / (x_), x\_Symbol] :> \text{Simp}[(\text{Log}[d * (e + f * x^m)^r] * (a + b * \text{Log}[c * x^n])^(p + 1)) / (b * n * (p + 1)), x] - \text{Dist}[(f * m * r) / (b * n * (p + 1)), \text{Int}[(x^(m - 1) * (a + b * \text{Log}[c * x^n])^(p + 1)) / (e + f * x^m), x], x] /;$  FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2317

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_)^(n_.)] * (b_.))^(p_.) / ((d_) + (e_.) * (x_)), x\_Symbol] :> \text{Simp}[(\text{Log}[1 + (e*x)/d] * (a + b * \text{Log}[c * x^n])^p) / e, x] - \text{Dist}[(b * n * p) / e, \text{Int}[(\text{Log}[1 + (e*x)/d] * (a + b * \text{Log}[c * x^n])^(p - 1)) / x, x], x] /;$  FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2425

$\text{Int}[(\text{Log}[(f_.) * (x_)^(m_.)] * ((a_.) + \text{Log}[(c_.) * ((d_) + (e_.) * (x_))^(n_.)] * (b_.))) / (x_), x\_Symbol] :> \text{Simp}[(\text{Log}[f * x^m]^2 * (a + b * \text{Log}[c * (d + e * x)^n])) / (2 * m), x] - \text{Dist}[(b * e * n) / (2 * m), \text{Int}[\text{Log}[f * x^m]^2 / (d + e * x), x], x] /;$  FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \tanh^{-1}(cx^3))^3 dx &= \int \left( \frac{1}{8} x^5 (2a - b \log(1 - cx^3))^3 + \frac{3}{8} b x^5 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) - \frac{3}{8} b^2 x^5 \right. \\
&= \frac{1}{8} \int x^5 (2a - b \log(1 - cx^3))^3 dx + \frac{1}{8} (3b) \int x^5 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) \\
&= \frac{1}{24} \text{Subst} \left( \int x (2a - b \log(1 - cx))^3 dx, x, x^3 \right) + \frac{1}{8} b \text{Subst} \left( \int x (-2a + b \log(1 - cx))^2 \right. \\
&= \frac{1}{16} b x^6 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{16} b^2 x^6 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= \frac{1}{16} b x^6 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{16} b^2 x^6 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= \frac{1}{16} b x^6 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{16} b^2 x^6 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) \\
&= -\frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c^2} + \frac{(1 - cx^3)^2 (2a - b \log(1 - cx^3))^3}{48c^2} - \frac{b(2a - b \log(1 - cx^3))}{48c^2} \\
&= -\frac{3b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{16c^2} + \frac{b(1 - cx^3)^2 (2a - b \log(1 - cx^3))^2}{32c^2} - \frac{(1 - cx^3)}{48c^2} \\
&= \frac{3ab^2 x^3}{4c} + \frac{3b^3 x^3}{8c} + \frac{b^3 (1 - cx^3)^2}{64c^2} - \frac{b^3 (1 + cx^3)^2}{64c^2} + \frac{b^2 (1 - cx^3)^2 (2a - b \log(1 - cx^3))}{32c^2} \\
&= \frac{3ab^2 x^3}{4c} + \frac{3b^3 x^3}{4c} + \frac{b^3 (1 - cx^3)^2}{64c^2} - \frac{b^3 (1 + cx^3)^2}{64c^2} + \frac{3b^3 (1 - cx^3) \log(1 - cx^3)}{8c^2} + \frac{b^2 (1 - cx^3)^2}{32c^2} \\
&= \frac{ab^2 x^3}{2c} + \frac{5b^3 x^3}{8c} + \frac{3b^3 (1 - cx^3) \log(1 - cx^3)}{8c^2} - \frac{b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{8c^2} - \frac{(1 - cx^3)}{32c^2} \\
&= \frac{ab^2 x^3}{2c} + \frac{b^3 x^3}{2c} + \frac{b^3 (1 - cx^3) \log(1 - cx^3)}{4c^2} - \frac{b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{8c^2} - \frac{(1 - cx^3)}{32c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.298934, size = 185, normalized size = 1.33

$$6b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(cx^3)}\right) + a\left(2a^2c^2x^6 + 6abcx^3 + 3ab \log(1 - cx^3) - 3ab \log(cx^3 + 1) + 6b^2 \log(1 - c^2x^6)\right) + 6b$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^5\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] (6\*b^2\*(-1 + c\*x^3)\*(a + b + a\*c\*x^3)\*ArcTanh[c\*x^3]^2 + 2\*b^3\*(-1 + c^2\*x^6)\*ArcTanh[c\*x^3]^3 + 6\*b\*ArcTanh[c\*x^3]\*(a\*c\*x^3\*(2\*b + a\*c\*x^3) - 2\*b^2\*Log[1 + E^(-2\*ArcTanh[c\*x^3])]) + a\*(6\*a\*b\*c\*x^3 + 2\*a^2\*c^2\*x^6 + 3\*a\*b\*Log[1 - c\*x^3] - 3\*a\*b\*Log[1 + c\*x^3] + 6\*b^2\*Log[1 - c^2\*x^6]) + 6\*b^3\*PolyLog[2, -E^(-2\*ArcTanh[c\*x^3])])/(12\*c^2)

**Maple [C]** time = 0.316, size = 750, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arctanh(c\*x^3))^3,x)

[Out]  $\frac{1}{48}b^3(c^2x^6-1)/c^2 \ln(c^2x^3+1)^3 + \frac{1}{16}b^2(-x^6b \ln(-cx^3+1)c^2 + 2a^2c^2x^6 + 2b^2cx^3 + b \ln(-cx^3+1) - 2a + 2b)/c^2 \ln(c^2x^3+1)^2 + (1/16)b^3(c^2x^6-1)/c^2 \ln(-cx^3+1)^2 - 1/4x^3b^2(a+c^2x^3+b)/c \ln(-cx^3+1) + 1/4b(a^2c^2x^6 + 2a^2cx^3 + b \ln(-cx^3+1)a + b^2 \ln(-cx^3+1))/c^2 \ln(c^2x^3+1) - 1/2ab^2/cx^3 \ln(-cx^3+1) + 1/4c^2b^3 \ln(-cx^3+1) - 1/4c^2b^3 \ln(c^2x^3-1) + 3/4b^2/c \sum(-2/3(\ln(x-\alpha)\ln(-cx^3+1)) + 3c(-1/3 \ln(x-\alpha))(\ln(\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=1) - x + \alpha)/\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=1)) + \ln(\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=2) - x + \alpha)/\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=2)) + \ln(1/2(x+\alpha)/\alpha))/c - 1/3(\text{dilog}(\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=1) - x + \alpha)/\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=1)) + \text{dilog}(\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=2) - x + \alpha)/\text{RootOf}(\_Z^2+\_Z\alpha+\alpha^2, \text{index}=2)) + \text{dilog}(1/2(x+\alpha)/\alpha))/c) * b/c, \alpha = \text{RootOf}(\_Z^3c+1) - 1/8b^3/c^2 + 1/8b^3/cx^3 \ln(-cx^3+1)^2 - 1/4a^2bx^6 \ln(-cx^3+1) + 1/4a^2b/c^2 \ln(c^2x^3-1) + 1/8ab^2x^6 \ln(-cx^3+1)^2 + 3/8ab^2/c^2 \ln(-cx^3+1) - 1/8ab^2/c^2 \ln(-cx^3+1)^2 + 1/6x^6a^3 - 1/48b^3x^6 \ln(-cx^3+1)^3 - 1/8b^3/c^2 \ln(-cx^3+1)^2 + 1/48b^3/c^2 \ln(-cx^3+1)^3 + 1/2ab^2/c^2 \ln(c^2x^3+1) + 1/8b^2/c^2 a \ln(c^2x^3-1) - 1/4a^2b/c^2 \ln(c^2x^3+1) + 1/2/c a^2bx^3$



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}ab^2x^6\operatorname{arctanh}(cx^3)^2 + \frac{1}{6}a^3x^6 + \frac{1}{4}(2x^6\operatorname{arctanh}(cx^3) + c(2x^3/c^2 - \log(cx^3 + 1)/c^3 + \log(cx^3 - 1)/c^3))a^2b + \frac{1}{8}(4c(2x^3/c^2 - \log(cx^3 + 1)/c^3 + \log(cx^3 - 1)/c^3)\operatorname{arctanh}(cx^3) - (2(\log(cx^3 - 1) - 2)\log(cx^3 + 1) - \log(cx^3 + 1)^2 - \log(cx^3 - 1)^2 - 4\log(cx^3 - 1))/c^2)a^2b^2 - \frac{1}{192}(4x^6\log(-cx^3 + 1)^3 + 3(x^6/c^3 + \log(c^2x^6 - 1)/c^5)c^3 - 6c((cx^6 + 2x^3)/c^2 + 2\log(cx^3 - 1)/c^3)\log(-cx^3 + 1)^2 + 21c^2(2x^3/c^3 - \log(cx^3 + 1)/c^4 + \log(cx^3 - 1)/c^4) + c(6(c^2x^6 + 6cx^3 + 2\log(cx^3 - 1)^2 + 6\log(cx^3 - 1))\log(-cx^3 + 1)/c^3 - (3c^2x^6 + 42cx^3 + 4\log(cx^3 - 1)^3 + 18\log(cx^3 - 1)^2 + 42\log(cx^3 - 1))/c^3) - 1728c\operatorname{integrate}(1/4x^5\log(cx^3 + 1)/(c^3x^6 - c), x) - 2(12cx^3\log(cx^3 + 1)^2 + 2(c^2x^6 - 1)\log(cx^3 + 1)^3 - 3(c^2x^6 - 2cx^3 - 2(c^2x^6 - 1)\log(cx^3 + 1) + 1)\log(-cx^3 + 1)^2 + 3(c^2x^6 + 6cx^3 - 2(c^2x^6 - 1)\log(cx^3 + 1)^2 - 8(cx^3 + 1)\log(cx^3 + 1))\log(-cx^3 + 1))/c^2 + 18\log(4c^3x^6 - 4c)/c^2 - 576\operatorname{integrate}(1/4x^2\log(cx^3 + 1)/(c^3x^6 - c), x))b^3$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x^5\operatorname{artanh}(cx^3)^3 + 3ab^2x^5\operatorname{artanh}(cx^3)^2 + 3a^2bx^5\operatorname{artanh}(cx^3) + a^3x^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="fricas")

[Out]  $\operatorname{integral}(b^3x^5\operatorname{arctanh}(cx^3)^3 + 3a^2bx^5\operatorname{arctanh}(cx^3)^2 + 3a^2b^2x^5\operatorname{arctanh}(cx^3) + a^3x^5, x)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^3 x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*x^5, x)
```

$$3.126 \quad \int x^2 \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$$

**Optimal.** Leaf size=130

$$\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx^3}\right) (a + b \tanh^{-1}(cx^3))}{c} + \frac{b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)}{2c} + \frac{1}{3} x^3 (a + b \tanh^{-1}(cx^3))^3 + \frac{(a + b \tanh^{-1}(cx^3))^3}{3}$$

[Out] (a + b\*ArcTanh[c\*x^3])^3/(3\*c) + (x^3\*(a + b\*ArcTanh[c\*x^3])^3)/3 - (b\*(a + b\*ArcTanh[c\*x^3])^2\*Log[2/(1 - c\*x^3)])/c - (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, 1 - 2/(1 - c\*x^3)])/c + (b^3\*PolyLog[3, 1 - 2/(1 - c\*x^3)])/(2\*c)

**Rubi [B]** time = 2.46745, antiderivative size = 390, normalized size of antiderivative = 3., number of steps used = 82, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}(1 - cx^3)\right) (2a - b \log(1 - cx^3))}{2c} - \frac{b^3 \text{PolyLog}\left(3, \frac{1}{2}(1 - cx^3)\right)}{2c} - \frac{b^3 \text{PolyLog}\left(3, \frac{1}{2}(cx^3 + 1)\right)}{2c} + \frac{b^3 \log(1 - cx^3)}{3}$$

Warning: Unable to verify antiderivative.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] -((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^3)/(24\*c) + (b\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[(1 + c\*x^3)/2])/(4\*c) - (b\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[1 + c\*x^3])/(8\*c) + (b\*x^3\*(2\*a - b\*Log[1 - c\*x^3])^2\*Log[1 + c\*x^3])/8 + (b^3\*Log[(1 - c\*x^3)/2]\*Log[1 + c\*x^3]^2)/(4\*c) + (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3]^2)/(8\*c) + (b^2\*x^3\*(2\*a - b\*Log[1 - c\*x^3])\*Log[1 + c\*x^3]^2)/8 + (b^3\*(1 + c\*x^3)\*Log[1 + c\*x^3]^3)/(24\*c) - (b^2\*(2\*a - b\*Log[1 - c\*x^3])\*PolyLog[2, (1 - c\*x^3)/2])/(2\*c) + (b^3\*Log[1 + c\*x^3]\*PolyLog[2, (1 + c\*x^3)/2])/(2\*c) - (b^3\*PolyLog[3, (1 - c\*x^3)/2])/(2\*c) - (b^3\*PolyLog[3, (1 + c\*x^3)/2])/(2\*c)

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)), x_Symbol] := Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))^(p_.), x_Symbol] := Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] := Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]
```

Rule 2430

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]*(g_.)), x_Symbol] := Simp[x*(a + b*Log[c*(d + e*x)^n])^p*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[g*j*m, Int[(x*(a + b*Log[c*(d + e*x)^n])^p)/(i + j*x), x], x] - Dist[b*e*n*p, Int[(x*(a + b*Log[c*(d + e*x)^n])^(p - 1)*(f + g*Log[h*(i + j*x)^m])]/(d + e*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((h_.)*(x_)^(m_.))*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((k\*x)/d)^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*(e\*h - d\*i)/e + (i\*x)/e]^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2346

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.))/(x\_), x\_Symbol] := Dist[d, Int[((d + e\*x)^(q - 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] + Dist[e, Int[(d + e\*x)^(q - 1)\*(a + b\*Log[c\*x^n])^p, x], x] /; Fre

$eQ[\{a, b, c, d, e, n\}, x] \&\& IGtQ[p, 0] \&\& GtQ[q, 0] \&\& IntegerQ[2*q]$

### Rule 2301

$Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x\_Symbol] \rightarrow Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[\{a, b, c, n\}, x]$

### Rule 6742

$Int[u_, x\_Symbol] \rightarrow With[\{v = ExpandIntegrand[u, x]\}, Int[v, x] /; SumQ[v]]$

### Rule 43

$Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x\_Symbol] \rightarrow Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[\{a, b, c, d, n\}, x] \&\& NeQ[b*c - a*d, 0] \&\& IGtQ[m, 0] \&\& (!IntegerQ[n] || (EqQ[c, 0] \&\& LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])$

### Rule 2394

$Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[\{a, b, c, d, e, f, g, n\}, x] \&\& NeQ[e*f - d*g, 0]$

### Rule 2393

$Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x\_Symbol] \rightarrow Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[\{a, b, c, d, e, f, g\}, x] \&\& NeQ[e*f - d*g, 0] \&\& EqQ[g + c*(e*f - d*g), 0]$

### Rule 2391

$Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x\_Symbol] \rightarrow -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[\{c, d, e, n\}, x] \&\& EqQ[c*d, 1]$

### Rule 2375

$Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]^(r_.))*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x\_Symbol] \rightarrow Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m - 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[\{a, b, c, d,$

$e, f, r, m, n\}, x]$  && IGtQ[p, 0] && NeQ[d\*e, 1]

### Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

### Rule 2425

Int[(Log[(f\_.)\*(x\_)^(m\_.)]\*((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.)))/(x\_), x\_Symbol] := Simp[(Log[f\*x^m]^2\*(a + b\*Log[c\*(d + e\*x)^n])/(2\*m), x] - Dist[(b\*e\*n)/(2\*m), Int[Log[f\*x^m]^2/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx^3))^3 dx &= \int \left( \frac{1}{8} x^2 (2a - b \log(1 - cx^3))^3 + \frac{3}{8} b x^2 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) - \frac{3}{8} b^2 x^2 \log(1 + cx^3) \right) dx \\
&= \frac{1}{8} \int x^2 (2a - b \log(1 - cx^3))^3 dx + \frac{1}{8} (3b) \int x^2 (-2a + b \log(1 - cx^3))^2 \log(1 + cx^3) dx - \frac{3}{8} \int b^2 x^2 \log(1 + cx^3) dx \\
&= \frac{1}{24} \text{Subst} \left( \int (2a - b \log(1 - cx))^3 dx, x, x^3 \right) + \frac{1}{8} b \text{Subst} \left( \int (-2a + b \log(1 - cx))^2 \log(1 + cx) dx, x, x^3 \right) - \frac{3}{8} \int b^2 x^2 \log(1 + cx^3) dx \\
&= \frac{1}{8} b x^3 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{8} b^2 x^3 (2a - b \log(1 - cx^3)) \log^2(1 + cx^3) - \frac{3}{8} b^2 x^3 \log^3(1 + cx^3) \\
&= -\frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c} + \frac{1}{8} b x^3 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{8} b^2 x^3 \log^2(1 + cx^3) - \frac{3}{8} b^2 x^3 \log^3(1 + cx^3) \\
&= -\frac{b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{8c} - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c} + \frac{1}{8} b x^3 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{8} b^2 x^3 \log^2(1 + cx^3) - \frac{3}{8} b^2 x^3 \log^3(1 + cx^3) \\
&= \frac{1}{2} a b^2 x^3 - \frac{b^3 x^3}{4} - \frac{b(1 - cx^3)(2a - b \log(1 - cx^3))^2}{8c} - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c} + \frac{1}{8} b x^3 (2a - b \log(1 - cx^3))^2 \log(1 + cx^3) + \frac{1}{8} b^2 x^3 \log^2(1 + cx^3) - \frac{3}{8} b^2 x^3 \log^3(1 + cx^3) \\
&= \frac{1}{2} a b^2 x^3 + \frac{b^3 (1 - cx^3) \log(1 - cx^3)}{4c} - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c} + \frac{b(2a - b \log(1 - cx^3))^2 \log\left(\frac{1}{2}(1 + cx^3)\right)}{4c} - \frac{b(2a - b \log(1 - cx^3)) \log^2\left(\frac{1}{2}(1 + cx^3)\right)}{4c} - \frac{b \log^3\left(\frac{1}{2}(1 + cx^3)\right)}{4c} \\
&= \frac{b^3 x^3}{4} + \frac{b^3 (1 - cx^3) \log(1 - cx^3)}{4c} - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24c} + \frac{b(2a - b \log(1 - cx^3))^2 \log\left(\frac{1}{2}(1 + cx^3)\right)}{4c} - \frac{b(2a - b \log(1 - cx^3)) \log^2\left(\frac{1}{2}(1 + cx^3)\right)}{4c} - \frac{b \log^3\left(\frac{1}{2}(1 + cx^3)\right)}{4c}
\end{aligned}$$

**Mathematica [A]** time = 0.281532, size = 191, normalized size = 1.47

$$\frac{6ab^2 \left( \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(cx^3)} \right) + \tanh^{-1}(cx^3) \left( (cx^3 - 1) \tanh^{-1}(cx^3) - 2 \log \left( e^{-2 \tanh^{-1}(cx^3)} + 1 \right) \right) \right) + b^3 \left( 6 \tanh^{-1}(cx^3) \right)}{24c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] (2\*a^3\*c\*x^3 + 6\*a^2\*b\*c\*x^3\*ArcTanh[c\*x^3] + 3\*a^2\*b\*Log[1 - c^2\*x^6] + 6\*a\*b^2\*(ArcTanh[c\*x^3]\*((-1 + c\*x^3)\*ArcTanh[c\*x^3] - 2\*Log[1 + E^(-2\*ArcTan



$\ln[c*x^3])) + \text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x^3])}] + b^3*(2*\text{ArcTanh}[c*x^3]^2 * ((-1 + c*x^3)*\text{ArcTanh}[c*x^3] - 3*\text{Log}[1 + E^{(-2*\text{ArcTanh}[c*x^3])}])) + 6*\text{ArcTanh}[c*x^3]*\text{PolyLog}[2, -E^{(-2*\text{ArcTanh}[c*x^3])}] + 3*\text{PolyLog}[3, -E^{(-2*\text{ArcTanh}[c*x^3])}])))/(6*c)$

**Maple [B]** time = 0.006, size = 295, normalized size = 2.3

$$\frac{x^3 a^3}{3} + \frac{b^3 x^3 (\text{Artanh}(cx^3))^3}{3} + \frac{b^3 (\text{Artanh}(cx^3))^3}{3c} - \frac{b^3 (\text{Artanh}(cx^3))^2}{c} \ln \left( \frac{(cx^3 + 1)^2}{-c^2 x^6 + 1} + 1 \right) - \frac{b^3 \text{Artanh}(cx^3)}{c} \text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^3))^3,x)

[Out]  $1/3*x^3*a^3 + 1/3*b^3*x^3*arctanh(c*x^3)^3 + 1/3/c*b^3*arctanh(c*x^3)^3 - 1/c*b^3*arctanh(c*x^3)^2*\ln((c*x^3+1)^2/(-c^2*x^6+1)+1) - 1/c*b^3*arctanh(c*x^3)*\text{polylog}(2, -(c*x^3+1)^2/(-c^2*x^6+1)) + 1/2/c*b^3*\text{polylog}(3, -(c*x^3+1)^2/(-c^2*x^6+1)) + arctanh(c*x^3)^2*x^3*a*b^2 - 2/c*\ln((c*x^3+1)^2/(-c^2*x^6+1))*arctanh(c*x^3)*a*b^2 + 1/c*a*b^2*arctanh(c*x^3)^2 - 1/c*\text{polylog}(2, -(c*x^3+1)^2/(-c^2*x^6+1))*a*b^2 + x^3*a^2*b*arctanh(c*x^3) + 1/2/c*a^2*b*\ln(-c^2*x^6+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^3 x^3 + \frac{(2cx^3 \text{artanh}(cx^3) + \log(-c^2x^6 + 1))a^2b}{2c} - \frac{(b^3cx^3 - b^3) \log(-cx^3 + 1)^3 - 3(2ab^2cx^3 + (b^3cx^3 + b^3) \log(cx^3 + 1))}{24c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="maxima")

[Out]  $1/3*a^3*x^3 + 1/2*(2*c*x^3*arctanh(c*x^3) + \log(-c^2*x^6 + 1))*a^2*b/c - 1/24*((b^3*c*x^3 - b^3)*\log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c*x^3 + (b^3*c*x^3 + b^3)*\log(c*x^3 + 1))*\log(-c*x^3 + 1)^2)/c - \text{integrate}(-1/8*((b^3*c*x^5 - b^3*x^2)*\log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^5 - a*b^2*x^2)*\log(c*x^3 + 1)^2 - 3*(4*a*b^2*c*x^5 + (b^3*c*x^5 - b^3*x^2)*\log(c*x^3 + 1)^2 + 2*((2*a*b^2*c + b^3*c)*x^5 - (2*a*b^2 - b^3)*x^2)*\log(c*x^3 + 1))*\log(-c*x^3 + 1))/(c*x^3 - 1), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2 \operatorname{artanh}(cx^3)^3 + 3ab^2x^2 \operatorname{artanh}(cx^3)^2 + 3a^2bx^2 \operatorname{artanh}(cx^3) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^2\*arctanh(c\*x^3)^3 + 3\*a\*b^2\*x^2\*arctanh(c\*x^3)^2 + 3\*a^2\*b\*x^2\*arctanh(c\*x^3) + a^3\*x^2, x)

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x\*\*3))\*\*3,x)

[Out] Exception raised: KeyError

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^3) + a)^3\*x^2, x)

$$3.127 \quad \int \frac{(a+b \tanh^{-1}(cx^3))^3}{x} dx$$

**Optimal.** Leaf size=210

$$\frac{1}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx^3} - 1\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b \text{PolyLog}\left(3, \frac{2}{1-cx^3} - 1\right)(a+b \tanh^{-1}(cx^3))$$

[Out] (2\*(a + b\*ArcTanh[c\*x^3])^3\*ArcTanh[1 - 2/(1 - c\*x^3)]/3 - (b\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, 1 - 2/(1 - c\*x^3)]/2 + (b\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, -1 + 2/(1 - c\*x^3)]/2 + (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, 1 - 2/(1 - c\*x^3)]/2 - (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, -1 + 2/(1 - c\*x^3)]/2 - (b^3\*PolyLog[4, 1 - 2/(1 - c\*x^3)]/4 + (b^3\*PolyLog[4, -1 + 2/(1 - c\*x^3)]/4

**Rubi [A]** time = 0.564568, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$\frac{1}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1-cx^3} - 1\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b \text{PolyLog}\left(3, 1 - \frac{2}{1-cx^3}\right)(a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b \text{PolyLog}\left(3, \frac{2}{1-cx^3} - 1\right)(a+b \tanh^{-1}(cx^3))$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^3])^3/x, x]

[Out] (2\*(a + b\*ArcTanh[c\*x^3])^3\*ArcTanh[1 - 2/(1 - c\*x^3)]/3 - (b\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, 1 - 2/(1 - c\*x^3)]/2 + (b\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, -1 + 2/(1 - c\*x^3)]/2 + (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, 1 - 2/(1 - c\*x^3)]/2 - (b^2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, -1 + 2/(1 - c\*x^3)]/2 - (b^3\*PolyLog[4, 1 - 2/(1 - c\*x^3)]/4 + (b^3\*PolyLog[4, -1 + 2/(1 - c\*x^3)]/4

### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_)/(x_), x_Symbol] := Simp[2*(a +
  b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
  *ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x))]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_) + (e_.)*(
  x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
  e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
  x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
  x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
  + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
  e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6062

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
  u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
  x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^3}{x} dx &= \frac{1}{3} \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^3}{x} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - (2bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \tanh^{-1}(cx)}{1 - c^2x^2} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) + (bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \log \left( \frac{1 + cx}{1 - cx} \right)}{1 - c^2x^2} dx, x, x^3 \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{2} b (a + b \tanh^{-1}(cx^3))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^3} \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{2} b (a + b \tanh^{-1}(cx^3))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^3} \right) \\
&= \frac{2}{3} (a + b \tanh^{-1}(cx^3))^3 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^3} \right) - \frac{1}{2} b (a + b \tanh^{-1}(cx^3))^2 \text{Li}_2 \left( 1 - \frac{2}{1 - cx^3} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.180065, size = 214, normalized size = 1.02

$$\frac{1}{4} b \left( 2 \text{PolyLog} \left( 2, \frac{cx^3 + 1}{1 - cx^3} \right) (a + b \tanh^{-1}(cx^3))^2 - 2 \text{PolyLog} \left( 2, \frac{cx^3 + 1}{cx^3 - 1} \right) (a + b \tanh^{-1}(cx^3))^2 + b \left( -2 \text{PolyLog} \left( 3, \frac{1 + cx^3}{1 - cx^3} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^3/x,x]

[Out] (2\*(a + b\*ArcTanh[c\*x^3])^3\*ArcTanh[1 + 2/(-1 + c\*x^3)]/3 + (b\*(2\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, (1 + c\*x^3)/(1 - c\*x^3)] - 2\*(a + b\*ArcTanh[c\*x^3])^2\*PolyLog[2, (1 + c\*x^3)/(-1 + c\*x^3)] + b\*(-2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, (1 + c\*x^3)/(1 - c\*x^3)] + 2\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[3, (1 + c\*x^3)/(-1 + c\*x^3)] + b\*(PolyLog[4, (1 + c\*x^3)/(1 - c\*x^3)] - PolyLog[4, (1 + c\*x^3)/(-1 + c\*x^3)])))/4

**Maple [F]** time = 0.151, size = 0, normalized size = 0.

$$\int \frac{(a + b \text{Artanh}(cx^3))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))^3/x,x)

[Out] int((a+b\*arctanh(c\*x^3))^3/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3 (\log(cx^3 + 1) - \log(-cx^3 + 1))^3}{8x} + \frac{3ab^2 (\log(cx^3 + 1) - \log(-cx^3 + 1))^2}{4x} + \frac{3a^2b (\log(cx^3 + 1) - \log(-cx^3 + 1))}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x,x, algorithm="maxima")

[Out] a^3\*log(x) + integrate(1/8\*b^3\*(log(c\*x^3 + 1) - log(-c\*x^3 + 1))^3/x + 3/4\*a\*b^2\*(log(c\*x^3 + 1) - log(-c\*x^3 + 1))^2/x + 3/2\*a^2\*b\*(log(c\*x^3 + 1) - log(-c\*x^3 + 1))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^3 \operatorname{artanh}(cx^3)^3 + 3ab^2 \operatorname{artanh}(cx^3)^2 + 3a^2b \operatorname{artanh}(cx^3) + a^3}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x^3)^3 + 3\*a\*b^2\*arctanh(c\*x^3)^2 + 3\*a^2\*b\*arctanh(c\*x^3) + a^3)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x, x)
```

**3.128** 
$$\int \frac{(a+b \tanh^{-1}(cx^3))^3}{x^4} dx$$

**Optimal.** Leaf size=120

$$-b^2c \operatorname{PolyLog}\left(2, \frac{2}{cx^3+1} - 1\right) (a+b \tanh^{-1}(cx^3)) - \frac{1}{2}b^3c \operatorname{PolyLog}\left(3, \frac{2}{cx^3+1} - 1\right) + \frac{1}{3}c (a+b \tanh^{-1}(cx^3))^3 - \frac{(a+b \tanh^{-1}(cx^3))^3}{3x^3}$$

[Out] (c\*(a + b\*ArcTanh[c\*x^3])^3)/3 - (a + b\*ArcTanh[c\*x^3])^3/(3\*x^3) + b\*c\*(a + b\*ArcTanh[c\*x^3])^2\*Log[2 - 2/(1 + c\*x^3)] - b^2\*c\*(a + b\*ArcTanh[c\*x^3])\*PolyLog[2, -1 + 2/(1 + c\*x^3)] - (b^3\*c\*PolyLog[3, -1 + 2/(1 + c\*x^3)])/2

**Rubi [F]** time = 0.778839, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(cx^3))^3}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^3])^3/x^4, x]

[Out] (b\*c\*Log[c\*x^3]\*(2\*a - b\*Log[1 - c\*x^3])^2)/8 - ((1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^3)/(24\*x^3) + (b^3\*c\*Log[-(c\*x^3)]\*Log[1 + c\*x^3]^2)/8 - (b^3\*(1 + c\*x^3)\*Log[1 + c\*x^3]^3)/(24\*x^3) - (b^2\*c\*(2\*a - b\*Log[1 - c\*x^3])\*PolyLog[2, 1 - c\*x^3])/4 + (b^3\*c\*Log[1 + c\*x^3]\*PolyLog[2, 1 + c\*x^3])/4 - (b^3\*c\*PolyLog[3, 1 - c\*x^3])/4 - (b^3\*c\*PolyLog[3, 1 + c\*x^3])/4 + (b\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])^2\*Log[1 + c\*x])/x^2, x], x, x^3])/8 - (b^2\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])\*Log[1 + c\*x]^2)/x^2, x], x, x^3])/8

Rubi steps



$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^3}{x^4} dx &= \int \left( \frac{(2a - b \log(1 - cx^3))^3}{8x^4} + \frac{3b(-2a + b \log(1 - cx^3))^2 \log(1 + cx^3)}{8x^4} - \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{8x^4} + \frac{b^3 \log^3(1 + cx^3)}{8x^4} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - cx^3))^3}{x^4} dx + \frac{1}{8} (3b) \int \frac{(-2a + b \log(1 - cx^3))^2 \log(1 + cx^3)}{x^4} dx - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= \frac{1}{24} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^3}{x^2} dx, x, x^3 \right) + \frac{1}{8} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^2} dx, x, x^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= -\frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24x^3} - \frac{b^3(1 + cx^3) \log^3(1 + cx^3)}{24x^3} + \frac{1}{8} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^2} dx, x, x^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= \frac{1}{8} bc \log(cx^3) (2a - b \log(1 - cx^3))^2 - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24x^3} + \frac{1}{8} b^3 c \log(-cx^3) \log^2(1 + cx^3) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= \frac{1}{8} bc \log(cx^3) (2a - b \log(1 - cx^3))^2 - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24x^3} + \frac{1}{8} b^3 c \log(-cx^3) \log^2(1 + cx^3) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= \frac{1}{8} bc \log(cx^3) (2a - b \log(1 - cx^3))^2 - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24x^3} + \frac{1}{8} b^3 c \log(-cx^3) \log^2(1 + cx^3) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx \\
&= \frac{1}{8} bc \log(cx^3) (2a - b \log(1 - cx^3))^2 - \frac{(1 - cx^3)(2a - b \log(1 - cx^3))^3}{24x^3} + \frac{1}{8} b^3 c \log(-cx^3) \log^2(1 + cx^3) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^4} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + cx^3)}{x^4} dx
\end{aligned}$$

**Mathematica [C]** time = 0.394616, size = 223, normalized size = 1.86

$$ab^2c \left( \tanh^{-1}(cx^3) \left( \left( 1 - \frac{1}{cx^3} \right) \tanh^{-1}(cx^3) + 2 \log \left( 1 - e^{-2 \tanh^{-1}(cx^3)} \right) \right) - \text{PolyLog} \left( 2, e^{-2 \tanh^{-1}(cx^3)} \right) \right) + \frac{1}{3} b^3 c \left( 3 \tanh^{-1}(cx^3) \log^2(1 + cx^3) - \log^3(1 + cx^3) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^3/x^4, x]

[Out]  $-\frac{a^3}{3x^3} - \frac{a^2 b \text{ArcTanh}[cx^3]}{x^3} + \frac{3a^2 b c \log[x] - (a^2 b c \log[1 - c^2 x^6])}{2} + \frac{a b^2 c (\text{ArcTanh}[cx^3] * ((1 - 1/(cx^3)) * \text{ArcTanh}[cx^3] + 2 * \log[1 - E^{(-2 * \text{ArcTanh}[cx^3])}])) - \text{PolyLog}[2, E^{(-2 * \text{ArcTanh}[cx^3])}])}{8} + \frac{b^3 c ((1/8) * \pi^3 - \text{ArcTanh}[cx^3]^3 - \text{ArcTanh}[cx^3]^3 / (cx^3) + 3 * \text{ArcTanh}[cx^3]^2 * \log[1 - E^{(2 * \text{ArcTanh}[cx^3])}] + 3 * \text{ArcTanh}[cx^3] * \text{PolyLog}[2, E^{(2 * \text{ArcTanh}[cx^3])}] - (3 * \text{PolyLog}[3, E^{(2 * \text{ArcTanh}[cx^3])}]) / 2))}{3}$

**Maple [F]** time = 0.208, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Artanh}(cx^3))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^3))^3/x^4,x)

[Out] int((a+b\*arctanh(c\*x^3))^3/x^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( c(\log(c^2x^6 - 1) - \log(x^6)) + \frac{2 \operatorname{artanh}(cx^3)}{x^3} \right) a^2 b - \frac{a^3}{3x^3} - \frac{(b^3cx^3 - b^3) \log(-cx^3 + 1)^3 + 3(2ab^2 + (b^3cx^3 + b^3) \log(-cx^3 + 1))}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x^4,x, algorithm="maxima")

[Out] -1/2\*(c\*(log(c^2\*x^6 - 1) - log(x^6)) + 2\*arctanh(c\*x^3)/x^3)\*a^2\*b - 1/3\*a^3/x^3 - 1/24\*((b^3\*c\*x^3 - b^3)\*log(-c\*x^3 + 1)^3 + 3\*(2\*a\*b^2 + (b^3\*c\*x^3 + b^3)\*log(c\*x^3 + 1))\*log(-c\*x^3 + 1)^2)/x^3 - integrate(-1/8\*((b^3\*c\*x^3 - b^3)\*log(c\*x^3 + 1)^3 + 6\*(a\*b^2\*c\*x^3 - a\*b^2)\*log(c\*x^3 + 1)^2 + 3\*(4\*a\*b^2\*c\*x^3 - (b^3\*c\*x^3 - b^3)\*log(c\*x^3 + 1)^2 + 2\*(b^3\*c^2\*x^6 - (2\*a\*b^2\*c - b^3\*c)\*x^3 + 2\*a\*b^2)\*log(c\*x^3 + 1))\*log(-c\*x^3 + 1))/(c\*x^7 - x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{b^3 \operatorname{artanh}(cx^3)^3 + 3ab^2 \operatorname{artanh}(cx^3)^2 + 3a^2b \operatorname{artanh}(cx^3) + a^3}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x^4,x, algorithm="fricas")

```
[Out] integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctanh
(c*x^3) + a^3)/x^4, x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**3))**3/x**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3/x^4, x)
```

$$3.129 \quad \int \frac{(a + b \tanh^{-1}(cx^3))^3}{x^7} dx$$

**Optimal.** Leaf size=136

$$-\frac{1}{2}b^3c^2\text{PolyLog}\left(2, \frac{2}{cx^3+1} - 1\right) + b^2c^2 \log\left(2 - \frac{2}{cx^3+1}\right) (a + b \tanh^{-1}(cx^3)) + \frac{1}{2}bc^2 (a + b \tanh^{-1}(cx^3))^2 + \frac{1}{6}c^2 (a + b \tanh^{-1}(cx^3))^3$$

[Out] (b\*c^2\*(a + b\*ArcTanh[c\*x^3])^2)/2 - (b\*c\*(a + b\*ArcTanh[c\*x^3])^2)/(2\*x^3) + (c^2\*(a + b\*ArcTanh[c\*x^3])^3)/6 - (a + b\*ArcTanh[c\*x^3])^3/(6\*x^6) + b^2\*c^2\*(a + b\*ArcTanh[c\*x^3])\*Log[2 - 2/(1 + c\*x^3)] - (b^3\*c^2\*PolyLog[2, -1 + 2/(1 + c\*x^3)])/2

**Rubi [F]** time = 1.57491, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^3))^3}{x^7} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^3])^3/x^7, x]

[Out] (3\*a\*b^2\*c^2\*Log[x])/4 - (b\*c\*(1 - c\*x^3)\*(2\*a - b\*Log[1 - c\*x^3])^2)/(16\*x^3) + (b\*c^2\*Log[c\*x^3]\*(2\*a - b\*Log[1 - c\*x^3])^2)/16 + (c^2\*(2\*a - b\*Log[1 - c\*x^3])^3)/48 - (2\*a - b\*Log[1 - c\*x^3])^3/(48\*x^6) - (b^3\*c\*(1 + c\*x^3)\*Log[1 + c\*x^3]^2)/(16\*x^3) - (b^3\*c^2\*Log[-(c\*x^3)]\*Log[1 + c\*x^3]^2)/16 + (b^3\*c^2\*Log[1 + c\*x^3]^3)/48 - (b^3\*Log[1 + c\*x^3]^3)/(48\*x^6) - (b^3\*c^2\*PolyLog[2, -(c\*x^3)])/8 + (b^3\*c^2\*PolyLog[2, c\*x^3])/8 - (b^2\*c^2\*(2\*a - b\*Log[1 - c\*x^3])\*PolyLog[2, 1 - c\*x^3])/8 - (b^3\*c^2\*Log[1 + c\*x^3]\*PolyLog[2, 1 + c\*x^3])/8 - (b^3\*c^2\*PolyLog[3, 1 - c\*x^3])/8 + (b^3\*c^2\*PolyLog[3, 1 + c\*x^3])/8 + (b\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])^2\*Log[1 + c\*x])/x^3, x], x, x^3])/8 - (b^2\*Defer[Subst][Defer[Int][((-2\*a + b\*Log[1 - c\*x])\*Log[1 + c\*x]^2)/x^3, x], x, x^3])/8

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^3))^3}{x^7} dx &= \int \left( \frac{(2a - b \log(1 - cx^3))^3}{8x^7} + \frac{3b(-2a + b \log(1 - cx^3))^2 \log(1 + cx^3)}{8x^7} - \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{8x^7} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - cx^3))^3}{x^7} dx + \frac{1}{8} (3b) \int \frac{(-2a + b \log(1 - cx^3))^2 \log(1 + cx^3)}{x^7} dx - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx^3)) \log^2(1 + cx^3)}{x^7} dx \\
&= \frac{1}{24} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^3}{x^3} dx, x, x^3 \right) + \frac{1}{8} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^3} dx, x, x^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= -\frac{(2a - b \log(1 - cx^3))^3}{48x^6} - \frac{b^3 \log^3(1 + cx^3)}{48x^6} + \frac{1}{8} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - cx))^2 \log(1 + cx)}{x^3} dx, x, x^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= -\frac{(2a - b \log(1 - cx^3))^3}{48x^6} - \frac{b^3 \log^3(1 + cx^3)}{48x^6} - \frac{1}{16} b \text{Subst} \left( \int \frac{(2a - b \log(x))^2}{x \left(\frac{1}{c} - \frac{x}{c}\right)^2} dx, x, 1 - cx^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= -\frac{(2a - b \log(1 - cx^3))^3}{48x^6} - \frac{b^3 \log^3(1 + cx^3)}{48x^6} - \frac{1}{16} b \text{Subst} \left( \int \frac{(2a - b \log(x))^2}{\left(\frac{1}{c} - \frac{x}{c}\right)^2} dx, x, 1 - cx^3 \right) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= -\frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))^2}{16x^3} - \frac{(2a - b \log(1 - cx^3))^3}{48x^6} - \frac{b^3 c(1 + cx^3) \log^2(1 + cx^3)}{16x^3} - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))^2}{16x^3} + \frac{1}{16} bc^2 \log(cx^3)(2a - b \log(1 - cx^3)) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))^2}{16x^3} + \frac{1}{16} bc^2 \log(cx^3)(2a - b \log(1 - cx^3)) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx \\
&= \frac{3}{4} ab^2 c^2 \log(x) - \frac{bc(1 - cx^3)(2a - b \log(1 - cx^3))^2}{16x^3} + \frac{1}{16} bc^2 \log(cx^3)(2a - b \log(1 - cx^3)) - \frac{1}{8} \int \frac{3b^2(-2a + b \log(1 - cx)) \log^2(1 + cx)}{x^7} dx
\end{aligned}$$

**Mathematica [A]** time = 0.27932, size = 218, normalized size = 1.6

$$-6b^3 c^2 x^6 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(cx^3)}\right) + a\left(-2a^2 - 3abc^2 x^6 \log(1 - cx^3) + 3abc^2 x^6 \log(cx^3 + 1) - 6abcx^3 + 12b^2 c^2 x^6 \log\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*x^3])^3/x^7, x]

```
[Out] (6*b^2*(-1 + c*x^3)*(a + a*c*x^3 + b*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 +
c^2*x^6)*ArcTanh[c*x^3]^3 - 6*b*ArcTanh[c*x^3]*(a^2 + 2*a*b*c*x^3 - 2*b^2*c
^2*x^6*Log[1 - E^(-2*ArcTanh[c*x^3])]) + a*(-2*a^2 - 6*a*b*c*x^3 - 3*a*b*c^
2*x^6*Log[1 - c*x^3] + 3*a*b*c^2*x^6*Log[1 + c*x^3] + 12*b^2*c^2*x^6*Log[(c
*x^3)/Sqrt[1 - c^2*x^6]]) - 6*b^3*c^2*x^6*PolyLog[2, E^(-2*ArcTanh[c*x^3])])
)/(12*x^6)
```

**Maple [F]** time = 0.23, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^3))^3}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

```
[Out] int((a+b*arctanh(c*x^3))^3/x^7,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( \left( c \log(cx^3 + 1) - c \log(cx^3 - 1) - \frac{2}{x^3} \right) c - \frac{2 \operatorname{artanh}(cx^3)}{x^6} \right) a^2 b + \frac{1}{8} \left( \left( 2(\log(cx^3 - 1) - 2) \log(cx^3 + 1) - \log(cx^3 + 1) \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^6
)*a^2*b + 1/8*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 -
log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1)
- c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*a*b^2 - 1/48*b^3*(((c^2*x^6 -
1)*log(-c*x^3 + 1)^3 + 3*(2*c*x^3 - (c^2*x^6 - 1)*log(c*x^3 + 1))*log(-c*x
^3 + 1)^2)/x^6 + 6*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^3 + 3*(2*c^2*x^6
- (c*x^3 - 1)*log(c*x^3 + 1)^2 - (c^3*x^9 - c*x^3)*log(c*x^3 + 1))*log(-c*x
^3 + 1))/(c*x^10 - x^7), x)) - 1/2*a*b^2*arctanh(c*x^3)^2/x^6 - 1/6*a^3/x^6
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(cx^3)^3 + 3ab^2 \operatorname{artanh}(cx^3)^2 + 3a^2b \operatorname{artanh}(cx^3) + a^3}{x^7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x^7,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x^3)^3 + 3\*a\*b^2\*arctanh(c\*x^3)^2 + 3\*a^2\*b\*arctanh(c\*x^3) + a^3)/x^7, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*3))\*\*3/x\*\*7,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^3))^3/x^7,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^3) + a)^3/x^7, x)

$$3.130 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^3, x]

**Rubi [A]** time = 0.0253039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] Defer[Int][(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$$

**Mathematica [A]** time = 1.80957, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^3,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^3, x]



**Maple [A]** time = 0.125, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^3))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^3))^3,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^3))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^3 \operatorname{artanh}(cx^3)^3 + 3ab^2 \operatorname{artanh}(cx^3)^2 + 3a^2b \operatorname{artanh}(cx^3) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3))^3,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*x^3)^3 + 3\*a\*b^2\*arctanh(c\*x^3)^2 + 3\*a^2\*b\*arctanh(c\*x^3) + a^3)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**3,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^3*(d*x)^m, x)
```

$$\mathbf{3.131} \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left((dx)^m (a + b \tanh^{-1}(cx^3))^2, x\right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^2, x]

**Rubi [A]** time = 0.025162, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^2, x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^2 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$$

**Mathematica [A]** time = 1.17361, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^2, x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3])^2, x]

**Maple [A]** time = 0.125, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^3))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^3))^2,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^3))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b^2 \operatorname{artanh}(cx^3)^2 + 2ab \operatorname{artanh}(cx^3) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^3)^2 + 2\*a\*b\*arctanh(c\*x^3) + a^2)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**3))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)^2*(d*x)^m, x)
```

### 3.132 $\int (dx)^m \left( a + b \tanh^{-1}(cx^3) \right) dx$

**Optimal.** Leaf size=74

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx^3) \right)}{d(m+1)} - \frac{3bc(dx)^{m+4} \text{Hypergeometric2F1} \left( 1, \frac{m+4}{6}, \frac{m+10}{6}, c^2x^6 \right)}{d^4(m+1)(m+4)}$$

[Out]  $((d*x)^{(1+m)*(a+b*\text{ArcTanh}[c*x^3])})/(d*(1+m)) - (3*b*c*(d*x)^{(4+m)*\text{Hypergeometric2F1}[1, (4+m)/6, (10+m)/6, c^2*x^6]})/(d^4*(1+m)*(4+m))$

**Rubi [A]** time = 0.0431427, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {6097, 16, 364}

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx^3) \right)}{d(m+1)} - \frac{3bc(dx)^{m+4} {}_2F_1 \left( 1, \frac{m+4}{6}; \frac{m+10}{6}; c^2x^6 \right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3]),x]

[Out]  $((d*x)^{(1+m)*(a+b*\text{ArcTanh}[c*x^3])})/(d*(1+m)) - (3*b*c*(d*x)^{(4+m)*\text{Hypergeometric2F1}[1, (4+m)/6, (10+m)/6, c^2*x^6]})/(d^4*(1+m)*(4+m))$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 16

Int[(u\_.)\*(v\_)^(m\_.)\*((b\_.)\*(v\_)^(n\_.)), x\_Symbol] :> Dist[1/b^m, Int[u\*(b\*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a

)]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \tanh^{-1}(cx^3)) dx &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^3))}{d(1+m)} - \frac{(3bc) \int \frac{x^2(dx)^{1+m}}{1-c^2x^6} dx}{d(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^3))}{d(1+m)} - \frac{(3bc) \int \frac{(dx)^{3+m}}{1-c^2x^6} dx}{d^3(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^3))}{d(1+m)} - \frac{3bc(dx)^{4+m} {}_2F_1\left(1, \frac{4+m}{6}; \frac{10+m}{6}; c^2x^6\right)}{d^4(1+m)(4+m)} \end{aligned}$$

**Mathematica [A]** time = 0.0623695, size = 64, normalized size = 0.86

$$\frac{x(dx)^m \left( 3bcx^3 \text{Hypergeometric2F1}\left(1, \frac{m+4}{6}, \frac{m+10}{6}, c^2x^6\right) - (m+4)(a + b \tanh^{-1}(cx^3)) \right)}{(m+1)(m+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^3]), x]

[Out] -((x\*(d\*x)^m\*(-((4 + m)\*(a + b\*ArcTanh[c\*x^3])) + 3\*b\*c\*x^3\*Hypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2\*x^6]))/((1 + m)\*(4 + m)))

**Maple [F]** time = 0.119, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \text{Artanh}(cx^3)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^3)), x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^3)), x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \operatorname{artanh}\left(cx^3\right) + a\right) (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^3)),x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^3) + a)\*(d\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*atanh(c\*x\*\*3)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^3) + a) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

$$3.133 \quad \int \frac{(dx)^m}{a + b \tanh^{-1}(cx^3)} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a + b \tanh^{-1}(cx^3)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^3]), x]

**Rubi [A]** time = 0.0280181, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a + b \tanh^{-1}(cx^3)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^3]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^3]), x]

Rubi steps

$$\int \frac{(dx)^m}{a + b \tanh^{-1}(cx^3)} dx = \int \frac{(dx)^m}{a + b \tanh^{-1}(cx^3)} dx$$

**Mathematica [A]** time = 0.349283, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \tanh^{-1}(cx^3)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^3]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^3]), x]

---

**Maple [A]** time = 0.07, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{Artanh}(cx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^3)),x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^3)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^3) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^3)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x^3) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx^3) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^3)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arctanh(c\*x^3) + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*atanh(c\*x\*\*3)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^3) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^3)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x^3) + a), x)

$$3.134 \quad \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^3])^2, x]

**Rubi [A]** time = 0.0274606, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^3])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^3])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2} dx$$

**Mathematica [A]** time = 0.361395, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^3)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^3])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^3])^2, x]

**Maple [A]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{Artanh}(cx^3))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^3))^2,x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^3))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2(c^2 d^m x^6 - d^m)x^m}{3(b^2 cx^2 \log(cx^3 + 1) - b^2 cx^2 \log(-cx^3 + 1) + 2 abcx^2)} + \int -\frac{2(c^2 d^m(m+4)x^6 - d^m(m-2))x^m}{3(b^2 cx^3 \log(cx^3 + 1) - b^2 cx^3 \log(-cx^3 + 1) + 2 abcx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^3))^2,x, algorithm="maxima")

[Out] 2/3\*(c^2\*d^m\*x^6 - d^m)\*x^m/(b^2\*c\*x^2\*log(c\*x^3 + 1) - b^2\*c\*x^2\*log(-c\*x^3 + 1) + 2\*a\*b\*c\*x^2) + integrate(-2/3\*(c^2\*d^m\*(m + 4)\*x^6 - d^m\*(m - 2))\*x^m/(b^2\*c\*x^3\*log(c\*x^3 + 1) - b^2\*c\*x^3\*log(-c\*x^3 + 1) + 2\*a\*b\*c\*x^3), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}(cx^3)^2 + 2 ab \operatorname{artanh}(cx^3) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^3))^2,x, algorithm="fricas")

[Out] `integral((d*x)^m/(b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m/(a+b*atanh(c*x**3))**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{artanh}(cx^3) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="giac")`

[Out] `integrate((d*x)^m/(b*arctanh(c*x^3) + a)^2, x)`

### 3.135 $\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx$

**Optimal.** Leaf size=50

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4}bc^3x - \frac{1}{4}bc^4 \tanh^{-1} \left( \frac{x}{c} \right) + \frac{1}{12}bcx^3$$

[Out] (b\*c^3\*x)/4 + (b\*c\*x^3)/12 + (x^4\*(a + b\*ArcTanh[c/x]))/4 - (b\*c^4\*ArcTanh[x/c])/4

**Rubi [A]** time = 0.0319407, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 302, 207}

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4}bc^3x - \frac{1}{4}bc^4 \tanh^{-1} \left( \frac{x}{c} \right) + \frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c/x]), x]

[Out] (b\*c^3\*x)/4 + (b\*c\*x^3)/12 + (x^4\*(a + b\*ArcTanh[c/x]))/4 - (b\*c^4\*ArcTanh[x/c])/4

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```



Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx &= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4} (bc) \int \frac{x^2}{1 - \frac{c^2}{x^2}} dx \\
 &= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4} (bc) \int \frac{x^4}{-c^2 + x^2} dx \\
 &= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4} (bc) \int \left( c^2 + x^2 + \frac{c^4}{-c^2 + x^2} \right) dx \\
 &= \frac{1}{4} bc^3 x + \frac{1}{12} bcx^3 + \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{4} (bc^5) \int \frac{1}{-c^2 + x^2} dx \\
 &= \frac{1}{4} bc^3 x + \frac{1}{12} bcx^3 + \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) - \frac{1}{4} bc^4 \tanh^{-1} \left( \frac{x}{c} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0099763, size = 67, normalized size = 1.34

$$\frac{ax^4}{4} + \frac{1}{4} bc^3 x + \frac{1}{8} bc^4 \log(x - c) - \frac{1}{8} bc^4 \log(c + x) + \frac{1}{12} bcx^3 + \frac{1}{4} bx^4 \tanh^{-1} \left( \frac{c}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c/x]),x]

[Out] (b\*c^3\*x)/4 + (b\*c\*x^3)/12 + (a\*x^4)/4 + (b\*x^4\*ArcTanh[c/x])/4 + (b\*c^4\*Log[-c + x])/8 - (b\*c^4\*Log[c + x])/8

**Maple [A]** time = 0.013, size = 62, normalized size = 1.2

$$\frac{x^4 a}{4} + \frac{bx^4}{4} \operatorname{Arctanh} \left( \frac{c}{x} \right) + \frac{c^4 b}{8} \ln \left( \frac{c}{x} - 1 \right) + \frac{bcx^3}{12} + \frac{bc^3 x}{4} - \frac{c^4 b}{8} \ln \left( 1 + \frac{c}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c/x)),x)

[Out]  $\frac{1}{4}ax^4 + \frac{1}{4}bx^4 \operatorname{arctanh}(c/x) + \frac{1}{8}c^4b \ln(c/x-1) + \frac{1}{12}bc^3x^3 + \frac{1}{4}bc^3x^3 - \frac{1}{8}c^4b \ln(1+c/x)$

**Maxima [A]** time = 0.978632, size = 77, normalized size = 1.54

$$\frac{1}{4}ax^4 + \frac{1}{24} \left( 6x^4 \operatorname{artanh}\left(\frac{c}{x}\right) - (3c^3 \log(c+x) - 3c^3 \log(-c+x) - 6c^2x - 2x^3)c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="maxima")`

[Out]  $\frac{1}{4}ax^4 + \frac{1}{24}(6x^4 \operatorname{arctanh}(c/x) - (3c^3 \log(c+x) - 3c^3 \log(-c+x) - 6c^2x - 2x^3)c) * b$

**Fricas [A]** time = 1.66774, size = 113, normalized size = 2.26

$$\frac{1}{4}bc^3x + \frac{1}{12}bcx^3 + \frac{1}{4}ax^4 - \frac{1}{8}(bc^4 - bx^4) \log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}bc^3x + \frac{1}{12}bcx^3 + \frac{1}{4}ax^4 - \frac{1}{8}(bc^4 - bx^4) \log(-(c+x)/(c-x))$

**Sympy [A]** time = 1.75908, size = 46, normalized size = 0.92

$$\frac{ax^4}{4} - \frac{bc^4 \operatorname{atanh}\left(\frac{c}{x}\right)}{4} + \frac{bc^3x}{4} + \frac{bcx^3}{12} + \frac{bx^4 \operatorname{atanh}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*atanh(c/x)),x)`

[Out]  $a*x**4/4 - b*c**4*atanh(c/x)/4 + b*c**3*x/4 + b*c*x**3/12 + b*x**4*atanh(c/x)/4$

---

**Giac [A]** time = 1.16809, size = 84, normalized size = 1.68

$$-\frac{1}{8}bc^4 \log(c+x) + \frac{1}{8}bc^4 \log(c-x) + \frac{1}{8}bx^4 \log\left(-\frac{c+x}{c-x}\right) + \frac{1}{4}bc^3x + \frac{1}{12}bcx^3 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x)),x, algorithm="giac")

[Out] -1/8\*b\*c^4\*log(c + x) + 1/8\*b\*c^4\*log(c - x) + 1/8\*b\*x^4\*log(-(c + x)/(c - x)) + 1/4\*b\*c^3\*x + 1/12\*b\*c\*x^3 + 1/4\*a\*x^4

### 3.136 $\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx$

**Optimal.** Leaf size=45

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{6}bc^3 \log(c^2 - x^2) + \frac{1}{6}bcx^2$$

[Out] (b\*c\*x^2)/6 + (x^3\*(a + b\*ArcTanh[c/x]))/3 + (b\*c^3\*Log[c^2 - x^2])/6

**Rubi [A]** time = 0.0327064, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 266, 43}

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{6}bc^3 \log(c^2 - x^2) + \frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c/x]), x]

[Out] (b\*c\*x^2)/6 + (x^3\*(a + b\*ArcTanh[c/x]))/3 + (b\*c^3\*Log[c^2 - x^2])/6

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx &= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{3} (bc) \int \frac{x}{1 - \frac{c^2}{x^2}} dx \\
&= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{3} (bc) \int \frac{x^3}{-c^2 + x^2} dx \\
&= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{6} (bc) \operatorname{Subst} \left( \int \frac{x}{-c^2 + x} dx, x, x^2 \right) \\
&= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{6} (bc) \operatorname{Subst} \left( \int \left( 1 - \frac{c^2}{c^2 - x} \right) dx, x, x^2 \right) \\
&= \frac{1}{6} bcx^2 + \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{6} bc^3 \log(c^2 - x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0079073, size = 50, normalized size = 1.11

$$\frac{ax^3}{3} + \frac{1}{6}bc^3 \log(x^2 - c^2) + \frac{1}{6}bcx^2 + \frac{1}{3}bx^3 \tanh^{-1}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTanh[c/x])/3 + (b*c^3*Log[-c^2 + x^2])/6
```

**Maple [A]** time = 0.013, size = 67, normalized size = 1.5

$$\frac{x^3 a}{3} + \frac{bx^3}{3} \operatorname{Arctanh}\left(\frac{c}{x}\right) + \frac{c^3 b}{6} \ln\left(\frac{c}{x} - 1\right) + \frac{bcx^2}{6} - \frac{c^3 b}{3} \ln\left(\frac{c}{x}\right) + \frac{c^3 b}{6} \ln\left(1 + \frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c/x)),x)
```

[Out]  $\frac{1}{3}x^3a + \frac{1}{3}bx^3\operatorname{arctanh}(c/x) + \frac{1}{6}c^3b\ln(c/x-1) + \frac{1}{6}bcx^2 - \frac{1}{3}c^3b\ln(c/x) + \frac{1}{6}c^3b\ln(1+c/x)$

**Maxima [A]** time = 0.961072, size = 57, normalized size = 1.27

$$\frac{1}{3}ax^3 + \frac{1}{6}\left(2x^3\operatorname{artanh}\left(\frac{c}{x}\right) + (c^2\log(-c^2 + x^2) + x^2)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="maxima")`

[Out]  $\frac{1}{3}ax^3 + \frac{1}{6}(2x^3\operatorname{arctanh}(c/x) + (c^2\log(-c^2 + x^2) + x^2)c)*b$

**Fricas [A]** time = 1.76935, size = 117, normalized size = 2.6

$$\frac{1}{6}bc^3\log(-c^2 + x^2) + \frac{1}{6}bx^3\log\left(-\frac{c+x}{c-x}\right) + \frac{1}{6}bcx^2 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="fricas")`

[Out]  $\frac{1}{6}bc^3\log(-c^2 + x^2) + \frac{1}{6}bx^3\log(-(c+x)/(c-x)) + \frac{1}{6}bcx^2 + \frac{1}{3}ax^3$

**Sympy [A]** time = 1.10867, size = 49, normalized size = 1.09

$$\frac{ax^3}{3} + \frac{bc^3\log(-c+x)}{3} + \frac{bc^3\operatorname{atanh}\left(\frac{c}{x}\right)}{3} + \frac{bcx^2}{6} + \frac{bx^3\operatorname{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c/x)),x)`

[Out]  $a*x**3/3 + b*c**3*\log(-c + x)/3 + b*c**3*\operatorname{atanh}(c/x)/3 + b*c*x**2/6 + b*x**3*\operatorname{atanh}(c/x)/3$

---

**Giac [A]** time = 1.1681, size = 66, normalized size = 1.47

$$\frac{1}{6}bc^3 \log(-c^2 + x^2) + \frac{1}{6}bx^3 \log\left(-\frac{c+x}{c-x}\right) + \frac{1}{6}bcx^2 + \frac{1}{3}ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x)),x, algorithm="giac")

[Out] 1/6\*b\*c^3\*log(-c^2 + x^2) + 1/6\*b\*x^3\*log(-(c + x)/(c - x)) + 1/6\*b\*c\*x^2 + 1/3\*a\*x^3

### 3.137 $\int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx$

**Optimal.** Leaf size=39

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) - \frac{1}{2}bc^2 \tanh^{-1} \left( \frac{x}{c} \right) + \frac{bcx}{2}$$

[Out] (b\*c\*x)/2 + (x^2\*(a + b\*ArcTanh[c/x]))/2 - (b\*c^2\*ArcTanh[x/c])/2

**Rubi [A]** time = 0.0202438, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6097, 193, 321, 207}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) - \frac{1}{2}bc^2 \tanh^{-1} \left( \frac{x}{c} \right) + \frac{bcx}{2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcTanh[c/x]),x]

[Out] (b\*c\*x)/2 + (x^2\*(a + b\*ArcTanh[c/x]))/2 - (b\*c^2\*ArcTanh[x/c])/2

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 193

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]



Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :-Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx &= \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{2} (bc) \int \frac{1}{1 - \frac{c^2}{x^2}} dx \\
 &= \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{2} (bc) \int \frac{x^2}{-c^2 + x^2} dx \\
 &= \frac{bcx}{2} + \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + \frac{1}{2} (bc^3) \int \frac{1}{-c^2 + x^2} dx \\
 &= \frac{bcx}{2} + \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) - \frac{1}{2} bc^2 \tanh^{-1} \left( \frac{x}{c} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.007661, size = 56, normalized size = 1.44

$$\frac{ax^2}{2} + \frac{1}{4}bc^2 \log(x-c) - \frac{1}{4}bc^2 \log(c+x) + \frac{1}{2}bx^2 \tanh^{-1} \left( \frac{c}{x} \right) + \frac{bcx}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c/x]), x]

[Out] (b\*c\*x)/2 + (a\*x^2)/2 + (b\*x^2\*ArcTanh[c/x])/2 + (b\*c^2\*Log[-c + x])/4 - (b\*c^2\*Log[c + x])/4

**Maple [A]** time = 0.01, size = 53, normalized size = 1.4

$$\frac{ax^2}{2} + \frac{bx^2}{2} \operatorname{Arctanh} \left( \frac{c}{x} \right) + \frac{xbc}{2} + \frac{c^2b}{4} \ln \left( \frac{c}{x} - 1 \right) - \frac{c^2b}{4} \ln \left( 1 + \frac{c}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c/x)), x)

[Out]  $\frac{1}{2}ax^2 + \frac{1}{2}\operatorname{arctanh}(c/x) * b * x^2 + \frac{1}{2}x * b * c + \frac{1}{4}c^2 * b * \ln(c/x - 1) - \frac{1}{4}c^2 * b * \ln(1 + c/x)$

**Maxima [A]** time = 0.969734, size = 59, normalized size = 1.51

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \operatorname{artanh}\left(\frac{c}{x}\right) - (c \log(c + x) - c \log(-c + x) - 2x)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c/x)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}ax^2 + \frac{1}{4}(2x^2 \operatorname{arctanh}(c/x) - (c \log(c + x) - c \log(-c + x) - 2x) * c) * b$

**Fricas [A]** time = 1.68662, size = 90, normalized size = 2.31

$$\frac{1}{2}bcx + \frac{1}{2}ax^2 - \frac{1}{4}(bc^2 - bx^2) \log\left(-\frac{c+x}{c-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c/x)),x, algorithm="fricas")`

[Out]  $\frac{1}{2}b * c * x + \frac{1}{2}a * x^2 - \frac{1}{4}(b * c^2 - b * x^2) * \log(-(c + x)/(c - x))$

**Sympy [A]** time = 0.486917, size = 36, normalized size = 0.92

$$\frac{ax^2}{2} - \frac{bc^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{bcx}{2} + \frac{bx^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c/x)),x)`

[Out]  $a * x^{**2} / 2 - b * c^{**2} * \operatorname{atanh}(c/x) / 2 + b * c * x / 2 + b * x^{**2} * \operatorname{atanh}(c/x) / 2$

---

**Giac [A]** time = 1.14094, size = 72, normalized size = 1.85

$$-\frac{1}{4}bc^2 \log(c+x) + \frac{1}{4}bc^2 \log(c-x) + \frac{1}{4}bx^2 \log\left(-\frac{c+x}{c-x}\right) + \frac{1}{2}bcx + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x)),x, algorithm="giac")

[Out] -1/4\*b\*c^2\*log(c + x) + 1/4\*b\*c^2\*log(c - x) + 1/4\*b\*x^2\*log(-(c + x)/(c - x)) + 1/2\*b\*c\*x + 1/2\*a\*x^2

### 3.138 $\int \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx$

**Optimal.** Leaf size=29

$$ax + \frac{1}{2}bc \log(c^2 - x^2) + bx \tanh^{-1} \left( \frac{c}{x} \right)$$

[Out] a\*x + b\*x\*ArcTanh[c/x] + (b\*c\*Log[c^2 - x^2])/2

**Rubi [A]** time = 0.0133345, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {6091, 263, 260}

$$ax + \frac{1}{2}bc \log(c^2 - x^2) + bx \tanh^{-1} \left( \frac{c}{x} \right)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c/x], x]

[Out] a\*x + b\*x\*ArcTanh[c/x] + (b\*c\*Log[c^2 - x^2])/2

#### Rule 6091

Int[ArcTanh[(c\_)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 263

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned}
\int \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) dx &= ax + b \int \tanh^{-1} \left( \frac{c}{x} \right) dx \\
&= ax + bx \tanh^{-1} \left( \frac{c}{x} \right) + (bc) \int \frac{1}{\left( 1 - \frac{c^2}{x^2} \right) x} dx \\
&= ax + bx \tanh^{-1} \left( \frac{c}{x} \right) + (bc) \int \frac{x}{-c^2 + x^2} dx \\
&= ax + bx \tanh^{-1} \left( \frac{c}{x} \right) + \frac{1}{2} bc \log(c^2 - x^2)
\end{aligned}$$

**Mathematica [A]** time = 0.0028142, size = 29, normalized size = 1.

$$ax + \frac{1}{2}bc \log(c^2 - x^2) + bx \tanh^{-1} \left( \frac{c}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c/x], x]

[Out] a\*x + b\*x\*ArcTanh[c/x] + (b\*c\*Log[c^2 - x^2])/2

**Maple [A]** time = 0.01, size = 48, normalized size = 1.7

$$ax + bx \operatorname{Artanh} \left( \frac{c}{x} \right) + \frac{bc}{2} \ln \left( \frac{c}{x} - 1 \right) - bc \ln \left( \frac{c}{x} \right) + \frac{bc}{2} \ln \left( 1 + \frac{c}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctanh(c/x), x)

[Out] a\*x+b\*x\*arctanh(c/x)+1/2\*b\*c\*ln(c/x-1)-b\*c\*ln(c/x)+1/2\*b\*c\*ln(1+c/x)

**Maxima [A]** time = 0.949895, size = 39, normalized size = 1.34

$$\frac{1}{2} \left( 2x \operatorname{artanh} \left( \frac{c}{x} \right) + c \log(-c^2 + x^2) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c/x),x, algorithm="maxima")

[Out] 1/2\*(2\*x\*arctanh(c/x) + c\*log(-c^2 + x^2))\*b + a\*x

**Fricas [A]** time = 1.67863, size = 85, normalized size = 2.93

$$\frac{1}{2}bc \log(-c^2 + x^2) + \frac{1}{2}bx \log\left(-\frac{c+x}{c-x}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c/x),x, algorithm="fricas")

[Out] 1/2\*b\*c\*log(-c^2 + x^2) + 1/2\*b\*x\*log(-(c + x)/(c - x)) + a\*x

**Sympy [A]** time = 0.317447, size = 24, normalized size = 0.83

$$ax + b\left(c \log(c - x) + c \operatorname{atanh}\left(\frac{c}{x}\right) + x \operatorname{atanh}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c/x),x)

[Out] a\*x + b\*(c\*log(c - x) + c\*atanh(c/x) + x\*atanh(c/x))

**Giac [A]** time = 1.13099, size = 57, normalized size = 1.97

$$\frac{1}{2}\left(x \log\left(-\frac{\frac{c}{x}+1}{\frac{c}{x}-1}\right) + c \log(|-c^2 + x^2|)\right)b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c/x),x, algorithm="giac")

[Out] 1/2\*(x\*log(-(c/x + 1)/(c/x - 1)) + c\*log(abs(-c^2 + x^2)))\*b + a\*x

$$3.139 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x} dx$$

**Optimal.** Leaf size=30

$$\frac{1}{2}b\text{PolyLog}\left(2, -\frac{c}{x}\right) - \frac{1}{2}b\text{PolyLog}\left(2, \frac{c}{x}\right) + a \log(x)$$

[Out] a\*Log[x] + (b\*PolyLog[2, -(c/x)])/2 - (b\*PolyLog[2, c/x])/2

**Rubi [A]** time = 0.031678, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6095, 5912}

$$\frac{1}{2}b\text{PolyLog}\left(2, -\frac{c}{x}\right) - \frac{1}{2}b\text{PolyLog}\left(2, \frac{c}{x}\right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x])/x, x]

[Out] a\*Log[x] + (b\*PolyLog[2, -(c/x)])/2 - (b\*PolyLog[2, c/x])/2

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x} dx &= -\text{Subst}\left(\int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, \frac{1}{x}\right) \\ &= a \log(x) + \frac{1}{2}b\text{Li}_2\left(-\frac{c}{x}\right) - \frac{1}{2}b\text{Li}_2\left(\frac{c}{x}\right) \end{aligned}$$

**Mathematica [A]** time = 0.012366, size = 28, normalized size = 0.93

$$\frac{1}{2}b \left( \text{PolyLog} \left( 2, -\frac{c}{x} \right) - \text{PolyLog} \left( 2, \frac{c}{x} \right) \right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x])/x,x]

[Out] a\*Log[x] + (b\*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2

**Maple [B]** time = 0.014, size = 63, normalized size = 2.1

$$-a \ln \left( \frac{c}{x} \right) - b \ln \left( \frac{c}{x} \right) \text{Artanh} \left( \frac{c}{x} \right) + \frac{b}{2} \text{dilog} \left( \frac{c}{x} \right) + \frac{b}{2} \text{dilog} \left( 1 + \frac{c}{x} \right) + \frac{b}{2} \ln \left( \frac{c}{x} \right) \ln \left( 1 + \frac{c}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x))/x,x)

[Out] -a\*ln(c/x)-b\*ln(c/x)\*arctanh(c/x)+1/2\*b\*dilog(c/x)+1/2\*b\*dilog(1+c/x)+1/2\*b\*ln(c/x)\*ln(1+c/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b \int \frac{\log \left( \frac{c}{x} + 1 \right) - \log \left( -\frac{c}{x} + 1 \right)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x,x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(c/x + 1) - log(-c/x + 1))/x, x) + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b \text{artanh} \left( \frac{c}{x} \right) + a}{x}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c/x) + a)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))/x,x)

[Out] Integral((a + b\*atanh(c/x))/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}\left(\frac{c}{x}\right) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)/x, x)

$$3.140 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x^2} dx$$

**Optimal.** Leaf size=35

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x} - \frac{b \log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

[Out]  $-\left(\left(a+b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right) / x\right) - \left(b \operatorname{Log}\left[1-\frac{c^2}{x^2}\right]\right) / \left(2 * c\right)$

**Rubi [A]** time = 0.0208549, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 260}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x} - \frac{b \log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(a+b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right) / x^2, x\right]$

[Out]  $-\left(\left(a+b \operatorname{ArcTanh}\left[\frac{c}{x}\right]\right) / x\right) - \left(b \operatorname{Log}\left[1-\frac{c^2}{x^2}\right]\right) / \left(2 * c\right)$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{x^2} dx = -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{x} - (bc) \int \frac{1}{\left(1 - \frac{c^2}{x^2}\right)x^3} dx$$

$$= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{x} - \frac{b \log\left(1 - \frac{c^2}{x^2}\right)}{2c}$$

**Mathematica [A]** time = 0.0082962, size = 38, normalized size = 1.09

$$-\frac{a}{x} - \frac{b \log\left(1 - \frac{c^2}{x^2}\right)}{2c} - \frac{b \tanh^{-1}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x])/x^2,x]

[Out] -(a/x) - (b\*ArcTanh[c/x])/x - (b\*Log[1 - c^2/x^2])/(2\*c)

**Maple [A]** time = 0.004, size = 37, normalized size = 1.1

$$-\frac{a}{x} - \frac{b}{x} \operatorname{Arctanh}\left(\frac{c}{x}\right) - \frac{b}{2c} \ln\left(1 - \frac{c^2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x))/x^2,x)

[Out] -a/x-b/x\*arctanh(c/x)-1/2\*b\*ln(1-c^2/x^2)/c

**Maxima [A]** time = 0.956861, size = 50, normalized size = 1.43

$$-\frac{b\left(\frac{2c \operatorname{artanh}\left(\frac{c}{x}\right)}{x} + \log\left(-\frac{c^2}{x^2} + 1\right)\right)}{2c} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x^2,x, algorithm="maxima")

[Out]  $-1/2*b*(2*c*arctanh(c/x)/x + \log(-c^2/x^2 + 1))/c - a/x$

**Fricas [A]** time = 1.69369, size = 115, normalized size = 3.29

$$\frac{bx \log(-c^2 + x^2) - 2bx \log(x) + bc \log\left(-\frac{c+x}{c-x}\right) + 2ac}{2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x^2,x, algorithm="fricas")

[Out]  $-1/2*(b*x*\log(-c^2 + x^2) - 2*b*x*\log(x) + b*c*\log(-(c + x)/(c - x)) + 2*a*c)/(c*x)$

**Sympy [A]** time = 1.8518, size = 39, normalized size = 1.11

$$\begin{cases} -\frac{a}{x} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{x} + \frac{b \log(x)}{c} - \frac{b \log(-c+x)}{c} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{c} & \text{for } c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))/x\*\*2,x)

[Out] Piecewise((-a/x - b\*atanh(c/x)/x + b\*log(x)/c - b\*log(-c + x)/c - b\*atanh(c/x)/c, Ne(c, 0)), (-a/x, True))

**Giac [A]** time = 1.13314, size = 66, normalized size = 1.89

$$-\frac{1}{2}b \left( \frac{\log\left(-\frac{\frac{c}{x}+1}{\frac{c}{x}-1}\right)}{x} + \frac{\log\left(\left|\frac{c^2}{x^2} - 1\right|\right)}{c} \right) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*b*(log(-(c/x + 1)/(c/x - 1)))/x + log(abs(c^2/x^2 - 1))/c - a/x
```

$$3.141 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x^3} dx$$

**Optimal.** Leaf size=43

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b \tanh^{-1}\left(\frac{x}{c}\right)}{2c^2} - \frac{b}{2cx}$$

[Out]  $-b/(2*c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)$

**Rubi [A]** time = 0.0266653, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 325, 207}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b \tanh^{-1}\left(\frac{x}{c}\right)}{2c^2} - \frac{b}{2cx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x])/x^3,x]

[Out]  $-b/(2*c*x) - (a + b*ArcTanh[c/x])/(2*x^2) + (b*ArcTanh[x/c])/(2*c^2)$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 325

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

x]

**Rule 207**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

**Rubi steps**

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{x^3} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc) \int \frac{1}{\left(1 - \frac{c^2}{x^2}\right)x^4} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{1}{2}(bc) \int \frac{1}{x^2(-c^2 + x^2)} dx \\
 &= -\frac{b}{2cx} - \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{b \int \frac{1}{-c^2 + x^2} dx}{2c} \\
 &= -\frac{b}{2cx} - \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} + \frac{b \tanh^{-1}\left(\frac{x}{c}\right)}{2c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.0090805, size = 60, normalized size = 1.4

$$-\frac{a}{2x^2} - \frac{b \log(x-c)}{4c^2} + \frac{b \log(c+x)}{4c^2} - \frac{b \tanh^{-1}\left(\frac{c}{x}\right)}{2x^2} - \frac{b}{2cx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x])/x^3, x]

[Out] -a/(2\*x^2) - b/(2\*c\*x) - (b\*ArcTanh[c/x])/(2\*x^2) - (b\*Log[-c + x])/(4\*c^2) + (b\*Log[c + x])/(4\*c^2)

**Maple [A]** time = 0.007, size = 57, normalized size = 1.3

$$-\frac{a}{2x^2} - \frac{b}{2x^2} \operatorname{Arctanh}\left(\frac{c}{x}\right) - \frac{b}{2cx} - \frac{b}{4c^2} \ln\left(\frac{c}{x} - 1\right) + \frac{b}{4c^2} \ln\left(1 + \frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x))/x^3,x)`

[Out]  $-1/2*a/x^2-1/2*b/x^2*arctanh(c/x)-1/2*b/c/x-1/4/c^2*b*\ln(c/x-1)+1/4/c^2*b*\ln(1+c/x)$

**Maxima [A]** time = 0.95463, size = 70, normalized size = 1.63

$$\frac{1}{4} \left( c \left( \frac{\log(c+x)}{c^3} - \frac{\log(-c+x)}{c^3} - \frac{2}{c^2 x} \right) - \frac{2 \operatorname{artanh}\left(\frac{c}{x}\right)}{x^2} \right) b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))/x^3,x, algorithm="maxima")`

[Out]  $1/4*(c*(\log(c+x)/c^3 - \log(-c+x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)*b - 1/2*a/x^2$

**Fricas [A]** time = 1.76097, size = 103, normalized size = 2.4

$$-\frac{2ac^2 + 2bcx + (bc^2 - bx^2) \log\left(-\frac{c+x}{c-x}\right)}{4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))/x^3,x, algorithm="fricas")`

[Out]  $-1/4*(2*a*c^2 + 2*b*c*x + (b*c^2 - b*x^2)*\log(-(c+x)/(c-x)))/(c^2*x^2)$

**Sympy [A]** time = 2.11399, size = 44, normalized size = 1.02

$$\begin{cases} -\frac{a}{2x^2} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{2x^2} - \frac{b}{2cx} + \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b*atanh(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atanh(c/x)/(2*x**2) - b/(2*c*x) + b*atanh(c/x)/(2*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

**Giac [A]** time = 1.16435, size = 77, normalized size = 1.79

$$\frac{b \log(c+x)}{4c^2} - \frac{b \log(c-x)}{4c^2} - \frac{b \log\left(-\frac{c+x}{c-x}\right)}{4x^2} - \frac{ac+bx}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b*log(c+x)/c^2 - 1/4*b*log(c-x)/c^2 - 1/4*b*log(-(c+x)/(c-x))/x^2 - 1/2*(a*c+b*x)/(c*x^2)
```

$$3.142 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{x^4} dx$$

**Optimal.** Leaf size=57

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{b \log(c^2-x^2)}{6c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{6cx^2}$$

[Out]  $-b/(6*c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2 - x^2])/(6*c^3)$

**Rubi [A]** time = 0.0407439, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 266, 44}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{b \log(c^2-x^2)}{6c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{6cx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x])/x^4, x]

[Out]  $-b/(6*c*x^2) - (a + b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2 - x^2])/(6*c^3)$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{x^4} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{3}(bc) \int \frac{1}{\left(1 - \frac{c^2}{x^2}\right)x^5} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{3}(bc) \int \frac{1}{x^3(-c^2 + x^2)} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{6}(bc) \operatorname{Subst}\left(\int \frac{1}{x^2(-c^2 + x)} dx, x, x^2\right) \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} - \frac{1}{6}(bc) \operatorname{Subst}\left(\int \left(-\frac{1}{c^4(c^2 - x)} - \frac{1}{c^2x^2} - \frac{1}{c^4x}\right) dx, x, x^2\right) \\
 &= -\frac{b}{6cx^2} - \frac{a + b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3} + \frac{b \log(x)}{3c^3} - \frac{b \log(c^2 - x^2)}{6c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.0093336, size = 62, normalized size = 1.09

$$-\frac{a}{3x^3} - \frac{b \log(x^2 - c^2)}{6c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{6cx^2} - \frac{b \tanh^{-1}\left(\frac{c}{x}\right)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x])/x^4, x]

[Out] -a/(3\*x^3) - b/(6\*c\*x^2) - (b\*ArcTanh[c/x])/(3\*x^3) + (b\*Log[x])/(3\*c^3) - (b\*Log[-c^2 + x^2])/(6\*c^3)

**Maple [A]** time = 0.007, size = 57, normalized size = 1.

$$-\frac{a}{3x^3} - \frac{b}{3x^3} \operatorname{Arctanh}\left(\frac{c}{x}\right) - \frac{b}{6cx^2} - \frac{b}{6c^3} \ln\left(\frac{c}{x} - 1\right) - \frac{b}{6c^3} \ln\left(1 + \frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x))/x^4,x)`

[Out] `-1/3*a/x^3-1/3*b/x^3*arctanh(c/x)-1/6*b/c/x^2-1/6/c^3*b*ln(c/x-1)-1/6/c^3*b*ln(1+c/x)`

**Maxima [A]** time = 0.96182, size = 74, normalized size = 1.3

$$-\frac{1}{6} \left( c \left( \frac{\log(-c^2 + x^2)}{c^4} - \frac{\log(x^2)}{c^4} + \frac{1}{c^2 x^2} \right) + \frac{2 \operatorname{artanh}\left(\frac{c}{x}\right)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))/x^4,x, algorithm="maxima")`

[Out] `-1/6*(c*(log(-c^2 + x^2)/c^4 - log(x^2)/c^4 + 1/(c^2*x^2)) + 2*arctanh(c/x)/x^3)*b - 1/3*a/x^3`

**Fricas [A]** time = 1.67192, size = 144, normalized size = 2.53

$$-\frac{bx^3 \log(-c^2 + x^2) - 2bx^3 \log(x) + bc^3 \log\left(-\frac{c+x}{c-x}\right) + 2ac^3 + bc^2x}{6c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))/x^4,x, algorithm="fricas")`

[Out] `-1/6*(b*x^3*log(-c^2 + x^2) - 2*b*x^3*log(x) + b*c^3*log(-(c + x)/(c - x)) + 2*a*c^3 + b*c^2*x)/(c^3*x^3)`

**Sympy [A]** time = 3.6407, size = 68, normalized size = 1.19

$$\begin{cases} -\frac{a}{3x^3} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{3x^3} - \frac{b}{6cx^2} + \frac{b \log(x)}{3c^3} - \frac{b \log(-c+x)}{3c^3} - \frac{b \operatorname{atanh}\left(\frac{c}{x}\right)}{3c^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))/x\*\*4,x)

[Out] Piecewise((-a/(3\*x\*\*3) - b\*atanh(c/x)/(3\*x\*\*3) - b/(6\*c\*x\*\*2) + b\*log(x)/(3\*c\*\*3) - b\*log(-c + x)/(3\*c\*\*3) - b\*atanh(c/x)/(3\*c\*\*3), Ne(c, 0)), (-a/(3\*x\*\*3), True))

**Giac [A]** time = 1.12926, size = 85, normalized size = 1.49

$$-\frac{b \log(-c^2 + x^2)}{6c^3} + \frac{b \log(x)}{3c^3} - \frac{b \log\left(-\frac{c+x}{c-x}\right)}{6x^3} - \frac{2ac^2 + bcx}{6c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))/x^4,x, algorithm="giac")

[Out] -1/6\*b\*log(-c^2 + x^2)/c^3 + 1/3\*b\*log(x)/c^3 - 1/6\*b\*log(-(c + x)/(c - x))/x^3 - 1/6\*(2\*a\*c^2 + b\*c\*x)/(c^2\*x^3)

### 3.143 $\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx$

**Optimal.** Leaf size=123

$$-\frac{1}{4}c^4 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 + \frac{1}{2}bc^3x \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) + \frac{1}{6}bcx^3 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) + \frac{1}{4}x^4 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 + \frac{1}{12}b^2c^2x$$

[Out] (b^2\*c^2\*x^2)/12 + (b\*c^3\*x\*(a + b\*ArcCoth[x/c]))/2 + (b\*c\*x^3\*(a + b\*ArcCoth[x/c]))/6 - (c^4\*(a + b\*ArcCoth[x/c])^2)/4 + (x^4\*(a + b\*ArcCoth[x/c])^2)/4 + (b^2\*c^4\*Log[1 - c^2/x^2])/3 + (2\*b^2\*c^4\*Log[x])/3

**Rubi [C]** time = 1.70313, antiderivative size = 812, normalized size of antiderivative = 6.6, number of steps used = 88, number of rules used = 34, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.125$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{16} \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 c^4 - \frac{1}{16} b^2 \log^2 \left( \frac{c+x}{x} \right) c^4 + \frac{5}{48} b^2 \log \left( 1 - \frac{c}{x} \right) c^4 + \frac{5}{48} b^2 \log(c-x) c^4 + \frac{1}{8} b^2 \log \left( \frac{c}{x} + 1 \right) \log(c -$$

Warning: Unable to verify antiderivative.

[In] Int[x^3\*(a + b\*ArcTanh[c/x])^2,x]

[Out] (a\*b\*c^3\*x)/4 - (a\*b\*c^2\*x^2)/8 + (b^2\*c^2\*x^2)/12 + (a\*b\*c\*x^3)/12 + (5\*b^2\*c^4\*Log[1 - c/x])/48 - (b^2\*c^3\*x\*Log[1 - c/x])/8 + (b^2\*c^2\*x^2\*Log[1 - c/x])/16 - (b^2\*c\*x^3\*Log[1 - c/x])/24 + (b\*c^3\*(1 - c/x)\*x\*(2\*a - b\*Log[1 - c/x]))/8 + (b\*c^2\*x^2\*(2\*a - b\*Log[1 - c/x]))/16 + (b\*c\*x^3\*(2\*a - b\*Log[1 - c/x]))/24 - (c^4\*(2\*a - b\*Log[1 - c/x])^2)/16 + (x^4\*(2\*a - b\*Log[1 - c/x])^2)/16 + (b^2\*c^3\*x\*Log[1 + c/x])/8 + (b^2\*c^2\*x^2\*Log[1 + c/x])/16 + (b^2\*c\*x^3\*Log[1 + c/x])/24 + (a\*b\*x^4\*Log[1 + c/x])/4 - (b^2\*x^4\*Log[1 - c/x]\*Log[1 + c/x])/8 + (5\*b^2\*c^4\*Log[c - x])/48 + (b^2\*c^4\*Log[1 + c/x]\*Log[c - x])/8 + (a\*b\*c^4\*Log[x])/4 + (11\*b^2\*c^4\*Log[x])/24 + (b^2\*c^4\*Log[c - x]\*Log[x/c])/8 - (a\*b\*c^4\*Log[c + x])/4 + (5\*b^2\*c^4\*Log[c + x])/48 + (b^2\*c^4\*Log[1 - c/x]\*Log[c + x])/8 - (b^2\*c^4\*Log[(c - x)/(2\*c)]\*Log[c + x])/8 + (b^2\*c^4\*Log[-(x/c)]\*Log[c + x])/8 - (b^2\*c^4\*Log[c - x]\*Log[(c + x)/(2\*c)])/8 + (11\*b^2\*c^4\*Log[(c + x)/x])/48 + (b^2\*c^3\*x\*Log[(c + x)/x])/8 - (b^2\*c^2\*x^2\*Log[(c + x)/x])/16 + (b^2\*c\*x^3\*Log[(c + x)/x])/24 - (b^2\*c^4\*Log[(c + x)/x]^2)/16 + (b^2\*x^4\*Log[(c + x)/x]^2)/16 - (b^2\*c^4\*PolyLog[2, (c - x)/(2\*c)])/8 - (b^2\*c^4\*PolyLog[2, -(c/x)])/8 - (b^2\*c^4\*PolyLog[2, c/x])/8 - (b^2\*c^4\*PolyLog[2, (c + x)/(2\*c)])/8 + (b^2\*c^4\*PolyLog[2, 1 - x/c])/8

$$8 + (b^2*c^4*PolyLog[2, 1 + x/c])/8$$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegerQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2347

Int[(((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(q\_)))/(x\_), x\_Symbol] :> Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

### Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[

$(a + b \cdot \log[c \cdot x^n])^p / (d + e \cdot x), x, x$  /; FreeQ[{a, b, c, d, e, n}, x] && I  
GtQ[p, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Lo  
g[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)\*(b\_.)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[  
((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/  
(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 -  
c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_), x  
\_Symbol] := Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b  
\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x  
] && EqQ[r\*(q + 1) + 1, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2319

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.),  
x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/(e\*(q + 1)), x]  
- Dist[(b\*n\*p)/(e\*(q + 1)), Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^(p -  
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,  
-1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] &&  
NeQ[q, 1]))

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[  
ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &  
& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m



+ n + 2, 0])

### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rule 2557

Int[Log[v\_]\*Log[w\_]\*(u\_), x\_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]\*Log[w], z, x] + (-Int[SimplifyIntegrand[(z\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p]]^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

#### Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

#### Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

#### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((h_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

$\text{Int}[\text{Log}[(c\_.) * ((d\_.) + (e\_.) * (x\_.)^{(n\_.)})] / (x\_.), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c * e * x^n)] / n, x] \text{ ; FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c * d, 1]$

Rule 193

$\text{Int}[(a\_.) + (b\_.) * (x\_.)^{(n\_.)}]^{(p\_.)}, x\_Symbol] \rightarrow \text{Int}[x^{(n * p)} * (b + a / x^n)^p, x] \text{ ; FreeQ}[\{a, b\}, x] \ \&\& \ \text{LtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$

Rule 2410

$\text{Int}[(\text{Log}[(c\_.) * ((d\_.) + (e\_.) * (x\_.)^{(m\_.)})] * (x\_.)^{(m\_.)}) / ((f\_.) + (g\_.) * (x\_.)^{(m\_.)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[\text{Log}[c * (d + e * x)], x^m / (f + g * x), x], x] \text{ ; FreeQ}[\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[e * f - d * g, 0] \ \&\& \ \text{EqQ}[c * d, 1] \ \&\& \ \text{IntegerQ}[m]$

Rule 2395

$\text{Int}[(a\_.) + \text{Log}[(c\_.) * ((d\_.) + (e\_.) * (x\_.)^{(n\_.)})] * (b\_.) * ((f\_.) + (g\_.) * (x\_.)^{(q\_.)})^{(q\_.)}, x\_Symbol] \rightarrow \text{Simp}[(f + g * x)^{(q + 1)} * (a + b * \text{Log}[c * (d + e * x)^n]) / (g * (q + 1)), x] - \text{Dist}[(b * e * n) / (g * (q + 1)), \text{Int}[(f + g * x)^{(q + 1)} / (d + e * x), x], x] \text{ ; FreeQ}[\{a, b, c, d, e, f, g, n, q\}, x] \ \&\& \ \text{NeQ}[e * f - d * g, 0] \ \&\& \ \text{NeQ}[q, -1]$

Rule 36

$\text{Int}[1 / (((a\_.) + (b\_.) * (x\_.)^{(n\_.)}) * ((c\_.) + (d\_.) * (x\_.)^{(m\_.)})), x\_Symbol] \rightarrow \text{Dist}[b / (b * c - a * d), \text{Int}[1 / (a + b * x), x], x] - \text{Dist}[d / (b * c - a * d), \text{Int}[1 / (c + d * x), x], x] \text{ ; FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b * c - a * d, 0]$

Rule 29

$\text{Int}[(x\_.)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

Rule 2390

$\text{Int}[(a\_.) + \text{Log}[(c\_.) * ((d\_.) + (e\_.) * (x\_.)^{(n\_.)})] * (b\_.)]^{(p\_.)} * ((f\_.) + (g\_.) * (x\_.)^{(q\_.)}), x\_Symbol] \rightarrow \text{Dist}[1 / e, \text{Subst}[\text{Int}[(f * x) / d]^{q * (a + b * \text{Log}[c * x^n])^p}, x], x, d + e * x], x] \text{ ; FreeQ}[\{a, b, c, d, e, f, g, n, p, q\}, x] \ \&\& \ \text{EqQ}[e * f - d * g, 0]$

Rubi steps



**Mathematica [A]** time = 0.0610989, size = 131, normalized size = 1.07

$$\frac{1}{12} \left( 3a^2x^4 + 2bx \tanh^{-1} \left( \frac{c}{x} \right) (3ax^3 + bc(3c^2 + x^2)) + 6abc^3x + bc^4(3a + 4b) \log(x - c) - 3abc^4 \log(c + x) + 2abcx^3 + b^2c^4 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c/x])^2,x]

[Out] (6\*a\*b\*c^3\*x + b^2\*c^2\*x^2 + 2\*a\*b\*c\*x^3 + 3\*a^2\*x^4 + 2\*b\*x\*(3\*a\*x^3 + b\*c\*(3\*c^2 + x^2))\*ArcTanh[c/x] + 3\*b^2\*(-c^4 + x^4)\*ArcTanh[c/x]^2 + b\*(3\*a + 4\*b)\*c^4\*Log[-c + x] - 3\*a\*b\*c^4\*Log[c + x] + 4\*b^2\*c^4\*Log[c + x])/12

**Maple [B]** time = 0.007, size = 328, normalized size = 2.7

$$\frac{a^2x^4}{4} + \frac{x^4b^2}{4} \left( \operatorname{Arctanh} \left( \frac{c}{x} \right) \right)^2 + \frac{c^4b^2}{4} \operatorname{Arctanh} \left( \frac{c}{x} \right) \ln \left( \frac{c}{x} - 1 \right) + \frac{b^2cx^3}{6} \operatorname{Arctanh} \left( \frac{c}{x} \right) + \frac{c^3b^2x}{2} \operatorname{Arctanh} \left( \frac{c}{x} \right) - \frac{c^4b^2}{4} \operatorname{Arctanh} \left( \frac{c}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c/x))^2,x)

[Out] 1/4\*a^2\*x^4+1/4\*b^2\*x^4\*arctanh(c/x)^2+1/4\*c^4\*b^2\*arctanh(c/x)\*ln(c/x-1)+1/6\*c\*b^2\*x^3\*arctanh(c/x)+1/2\*c^3\*b^2\*x\*arctanh(c/x)-1/4\*c^4\*b^2\*arctanh(c/x)\*ln(1+c/x)+1/16\*c^4\*b^2\*ln(c/x-1)^2-1/8\*c^4\*b^2\*ln(c/x-1)\*ln(1/2+1/2\*c/x)+1/8\*c^4\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1/2+1/2\*c/x)-1/8\*c^4\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1+c/x)+1/16\*c^4\*b^2\*ln(1+c/x)^2+1/3\*c^4\*b^2\*ln(c/x-1)+1/12\*b^2\*c^2\*x^2-2/3\*c^4\*b^2\*ln(c/x)+1/3\*c^4\*b^2\*ln(1+c/x)+1/2\*a\*b\*x^4\*arctanh(c/x)+1/4\*c^4\*a\*b\*ln(c/x-1)+1/6\*a\*b\*c\*x^3+1/2\*c^3\*a\*b\*x-1/4\*c^4\*a\*b\*ln(1+c/x)

**Maxima [A]** time = 0.981755, size = 255, normalized size = 2.07

$$\frac{1}{4} b^2 x^4 \operatorname{artanh} \left( \frac{c}{x} \right)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{12} \left( 6 x^4 \operatorname{artanh} \left( \frac{c}{x} \right) - (3 c^3 \log(c + x) - 3 c^3 \log(-c + x) - 6 c^2 x - 2 x^3) c \right) a b + \frac{1}{48} \left( (3 c^4 x^4 + 2 b x \operatorname{artanh} \left( \frac{c}{x} \right) (3 a x^3 + b c (3 c^2 + x^2)) + 6 a b c^3 x + b c^4 (3 a + 4 b) \log(x - c) - 3 a b c^4 \log(c + x) + 2 a b c x^3 + b^2 c^4) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}b^2x^4\operatorname{arctanh}(c/x)^2 + \frac{1}{4}a^2x^4 + \frac{1}{12}(6x^4\operatorname{arctanh}(c/x) - (3c^3\log(c+x) - 3c^3\log(-c+x) - 6c^2x - 2x^3)c)ab + \frac{1}{48}((3c^2\log(c+x)^2 + 3c^2\log(-c+x)^2 + 16c^2\log(c+x) + 4x^2 - 2(3c^2\log(c+x) - 8c^2)\log(-c+x))c^2 - 4(3c^3\log(c+x) - 3c^3\log(-c+x) - 6c^2x - 2x^3)c\operatorname{arctanh}(c/x))b^2$

**Fricas [A]** time = 1.67716, size = 350, normalized size = 2.85

$$\frac{1}{2}abc^3x + \frac{1}{12}b^2c^2x^2 + \frac{1}{6}abcx^3 + \frac{1}{4}a^2x^4 - \frac{1}{12}(3ab - 4b^2)c^4\log(c+x) + \frac{1}{12}(3ab + 4b^2)c^4\log(-c+x) - \frac{1}{16}(b^2c^4 - b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2}abc^3x + \frac{1}{12}b^2c^2x^2 + \frac{1}{6}abcx^3 + \frac{1}{4}a^2x^4 - \frac{1}{12}(3ab - 4b^2)c^4\log(c+x) + \frac{1}{12}(3ab + 4b^2)c^4\log(-c+x) - \frac{1}{16}(b^2c^4 - b^2x^4)\log(-(c+x)/(c-x))^2 + \frac{1}{12}(3b^2c^3x + b^2cx^3 + 3abx^4)\log(-(c+x)/(c-x))$

**Sympy [A]** time = 3.44274, size = 158, normalized size = 1.28

$$\frac{a^2x^4}{4} - \frac{abc^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{abc^3x}{2} + \frac{abcx^3}{6} + \frac{abx^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{2b^2c^4\log(-c+x)}{3} - \frac{b^2c^4\operatorname{atanh}^2\left(\frac{c}{x}\right)}{4} + \frac{2b^2c^4\operatorname{atanh}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*atanh(c/x))**2,x)`

[Out]  $a**2*x**4/4 - a*b*c**4*\operatorname{atanh}(c/x)/2 + a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**4*\operatorname{atanh}(c/x)/2 + 2*b**2*c**4*\log(-c+x)/3 - b**2*c**4*\operatorname{atanh}(c/x)**2/4 + 2*b**2*c**4*\operatorname{atanh}(c/x)/3 + b**2*c**3*x*\operatorname{atanh}(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*\operatorname{atanh}(c/x)/6 + b**2*x**4*\operatorname{atanh}(c/x)**2/4$

**Giac [A]** time = 1.22244, size = 209, normalized size = 1.7

$$\frac{1}{2}abc^3x + \frac{1}{12}b^2c^2x^2 + \frac{1}{6}abcx^3 + \frac{1}{4}a^2x^4 - \frac{1}{16}(b^2c^4 - b^2x^4)\log\left(\frac{-c+x}{c-x}\right)^2 - \frac{1}{12}(3abc^4 - 4b^2c^4)\log(c+x) + \frac{1}{12}(3abc^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/16*(b^2*c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 - 1/12*(3*a*b*c^4 - 4*b^2*c^4)*log(c + x) + 1/12*(3*a*b*c^4 + 4*b^2*c^4)*log(c - x) + 1/12*(3*b^2*c^3*x + b^2*c*x^3 + 3*a*b*x^4)*log(-(c + x)/(c - x))
```

### 3.144 $\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx$

**Optimal.** Leaf size=142

$$\frac{1}{3}b^2c^3\text{PolyLog}\left(2, \frac{2}{\frac{c}{x}+1} - 1\right) - \frac{1}{3}c^3\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{2}{3}bc^3 \log\left(2 - \frac{2}{\frac{c}{x}+1}\right)\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{3}bcx^2\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)$$

[Out] (b^2\*c^2\*x)/3 - (b^2\*c^3\*ArcCoth[x/c])/3 + (b\*c\*x^2\*(a + b\*ArcCoth[x/c]))/3 - (c^3\*(a + b\*ArcCoth[x/c])^2)/3 + (x^3\*(a + b\*ArcCoth[x/c])^2)/3 - (2\*b\*c^3\*(a + b\*ArcCoth[x/c])\*Log[2 - 2/(1 + c/x)])/3 + (b^2\*c^3\*PolyLog[2, -1 + 2/(1 + c/x)])/3

**Rubi [B]** time = 1.38924, antiderivative size = 695, normalized size of antiderivative = 4.89, number of steps used = 73, number of rules used = 34, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.125$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2319, 44, 2455, 263, 43, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 193, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c-x}{2c}\right) + \frac{1}{6}b^2c^3\text{PolyLog}\left(2, -\frac{c}{x}\right) - \frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c}{x}\right) + \frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c+x}{2c}\right) + \frac{1}{6}b^2c^3\text{PolyLog}\left(2, \frac{c+x}{2c}\right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^2\*(a + b\*ArcTanh[c/x])^2,x]

[Out] -(a\*b\*c^2\*x)/3 + (b^2\*c^2\*x)/3 + (a\*b\*c\*x^2)/6 + (b^2\*c^3\*Log[1 - c/x])/12 + (b^2\*c^2\*x\*Log[1 - c/x])/6 - (b^2\*c\*x^2\*Log[1 - c/x])/12 + (b\*c^2\*(1 - c/x)\*x\*(2\*a - b\*Log[1 - c/x]))/6 + (b\*c\*x^2\*(2\*a - b\*Log[1 - c/x]))/12 - (c^3\*(2\*a - b\*Log[1 - c/x])^2)/12 + (x^3\*(2\*a - b\*Log[1 - c/x])^2)/12 + (b^2\*c^2\*x\*Log[1 + c/x])/6 + (b^2\*c\*x^2\*Log[1 + c/x])/12 + (a\*b\*x^3\*Log[1 + c/x])/3 - (b^2\*x^3\*Log[1 - c/x]\*Log[1 + c/x])/6 - (b^2\*c^3\*Log[c - x])/12 + (b^2\*c^3\*Log[1 + c/x]\*Log[c - x])/6 + (a\*b\*c^3\*Log[x])/3 + (b^2\*c^3\*Log[c - x]\*Log[x/c])/6 + (a\*b\*c^3\*Log[c + x])/3 + (b^2\*c^3\*Log[c + x])/12 - (b^2\*c^3\*Log[1 - c/x]\*Log[c + x])/6 + (b^2\*c^3\*Log[(c - x)/(2\*c)]\*Log[c + x])/6 - (b^2\*c^3\*Log[-(x/c)]\*Log[c + x])/6 - (b^2\*c^3\*Log[c - x]\*Log[(c + x)/(2\*c)])/6 - (b^2\*c^3\*Log[(c + x)/x])/4 - (b^2\*c^2\*x\*Log[(c + x)/x])/6 + (b^2\*c\*x^2\*Log[(c + x)/x])/12 + (b^2\*c^3\*Log[(c + x)/x]^2)/12 + (b^2\*x^3\*Log[(c + x)/x]^2)/12 - (b^2\*c^3\*PolyLog[2, (c - x)/(2\*c)])/6 + (b^2\*c^3\*PolyLog[2, -(c/x)])/6 - (b^2\*c^3\*PolyLog[2, c/x])/6 + (b^2\*c^3\*PolyLog[2, (c + x)/(2\*c)])/6 + (b^2\*c^3\*PolyLog[2, 1 - x/c])/6 - (b^2\*c^3\*PolyLog[2, 1 + x/c])/6



Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

Rule 2398

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(g\*(q + 1)), x] - Dist[(b\*e\*n\*p)/(g\*(q + 1)), Int[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && GtQ[p, 0] && NeQ[q, -1] && IntegersQ[2\*p, 2\*q] && (!IGtQ[q, 0] || (EqQ[p, 2] && NeQ[q, 1]))

Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

Rule 2347

Int((((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_))/ (x\_), x\_Symbol] :> Dist[1/d, Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/x, x], x] - Dist[e/d, Int[(d + e\*x)^q\*(a + b\*Log[c\*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2\*q]

Rule 2344

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] :> Dist[1/d, Int[(a + b\*Log[c\*x^n])^p/x, x], x] - Dist[e/d, Int[(a + b\*Log[c\*x^n])^p/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && I

GtQ[p, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)\*(b\_.)])/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_), x\_Symbol] := Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r\*(q + 1) + 1, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2319

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^p)/(e\*(q + 1)), x] - Dist[(b\*n\*p)/(e\*(q + 1)), Int[((d + e\*x)^(q + 1)\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q, -1] && (EqQ[p, 1] || (IntegersQ[2\*p, 2\*q] && !IGtQ[q, 0]) || (EqQ[p, 2] && NeQ[q, 1]))

### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m

+ n + 2, 0])

### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rule 2557

Int[Log[v\_]\*Log[w\_]\*(u\_), x\_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]\*Log[w], z, x] + (-Int[SimplifyIntegrand[(z\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2466

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p]]^q, x^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]

#### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

#### Rule 2462

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[f + g\*x]\*(a + b\*Log[c\*(d + e\*x^n)^p])/g, x] - Dist[(b\*e\*n\*p)/g, Int[(x^(n - 1)\*Log[f + g\*x])/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*((h\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]]^p, (h\*x)^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 193

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(x\_)^(m\_.))/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n])/(g\*(q + 1)), x] - Dist[(b\*e^n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.))\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rubi steps

$$\begin{aligned}
\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx &= \int \left( \frac{1}{4} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{2} b x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{4} b^2 x^2 \log^2 \left( 1 + \frac{c}{x} \right) \right) dx \\
&= \frac{1}{4} \int x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 dx + \frac{1}{2} b \int x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) dx + \frac{1}{4} b^2 \int x^2 \log^2 \left( 1 + \frac{c}{x} \right) dx \\
&= - \left( \frac{1}{4} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^4} dx, x, \frac{1}{x} \right) \right) + \frac{1}{2} b \int \left( 2ax^2 \log \left( 1 + \frac{c}{x} \right) - bx^2 \log \left( 1 - \frac{c}{x} \right) \right) dx \\
&= \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{12} b^2 x^3 \log^2 \left( \frac{c+x}{x} \right) + (ab) \int x^2 \log \left( 1 + \frac{c}{x} \right) dx - \frac{1}{2} b^2 \int x^2 \log \left( 1 - \frac{c}{x} \right) dx \\
&= \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{3} abx^3 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{6} b^2 x^3 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{12} b^2 x^3 \log^2 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{3} abx^3 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{6} b^2 x^3 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{12} b^2 x^3 \log^2 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{12} bcx^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) + \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{3} abx^3 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{6} b^2 x^3 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} abcx^2 + \frac{1}{6} bc^2 \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) + \frac{1}{12} bcx^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right) \\
&= -\frac{1}{3} abc^2 x + \frac{1}{6} b^2 c^2 x + \frac{1}{6} abcx^2 + \frac{1}{12} b^2 c^3 \log \left( 1 - \frac{c}{x} \right) + \frac{1}{6} b^2 c^2 x \log \left( 1 - \frac{c}{x} \right) - \frac{1}{12} b^2 cx^2 \log \left( 1 - \frac{c}{x} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.303487, size = 145, normalized size = 1.02

$$\frac{1}{3} \left( b^2 c^3 \text{PolyLog} \left( 2, e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) + a^2 x^3 + abc^3 \log \left( 1 - \frac{c^2}{x^2} \right) + b \tanh^{-1} \left( \frac{c}{x} \right) \left( 2ax^3 - 2bc^3 \log \left( 1 - e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) - bc^3 + \dots \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c/x])^2,x]

[Out] (b^2\*c^2\*x + a\*b\*c\*x^2 + a^2\*x^3 + b^2\*(-c^3 + x^3)\*ArcTanh[c/x]^2 + b\*ArcTanh[c/x]\*(-b\*c^3) + b\*c\*x^2 + 2\*a\*x^3 - 2\*b\*c^3\*Log[1 - E^(-2\*ArcTanh[c/x])]) + a\*b\*c^3\*Log[1 - c^2/x^2] - 2\*a\*b\*c^3\*Log[c/x] + b^2\*c^3\*PolyLog[2, E^(-2\*ArcTanh[c/x])])/3

**Maple [B]** time = 0.032, size = 391, normalized size = 2.8

$$\frac{x^3 a^2}{3} + \frac{x^3 b^2}{3} \left( \operatorname{Arctanh}\left(\frac{c}{x}\right) \right)^2 + \frac{c^3 b^2}{3} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x} - 1\right) + \frac{b^2 c x^2}{3} \operatorname{Arctanh}\left(\frac{c}{x}\right) - \frac{2 c^3 b^2}{3} \ln\left(\frac{c}{x}\right) \operatorname{Arctanh}\left(\frac{c}{x}\right) + \frac{c^3 b^2}{3} \operatorname{Arctanh}\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c/x))^2,x)

[Out] 1/3\*x^3\*a^2+1/3\*b^2\*x^3\*arctanh(c/x)^2+1/3\*c^3\*b^2\*arctanh(c/x)\*ln(c/x-1)+1/3\*c\*b^2\*arctanh(c/x)\*x^2-2/3\*c^3\*b^2\*ln(c/x)\*arctanh(c/x)+1/3\*c^3\*b^2\*arctanh(c/x)\*ln(1+c/x)+1/3\*b^2\*c^2\*x+1/6\*c^3\*b^2\*ln(c/x-1)-1/6\*c^3\*b^2\*ln(1+c/x)-1/3\*c^3\*b^2\*dilog(1/2+1/2\*c/x)-1/6\*c^3\*b^2\*ln(c/x-1)\*ln(1/2+1/2\*c/x)+1/12\*c^3\*b^2\*ln(c/x-1)^2-1/6\*c^3\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1/2+1/2\*c/x)+1/6\*c^3\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1+c/x)-1/12\*c^3\*b^2\*ln(1+c/x)^2+1/3\*c^3\*b^2\*dilog(c/x)+1/3\*c^3\*b^2\*dilog(1+c/x)+1/3\*c^3\*b^2\*ln(c/x)\*ln(1+c/x)+2/3\*a\*b\*x^3\*arctanh(c/x)+1/3\*c^3\*a\*b\*ln(c/x-1)+1/3\*a\*b\*c\*x^2-2/3\*c^3\*a\*b\*ln(c/x)+1/3\*c^3\*a\*b\*ln(1+c/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^2 x^3 + \frac{1}{3} \left( 2x^3 \operatorname{artanh}\left(\frac{c}{x}\right) + (c^2 \log(-c^2 + x^2) + x^2)c \right) ab + \frac{1}{12} \left( 6c^4 \int -\frac{\log(c+x)}{3(c^2-x^2)} dx + x^3 \log(c+x)^2 + 6c^3 \int -\frac{x}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x))^2,x, algorithm="maxima")

[Out] 1/3\*a^2\*x^3 + 1/3\*(2\*x^3\*arctanh(c/x) + (c^2\*log(-c^2 + x^2) + x^2)\*c)\*a\*b + 1/12\*(6\*c^4\*integrate(-1/3\*log(c + x)/(c^2 - x^2), x) + x^3\*log(c + x)^2

```
+ 6*c^3*integrate(-1/3*x*log(c + x)/(c^2 - x^2), x) - (c*log(c + x) - c*log
(-c + x) - 2*x)*c^2 - (c^3 - x^3)*log(-c + x)^2 + (c^2*log(-c^2 + x^2) + x^
2)*c + 12*c*integrate(-1/3*x^3*log(c + x)/(c^2 - x^2), x) - 2*(c*x^2 + (c^3
+ x^3)*log(c + x))*log(-c + x)*b^2
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2x^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2abx^2 \operatorname{artanh}\left(\frac{c}{x}\right) + a^2x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctanh(c/x)^2 + 2*a*b*x^2*arctanh(c/x) + a^2*x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atanh(c/x))**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2*x^2, x)
```



### 3.145 $\int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx$

**Optimal.** Leaf size=83

$$-\frac{1}{2}c^2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 + \frac{1}{2}x^2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 + bcx \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) + \frac{1}{2}b^2c^2 \log \left( 1 - \frac{c^2}{x^2} \right) + b^2c^2 \log(x)$$

[Out] b\*c\*x\*(a + b\*ArcCoth[x/c]) - (c^2\*(a + b\*ArcCoth[x/c])^2)/2 + (x^2\*(a + b\*ArcCoth[x/c])^2)/2 + (b^2\*c^2\*Log[1 - c^2/x^2])/2 + b^2\*c^2\*Log[x]

**Rubi [C]** time = 1.04099, antiderivative size = 574, normalized size of antiderivative = 6.92, number of steps used = 58, number of rules used = 32, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.286$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 193, 43, 6742, 30, 2557, 12, 2466, 2448, 263, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{4}b^2c^2 \text{PolyLog} \left( 2, \frac{c-x}{2c} \right) - \frac{1}{4}b^2c^2 \text{PolyLog} \left( 2, -\frac{c}{x} \right) - \frac{1}{4}b^2c^2 \text{PolyLog} \left( 2, \frac{c}{x} \right) - \frac{1}{4}b^2c^2 \text{PolyLog} \left( 2, \frac{c+x}{2c} \right) + \frac{1}{4}b^2c^2 \text{PolyLog} \left( 2, \frac{c-x}{2c} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[x\*(a + b\*ArcTanh[c/x])^2,x]

[Out] (a\*b\*c\*x)/2 - (b^2\*c\*x\*Log[1 - c/x])/4 + (b\*c\*(1 - c/x)\*x\*(2\*a - b\*Log[1 - c/x]))/4 - (c^2\*(2\*a - b\*Log[1 - c/x])^2)/8 + (x^2\*(2\*a - b\*Log[1 - c/x])^2)/8 + (b^2\*c\*x\*Log[1 + c/x])/4 + (a\*b\*x^2\*Log[1 + c/x])/2 - (b^2\*x^2\*Log[1 - c/x]\*Log[1 + c/x])/4 + (b^2\*c^2\*Log[c - x])/4 + (b^2\*c^2\*Log[1 + c/x]\*Log[c - x])/4 + (a\*b\*c^2\*Log[x])/2 + (b^2\*c^2\*Log[x])/2 + (b^2\*c^2\*Log[c - x]\*Log[x/c])/4 - (a\*b\*c^2\*Log[c + x])/2 + (b^2\*c^2\*Log[c + x])/4 + (b^2\*c^2\*Log[1 - c/x]\*Log[c + x])/4 - (b^2\*c^2\*Log[(c - x)/(2\*c)]\*Log[c + x])/4 + (b^2\*c^2\*Log[-(x/c)]\*Log[c + x])/4 - (b^2\*c^2\*Log[c - x]\*Log[(c + x)/(2\*c)])/4 + (b^2\*c^2\*Log[(c + x)/x])/4 + (b^2\*c\*x\*Log[(c + x)/x])/4 - (b^2\*c^2\*Log[(c + x)/x]^2)/8 + (b^2\*x^2\*Log[(c + x)/x]^2)/8 - (b^2\*c^2\*PolyLog[2, (c - x)/(2\*c)])/4 - (b^2\*c^2\*PolyLog[2, -(c/x)])/4 - (b^2\*c^2\*PolyLog[2, c/x])/4 - (b^2\*c^2\*PolyLog[2, (c + x)/(2\*c)])/4 + (b^2\*c^2\*PolyLog[2, 1 - x/c])/4 + (b^2\*c^2\*PolyLog[2, 1 + x/c])/4

#### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 -

$c*x^n)/2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

#### Rule 2454

$\text{Int}[(a_.) + \text{Log}[c_.)*((d_.) + (e_.)*(x_.)^{n_.})^{p_.}*(b_.)^{q_.}*(x_.)^{m_.}], x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \ \&\& \ (\text{GtQ}[(m+1)/n, 0] \ || \ \text{IGtQ}[q, 0]) \ \&\& \ !(\text{EqQ}[q, 1] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IGtQ}[m, 0])$

#### Rule 2398

$\text{Int}[(a_.) + \text{Log}[c_.)*((d_.) + (e_.)*(x_.)^{n_.})^{p_.}*(f_.) + (g_.)*(x_.)^{q_.}], x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{q+1}*(a + b*\text{Log}[c*(d + e*x)^n])^p)/(g*(q+1)), x] - \text{Dist}[(b*e*n*p)/(g*(q+1)), \text{Int}[(f + g*x)^{q+1}*(a + b*\text{Log}[c*(d + e*x)^n])^{p-1})/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\} \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ \text{NeQ}[q, -1] \ \&\& \ \text{IntegerQ}[2*p, 2*q] \ \&\& \ (!\text{IGtQ}[q, 0] \ || \ (\text{EqQ}[p, 2] \ \&\& \ \text{NeQ}[q, 1]))$

#### Rule 2411

$\text{Int}[(a_.) + \text{Log}[c_.)*((d_.) + (e_.)*(x_.)^{n_.})^{p_.}*(f_.) + (g_.)*(x_.)^{q_.}*(h_.) + (i_.)*(x_.)^{r_.}], x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x\} \ \&\& \ \text{EqQ}[e*f - d*g, 0] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{IGtQ}[r, 0]) \ \&\& \ \text{IntegerQ}[2*r]$

#### Rule 2347

$\text{Int}[(a_.) + \text{Log}[c_.)*(x_.)^{n_.}*(b_.)^{p_.}*(d_.) + (e_.)*(x_.)^{q_.}]/(x_.), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(d + e*x)^{q+1}*(a + b*\text{Log}[c*x^n])^p)/x, x], x] - \text{Dist}[e/d, \text{Int}[(d + e*x)^q*(a + b*\text{Log}[c*x^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{LtQ}[q, -1] \ \&\& \ \text{IntegerQ}[2*q]$

#### Rule 2344

$\text{Int}[(a_.) + \text{Log}[c_.)*(x_.)^{n_.}*(b_.)^{p_.}]/((x_.)*((d_.) + (e_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/x, x], x] - \text{Dist}[e/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \ \&\& \ \text{IGtQ}[p, 0]$

#### Rule 2301

$\text{Int}[\frac{(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}](b_.)}{(x_)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x]$

### Rule 2316

$\text{Int}[\frac{(a_.) + \text{Log}[(c_.)*(x_)](b_.)}{((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow \text{Simp}[\frac{(a + b*\text{Log}[-((c*d)/e)])*\text{Log}[d + e*x]}{e}, x] + \text{Dist}[b, \text{Int}[\frac{\text{Log}[-((e*x)/d)]}{(d + e*x)}, x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{GtQ}[-((c*d)/e), 0]$

### Rule 2315

$\text{Int}[\frac{\text{Log}[(c_.)*(x_)]}{((d_.) + (e_.)*(x_))}, x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

### Rule 2314

$\text{Int}[\frac{(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}](b_.)*((d_.) + (e_.)*(x_)^{(r_.)})^{(q_.)}}{(x\_Symbol)}, x] \rightarrow \text{Simp}[\frac{x*(d + e*x^r)^{(q+1)}*(a + b*\text{Log}[c*x^n])}{d}, x] - \text{Dist}[(b*n)/d, \text{Int}[(d + e*x^r)^{(q+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, q, r\}, x] \&\& \text{EqQ}[r*(q+1) + 1, 0]$

### Rule 31

$\text{Int}[\frac{(a_.) + (b_.)*(x_)^{(-1)}}{(x\_Symbol)}, x] \rightarrow \text{Simp}[\frac{\text{Log}[\text{RemoveContent}[a + b*x, x]]}{b}, x] /; \text{FreeQ}\{a, b\}, x]$

### Rule 2455

$\text{Int}[\frac{(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_)^{(n_.)})^{(p_.)}](b_.)*((f_.)*(x_)^{(m_.)})}{(x\_Symbol)}, x] \rightarrow \text{Simp}[\frac{(f*x)^{(m+1)}*(a + b*\text{Log}[c*(d + e*x^n)^p])}{f*(m+1)}, x] - \text{Dist}[(b*e*n*p)/(f*(m+1)), \text{Int}[(x^{(n-1)}*(f*x)^{(m+1)})/(d + e*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& \text{NeQ}[m, -1]$

### Rule 193

$\text{Int}[\frac{(a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}}{(x\_Symbol)}, x] \rightarrow \text{Int}[x^{(n*p)}*(b + a/x^n)^p, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{LtQ}[n, 0] \&\& \text{IntegerQ}[p]$

### Rule 43

$\text{Int}[\frac{(a_.) + (b_.)*(x_)^{(m_.)})*((c_.) + (d_.)*(x_)^{(n_.)}}{(x\_Symbol)}, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) || \text{LtQ}[9*m + 5*(n+1), 0]) || \text{GtQ}[m + n + 2, 0])$

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))^(q_.)*(x_)^(m
_.)*((f_.) + (g_.)*(x_)^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log
[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g
, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.), x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p]))/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
```

reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

### Rule 260

Int[(x\_)^m\_./((a\_) + (b\_)\*(x\_)^n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2416

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)])\*(b\_)^p\_)\*((h\_)\*(x\_)^m\_)\*((f\_) + (g\_)\*(x\_)^r\_)^q\_, x\_Symbol] :> Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2410

Int[(Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)])\*(x\_)^m\_./((f\_) + (g\_)\*(x\_)), x\_Symbol] :> Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ[{c, d, e, f, g}, x] && EqQ[e\*f - d\*g, 0] && EqQ[c\*d, 1] && IntegerQ[m]

### Rule 2395

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^n\_)])\*(b\_)\*((f\_) + (g\_)\*(x\_)^q\_), x\_Symbol] :> Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && N

eQ[q, -1]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 29

Int[(x\_)^(-1), x\_Symbol] := Simp[Log[x], x]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rubi steps

$$\begin{aligned}
\int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx &= \int \left( \frac{1}{4} x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{2} b x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{4} b^2 x \log^2 \left( 1 + \frac{c}{x} \right) \right) dx \\
&= \frac{1}{4} \int x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 dx + \frac{1}{2} b \int x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) dx + \frac{1}{4} b^2 \int x \log^2 \left( 1 + \frac{c}{x} \right) dx \\
&= - \left( \frac{1}{4} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^3} dx, x, \frac{1}{x} \right) \right) + \frac{1}{2} b \int \left( 2ax \log \left( 1 + \frac{c}{x} \right) - bx \log \left( 1 - \frac{c}{x} \right) \right) dx \\
&= \frac{1}{8} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{8} b^2 x^2 \log^2 \left( \frac{c+x}{x} \right) + (ab) \int x \log \left( 1 + \frac{c}{x} \right) dx - \frac{1}{2} b^2 \int x \log \left( 1 - \frac{c}{x} \right) dx \\
&= \frac{1}{8} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{8} b^2 x^2 \log^2 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{8} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{8} b^2 x^2 \log^2 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) + \frac{1}{8} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{2} abcx + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{8} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \\
&= \frac{1}{2} abcx - \frac{1}{4} b^2 cx \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} bc \left( 1 - \frac{c}{x} \right) x \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) - \frac{1}{8} c^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2
\end{aligned}$$

**Mathematica [A]** time = 0.0472649, size = 92, normalized size = 1.11

$$\frac{1}{2} \left( a^2 x^2 + bc^2(a+b) \log(x-c) - abc^2 \log(c+x) + 2abcx + 2bx \tanh^{-1} \left( \frac{c}{x} \right) (ax+bc) + b^2(x^2-c^2) \tanh^{-1} \left( \frac{c}{x} \right)^2 + b^2c^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c/x])^2,x]

[Out] (2\*a\*b\*c\*x + a^2\*x^2 + 2\*b\*x\*(b\*c + a\*x)\*ArcTanh[c/x] + b^2\*(-c^2 + x^2)\*ArcTanh[c/x]^2 + b\*(a + b)\*c^2\*Log[-c + x] - a\*b\*c^2\*Log[c + x] + b^2\*c^2\*Log[c + x])/2

**Maple [B]** time = 0.025, size = 287, normalized size = 3.5

$$\frac{a^2x^2}{2} + \frac{b^2x^2}{2} \left( \operatorname{Arctanh}\left(\frac{c}{x}\right) \right)^2 + cb^2x \operatorname{Arctanh}\left(\frac{c}{x}\right) + \frac{b^2c^2}{2} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x} - 1\right) - \frac{b^2c^2}{2} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left(1 + \frac{c}{x}\right) - \frac{b^2c^2}{4} \ln\left(\frac{c}{x} - 1\right) \ln\left(1 + \frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c/x))^2,x)

[Out] 1/2\*a^2\*x^2+1/2\*b^2\*x^2\*arctanh(c/x)^2+c\*b^2\*x\*arctanh(c/x)+1/2\*c^2\*b^2\*arctanh(c/x)\*ln(c/x-1)-1/2\*c^2\*b^2\*arctanh(c/x)\*ln(1+c/x)-1/4\*c^2\*b^2\*ln(c/x-1)\*ln(1/2+1/2\*c/x)+1/8\*c^2\*b^2\*ln(c/x-1)^2+1/2\*c^2\*b^2\*ln(c/x-1)-c^2\*b^2\*ln(c/x)+1/2\*c^2\*b^2\*ln(1+c/x)-1/4\*c^2\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1+c/x)+1/4\*c^2\*b^2\*ln(-1/2\*c/x+1/2)\*ln(1/2+1/2\*c/x)+1/8\*c^2\*b^2\*ln(1+c/x)^2+a\*b\*x^2\*arctanh(c/x)+a\*b\*c\*x+1/2\*c^2\*a\*b\*ln(c/x-1)-1/2\*c^2\*a\*b\*ln(1+c/x)

**Maxima [A]** time = 0.975595, size = 184, normalized size = 2.22

$$\frac{1}{2} b^2 x^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{2} a^2 x^2 + \frac{1}{2} \left( 2 x^2 \operatorname{artanh}\left(\frac{c}{x}\right) - (c \log(c+x) - c \log(-c+x) - 2x)c \right) a b + \frac{1}{8} \left( (\log(c+x))^2 - 2(\log(c+x) - \log(-c+x)) \log(c+x) + \log(-c+x)^2 + 4 \log(c+x) \right) c^2 - 4(c \log(c+x) - c \log(-c+x) - 2x) c \operatorname{artanh}\left(\frac{c}{x}\right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x))^2,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2\*arctanh(c/x)^2 + 1/2\*a^2\*x^2 + 1/2\*(2\*x^2\*arctanh(c/x) - (c\*log(c + x) - c\*log(-c + x) - 2\*x)\*c)\*a\*b + 1/8\*((log(c + x))^2 - 2\*(log(c + x) - log(-c + x))\*log(c + x) + log(-c + x)^2 + 4\*log(c + x))\*c^2 - 4\*(c\*log(c + x) - c\*log(-c + x) - 2\*x)\*c\*arctanh(c/x)\*b^2



**Fricas [A]** time = 1.76187, size = 254, normalized size = 3.06

$$abcx + \frac{1}{2}a^2x^2 - \frac{1}{2}(ab - b^2)c^2 \log(c + x) + \frac{1}{2}(ab + b^2)c^2 \log(-c + x) - \frac{1}{8}(b^2c^2 - b^2x^2) \log\left(-\frac{c+x}{c-x}\right)^2 + \frac{1}{2}(b^2cx + abx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x))^2,x, algorithm="fricas")

[Out] a\*b\*c\*x + 1/2\*a^2\*x^2 - 1/2\*(a\*b - b^2)\*c^2\*log(c + x) + 1/2\*(a\*b + b^2)\*c^2\*log(-c + x) - 1/8\*(b^2\*c^2 - b^2\*x^2)\*log(-(c + x)/(c - x))^2 + 1/2\*(b^2\*c\*x + a\*b\*x^2)\*log(-(c + x)/(c - x))

**Sympy [A]** time = 1.05, size = 104, normalized size = 1.25

$$\frac{a^2x^2}{2} - abc^2 \operatorname{atanh}\left(\frac{c}{x}\right) + abcx + abx^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2c^2 \log(-c + x) - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2} + b^2c^2 \operatorname{atanh}\left(\frac{c}{x}\right) + b^2cx \operatorname{atanh}\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c/x))\*\*2,x)

[Out] a\*\*2\*x\*\*2/2 - a\*b\*c\*\*2\*atanh(c/x) + a\*b\*c\*x + a\*b\*x\*\*2\*atanh(c/x) + b\*\*2\*c\*\*2\*log(-c + x) - b\*\*2\*c\*\*2\*atanh(c/x)\*\*2/2 + b\*\*2\*c\*\*2\*atanh(c/x) + b\*\*2\*c\*x\*atanh(c/x) + b\*\*2\*x\*\*2\*atanh(c/x)\*\*2/2

**Giac [A]** time = 1.21089, size = 159, normalized size = 1.92

$$abcx + \frac{1}{2}a^2x^2 - \frac{1}{8}(b^2c^2 - b^2x^2) \log\left(-\frac{c+x}{c-x}\right)^2 - \frac{1}{2}(abc^2 - b^2c^2) \log(c + x) + \frac{1}{2}(abc^2 + b^2c^2) \log(c - x) + \frac{1}{2}(b^2cx + abx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x))^2,x, algorithm="giac")

[Out] a\*b\*c\*x + 1/2\*a^2\*x^2 - 1/8\*(b^2\*c^2 - b^2\*x^2)\*log(-(c + x)/(c - x))^2 - 1/2\*(a\*b\*c^2 - b^2\*c^2)\*log(c + x) + 1/2\*(a\*b\*c^2 + b^2\*c^2)\*log(c - x) + 1/2\*(b^2\*c\*x + a\*b\*x^2)\*log(-(c + x)/(c - x))

### 3.146 $\int \left(a + b \tanh^{-1} \left(\frac{c}{x}\right)\right)^2 dx$

**Optimal.** Leaf size=74

$$b^2(-c)\text{PolyLog}\left(2, -\frac{c+x}{c-x}\right) + c\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2 + x\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2 - 2bc \log\left(\frac{2c}{c-x}\right)\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)$$

[Out]  $c*(a + b*\text{ArcCoth}[x/c])^2 + x*(a + b*\text{ArcCoth}[x/c])^2 - 2*b*c*(a + b*\text{ArcCoth}[x/c])*Log[(2*c)/(c - x)] - b^2*c*\text{PolyLog}[2, -((c + x)/(c - x))]$

**Rubi [B]** time = 0.402079, antiderivative size = 370, normalized size of antiderivative = 5., number of steps used = 31, number of rules used = 14, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$ , Rules used = {6093, 2448, 263, 31, 2449, 2391, 2556, 12, 2462, 260, 2416, 2394, 2315, 2393}

$$-\frac{1}{2}b^2c\text{PolyLog}\left(2, \frac{c-x}{2c}\right) + \frac{1}{2}b^2c\text{PolyLog}\left(2, -\frac{c}{x}\right) - \frac{1}{2}b^2c\text{PolyLog}\left(2, \frac{c}{x}\right) + \frac{1}{2}b^2c\text{PolyLog}\left(2, \frac{c+x}{2c}\right) + \frac{1}{2}b^2c\text{PolyLog}\left(2, \frac{c+x}{2c}\right)$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c/x])^2, x]$

[Out]  $a^2*x - a*b*x*Log[1 - c/x] - (b^2*(c - x)*Log[1 - c/x]^2)/4 + a*b*x*Log[1 + c/x] - (b^2*x*Log[1 - c/x]*Log[1 + c/x])/2 + (b^2*(c + x)*Log[1 + c/x]^2)/4 - (b^2*c*Log[1 - c/x]*Log[-c - x])/2 + a*b*c*Log[c - x] + (b^2*c*Log[-c - x]*Log[(c - x)/(2*c)])/2 - (b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (b^2*c*Log[1 + c/x]*Log[-c + x])/2 + (b^2*c*Log[x/c]*Log[-c + x])/2 + a*b*c*Log[c + x] - (b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (b^2*c*PolyLog[2, (c - x)/(2*c)])/2 + (b^2*c*PolyLog[2, -(c/x)])/2 - (b^2*c*PolyLog[2, c/x])/2 + (b^2*c*PolyLog[2, (c + x)/(2*c)])/2 + (b^2*c*PolyLog[2, 1 - x/c])/2 - (b^2*c*PolyLog[2, 1 + x/c])/2$

#### Rule 6093

$\text{Int}[(a + b*\text{ArcTanh}[c/x])^2, x] \rightarrow \text{Int}[\text{Expand}[\text{Integrand}[(a + (b*\text{Log}[1 + c*x^n])/2 - (b*\text{Log}[1 - c*x^n])/2]^p, x], x] /; \text{FreeQ}\{a, b, c, n\}, x] \&\amp; \text{IGtQ}[p, 0] \&\amp; \text{IntegerQ}[n]$

#### Rule 2448

$\text{Int}[\text{Log}[(c + d*x + e*x^n)^p], x] \rightarrow \text{Simp}[x*\text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n/(d + e*x^n), x], x] /; \text{FreeQ}\{c, d,$

e, n, p}, x]

### Rule 263

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*  
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2449

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)/(x\_))^(p\_)]\*(b\_))^(q\_), x\_Symbol] :=  
Simp[((e + d\*x)\*(a + b\*Log[c\*(d + e/x)^p])^q)/d, x] + Dist[(b\*e\*p\*q)/d, In  
t[(a + b\*Log[c\*(d + e/x)^p])^(q - 1)/x, x], x] /; FreeQ[{a, b, c, d, e, p},  
x] && IGtQ[q, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2556

Int[Log[v\_]\*Log[w\_], x\_Symbol] := Simp[x\*Log[v]\*Log[w], x] + (-Int[Simplify  
Integrand[(x\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x\*Log[v]\*D[  
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,  
x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match  
Q[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2462

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)]\*(b\_))/((f\_) + (g\_  
)\*(x\_)), x\_Symbol] := Simp[(Log[f + g\*x]\*(a + b\*Log[c\*(d + e\*x^n)^p])/g, x  
] - Dist[(b\*e\*n\*p)/g, Int[(x^(n - 1)\*Log[f + g\*x])/(d + e\*x^n), x], x] /; F  
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 2416

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))^(p_)*((h_)*(x_)^(m_))*((f_) + (g_)*(x_)^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2394

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))]*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2315

```
Int[Log[(c_)*(x_)]/((d_) + (e_)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2393

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))]*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rubi steps

$$\begin{aligned}
\int \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 dx &= \int \left( a^2 - ab \log \left( 1 - \frac{c}{x} \right) + \frac{1}{4} b^2 \log^2 \left( 1 - \frac{c}{x} \right) + ab \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \right) dx \\
&= a^2 x - (ab) \int \log \left( 1 - \frac{c}{x} \right) dx + (ab) \int \log \left( 1 + \frac{c}{x} \right) dx + \frac{1}{4} b^2 \int \log^2 \left( 1 - \frac{c}{x} \right) dx + \frac{1}{4} b^2 \int \log^2 \left( 1 + \frac{c}{x} \right) dx \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= a^2 x - abx \log \left( 1 - \frac{c}{x} \right) - \frac{1}{4} b^2 (c-x) \log^2 \left( 1 - \frac{c}{x} \right) + abx \log \left( 1 + \frac{c}{x} \right) - \frac{1}{2} b^2 x \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.121809, size = 97, normalized size = 1.31

$$b^2 c \text{PolyLog} \left( 2, e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) + a \left( ax + bc \log \left( 1 - \frac{c^2}{x^2} \right) - 2bc \log \left( \frac{c}{x} \right) \right) + 2b \tanh^{-1} \left( \frac{c}{x} \right) \left( ax - bc \log \left( 1 - e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^2, x]

[Out] b^2\*(-c + x)\*ArcTanh[c/x]^2 + 2\*b\*ArcTanh[c/x]\*(a\*x - b\*c\*Log[1 - E^(-2\*ArcTanh[c/x])]) + a\*(a\*x + b\*c\*Log[1 - c^2/x^2] - 2\*b\*c\*Log[c/x]) + b^2\*c\*PolyLog[2, E^(-2\*ArcTanh[c/x])]

**Maple [B]** time = 0.012, size = 282, normalized size = 3.8

$$a^2 x + b^2 x \left( \text{Artanh} \left( \frac{c}{x} \right) \right)^2 + cb^2 \text{Artanh} \left( \frac{c}{x} \right) \ln \left( \frac{c}{x} - 1 \right) - 2cb^2 \ln \left( \frac{c}{x} \right) \text{Artanh} \left( \frac{c}{x} \right) + cb^2 \text{Artanh} \left( \frac{c}{x} \right) \ln \left( 1 + \frac{c}{x} \right) - cb^2 \text{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x))^2,x)`

[Out]  $a^2x + b^2x \operatorname{arctanh}(c/x)^2 + c b^2 \operatorname{arctanh}(c/x) \ln(c/x-1) - 2 c b^2 \ln(c/x) \operatorname{arctanh}(c/x) + c b^2 \operatorname{arctanh}(c/x) \ln(1+c/x) - c b^2 \operatorname{dilog}(1/2+1/2*c/x) - 1/2 c b^2 \ln(c/x-1) \ln(1/2+1/2*c/x) + 1/4 c b^2 \ln(c/x-1)^2 - 1/2 c b^2 \ln(-1/2*c/x+1/2) \ln(1/2+1/2*c/x) + 1/2 c b^2 \ln(-1/2*c/x+1/2) \ln(1+c/x) - 1/4 c b^2 \ln(1+c/x)^2 + c b^2 \operatorname{dilog}(c/x) + c b^2 \operatorname{dilog}(1+c/x) + c b^2 \ln(c/x) \ln(1+c/x) + 2 a b x \operatorname{arctanh}(c/x) + c a b \ln(c/x-1) - 2 c a b \ln(c/x) + c a b \ln(1+c/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left(2x \operatorname{artanh}\left(\frac{c}{x}\right) + c \log(-c^2 + x^2)\right)ab + \frac{1}{4} \left(x \log(c+x)^2 - 2(c+x) \log(c+x) \log(-c+x) - (c-x) \log(-c+x)^2 + \int -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^2,x, algorithm="maxima")`

[Out]  $(2x \operatorname{arctanh}(c/x) + c \log(-c^2 + x^2)) * a * b + 1/4 * (x * \log(c + x)^2 - 2 * (c + x) * \log(c + x) * \log(-c + x) - (c - x) * \log(-c + x)^2 + \operatorname{integrate}(-2 * (c^2 + 3 * c * x) * \log(c + x) / (c^2 - x^2), x)) * b^2 + a^2 * x$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^2,x, algorithm="fricas")`

[Out] `integral(b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( a + b \operatorname{atanh} \left( \frac{c}{x} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*\*2,x)

[Out] Integral((a + b\*atanh(c/x))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x} \right) + a \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^2, x)

$$3.147 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x} dx$$

**Optimal.** Leaf size=133

$$b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right) - b \operatorname{PolyLog}\left(2, \frac{2}{1 - \frac{c}{x}} - 1\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x}}\right)$$

[Out] -2\*(a + b\*ArcCoth[x/c])^2\*ArcTanh[1 - 2/(1 - c/x)] + b\*(a + b\*ArcCoth[x/c])  
\*PolyLog[2, 1 - 2/(1 - c/x)] - b\*(a + b\*ArcCoth[x/c])\*PolyLog[2, -1 + 2/(1  
- c/x)] - (b^2\*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2\*PolyLog[3, -1 + 2/(1 -  
c/x)])/2

**Rubi [A]** time = 0.314473, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right) - b \operatorname{PolyLog}\left(2, \frac{2}{1 - \frac{c}{x}} - 1\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x])^2/x, x]

[Out] -2\*(a + b\*ArcCoth[x/c])^2\*ArcTanh[1 - 2/(1 - c/x)] + b\*(a + b\*ArcCoth[x/c])  
\*PolyLog[2, 1 - 2/(1 - c/x)] - b\*(a + b\*ArcCoth[x/c])\*PolyLog[2, -1 + 2/(1  
- c/x)] - (b^2\*PolyLog[3, 1 - 2/(1 - c/x)])/2 + (b^2\*PolyLog[3, -1 + 2/(1 -  
c/x)])/2

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Dist[  
1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,  
n}, x] && IGtQ[p, 0]

#### Rule 5914

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Simp[2\*(a +  
b\*ArcTanh[c\*x])^p\*ArcTanh[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[(a + b  
\*ArcTanh[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 - c\*x)]/(1 - c^2\*x^2), x], x] /; F



reeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 6052

Int[(ArcTanh[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[1 + u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6058

Int[(Log[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^2}{x} dx &= -\text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx, x, \frac{1}{x} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + (4bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \tanh^{-1}(1 - c^2 x^2)}{1 - c^2 x^2} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - (2bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \log \left( \frac{2}{1 - cx} \right)}{1 - c^2 x^2} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \text{Li}_2 \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \text{Li}_2 \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0921291, size = 114, normalized size = 0.86

$$\frac{1}{2}b \left( 2\text{PolyLog} \left( 2, \frac{c+x}{c-x} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) - 2\text{PolyLog} \left( 2, \frac{c+x}{x-c} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + b \left( \text{PolyLog} \left( 3, \frac{c+x}{x-c} \right) - \text{PolyLog} \left( 3, \frac{c+x}{-c+x} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^2/x,x]

[Out] -2\*(a + b\*ArcTanh[c/x])^2\*ArcTanh[(c + x)/(c - x)] + (b\*(2\*(a + b\*ArcTanh[c/x])\*PolyLog[2, (c + x)/(c - x)] - 2\*(a + b\*ArcTanh[c/x])\*PolyLog[2, (c + x)/(-c + x)] + b\*(-PolyLog[3, (c + x)/(c - x)] + PolyLog[3, (c + x)/(-c + x)])))/2

**Maple [C]** time = 0.162, size = 780, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x))^2/x,x)

```
[Out] -a^2*ln(c/x)-b^2*ln(c/x)*arctanh(c/x)^2+b^2*arctanh(c/x)*polylog(2,-(1+c/x)
^2/(1-c^2/x^2))-1/2*b^2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+b^2*arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)-1)-b^2*arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylog
(3,(1+c/x)/(1-c^2/x^2)^(1/2))-b^2*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(
1/2))-2*b^2*arctanh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+2*b^2*polylo
g(3,-(1+c/x)/(1-c^2/x^2)^(1/2))+1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-
1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c
/x)^2-1/2*I*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+
1))^3*arctanh(c/x)^2+1/2*I*b^2*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*
((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*arctanh(c/x)^2-1/2*I
*b^2*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))
*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*arctanh(c/x)^2
-2*a*b*ln(c/x)*arctanh(c/x)+a*b*ln(c/x)*ln(1+c/x)+a*b*dilog(c/x)+a*b*dilog(
1+c/x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2 \left( \log\left(\frac{c}{x} + 1\right) - \log\left(-\frac{c}{x} + 1\right) \right)^2}{4x} + \frac{ab \left( \log\left(\frac{c}{x} + 1\right) - \log\left(-\frac{c}{x} + 1\right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate(1/4*b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + a*b*(lo
g(c/x + 1) - log(-c/x + 1))/x, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x}\right) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*\*2/x,x)

[Out] Integral((a + b\*atanh(c/x))\*\*2/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^2/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^2/x, x)

$$3.148 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2} dx$$

**Optimal.** Leaf size=87

$$\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right)}{c} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x} + \frac{2b \log\left(\frac{2}{1 - \frac{c}{x}}\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)}{c}$$

[Out] -((a + b\*ArcCoth[x/c])^2/c) - (a + b\*ArcCoth[x/c])^2/x + (2\*b\*(a + b\*ArcCoth[x/c])\*Log[2/(1 - c/x)])/c + (b^2\*PolyLog[2, 1 - 2/(1 - c/x)])/c

**Rubi [B]** time = 0.506839, antiderivative size = 205, normalized size of antiderivative = 2.36, number of steps used = 28, number of rules used = 12, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, -\frac{c-x}{2x}\right)}{2c} - \frac{b^2 \text{PolyLog}\left(2, \frac{c+x}{2x}\right)}{2c} - \frac{b \log\left(\frac{c+x}{2x}\right) \left(2a - b \log\left(1 - \frac{c}{x}\right)\right)}{2c} - \frac{b \log\left(\frac{c+x}{x}\right) \left(2a - b \log\left(1 - \frac{c}{x}\right)\right)}{2x} + \frac{(1 - c/x) \left(2a - b \log\left(1 - \frac{c}{x}\right)\right)^2}{4c} - \frac{b \left(2a - b \log\left(1 - \frac{c}{x}\right)\right) \log\left(\frac{c+x}{2x}\right)}{2c} - \frac{b \left(2a - b \log\left(1 - \frac{c}{x}\right)\right) \log\left(\frac{c+x}{x}\right)}{2x} - \frac{b^2 \log\left(-\frac{c-x}{2x}\right) \log\left(\frac{c+x}{x}\right)}{2c} - \frac{b^2 (1 + c/x) \log\left(\frac{c+x}{x}\right)^2}{4c} + \frac{b^2 \text{PolyLog}\left[2, -\frac{c-x}{2x}\right]}{2c} - \frac{b^2 \text{PolyLog}\left[2, \frac{c+x}{2x}\right]}{2c}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c/x])^2/x^2, x]

[Out] ((1 - c/x)\*(2\*a - b\*Log[1 - c/x])^2)/(4\*c) - (b\*(2\*a - b\*Log[1 - c/x])\*Log[(c + x)/(2\*x)])/(2\*c) - (b\*(2\*a - b\*Log[1 - c/x])\*Log[(c + x)/x])/(2\*x) - (b^2\*Log[-(c - x)/(2\*x)]\*Log[(c + x)/x])/(2\*c) - (b^2\*(1 + c/x)\*Log[(c + x)/x]^2)/(4\*c) + (b^2\*PolyLog[2, -(c - x)/(2\*x)])/(2\*c) - (b^2\*PolyLog[2, (c + x)/(2\*x)])/(2\*c)

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},

x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] :> Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.)), x\_Symbol] :> Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m))]/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^2}{x^2} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x}))^2}{4x^2} + \frac{b(2a - b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{2x^2} + \frac{b^2 \log^2(1 + \frac{c}{x})}{4x^2} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - \frac{c}{x}))^2}{x^2} dx + \frac{1}{2} b \int \frac{(2a - b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{x^2} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + \frac{c}{x})}{x^2} dx \\
&= -\left( \frac{1}{4} \text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, \frac{1}{x} \right) \right) - \frac{1}{2} b \text{Subst} \left( \int (2a - b \log(1 - cx)) \log(1 + cx) dx, x, \frac{1}{x} \right) - \frac{1}{4} b^2 \text{Subst} \left( \int \log^2(1 + cx) dx, x, \frac{1}{x} \right) \\
&= -\frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2x} + \frac{\text{Subst} \left( \int (2a - b \log(x))^2 dx, x, 1 - \frac{c}{x} \right)}{4c} - \frac{b^2 \text{Subst} \left( \int \log^2(x) dx, x, 1 + \frac{c}{x} \right)}{4c} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2x} - \frac{b^2(1 + \frac{c}{x}) \log^2(\frac{c+x}{x})}{4c} + \frac{b^2 \log^2(1 + \frac{c}{x})}{4c} \\
&= -\frac{ab}{x} - \frac{b^2}{2x} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c} + \frac{b^2(1 + \frac{c}{x}) \log(\frac{c+x}{x})}{2c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2x} \\
&= -\frac{b^2}{x} - \frac{b^2(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{2c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2c} \\
&= -\frac{b^2}{2x} - \frac{b^2(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{2c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2c} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2c} - \frac{b(2a - b \log(1 - \frac{c}{x})) \log(\frac{c+x}{x})}{2c}
\end{aligned}$$

**Mathematica [A]** time = 0.0965751, size = 101, normalized size = 1.16

$$-b^2 x \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(\frac{c}{x})} \right) + a \left( 2bx \log \left( \frac{1}{\sqrt{1 - \frac{c^2}{x^2}}} \right) - ac \right) + 2b \tanh^{-1} \left( \frac{c}{x} \right) \left( bx \log \left( e^{-2 \tanh^{-1}(\frac{c}{x})} + 1 \right) - ac \right) + b^2(x - c)$$


---

$cx$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^2/x^2, x]

[Out] (b^2\*(-c + x)\*ArcTanh[c/x]^2 + 2\*b\*ArcTanh[c/x]\*(-(a\*c) + b\*x\*Log[1 + E^(-2\*ArcTanh[c/x])]) + a\*(-(a\*c) + 2\*b\*x\*Log[1/Sqrt[1 - c^2/x^2]]) - b^2\*x\*PolyLog[2, -E^(-2\*ArcTanh[c/x])])/(c\*x)



---

**Maple [A]** time = 0.004, size = 144, normalized size = 1.7

$$-\frac{a^2}{x} - \frac{b^2}{x} \left( \operatorname{Arctanh}\left(\frac{c}{x}\right) \right)^2 - \frac{b^2}{c} \left( \operatorname{Arctanh}\left(\frac{c}{x}\right) \right)^2 + 2 \frac{b^2}{c} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left( \left(1 + \frac{c}{x}\right)^2 \left(1 - \frac{c^2}{x^2}\right)^{-1} + 1 \right) + \frac{b^2}{c} \operatorname{polylog}\left(2, -\left(1 + \frac{c}{x}\right)^2 \left(1 - \frac{c^2}{x^2}\right)^{-1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x))^2/x^2,x)`

[Out] `-a^2/x-arctanh(c/x)^2/x*b^2-1/c*b^2*arctanh(c/x)^2+2/c*arctanh(c/x)*ln((1+c/x)^2/(1-c^2/x^2)+1)*b^2+1/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*b^2-2*a*b/x*arctanh(c/x)-1/c*a*b*ln(1-c^2/x^2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} \left( c^3 \int -\frac{\log(x)^2}{c^3 x^2 - c x^4} dx + c^2 \left( \frac{\log(-c^2 + x^2)}{c^3} - \frac{\log(x^2)}{c^3} \right) - 4c^2 \int -\frac{x \log(c+x)}{c^3 x^2 - c x^4} dx + 2c^2 \int -\frac{x \log(x)}{c^3 x^2 - c x^4} dx + 2c \left( \frac{\log(-c^2 + x^2)}{c^3} - \frac{\log(x^2)}{c^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="maxima")`

[Out] `1/4*(c^3*integrate(-log(x)^2/(c^3*x^2 - c*x^4), x) + c^2*(log(-c^2 + x^2)/c^3 - log(x^2)/c^3) - 4*c^2*integrate(-x*log(c + x)/(c^3*x^2 - c*x^4), x) + 2*c^2*integrate(-x*log(x)/(c^3*x^2 - c*x^4), x) + 2*c*(log(-c + x)/c^2 - log(x)/c^2 + 1/(c*x))*log(-c/x + 1) - c*(log(c + x)/c^2 - log(-c + x)/c^2) - c*integrate(-x^2*log(x)^2/(c^3*x^2 - c*x^4), x) - 2*c*integrate(-x^2*log(c + x)/(c^3*x^2 - c*x^4), x) + 4*c*integrate(-x^2*log(x)/(c^3*x^2 - c*x^4), x) - log(-c/x + 1)^2/x - (c*log(c + x)^2 - 2*((c + x)*log(c + x) - (c + x)*log(x) - c)*log(-c + x))/(c*x) - (x*log(-c + x)^2 + x*log(x)^2 - 2*(x*log(x) - x)*log(-c + x) - 2*x*log(x) + 2*c)/(c*x) - 2*integrate(-x^3*log(c + x)/(c^3*x^2 - c*x^4), x) + 2*integrate(-x^3*log(x)/(c^3*x^2 - c*x^4), x))*b^2 - a*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^2/x`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x}\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^2/x^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c/x)^2 + 2\*a\*b\*arctanh(c/x) + a^2)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}\left(\frac{c}{x}\right))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*\*2/x\*\*2,x)

[Out] Integral((a + b\*atanh(c/x))\*\*2/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}\left(\frac{c}{x}\right) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^2/x^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^2/x^2, x)

$$3.149 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3} dx$$

**Optimal.** Leaf size=87

$$\frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} - \frac{ab}{cx} - \frac{b^2 \log\left(1 - \frac{c^2}{x^2}\right)}{2c^2} - \frac{b^2 \coth^{-1}\left(\frac{x}{c}\right)}{cx}$$

[Out]  $-\left(\frac{a*b}{c*x}\right) - \left(\frac{b^2*ArcCoth[x/c]}{c*x}\right) + \left(a + b*ArcCoth[x/c]\right)^2/(2*c^2) - \left(a + b*ArcCoth[x/c]\right)^2/(2*x^2) - \left(\frac{b^2*Log[1 - c^2/x^2]}{2*c^2}\right)$

**Rubi [C]** time = 1.24698, antiderivative size = 707, normalized size of antiderivative = 8.13, number of steps used = 66, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{c-x}{2c}\right)}{4c^2} + \frac{b^2 \text{PolyLog}\left(2, -\frac{c}{x}\right)}{4c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{c}{x}\right)}{4c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{c+x}{2c}\right)}{4c^2} - \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{x}{c}\right)}{4c^2} - \frac{b^2 \text{PolyLog}\left(2, 1 + \frac{x}{c}\right)}{4c^2}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c/x])^2/x^3,x]

[Out]  $-\left(\frac{b^2*(1 - c/x)^2}{16*c^2}\right) - \left(\frac{b^2*(1 + c/x)^2}{16*c^2}\right) + \left(\frac{a*b}{4*x^2}\right) + \frac{b^2}{8*x^2} - \left(\frac{3*a*b}{2*c*x}\right) + \left(\frac{b^2*Log[1 - c/x]}{8*c^2}\right) - \left(\frac{3*b^2*(1 - c/x)*Log[1 - c/x]}{4*c^2}\right) - \left(\frac{b^2*Log[1 - c/x]}{8*x^2}\right) - \left(\frac{b*(1 - c/x)^2*(2*a - b*Log[1 - c/x])}{8*c^2}\right) + \left(\frac{(1 - c/x)*(2*a - b*Log[1 - c/x])^2}{4*c^2}\right) - \left(\frac{(1 - c/x)^2*(2*a - b*Log[1 - c/x])^2}{8*c^2}\right) + \left(\frac{b^2*Log[1 - c/x]*Log[1 + c/x]}{4*x^2}\right) - \left(\frac{b^2*Log[1 + c/x]*Log[c - x]}{4*c^2}\right) - \left(\frac{b^2*Log[c - x]*Log[x/c]}{4*c^2}\right) - \left(\frac{b^2*Log[1 - c/x]*Log[c + x]}{4*c^2}\right) + \left(\frac{b^2*Log[(c - x)/(2*c)]*Log[c + x]}{4*c^2}\right) - \left(\frac{b^2*Log[-(x/c)]*Log[c + x]}{4*c^2}\right) + \left(\frac{b^2*Log[c - x]*Log[(c + x)/(2*c)]}{4*c^2}\right) + \left(\frac{a*b*Log[(c + x)/x]}{2*c^2}\right) + \left(\frac{b^2*Log[(c + x)/x]}{8*c^2}\right) - \left(\frac{3*b^2*(1 + c/x)*Log[(c + x)/x]}{4*c^2}\right) + \left(\frac{b^2*(1 + c/x)^2*Log[(c + x)/x]}{8*c^2}\right) - \left(\frac{a*b*Log[(c + x)/x]}{2*x^2}\right) - \left(\frac{b^2*Log[(c + x)/x]}{8*x^2}\right) + \left(\frac{b^2*(1 + c/x)*Log[(c + x)/x]^2}{4*c^2}\right) - \left(\frac{b^2*(1 + c/x)^2*Log[(c + x)/x]^2}{8*c^2}\right) + \left(\frac{b^2*PolyLog[2, (c - x)/(2*c)]}{4*c^2}\right) + \left(\frac{b^2*PolyLog[2, -(c/x)]}{4*c^2}\right) + \left(\frac{b^2*PolyLog[2, c/x]}{4*c^2}\right) + \left(\frac{b^2*PolyLog[2, (c + x)/(2*c)]}{4*c^2}\right) - \left(\frac{b^2*PolyLog[2, 1 - x/c]}{4*c^2}\right) - \left(\frac{b^2*PolyLog[2, 1 + x/c]}{4*c^2}\right)$

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2401

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_.) + (g_.)*(x_)^(q_.), x_Symbol]
:> Int[ExpandIntegrand[(f + g*x)^q*(a + b*Log[c*(d + e*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[q, 0]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.), x_Symbol]
:> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)])*(b_.))^(p_.), x_Symbol]
:> Simp[x*(a + b*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_)], x_Symbol]
:> Simp[x*Log[c*x^n], x] - Simp[n*x, x] /; FreeQ[{c, n}, x]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_) + (g_.)*(x_)^(q_.), x_Symbol]
:> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
```

qQ[e\*f - d\*g, 0]

### Rule 2305

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n])^p)/(d\*(m + 1)), x] - Dist[(b\*n\*p)/(m + 1), Int[(d\*x)^m\*(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

### Rule 2304

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*Log[c\*x^n]))/(d\*(m + 1)), x] - Simp[(b\*n\*(d\*x)^(m + 1))/(d\*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 2395

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Simp[((f + g\*x)^(q + 1)\*(a + b\*Log[c\*(d + e\*x)^n]))/(g\*(q + 1)), x] - Dist[(b\*e\*n)/(g\*(q + 1)), Int[(f + g\*x)^(q + 1)/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e\*f - d\*g, 0] && NeQ[q, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

### Rule 2557

Int[Log[v\_]\*Log[w\_]\*(u\_), x\_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]\*Log[w], z, x] + (-Int[SimplifyIntegrand[(z\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z

, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2466

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p]]^q, x^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]

### Rule 2462

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[f + g\*x]\*(a + b\*Log[c\*(d + e\*x^n)^p])/g, x] - Dist[(b\*e\*n\*p)/g, Int[(x^(n - 1)\*Log[f + g\*x])/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))^(q\_.)\*((h\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)]^p, (h\*x)^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^2}{x^3} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x}))^2}{4x^3} + \frac{b(2a - b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{2x^3} + \frac{b^2 \log^2(1 + \frac{c}{x})}{4x^3} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - \frac{c}{x}))^2}{x^3} dx + \frac{1}{2} b \int \frac{(2a - b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{x^3} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + \frac{c}{x})}{x^3} dx \\
&= -\left( \frac{1}{4} \text{Subst} \left( \int x(2a - b \log(1 - cx))^2 dx, x, \frac{1}{x} \right) \right) + \frac{1}{2} b \int \left( \frac{2a \log(1 + \frac{c}{x})}{x^3} - \frac{b \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{x^3} \right) dx \\
&= -\left( \frac{1}{4} \text{Subst} \left( \int \left( \frac{(2a - b \log(1 - cx))^2}{c} - \frac{(1 - cx)(2a - b \log(1 - cx))^2}{c} \right) dx, x, \frac{1}{x} \right) \right) + (ab) \int \frac{\log^2(1 + \frac{c}{x})}{x^3} dx \\
&= \frac{b^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} - (ab) \text{Subst} \left( \int x \log(1 + cx) dx, x, \frac{1}{x} \right) + \frac{1}{2} b^2 \int \frac{c \log(1 - \frac{c}{x})}{2x^3(c+x)} dx \\
&= \frac{b^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} - \frac{ab \log(\frac{c+x}{x})}{2x^2} + \frac{\text{Subst}(\int (2a - b \log(x))^2 dx, x, 1 - \frac{c}{x})}{4c^2} - \frac{\text{Subst}(\int \log^2(x) dx, x, 1 - \frac{c}{x})}{4c^2} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{4c^2} - \frac{(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))^2}{8c^2} + \frac{b^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} - \frac{3ab}{2cx} + \frac{b^2}{2cx} - \frac{b(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))}{8c^2} + \frac{(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} - \frac{3ab}{2cx} - \frac{b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{2c^2} - \frac{b(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))}{8c^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} - \frac{3ab}{2cx} - \frac{b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{2c^2} - \frac{b^2 \log(1 - \frac{c}{x})}{8x^2} - \frac{b(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} - \frac{3ab}{2cx} - \frac{3b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2} - \frac{b^2 \log(1 - \frac{c}{x})}{8x^2} - \frac{b(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} + \frac{b^2}{8x^2} - \frac{3ab}{2cx} + \frac{b^2 \log(1 - \frac{c}{x})}{8c^2} - \frac{3b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} + \frac{b^2}{8x^2} - \frac{3ab}{2cx} + \frac{b^2 \log(1 - \frac{c}{x})}{8c^2} - \frac{3b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2} \\
&= -\frac{b^2 (1 - \frac{c}{x})^2}{16c^2} - \frac{b^2 (1 + \frac{c}{x})^2}{16c^2} + \frac{ab}{4x^2} + \frac{b^2}{8x^2} - \frac{3ab}{2cx} + \frac{b^2 \log(1 - \frac{c}{x})}{8c^2} - \frac{3b^2 (1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2}
\end{aligned}$$



**Mathematica [A]** time = 0.067157, size = 119, normalized size = 1.37

$$\frac{a^2c^2 + abx^2 \log(x - c) - abx^2 \log(c + x) + 2abcx + 2bc \tanh^{-1}\left(\frac{c}{x}\right)(ac + bx) + b^2(c^2 - x^2) \tanh^{-1}\left(\frac{c}{x}\right)^2 + b^2x^2 \log(x - c)}{2c^2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x])^2/x^3,x]

[Out]  $-(a^2c^2 + 2abcx + 2b^2c^2 \operatorname{ArcTanh}[c/x] + b^2(c^2 - x^2) \operatorname{ArcTanh}[c/x]^2 - 2b^2x^2 \operatorname{Log}[x] + abx^2 \operatorname{Log}[-c + x] + b^2x^2 \operatorname{Log}[-c + x] - abx^2 \operatorname{Log}[c + x] + b^2x^2 \operatorname{Log}[c + x]) / (2c^2x^2)$

**Maple [B]** time = 0.016, size = 284, normalized size = 3.3

$$-\frac{a^2}{2x^2} - \frac{b^2}{2x^2} \left( \operatorname{Arctanh}\left(\frac{c}{x}\right) \right)^2 - \frac{b^2}{cx} \operatorname{Arctanh}\left(\frac{c}{x}\right) - \frac{b^2}{2c^2} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x} - 1\right) + \frac{b^2}{2c^2} \operatorname{Arctanh}\left(\frac{c}{x}\right) \ln\left(1 + \frac{c}{x}\right) + \frac{b^2}{4c^2} \ln\left(\frac{c}{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x))^2/x^3,x)

[Out]  $-1/2*a^2/x^2 - 1/2*b^2/x^2*arctanh(c/x)^2 - 1/c*b^2*arctanh(c/x)/x - 1/2/c^2*b^2*arctanh(c/x)*\ln(c/x-1) + 1/2/c^2*b^2*arctanh(c/x)*\ln(1+c/x) + 1/4/c^2*b^2*\ln(c/x-1)*\ln(1/2+1/2*c/x) - 1/8/c^2*b^2*\ln(c/x-1)^2 - 1/2/c^2*b^2*\ln(c/x-1) - 1/2/c^2*b^2*\ln(1+c/x) + 1/4/c^2*b^2*\ln(-1/2*c/x+1/2)*\ln(1+c/x) - 1/4/c^2*b^2*\ln(-1/2*c/x+1/2)*\ln(1/2+1/2*c/x) - 1/8/c^2*b^2*\ln(1+c/x)^2 - a*b/x^2*arctanh(c/x) - a*b/c/x - 1/2/c^2*a*b*\ln(c/x-1) + 1/2/c^2*a*b*\ln(1+c/x)$

**Maxima [B]** time = 0.989422, size = 223, normalized size = 2.56

$$\frac{1}{2} \left( c \left( \frac{\log(c+x)}{c^3} - \frac{\log(-c+x)}{c^3} - \frac{2}{c^2x} \right) - \frac{2 \operatorname{artanh}\left(\frac{c}{x}\right)}{x^2} \right) ab - \frac{1}{8} \left( c^2 \left( \frac{\log(c+x)^2 - 2(\log(c+x) - 2) \log(-c+x) + \log(-c+x)^2}{c^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^2/x^3,x, algorithm="maxima")

```
[Out] 1/2*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2)
*a*b - 1/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c +
x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c +
x)/c^3 - 2/(c^2*x))*arctanh(c/x)*b^2 - 1/2*b^2*arctanh(c/x)^2/x^2 - 1/2*a^2
/x^2
```

**Fricas [A]** time = 1.86745, size = 288, normalized size = 3.31

$$\frac{8b^2x^2 \log(x) - 4a^2c^2 - 8abcx + 4(ab - b^2)x^2 \log(c + x) - 4(ab + b^2)x^2 \log(-c + x) - (b^2c^2 - b^2x^2) \log\left(-\frac{c+x}{c-x}\right)^2 - 4(ab - b^2)x^2 \log\left(\frac{c+x}{c-x}\right)}{8c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^2*x^2*log(x) - 4*a^2*c^2 - 8*a*b*c*x + 4*(a*b - b^2)*x^2*log(c + x)
) - 4*(a*b + b^2)*x^2*log(-c + x) - (b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x
))^2 - 4*(a*b*c^2 + b^2*c*x)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

**Sympy [A]** time = 1.96205, size = 124, normalized size = 1.43

$$\left\{ \begin{array}{l} -\frac{a^2}{2x^2} - \frac{ab \operatorname{atanh}\left(\frac{c}{x}\right)}{x^2} - \frac{ab}{cx} + \frac{ab \operatorname{atanh}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2x^2} - \frac{b^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{cx} + \frac{b^2 \log(x)}{c^2} - \frac{b^2 \log(-c+x)}{c^2} + \frac{b^2 \operatorname{atanh}^2\left(\frac{c}{x}\right)}{2c^2} - \frac{b^2 \operatorname{atanh}\left(\frac{c}{x}\right)}{c^2} \end{array} \right. \text{for } \text{ot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x))^2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atanh(c/x)/x**2 - a*b/(c*x) + a*b*atanh(c/x)
)/c**2 - b**2*atanh(c/x)**2/(2*x**2) - b**2*atanh(c/x)/(c*x) + b**2*log(x)/
c**2 - b**2*log(-c + x)/c**2 + b**2*atanh(c/x)**2/(2*c**2) - b**2*atanh(c/x)
)/c**2, Ne(c, 0)), (-a**2/(2*x**2), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^2/x^3, x)
```

$$3.150 \quad \int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$$

**Optimal.** Leaf size=203

$$b^3 c^4 \text{PolyLog} \left( 2, \frac{2}{\frac{c}{x} + 1} - 1 \right) + \frac{1}{4} b^2 c^2 x^2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) - 2b^2 c^4 \log \left( 2 - \frac{2}{\frac{c}{x} + 1} \right) \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) - \frac{1}{4} c^4 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3$$

$$\begin{aligned} \text{[Out]} & (b^3 c^3 x)/4 - (b^3 c^4 \text{ArcCoth}[x/c])/4 + (b^2 c^2 x^2 (a + b \text{ArcCoth}[x/c]))/4 - b c^4 (a + b \text{ArcCoth}[x/c])^2 \\ & + (3 b c^3 x (a + b \text{ArcCoth}[x/c])^2)/4 + (b c x^3 (a + b \text{ArcCoth}[x/c])^2)/4 - (c^4 (a + b \text{ArcCoth}[x/c])^3)/4 + (x^4 (a + b \text{ArcCoth}[x/c])^3)/4 \\ & - 2 b^2 c^4 (a + b \text{ArcCoth}[x/c]) \text{Log}[2 - 2/(1 + c/x)] + b^3 c^4 \text{PolyLog}[2, -1 + 2/(1 + c/x)] \end{aligned}$$

**Rubi [F]** time = 4.39036, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$$

Verification is Not applicable to the result.

$$\text{[In]} \quad \text{Int}[x^3 (a + b \text{ArcTanh}[c/x])^3, x]$$

$$\begin{aligned} \text{[Out]} & (3 a^2 b c^3 x)/8 - (5 a b^2 c^3 x)/16 + (b^3 c^3 x)/16 - (3 a^2 b c^2 x^2)/16 + (3 a b^2 c^2 x^2)/16 + (a^2 b c x^3)/8 + (b^3 c^4 \text{Log}[1 - c/x])/32 - \\ & (3 a b^2 c^3 x \text{Log}[1 - c/x])/8 + (3 a b^2 c^2 x^2 \text{Log}[1 - c/x])/16 - (a b^2 c x^3 \text{Log}[1 - c/x])/8 + (5 b^2 c^3 (1 - c/x) x (2 a - b \text{Log}[1 - c/x]))/32 \\ & + (b^2 c^2 x^2 (2 a - b \text{Log}[1 - c/x]))/32 - (5 b c^4 (2 a - b \text{Log}[1 - c/x])^2)/64 + (3 b c^3 (1 - c/x) x (2 a - b \text{Log}[1 - c/x])^2)/32 \\ & + (3 b c^2 x^2 (2 a - b \text{Log}[1 - c/x])^2)/64 + (b c x^3 (2 a - b \text{Log}[1 - c/x])^2)/32 - (c^4 (2 a - b \text{Log}[1 - c/x])^3)/32 \\ & + (x^4 (2 a - b \text{Log}[1 - c/x])^3)/32 + (3 a b^2 c^3 x \text{Log}[1 + c/x])/8 + (3 a b^2 c^2 x^2 \text{Log}[1 + c/x])/16 + (a b^2 c x^3 \text{Log}[1 + c/x])/8 \\ & + (3 a^2 b x^4 \text{Log}[1 + c/x])/8 - (3 a b^2 x^4 \text{Log}[1 - c/x] \text{Log}[1 + c/x])/8 + (5 a b^2 c^4 \text{Log}[c - x])/16 + (3 a b^2 c^4 \text{Log}[1 + c/x] \text{Log}[c - x])/8 \\ & - (3 b c^4 (2 a - b \text{Log}[1 - c/x])^2 \text{Log}[c/x])/32 + (11 a b^2 c^4 \text{Log}[x])/8 + (3 a b^2 c^4 \text{Log}[c - x] \text{Log}[x/c])/8 - (3 a^2 b c^4 \text{Log}[c + x])/8 \\ & + (5 a b^2 c^4 \text{Log}[c + x])/16 + (3 a b^2 c^4 \text{Log}[1 - c/x] \text{Log}[c + x])/8 - (3 a b^2 c^4 \text{Log}[(c - x)/(2 c)] \text{Log}[c + x])/8 + (3 a b^2 c^4 \text{Log}[-(x/c)] \text{Log}[c + x])/8 \\ & - (3 a b^2 c^4 \text{Log}[c - x] \text{Log}[(c + x)/(2 c)])/8 + (11 a b^2 c^4 \text{Log}[(c + x)/x])/16 - (b^3 c^4 \text{Log}[(c + x)/x])/32 + (3 a b^2 c^3 x \text{Log}[(c + x)/x])/8 \end{aligned}$$

$$\begin{aligned}
& (c + x)/x)/8 - (5*b^3*c^3*(1 + c/x)*x*\text{Log}[(c + x)/x])/32 - (3*a*b^2*c^2*x^2 \\
& * \text{Log}[(c + x)/x])/16 + (b^3*c^2*x^2*\text{Log}[(c + x)/x])/32 + (a*b^2*c*x^3*\text{Log}[(c \\
& + x)/x])/8 - (3*a*b^2*c^4*\text{Log}[(c + x)/x]^2)/16 + (5*b^3*c^4*\text{Log}[(c + x)/x] \\
& ^2)/64 + (3*b^3*c^3*(1 + c/x)*x*\text{Log}[(c + x)/x]^2)/32 - (3*b^3*c^2*x^2*\text{Log}[(c \\
& + x)/x]^2)/64 + (b^3*c*x^3*\text{Log}[(c + x)/x]^2)/32 + (3*a*b^2*x^4*\text{Log}[(c + x) \\
& )/x]^2)/16 + (3*b^3*c^4*\text{Log}[-(c/x)]*\text{Log}[(c + x)/x]^2)/32 - (b^3*c^4*\text{Log}[(c \\
& + x)/x]^3)/32 + (b^3*x^4*\text{Log}[(c + x)/x]^3)/32 + (3*b^2*c^4*(2*a - b*\text{Log}[1 - \\
& c/x])* \text{PolyLog}[2, 1 - c/x])/16 - (3*a*b^2*c^4*\text{PolyLog}[2, (c - x)/(2*c)])/8 \\
& - (3*a*b^2*c^4*\text{PolyLog}[2, -(c/x)])/8 + (11*b^3*c^4*\text{PolyLog}[2, -(c/x)])/32 - \\
& (11*b^3*c^4*\text{PolyLog}[2, c/x])/32 - (3*a*b^2*c^4*\text{PolyLog}[2, (c + x)/(2*c)])/ \\
& 8 + (3*b^3*c^4*\text{Log}[(c + x)/x]* \text{PolyLog}[2, (c + x)/x])/16 + (3*a*b^2*c^4*\text{Poly} \\
& \text{Log}[2, 1 - x/c])/8 + (3*a*b^2*c^4*\text{PolyLog}[2, 1 + x/c])/8 + (3*b^3*c^4*\text{PolyL} \\
& \text{og}[3, 1 - c/x])/16 - (3*b^3*c^4*\text{PolyLog}[3, (c + x)/x])/16 + (3*b^3*\text{Defer}[\text{In} \\
& \text{t}][x^3*\text{Log}[1 - c/x]^2*\text{Log}[1 + c/x], x])/8 - (3*b^3*\text{Defer}[\text{Int}][x^3*\text{Log}[1 - c \\
& /x]*\text{Log}[1 + c/x]^2, x])/8
\end{aligned}$$

Rubi steps

$$\begin{aligned}
\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx &= \int \left( \frac{1}{8} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 + \frac{3}{8} b x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \log \left( 1 + \frac{c}{x} \right) + \frac{3}{8} b^2 x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log^2 \left( 1 + \frac{c}{x} \right) + \frac{1}{8} b^3 x^3 \log^3 \left( 1 + \frac{c}{x} \right) \right) dx \\
&= \frac{1}{8} \int x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 dx + \frac{1}{8} (3b) \int x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 \log \left( 1 + \frac{c}{x} \right) dx + \frac{1}{8} (3b^2) \int x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right) \log^2 \left( 1 + \frac{c}{x} \right) dx + \frac{1}{8} b^3 \int x^3 \log^3 \left( 1 + \frac{c}{x} \right) dx \\
&= - \left( \frac{1}{8} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^3}{x^5} dx, x, \frac{1}{x} \right) \right) + \frac{1}{8} (3b) \int \left( 4a^2 x^3 \log \left( 1 + \frac{c}{x} \right) - 4abx^3 \log^2 \left( 1 + \frac{c}{x} \right) + b^2 x^3 \log^3 \left( 1 + \frac{c}{x} \right) \right) dx \\
&= \frac{1}{32} x^4 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 + \frac{1}{32} b^3 x^4 \log^3 \left( \frac{c+x}{x} \right) + \frac{1}{2} (3a^2 b) \int x^3 \log \left( 1 + \frac{c}{x} \right) dx + \frac{1}{4} (3ab^2) \int x^3 \log^2 \left( 1 + \frac{c}{x} \right) dx \\
&= \frac{1}{32} x^4 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 + \frac{3}{8} a^2 b x^4 \log \left( 1 + \frac{c}{x} \right) - \frac{3}{8} a b^2 x^4 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{1}{32} b^3 x^4 \log^3 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{32} x^4 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 + \frac{3}{8} a^2 b x^4 \log \left( 1 + \frac{c}{x} \right) - \frac{3}{8} a b^2 x^4 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) + \frac{3}{16} a b^2 x^4 \log^2 \left( 1 + \frac{c}{x} \right) \\
&= \frac{1}{32} b c x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{32} x^4 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 + \frac{3}{8} a^2 b x^4 \log \left( 1 + \frac{c}{x} \right) - \frac{3}{8} a b^2 x^4 \log \left( 1 - \frac{c}{x} \right) \log \left( 1 + \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{3}{64} b c^2 x^2 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^2 + \frac{1}{32} b c x^3 \left( 2a - b \log \left( 1 - \frac{c}{x} \right) \right)^3 \\
&= \frac{3}{8} a^2 b c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{1}{8} a^2 b c x^3 - \frac{3}{8} a b^2 c^3 x \log \left( 1 - \frac{c}{x} \right) + \frac{3}{16} a b^2 c^2 x^2 \log \left( 1 - \frac{c}{x} \right) - \frac{1}{8} a b^2 c^2 x^2 \log^2 \left( 1 - \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{1}{8} a^2 b c x^3 - \frac{3}{8} a b^2 c^3 x \log \left( 1 - \frac{c}{x} \right) + \frac{3}{16} a b^2 c^2 x^2 \log \left( 1 - \frac{c}{x} \right) - \frac{1}{8} a b^2 c^2 x^2 \log^2 \left( 1 - \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{1}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{1}{32} b^3 c^4 \log \left( 1 - \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{1}{32} b^3 c^4 \log \left( 1 - \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{1}{32} b^3 c^4 \log \left( 1 - \frac{c}{x} \right) \\
&= \frac{3}{8} a^2 b c^3 x - \frac{5}{16} a b^2 c^3 x + \frac{1}{16} b^3 c^3 x - \frac{3}{16} a^2 b c^2 x^2 + \frac{3}{16} a b^2 c^2 x^2 + \frac{1}{8} a^2 b c x^3 + \frac{1}{32} b^3 c^4 \log \left( 1 - \frac{c}{x} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.603465, size = 286, normalized size = 1.41

$$\frac{1}{8} \left( 8b^3c^4 \text{PolyLog} \left( 2, e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) + 2b \tanh^{-1} \left( \frac{c}{x} \right) \left( 3a^2x^4 + 2abcx(3c^2 + x^2) + b^2(c^2x^2 - c^4) - 8b^2c^4 \log \left( 1 - e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcTanh[c/x])^3,x]

[Out]  $(-2ab^2c^4 + 6a^2bc^3x + 2b^3c^3x + 2ab^2c^2x^2 + 2a^2b^2cx^3 + 2a^3x^4 + 2b^2(bc(-4c^3 + 3c^2x + x^3) + 3a(-c^4 + x^4)) \text{ArcTanh}[c/x]^2 + 2b^3(-c^4 + x^4) \text{ArcTanh}[c/x]^3 + 2b \text{ArcTanh}[c/x] (3a^2x^4 + 2ab^2cx(3c^2 + x^2) + b^2(-c^4 + c^2x^2) - 8b^2c^4 \text{Log}[1 - E^{-2 \text{ArcTanh}[c/x]}]) + 3a^2bc^4 \text{Log}[1 - c/x] - 16ab^2c^4 \text{Log}[c/\sqrt{1 - c^2/x^2}x] - 3a^2bc^4 \text{Log}[(c + x)/x] + 8b^3c^4 \text{PolyLog}[2, E^{-2 \text{ArcTanh}[c/x]}]) / 8$

**Maple [C]** time = 0.332, size = 1410, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c/x))^3,x)

[Out]  $3/4a^2bx^4 \arctanh(c/x) + 3/8c^4b^3 \arctanh(c/x)^2 \ln(c/x-1) + 1/4c^4b^3 / (c/x+1 - (1-c^2/x^2)^{1/2}) * (1-c^2/x^2)^{1/2} + c^4ab^2 \ln(c/x-1) + c^4ab^2 \ln(1+c/x) - 3/16Ic^4b^3\text{Pi} * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)) * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2 * \arctanh(c/x)^2 - 2c^4b^3 \text{dilog}(1+(1+c/x)/(1-c^2/x^2)^{1/2}) + 2c^4b^3 \text{dilog}((1+c/x)/(1-c^2/x^2)^{1/2}) + 3/16 * Ic^4b^3\text{Pi} * \text{csgn}(I(1+c/x)/(1-c^2/x^2)^{1/2})^2 * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)) * \arctanh(c/x)^2 + 3/8Ic^4b^3\text{Pi} * \text{csgn}(I(1+c/x)/(1-c^2/x^2)^{1/2}) * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2))^2 * \arctanh(c/x)^2 + 3/16Ic^4b^3\text{Pi} * \text{csgn}(I/((1+c/x)^2/(1-c^2/x^2)+1)) * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2 * \arctanh(c/x)^2 - 2c^4b^3 \arctanh(c/x) * \ln(1+(1+c/x)/(1-c^2/x^2)^{1/2}) + 3/4c^4b^3 \arctanh(c/x)^2 \ln((1+c/x)/(1-c^2/x^2)^{1/2}) + 3/4c^3b^3 \arctanh(c/x)^2 * x + 1/4c^3b^3 \arctanh(c/x)^2 * x^3 - 3/16Ic^4b^3\text{Pi} * \text{csgn}(I/((1+c/x)^2/(1-c^2/x^2)+1)) * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)) * \text{csgn}(I(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1)) * \arctanh(c/x)^2 + 3/16c^4ab^2 \ln(1+c/x)^2 + 1/4b^3x^4 \arctanh(c/x)^3 - 1/4c^4b^3 \arctanh(c/x) + 3/4c^3a^2bx - 1/4c^4b^3$





Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="fricas")`

[Out] `integral(b^3*x^3*arctanh(c/x)^3 + 3*a*b^2*x^3*arctanh(c/x)^2 + 3*a^2*b*x^3*arctanh(c/x) + a^3*x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \left( a + b \operatorname{atanh} \left( \frac{c}{x} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*atanh(c/x))**3,x)`

[Out] `Integral(x**3*(a + b*atanh(c/x))**3, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x} \right) + a \right)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c/x) + a)^3*x^3, x)`

$$3.151 \quad \int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$$

**Optimal.** Leaf size=217

$$b^2 c^3 \text{PolyLog} \left( 2, \frac{2}{\frac{c}{x} + 1} - 1 \right) \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) + \frac{1}{2} b^3 c^3 \text{PolyLog} \left( 3, \frac{2}{\frac{c}{x} + 1} - 1 \right) + b^2 c^2 x \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) - \frac{1}{3} c^3 \left( a + b \right)$$

[Out]  $b^2 c^2 x (a + b \text{ArcCoth}[x/c]) - (b^3 c^3 (a + b \text{ArcCoth}[x/c])^2) / 2 + (b^2 c^2 x^2 (a + b \text{ArcCoth}[x/c])^2) / 2 - (c^3 (a + b \text{ArcCoth}[x/c])^3) / 3 + (x^3 (a + b \text{ArcCoth}[x/c])^3) / 3 - b^3 c^3 (a + b \text{ArcCoth}[x/c])^2 \text{Log}[2 - 2/(1 + c/x)] + (b^3 c^3 \text{Log}[1 - c^2/x^2]) / 2 + b^3 c^3 \text{Log}[x] + b^2 c^3 (a + b \text{ArcCoth}[x/c]) \text{PolyLog}[2, -1 + 2/(1 + c/x)] + (b^3 c^3 \text{PolyLog}[3, -1 + 2/(1 + c/x)]) / 2$

**Rubi [F]** time = 3.30156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x^2 (a + b \text{ArcTanh}[c/x])^3, x]$

[Out]  $-(a^2 b c^2 x) / 2 + (3 a b^2 c^2 x) / 4 + (a^2 b c x^2) / 4 + (a b^2 c^2 x \text{Log}[1 - c/x]) / 2 - (a b^2 c x^2 \text{Log}[1 - c/x]) / 4 + (b^2 c^2 (1 - c/x) x (2 a - b \text{Log}[1 - c/x])) / 8 - (b^3 c^3 (2 a - b \text{Log}[1 - c/x])^2) / 16 + (b^2 c^2 (1 - c/x) x (2 a - b \text{Log}[1 - c/x])^2) / 8 + (b^2 c x^2 (2 a - b \text{Log}[1 - c/x])^2) / 16 - (c^3 (2 a - b \text{Log}[1 - c/x])^3) / 24 + (x^3 (2 a - b \text{Log}[1 - c/x])^3) / 24 + (a b^2 c^2 x \text{Log}[1 + c/x]) / 2 + (a b^2 c x^2 \text{Log}[1 + c/x]) / 4 + (a^2 b x^3 \text{Log}[1 + c/x]) / 2 - (a b^2 x^3 \text{Log}[1 - c/x] \text{Log}[1 + c/x]) / 2 - (a b^2 c^3 \text{Log}[c - x]) / 4 + (a b^2 c^3 \text{Log}[1 + c/x] \text{Log}[c - x]) / 2 - (b^3 c^3 (2 a - b \text{Log}[1 - c/x])^2 \text{Log}[c/x]) / 8 + (b^3 c^3 \text{Log}[x]) / 4 + (a b^2 c^3 \text{Log}[c - x] \text{Log}[x/c]) / 2 + (a^2 b c^3 \text{Log}[c + x]) / 2 + (a b^2 c^3 \text{Log}[c + x]) / 4 - (a b^2 c^3 \text{Log}[1 - c/x] \text{Log}[c + x]) / 2 + (a b^2 c^3 \text{Log}[(c - x) / (2 c)] \text{Log}[c + x]) / 2 - (a b^2 c^3 \text{Log}[-(x/c)] \text{Log}[c + x]) / 2 - (a b^2 c^3 \text{Log}[c - x] \text{Log}[(c + x) / (2 c)]) / 2 - (3 a b^2 c^3 \text{Log}[(c + x) / x]) / 4 - (a b^2 c^2 x \text{Log}[(c + x) / x]) / 2 + (b^3 c^2 (1 + c/x) x \text{Log}[(c + x) / x]) / 8 + (a b^2 c x^2 \text{Log}[(c + x) / x]) / 4 + (a b^2 c^3 \text{Log}[(c + x) / x]^2) / 4 - (b^3 c^3 \text{Log}[(c + x) / x]^2) / 16 - (b^3 c^2 (1 + c/x) x \text{Log}[(c + x) / x]^2) / 8 + (b^3 c x^2 \text{Log}[(c + x) / x]^2) / 16 + (a b^2 x^3 \text{Log}[(c + x) / x]^2) / 4 - (b^3 c^3 \text{Log}[-(c/x)] \text{Log}[(c + x) / x]^2) / 8 + (b^3 c^3 \text{Log}[(c + x) / x]$

$$\begin{aligned}
& ]^3)/24 + (b^3*x^3*\text{Log}[(c+x)/x]^3)/24 + (b^2*c^3*(2*a - b*\text{Log}[1 - c/x])* \\
& \text{PolyLog}[2, 1 - c/x])/4 - (a*b^2*c^3*\text{PolyLog}[2, (c-x)/(2*c)])/2 + (a*b^2*c^ \\
& 3*\text{PolyLog}[2, -(c/x)])/2 - (3*b^3*c^3*\text{PolyLog}[2, -(c/x)])/8 - (3*b^3*c^3*\text{Pol} \\
& \text{yLog}[2, c/x])/8 + (a*b^2*c^3*\text{PolyLog}[2, (c+x)/(2*c)])/2 - (b^3*c^3*\text{Log}[(c \\
& + x)/x]*\text{PolyLog}[2, (c+x)/x])/4 + (a*b^2*c^3*\text{PolyLog}[2, 1 - x/c])/2 - (a* \\
& b^2*c^3*\text{PolyLog}[2, 1 + x/c])/2 + (b^3*c^3*\text{PolyLog}[3, 1 - c/x])/4 + (b^3*c^3 \\
& *\text{PolyLog}[3, (c+x)/x])/4 + (3*b^3*\text{Defer}[\text{Int}][x^2*\text{Log}[1 - c/x]^2*\text{Log}[1 + c/ \\
& x], x])/8 - (3*b^3*\text{Defer}[\text{Int}][x^2*\text{Log}[1 - c/x]*\text{Log}[1 + c/x]^2, x])/8
\end{aligned}$$

Rubi steps



**Mathematica [C]** time = 0.744349, size = 316, normalized size = 1.46

$$\frac{1}{6} \left( 6ab^2 \left( c^3 \text{PolyLog} \left( 2, e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) + (x^3 - c^3) \tanh^{-1} \left( \frac{c}{x} \right)^2 + c \tanh^{-1} \left( \frac{c}{x} \right) \left( -2c^2 \log \left( 1 - e^{-2 \tanh^{-1} \left( \frac{c}{x} \right)} \right) - c^2 + x^2 \right) + \right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c/x])^3,x]

[Out] (3\*a^2\*b\*c\*x^2 + 2\*a^3\*x^3 + 6\*a^2\*b\*x^3\*ArcTanh[c/x] + 3\*a^2\*b\*c^3\*Log[-c^2 + x^2] + 6\*a\*b^2\*(c^2\*x + (-c^3 + x^3)\*ArcTanh[c/x]^2 + c\*ArcTanh[c/x]\*(-c^2 + x^2 - 2\*c^2\*Log[1 - E^(-2\*ArcTanh[c/x])])) + c^3\*PolyLog[2, E^(-2\*ArcTanh[c/x])]) + (b^3\*((-I)\*c^3\*Pi^3 + 24\*c^2\*x\*ArcTanh[c/x] - 12\*c^3\*ArcTanh[c/x]^2 + 12\*c\*x^2\*ArcTanh[c/x]^2 + 8\*c^3\*ArcTanh[c/x]^3 + 8\*x^3\*ArcTanh[c/x]^3 - 24\*c^3\*ArcTanh[c/x]^2\*Log[1 - E^(2\*ArcTanh[c/x])] - 24\*c^3\*Log[c/(Sqrt[1 - c^2/x^2]\*x)] - 24\*c^3\*ArcTanh[c/x]\*PolyLog[2, E^(2\*ArcTanh[c/x])] + 12\*c^3\*PolyLog[3, E^(2\*ArcTanh[c/x])]))/4)/6

**Maple [C]** time = 0.02, size = 2033, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c/x))^3,x)

[Out] 1/4\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2))\*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))\*Pi+1/4\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2))\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2\*Pi+1/2\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))\*csgn(I\*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2\*Pi-1/2\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I\*(1+c/x)/(1-c^2/x^2)^(1/2))\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2))^2\*Pi-1/2\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))\*csgn(I\*((1+c/x)^2/(1-c^2/x^2)-1))\*csgn(I\*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))\*Pi-1/4\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I\*(1+c/x)/(1-c^2/x^2)^(1/2))^2\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2))\*Pi-1/4\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))\*csgn(I\*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2\*Pi+1/3\*x^3\*a^3+1/2\*I\*c^3\*b^3\*arctanh(c/x)^2\*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^2\*Pi+1/2\*c\*b^3\*arctanh(c/x)^2\*x^2+1/4\*c^3\*a\*b^2\*ln(c/x-1)^2-1/2\*c^3\*a\*b^2\*ln(1+c/x)+1/2\*



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^2 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3ab^2x^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3a^2bx^2 \operatorname{artanh}\left(\frac{c}{x}\right) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^2\*arctanh(c/x)^3 + 3\*a\*b^2\*x^2\*arctanh(c/x)^2 + 3\*a^2\*b\*x^2\*arctanh(c/x) + a^3\*x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \left( a + b \operatorname{atanh}\left(\frac{c}{x}\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c/x))\*\*3,x)

[Out] Integral(x\*\*2\*(a + b\*atanh(c/x))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh}\left(\frac{c}{x}\right) + a \right)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^3\*x^2, x)

### 3.152 $\int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$

**Optimal.** Leaf size=135

$$\frac{3}{2}b^3c^2\text{PolyLog}\left(2, \frac{2}{\frac{c}{x}+1} - 1\right) - 3b^2c^2\log\left(2 - \frac{2}{\frac{c}{x}+1}\right)\left(a + b\coth^{-1}\left(\frac{x}{c}\right)\right) - \frac{3}{2}bc^2\left(a + b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{2}c^2\left(a + b\coth^{-1}\left(\frac{x}{c}\right)\right)^3$$

[Out]  $(-3*b*c^2*(a + b*\text{ArcCoth}[x/c])^2)/2 + (3*b*c*x*(a + b*\text{ArcCoth}[x/c])^2)/2 - (c^2*(a + b*\text{ArcCoth}[x/c])^3)/2 + (x^2*(a + b*\text{ArcCoth}[x/c])^3)/2 - 3*b^2*c^2*(a + b*\text{ArcCoth}[x/c])*Log[2 - 2/(1 + c/x)] + (3*b^3*c^2*\text{PolyLog}[2, -1 + 2/(1 + c/x)])/2$

**Rubi [F]** time = 2.24482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[x\*(a + b\*ArcTanh[c/x])^3,x]

[Out]  $(3*a^2*b*c*x)/4 - (3*a*b^2*c*x*Log[1 - c/x])/4 + (3*b*c*(1 - c/x)*x*(2*a - b*Log[1 - c/x])^2)/16 - (c^2*(2*a - b*Log[1 - c/x])^3)/16 + (x^2*(2*a - b*Log[1 - c/x])^3)/16 + (3*a*b^2*c*x*Log[1 + c/x])/4 + (3*a^2*b*x^2*Log[1 + c/x])/4 - (3*a*b^2*x^2*Log[1 - c/x]*Log[1 + c/x])/4 + (3*a*b^2*c^2*Log[c - x])/4 + (3*a*b^2*c^2*Log[1 + c/x]*Log[c - x])/4 - (3*b*c^2*(2*a - b*Log[1 - c/x])^2*Log[c/x])/16 + (3*a*b^2*c^2*Log[x])/2 + (3*a*b^2*c^2*Log[c - x]*Log[x/c])/4 - (3*a^2*b*c^2*Log[c + x])/4 + (3*a*b^2*c^2*Log[c + x])/4 + (3*a*b^2*c^2*Log[1 - c/x]*Log[c + x])/4 - (3*a*b^2*c^2*Log[(c - x)/(2*c)]*Log[c + x])/4 + (3*a*b^2*c^2*Log[-(x/c)]*Log[c + x])/4 - (3*a*b^2*c^2*Log[c - x]*Log[(c + x)/(2*c)])/4 + (3*a*b^2*c^2*Log[(c + x)/x])/4 + (3*a*b^2*c*x*Log[(c + x)/x])/4 - (3*a*b^2*c^2*Log[(c + x)/x]^2)/8 + (3*b^3*c*(1 + c/x)*x*Log[(c + x)/x]^2)/16 + (3*a*b^2*x^2*Log[(c + x)/x]^2)/8 + (3*b^3*c^2*Log[-(c/x)]*Log[(c + x)/x]^2)/16 - (b^3*c^2*Log[(c + x)/x]^3)/16 + (b^3*x^2*Log[(c + x)/x]^3)/16 + (3*b^2*c^2*(2*a - b*Log[1 - c/x])*PolyLog[2, 1 - c/x])/8 - (3*a*b^2*c^2*PolyLog[2, (c - x)/(2*c)]/4 - (3*a*b^2*c^2*PolyLog[2, -(c/x)]/4 + (3*b^3*c^2*PolyLog[2, -(c/x)]/8 - (3*b^3*c^2*PolyLog[2, c/x])/8 - (3*a*b^2*c^2*PolyLog[2, (c + x)/(2*c)]/4 + (3*b^3*c^2*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/8 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4 + (3*a*b^2*c^2*PolyLog[2, 1 - x/c])/4$



$$2, 1 + x/c])/4 + (3*b^3*c^2*PolyLog[3, 1 - c/x])/8 - (3*b^3*c^2*PolyLog[3, (c + x)/x])/8 + (3*b^3*Defer[Int][x*Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][x*Log[1 - c/x]*Log[1 + c/x]^2, x])/8$$

Rubi steps



Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c/x])^3,x]
```

```
[Out] (6*b^2*(-c + x)*(b*c + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcTanh[c/x]^3 + 6*b*ArcTanh[c/x]*(a*x*(2*b*c + a*x) - 2*b^2*c^2*Log[1 - E^(-2*ArcTanh[c/x])]) + a*(3*a*b*c^2*Log[1 - c/x] - 12*b^2*c^2*Log[c/(Sqrt[1 - c^2/x^2]*x)]) + a*(6*b*c*x + 2*a*x^2 - 3*b*c^2*Log[(c + x)/x])) + 6*b^3*c^2*PolyLog[2, E^(-2*ArcTanh[c/x])])/4
```

**Maple [C]** time = 0.278, size = 5536, normalized size = 41.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x))^3,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} ab^2 x^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{4} \left(2x^2 \operatorname{artanh}\left(\frac{c}{x}\right) - (c \log(c+x) - c \log(-c+x) - 2x)c\right) a^2 b + \frac{3}{8} \left(\log(c+x)^2 - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctanh(c/x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c/x) - (c*log(c + x) - c*log(-c + x) - 2*x)*c)*a^2*b + 3/8*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x) - c*log(-c + x) - 2*x)*c*arctanh(c/x))*a*b^2 + 1/16*(6*c*x*log(c + x)^2 - (c^2 - x^2)*log(c + x)^3 + (c^2 - x^2)*log(-c + x)^3 - 3*(2*c^2 - 2*c*x + (c^2 - x^2)*log(c + x))*log(-c + x)^2 + 3*((c^2 - x^2)*log(c + x)^2 - 4*(c^2 + c*x)*log(c + x))*log(-c + x) + 2*integrate(-6*(c^3 + 3*c^2*x)*log(c + x)/(c^2 - x^2), x))*b^3
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3ab^2x \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3a^2bx \operatorname{artanh}\left(\frac{c}{x}\right) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x\*arctanh(c/x)^3 + 3\*a\*b^2\*x\*arctanh(c/x)^2 + 3\*a^2\*b\*x\*arctanh(c/x) + a^3\*x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \left( a + b \operatorname{atanh}\left(\frac{c}{x}\right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c/x))\*\*3,x)

[Out] Integral(x\*(a + b\*atanh(c/x))\*\*3, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh}\left(\frac{c}{x}\right) + a \right)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^3\*x, x)

### 3.153 $\int \left(a + b \tanh^{-1} \left(\frac{c}{x}\right)\right)^3 dx$

**Optimal.** Leaf size=108

$$-3b^2c \operatorname{PolyLog}\left(2, 1 - \frac{2c}{c-x}\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}b^3c \operatorname{PolyLog}\left(3, 1 - \frac{2c}{c-x}\right) + c \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right)^3 + x \left(a + b \operatorname{coth}^{-1}\left(\frac{x}{c}\right)\right)^2$$

```
[Out] c*(a + b*ArcCoth[x/c])^3 + x*(a + b*ArcCoth[x/c])^3 - 3*b*c*(a + b*ArcCoth[x/c])^2*Log[(2*c)/(c - x)] - 3*b^2*c*(a + b*ArcCoth[x/c])*PolyLog[2, 1 - (2*c)/(c - x)] + (3*b^3*c*PolyLog[3, 1 - (2*c)/(c - x)])/2
```

**Rubi [F]** time = 0.700751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \left(a + b \tanh^{-1} \left(\frac{c}{x}\right)\right)^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c/x])^3, x]
```

```
[Out] a^3*x - (3*a^2*b*x*Log[1 - c/x])/2 - (3*a*b^2*(c - x)*Log[1 - c/x]^2)/4 + (b^3*(c - x)*Log[1 - c/x]^3)/8 + (3*a^2*b*x*Log[1 + c/x])/2 - (3*a*b^2*x*Log[1 - c/x]*Log[1 + c/x])/2 + (3*a*b^2*(c + x)*Log[1 + c/x]^2)/4 + (b^3*(c + x)*Log[1 + c/x]^3)/8 - (3*a*b^2*c*Log[1 - c/x]*Log[-c - x])/2 + (3*a^2*b*c*Log[c - x])/2 + (3*a*b^2*c*Log[-c - x]*Log[(c - x)/(2*c)])/2 - (3*b^3*c*Log[1 - c/x]^2*Log[c/x])/8 - (3*a*b^2*c*Log[-c - x]*Log[-(x/c)])/2 + (3*a*b^2*c*Log[1 + c/x]*Log[-c + x])/2 + (3*a*b^2*c*Log[x/c]*Log[-c + x])/2 + (3*a^2*b*c*Log[c + x])/2 - (3*a*b^2*c*Log[-c + x]*Log[(c + x)/(2*c)])/2 - (3*b^3*c*Log[-(c/x)]*Log[(c + x)/x]^2)/8 - (3*b^3*c*Log[1 - c/x]*PolyLog[2, 1 - c/x])/4 - (3*a*b^2*c*PolyLog[2, (c - x)/(2*c)])/2 + (3*a*b^2*c*PolyLog[2, -(c/x)])/2 - (3*a*b^2*c*PolyLog[2, c/x])/2 + (3*a*b^2*c*PolyLog[2, (c + x)/(2*c)])/2 - (3*b^3*c*Log[(c + x)/x]*PolyLog[2, (c + x)/x])/4 + (3*a*b^2*c*PolyLog[2, 1 - x/c])/2 - (3*a*b^2*c*PolyLog[2, 1 + x/c])/2 + (3*b^3*c*PolyLog[3, 1 - c/x])/4 + (3*b^3*c*PolyLog[3, (c + x)/x])/4 + (3*b^3*Defer[Int][Log[1 - c/x]^2*Log[1 + c/x], x])/8 - (3*b^3*Defer[Int][Log[1 - c/x]*Log[1 + c/x]^2, x])/8
```

Rubi steps

$$\begin{aligned}
\int \left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3 dx &= \int \left(a^3 - \frac{3}{2}a^2b \log\left(1 - \frac{c}{x}\right) + \frac{3}{4}ab^2 \log^2\left(1 - \frac{c}{x}\right) - \frac{1}{8}b^3 \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2b \log\left(1 + \frac{c}{x}\right) - \frac{3}{2}a^2b \log\left(1 - \frac{c}{x}\right)\right) dx \\
&= a^3x - \frac{1}{2}(3a^2b) \int \log\left(1 - \frac{c}{x}\right) dx + \frac{1}{2}(3a^2b) \int \log\left(1 + \frac{c}{x}\right) dx + \frac{1}{4}(3ab^2) \int \log^2\left(1 - \frac{c}{x}\right) dx \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right) \\
&= a^3x - \frac{3}{2}a^2bx \log\left(1 - \frac{c}{x}\right) - \frac{3}{4}ab^2(c-x) \log^2\left(1 - \frac{c}{x}\right) + \frac{1}{8}b^3(c-x) \log^3\left(1 - \frac{c}{x}\right) + \frac{3}{2}a^2bx \log\left(1 + \frac{c}{x}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.263006, size = 198, normalized size = 1.83

$$-3ab^2 \left(\tanh^{-1}\left(\frac{c}{x}\right)\right) \left((c-x) \tanh^{-1}\left(\frac{c}{x}\right) + 2c \log\left(1 - e^{-2 \tanh^{-1}\left(\frac{c}{x}\right)}\right)\right) - c \text{PolyLog}\left(2, e^{-2 \tanh^{-1}\left(\frac{c}{x}\right)}\right) + \frac{1}{8}b^3 \left(-24c \tanh^{-1}\left(\frac{c}{x}\right) + \dots\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^3,x]

[Out] a^3\*x + 3\*a^2\*b\*x\*ArcTanh[c/x] + (3\*a^2\*b\*c\*Log[-c^2 + x^2])/2 - 3\*a\*b^2\*(ArcTanh[c/x]\*((c-x)\*ArcTanh[c/x] + 2\*c\*Log[1 - E^(-2\*ArcTanh[c/x])]) - c\*PolyLog[2, E^(-2\*ArcTanh[c/x])]) + (b^3\*((-1)\*c\*Pi^3 + 8\*c\*ArcTanh[c/x]^3 + 8\*x\*ArcTanh[c/x]^3 - 24\*c\*ArcTanh[c/x]^2\*Log[1 - E^(2\*ArcTanh[c/x])] - 24\*c\*ArcTanh[c/x]\*PolyLog[2, E^(2\*ArcTanh[c/x])] + 12\*c\*PolyLog[3, E^(2\*ArcTanh[c/x])]))/8

**Maple [C]** time = 0.121, size = 1756, normalized size = 16.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arctanh}(c/x))^3, x)$

[Out] 
$$\begin{aligned} & -6*c*a*b^2*\ln(c/x)*\text{arctanh}(c/x)+3*c*a*b^2*\ln(c/x)*\ln(1+c/x)+3/2*c*a*b^2*\ln(-1/2*c/x+1/2)*\ln(1+c/x)-3/2*c*a*b^2*\ln(-1/2*c/x+1/2)*\ln(1/2+1/2*c/x)+3*c*a*b^2*\text{arctanh}(c/x)*\ln(c/x-1)+3*c*a*b^2*\text{arctanh}(c/x)*\ln(1+c/x)-3/2*c*a*b^2*\ln(c/x-1)*\ln(1/2+1/2*c/x)-3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*\text{arctanh}(c/x)^2+6*c*b^3*\text{polylog}(3, -(1+c/x)/(1-c^2/x^2)^{(1/2)})+6*c*b^3*\text{polylog}(3, (1+c/x)/(1-c^2/x^2)^{(1/2)})+c*b^3*\text{arctanh}(c/x)^3+b^3*x*a*\text{rctanh}(c/x)^3+3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2+3/2*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*\text{arctanh}(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*((1+c/x)/(1-c^2/x^2)^{(1/2)})^2*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*\text{arctanh}(c/x)^2-3/4*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2-3/2*I*c*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^{(1/2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^2*\text{arctanh}(c/x)^2+3/4*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*(1+c/x)^2/(-1+c^2/x^2))*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))*\text{arctanh}(c/x)^2+x*a^3-6*c*b^3*\text{arctanh}(c/x)*\text{polylog}(2, (1+c/x)/(1-c^2/x^2)^{(1/2)})-6*c*b^3*a*\text{rctanh}(c/x)*\text{polylog}(2, -(1+c/x)/(1-c^2/x^2)^{(1/2)})-3*c*b^3*\text{arctanh}(c/x)^2*\ln(1-(1+c/x)/(1-c^2/x^2)^{(1/2)})+3*c*a*b^2*\text{dilog}(c/x)+3/2*c*a^2*b*\ln(1+c/x)+3/4*c*a*b^2*\ln(c/x-1)^2-3/4*c*a*b^2*\ln(1+c/x)^2-3*c*b^3*\text{arctanh}(c/x)^2*\ln(2)-3*c*a^2*b*\ln(c/x)+3*c*a*b^2*\text{dilog}(1+c/x)-3*c*a*b^2*\text{dilog}(1/2+1/2*c/x)+3*a*b^2*x*\text{arctanh}(c/x)^2+3*a^2*b*x*\text{arctanh}(c/x)+3/2*c*b^3*\text{arctanh}(c/x)^2*\ln(c/x-1)+3/2*c*b^3*\text{arctanh}(c/x)^2*\ln(1+c/x)-3*c*b^3*\text{arctanh}(c/x)^2*\ln((1+c/x)/(1-c^2/x^2)^{(1/2)})-3*c*b^3*\text{arctanh}(c/x)^2*\ln(1+(1+c/x)/(1-c^2/x^2)^{(1/2)})-3*c*b^3*\ln(c/x)*\text{arctanh}(c/x)^2+3*c*b^3*\text{arctanh}(c/x)^2*\ln((1+c/x)^2/(1-c^2/x^2)-1)-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2)/((1+c/x)^2/(1-c^2/x^2)+1))^3*\text{arctanh}(c/x)^2-3/4*I*c*b^3*Pi*csgn(I*(1+c/x)^2/(-1+c^2/x^2))^3*\text{arctanh}(c/x)^2-3/2*I*c*b^3*Pi*\text{arctanh}(c/x)^2+3/2*I*c*b^3*Pi*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2+3/2*I*c*b^3*Pi*csgn(I/((1+c/x)^2/(1-c^2/x^2)+1))*csgn(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2 \end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} \left( 2x \operatorname{artanh} \left( \frac{c}{x} \right) + c \log(-c^2 + x^2) \right) a^2 b + a^3 x + \frac{1}{8} (b^3 c - b^3 x) \log(-c + x)^3 + \frac{3}{8} (2ab^2 x + (b^3 c + b^3 x) \log(c + x)) \log(-c + x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3,x, algorithm="maxima")

[Out] 3/2\*(2\*x\*arctanh(c/x) + c\*log(-c^2 + x^2))\*a^2\*b + a^3\*x + 1/8\*(b^3\*c - b^3\*x)\*log(-c + x)^3 + 3/8\*(2\*a\*b^2\*x + (b^3\*c + b^3\*x)\*log(c + x))\*log(-c + x)^2 - integrate(-1/8\*((b^3\*c - b^3\*x)\*log(c + x)^3 + 6\*(a\*b^2\*c - a\*b^2\*x)\*log(c + x)^2 + 3\*(4\*a\*b^2\*x - (b^3\*c - b^3\*x)\*log(c + x)^2 - 2\*(2\*a\*b^2\*c - b^3\*c - (2\*a\*b^2 + b^3)\*x)\*log(c + x))\*log(-c + x))/(c - x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( b^3 \operatorname{artanh} \left( \frac{c}{x} \right)^3 + 3ab^2 \operatorname{artanh} \left( \frac{c}{x} \right)^2 + 3a^2b \operatorname{artanh} \left( \frac{c}{x} \right) + a^3, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3,x, algorithm="fricas")

[Out] integral(b^3\*arctanh(c/x)^3 + 3\*a\*b^2\*arctanh(c/x)^2 + 3\*a^2\*b\*arctanh(c/x) + a^3, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( a + b \operatorname{atanh} \left( \frac{c}{x} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*\*3,x)

[Out] Integral((a + b\*atanh(c/x))\*\*3, x)



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh}\left(\frac{c}{x}\right) + a \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^3, x)

$$3.154 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x} dx$$

**Optimal.** Leaf size=208

$$-\frac{3}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x}}\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1 - \frac{c}{x}} - 1\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}b \text{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right)$$

```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[2, 1 - 2/(1 - c/x)]/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[2, -1 + 2/(1 - c/x)]/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 - c/x)]/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)]/2 + (3*b^3*PolyLog[4, 1 - 2/(1 - c/x)]/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)]/4
```

**Rubi [A]** time = 0.508605, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$-\frac{3}{2}b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x}}\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}b^2 \text{PolyLog}\left(3, \frac{2}{1 - \frac{c}{x}} - 1\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}b \text{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c/x])^3/x, x]
```

```
[Out] -2*(a + b*ArcCoth[x/c])^3*ArcTanh[1 - 2/(1 - c/x)] + (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[2, 1 - 2/(1 - c/x)]/2 - (3*b*(a + b*ArcCoth[x/c])^2*PolyLog[2, -1 + 2/(1 - c/x)]/2 - (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, 1 - 2/(1 - c/x)]/2 + (3*b^2*(a + b*ArcCoth[x/c])*PolyLog[3, -1 + 2/(1 - c/x)]/2 + (3*b^3*PolyLog[4, 1 - 2/(1 - c/x)]/4 - (3*b^3*PolyLog[4, -1 + 2/(1 - c/x)]/4
```

**Rule 6095**

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

**Rule 5914**

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] := Simp[2*(a +
b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x))]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^3}{x} dx &= -\text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^3}{x} dx, x, \frac{1}{x} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + (6bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \tanh^{-1}(cx)}{1 - c^2 x^2} dx, x, \frac{1}{x} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - (3bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \log \left( \frac{2}{1 - cx} \right)}{1 - c^2 x^2} dx, x, \frac{1}{x} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \text{Li}_2 \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \log \left( \frac{2}{1 - \frac{c}{x}} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \text{Li}_2 \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \log \left( \frac{2}{1 - \frac{c}{x}} \right) \\
&= -2 \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^3 \tanh^{-1} \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) + \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right)^2 \text{Li}_2 \left( 1 - \frac{2}{1 - \frac{c}{x}} \right) - \frac{3}{2} b \left( a + b \coth^{-1} \left( \frac{x}{c} \right) \right) \log \left( \frac{2}{1 - \frac{c}{x}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.191137, size = 171, normalized size = 0.82

$$\frac{3}{4} b \left( 2 \text{PolyLog} \left( 2, \frac{c+x}{c-x} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 - 2 \text{PolyLog} \left( 2, \frac{c+x}{x-c} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right)^2 + b \left( -2 \text{PolyLog} \left( 3, \frac{c+x}{c-x} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + 2 \text{PolyLog} \left( 3, \frac{c+x}{-c+x} \right) \left( a + b \tanh^{-1} \left( \frac{c}{x} \right) \right) + b \left( \text{PolyLog} \left[ 4, \frac{c+x}{c-x} \right] - \text{PolyLog} \left[ 4, \frac{c+x}{-c+x} \right] \right) \right) / 4$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTanh[c/x])^3*ArcTanh[(c + x)/(c - x)] + (3*b*(2*(a + b*ArcTanh[c/x])^2*PolyLog[2, (c + x)/(c - x)] - 2*(a + b*ArcTanh[c/x])^2*PolyLog[2, (c + x)/(-c + x)] + b*(-2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(c - x)] + 2*(a + b*ArcTanh[c/x])*PolyLog[3, (c + x)/(-c + x)] + b*(PolyLog[4, (c + x)/(c - x)] - PolyLog[4, (c + x)/(-c + x)])))/4
```

**Maple [C]** time = 0.07, size = 1631, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arctanh}(c/x))^3/x,x)$

[Out]  $3/2*a^2*b*\text{dilog}(1+c/x)-3/2*a*b^2*\text{polylog}(3,-(1+c/x)^2/(1-c^2/x^2))+6*a*b^2*\text{polylog}(3,(1+c/x)/(1-c^2/x^2)^{(1/2)})+6*a*b^2*\text{polylog}(3,-(1+c/x)/(1-c^2/x^2)^{(1/2)})-b^3*\text{arctanh}(c/x)^3*\ln(1-(1+c/x)/(1-c^2/x^2)^{(1/2)})-3*b^3*\text{arctanh}(c/x)^2*\text{polylog}(2,(1+c/x)/(1-c^2/x^2)^{(1/2)})+6*b^3*\text{arctanh}(c/x)*\text{polylog}(3,(1+c/x)/(1-c^2/x^2)^{(1/2)})-b^3*\text{arctanh}(c/x)^3*\ln(1+(1+c/x)/(1-c^2/x^2)^{(1/2)})-3*b^3*\text{arctanh}(c/x)^2*\text{polylog}(2,-(1+c/x)/(1-c^2/x^2)^{(1/2)})+6*b^3*\text{arctanh}(c/x)*\text{polylog}(3,-(1+c/x)/(1-c^2/x^2)^{(1/2)})-b^3*\ln(c/x)*\text{arctanh}(c/x)^3+b^3*\text{arctanh}(c/x)^3*\ln((1+c/x)^2/(1-c^2/x^2)-1)+3/2*b^3*\text{arctanh}(c/x)^2*\text{polylog}(2,-(1+c/x)^2/(1-c^2/x^2))-3/2*b^3*\text{arctanh}(c/x)*\text{polylog}(3,-(1+c/x)^2/(1-c^2/x^2))+3/2*a^2*b*\text{dilog}(c/x)-3/2*I*a*b^2*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1))*c\text{sgn}(I/((1+c/x)^2/(1-c^2/x^2)+1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*\text{arctanh}(c/x)^2-3*a*b^2*\text{arctanh}(c/x)^2*\ln(1+(1+c/x)/(1-c^2/x^2)^{(1/2)})-6*a*b^2*\text{arctanh}(c/x)*\text{polylog}(2,-(1+c/x)/(1-c^2/x^2)^{(1/2)})-3*a*b^2*\ln(c/x)*\text{arctanh}(c/x)^2+3*a*b^2*\text{arctanh}(c/x)*\text{polylog}(2,-(1+c/x)^2/(1-c^2/x^2))-3*a^2*b*\ln(c/x)*\text{arctanh}(c/x)+3/2*a^2*b*\ln(c/x)*\ln(1+c/x)+3*a*b^2*\text{arctanh}(c/x)^2*\ln((1+c/x)^2/(1-c^2/x^2)-1)-3*a*b^2*\text{arctanh}(c/x)^2*\ln(1-(1+c/x)/(1-c^2/x^2)^{(1/2)})-6*a*b^2*\text{arctanh}(c/x)*\text{polylog}(2,(1+c/x)/(1-c^2/x^2)^{(1/2)})+1/2*I*b^3*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^3+1/2*I*b^3*\text{Pi}*c\text{sgn}(I/((1+c/x)^2/(1-c^2/x^2)+1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^3-3/2*I*a*b^2*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*\text{arctanh}(c/x)^2+3/2*I*a*b^2*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2+3/2*I*a*b^2*\text{Pi}*c\text{sgn}(I/((1+c/x)^2/(1-c^2/x^2)+1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^2*\text{arctanh}(c/x)^2-1/2*I*b^3*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1))*c\text{sgn}(I/((1+c/x)^2/(1-c^2/x^2)+1))*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))*\text{arctanh}(c/x)^3-a^3*\ln(c/x)+3/4*b^3*\text{polylog}(4,-(1+c/x)^2/(1-c^2/x^2))-6*b^3*\text{polylog}(4,(1+c/x)/(1-c^2/x^2)^{(1/2)})-6*b^3*\text{polylog}(4,-(1+c/x)/(1-c^2/x^2)^{(1/2)})-1/2*I*b^3*\text{Pi}*c\text{sgn}(I*((1+c/x)^2/(1-c^2/x^2)-1)/((1+c/x)^2/(1-c^2/x^2)+1))^3*\text{arctanh}(c/x)^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3 \left( \log\left(\frac{c}{x} + 1\right) - \log\left(-\frac{c}{x} + 1\right) \right)^3}{8x} + \frac{3ab^2 \left( \log\left(\frac{c}{x} + 1\right) - \log\left(-\frac{c}{x} + 1\right) \right)^2}{4x} + \frac{3a^2b \left( \log\left(\frac{c}{x} + 1\right) - \log\left(-\frac{c}{x} + 1\right) \right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3/x,x, algorithm="maxima")

[Out] a^3\*log(x) + integrate(1/8\*b^3\*(log(c/x + 1) - log(-c/x + 1))^3/x + 3/4\*a\*b^2\*(log(c/x + 1) - log(-c/x + 1))^2/x + 3/2\*a^2\*b\*(log(c/x + 1) - log(-c/x + 1))/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3ab^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3a^2b \operatorname{artanh}\left(\frac{c}{x}\right) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c/x)^3 + 3\*a\*b^2\*arctanh(c/x)^2 + 3\*a^2\*b\*arctanh(c/x) + a^3)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*3/x,x)

[Out] Integral((a + b\*atanh(c/x))\*3/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x) + a)^3/x, x)
```

$$3.155 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2} dx$$

**Optimal.** Leaf size=126

$$\frac{3b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-\frac{c}{x}}\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)}{c} - \frac{3b^3 \text{PolyLog}\left(3, 1 - \frac{2}{1-\frac{c}{x}}\right)}{2c} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x} +$$

[Out]  $-\left((a + b \text{ArcCoth}[x/c])^3/c\right) - (a + b \text{ArcCoth}[x/c])^3/x + (3*b*(a + b \text{ArcCoth}[x/c])^2*\text{Log}[2/(1 - c/x)])/c + (3*b^2*(a + b \text{ArcCoth}[x/c])* \text{PolyLog}[2, 1 - 2/(1 - c/x)])/c - (3*b^3*\text{PolyLog}[3, 1 - 2/(1 - c/x)])/(2*c)$

**Rubi [B]** time = 2.23691, antiderivative size = 387, normalized size of antiderivative = 3.07, number of steps used = 82, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 2416, 2396, 2433, 2374, 6589, 2411, 2346, 2301, 6742, 43, 2394, 2393, 2391, 2375, 2317, 2425}

$$\frac{3b^2 \text{PolyLog}\left(2, -\frac{c-x}{2x}\right) \left(2a - b \log\left(1 - \frac{c}{x}\right)\right)}{2c} + \frac{3b^3 \text{PolyLog}\left(3, -\frac{c-x}{2x}\right)}{2c} + \frac{3b^3 \text{PolyLog}\left(3, \frac{c+x}{2x}\right)}{2c} - \frac{3b^3 \log\left(\frac{c+x}{x}\right) \text{PolyLog}\left(2, \frac{c+x}{2x}\right)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c/x])^3/x^2, x]

[Out]  $((1 - c/x)*(2*a - b*\text{Log}[1 - c/x])^3)/(8*c) - (3*b*(2*a - b*\text{Log}[1 - c/x])^2*\text{Log}[(c + x)/(2*x)])/(4*c) + (3*b*(2*a - b*\text{Log}[1 - c/x])^2*\text{Log}[(c + x)/x])/(8*c) - (3*b*(2*a - b*\text{Log}[1 - c/x])^2*\text{Log}[(c + x)/x])/(8*x) - (3*b^2*(2*a - b*\text{Log}[1 - c/x])* \text{Log}[(c + x)/x]^2)/(8*c) - (3*b^2*(2*a - b*\text{Log}[1 - c/x])* \text{Log}[(c + x)/x]^2)/(8*x) - (3*b^3*\text{Log}[-(c - x)/(2*x)]*\text{Log}[(c + x)/x]^2)/(4*c) - (b^3*(1 + c/x)* \text{Log}[(c + x)/x]^3)/(8*c) + (3*b^2*(2*a - b*\text{Log}[1 - c/x])* \text{PolyLog}[2, -(c - x)/(2*x)])/(2*c) - (3*b^3*\text{Log}[(c + x)/x]* \text{PolyLog}[2, (c + x)/(2*x)])/(2*c) + (3*b^3*\text{PolyLog}[3, -(c - x)/(2*x)])/(2*c) + (3*b^3*\text{PolyLog}[3, (c + x)/(2*x)])/(2*c)$

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && Inte



gerQ[m] && IntegerQ[n]

#### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

#### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

#### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

#### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

#### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function0 fQ[x^(m + 1), u, x]

#### Rule 2430

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.)]\*(g\_.)), x\_Symbol] := Simp[x\*(a + b\*Log[c\*(d + e\*x)^n])^p\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[g\*j\*m, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^p)/(i + j\*x), x], x] - Dist[b\*e\*n\*p, Int[(x\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1)\*(f + g\*Log[h\*(i + j\*x)^m))]/(d + e\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && IGtQ[p, 0]

#### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((h\_.)\*(x\_))^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

### Rule 2374

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(q_.))
/(x_), x_Symbol] := Dist[d, Int[((d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p)/x,
x], x] + Dist[e, Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; Free
eQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*(a_.) + Log[(c_.)*(x_)^(n_
.))]*(b_.))^(p_.)/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
```

```
c*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
  Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2425

```
Int[(Log[(f_.)*(x_)^(m_.)]*((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.)]*(b
_.)))/(x_), x_Symbol] :> Simp[(Log[f*x^m]^2*(a + b*Log[c*(d + e*x)^n])/(2*
m), x] - Dist[(b*e*n)/(2*m), Int[Log[f*x^m]^2/(d + e*x), x], x] /; FreeQ[{a
, b, c, d, e, f, m, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^3}{x^2} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x}))^3}{8x^2} + \frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(1 + \frac{c}{x})}{8x^2} + \frac{3b^2(2a - b \log(1 - \frac{c}{x}))}{8x^2} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - \frac{c}{x}))^3}{x^2} dx + \frac{1}{8}(3b) \int \frac{(2a - b \log(1 - \frac{c}{x}))^2 \log(1 + \frac{c}{x})}{x^2} dx + \frac{1}{8}(3b^2) \int \frac{1}{x^2} dx \\
&= -\left(\frac{1}{8} \text{Subst} \left( \int (2a - b \log(1 - cx))^3 dx, x, \frac{1}{x} \right)\right) - \frac{1}{8}(3b) \text{Subst} \left( \int (2a - b \log(1 - cx))^2 \log(1 + cx) dx, x, \frac{1}{x} \right) + \frac{1}{8}(3b^2) \text{Subst} \left( \int \frac{1}{x^2} dx, x, \frac{1}{x} \right) \\
&= -\frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(\frac{c+x}{x})}{8x} - \frac{3b^2(2a - b \log(1 - \frac{c}{x})) \log^2(\frac{c+x}{x})}{8x} + \frac{\text{Subst}(\int (2a - b \log(1 - cx))^3 dx, x, \frac{1}{x})}{8} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(\frac{c+x}{x})}{8x} - \frac{3b^2(2a - b \log(1 - \frac{c}{x}))}{8x} \\
&= \frac{3b(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{8c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2}{8x} \\
&= -\frac{3ab^2}{2x} + \frac{3b^3}{4x} + \frac{3b(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{8c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2}{8x} \\
&= -\frac{3ab^2}{2x} - \frac{3b^3(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2}{8c} \\
&= -\frac{3b^3}{4x} - \frac{3b^3(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c} + \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2}{4c} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(\frac{c+x}{2x})}{4c} + \frac{3b(2a - b \log(1 - \frac{c}{x}))}{8c} \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c} - \frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(\frac{c+x}{2x})}{4c} + \frac{3b(2a - b \log(1 - \frac{c}{x}))}{8c}
\end{aligned}$$

**Mathematica [A]** time = 0.122466, size = 205, normalized size = 1.63

$$\frac{3ab^2 \left( \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(\frac{c}{x})} \right) + \tanh^{-1} \left( \frac{c}{x} \right) \left( \frac{c \tanh^{-1}(\frac{c}{x})}{x} - \tanh^{-1} \left( \frac{c}{x} \right) - 2 \log \left( e^{-2 \tanh^{-1}(\frac{c}{x})} + 1 \right) \right) \right)}{c} - b^3 \left( 3 \tanh^{-1} \left( \frac{c}{x} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^3/x^2, x]

```
[Out] -(a^3/x) - (3*a^2*b*ArcTanh[c/x])/x - (3*a^2*b*Log[1 - c^2/x^2])/(2*c) - (3
*a*b^2*(ArcTanh[c/x]*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 2*Log[1 + E^(-2*
ArcTanh[c/x])])) + PolyLog[2, -E^(-2*ArcTanh[c/x])])/c - (b^3*(ArcTanh[c/x]
^2*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 3*Log[1 + E^(-2*ArcTanh[c/x])]) +
3*ArcTanh[c/x]*PolyLog[2, -E^(-2*ArcTanh[c/x])] + (3*PolyLog[3, -E^(-2*ArcT
anh[c/x])]))/2)/c
```

**Maple [B]** time = 0.004, size = 298, normalized size = 2.4

$$-\frac{a^3}{x} - \frac{b^3}{x} \left( \operatorname{Artanh}\left(\frac{c}{x}\right) \right)^3 - \frac{b^3}{c} \left( \operatorname{Artanh}\left(\frac{c}{x}\right) \right)^3 + 3 \frac{b^3}{c} \left( \operatorname{Artanh}\left(\frac{c}{x}\right) \right)^2 \ln \left( \left(1 + \frac{c}{x}\right)^2 \left(1 - \frac{c^2}{x^2}\right)^{-1} + 1 \right) + 3 \frac{b^3}{c} \operatorname{Artanh}\left(\frac{c}{x}\right) \operatorname{PolyLog}\left(2, -E^{-2 \operatorname{Artanh}\left(\frac{c}{x}\right)}\right) + \frac{3}{2} \frac{b^3}{c} \operatorname{PolyLog}\left(3, -E^{-2 \operatorname{Artanh}\left(\frac{c}{x}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c/x))^3/x^2,x)
```

```
[Out] -a^3/x-b^3/x*arctanh(c/x)^3-1/c*b^3*arctanh(c/x)^3+3/c*b^3*arctanh(c/x)^2*ln
((1+c/x)^2/(1-c^2/x^2)+1)+3/c*b^3*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2
/x^2))-3/2/c*b^3*polylog(3,-(1+c/x)^2/(1-c^2/x^2))-3*arctanh(c/x)^2/x*a*b^2
+6/c*ln((1+c/x)^2/(1-c^2/x^2)+1)*arctanh(c/x)*a*b^2-3/c*a*b^2*arctanh(c/x)^
2+3/c*polylog(2,-(1+c/x)^2/(1-c^2/x^2))*a*b^2-3/x*a^2*b*arctanh(c/x)-3/2/c*
a^2*b*ln(1-c^2/x^2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{3a^2b \left( \frac{2c \operatorname{artanh}\left(\frac{c}{x}\right)}{x} + \log\left(-\frac{c^2}{x^2} + 1\right) \right)}{2c} - \frac{a^3}{x} + \frac{(b^3c - b^3x) \log(-c + x)^3 - 3(2ab^2c + (b^3c + b^3x) \log(c + x)) \log(-c + x)^2}{8cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^3/x + 1/8*((b^3*c
- b^3*x)*log(-c + x)^3 - 3*(2*a*b^2*c + (b^3*c + b^3*x)*log(c + x))*log(-c
+ x)^2)/(c*x) - integrate(-1/8*((b^3*c^2 - b^3*c*x)*log(c + x)^3 + 6*(a*b^
2*c^2 - a*b^2*c*x)*log(c + x)^2 - 3*(4*a*b^2*c*x + (b^3*c^2 - b^3*c*x)*log(
c + x)^2 + 2*(2*a*b^2*c^2 + b^3*x^2 - (2*a*b^2*c - b^3*c)*x)*log(c + x))*lo
```

$g(-c + x)/(c^2*x^2 - c*x^3), x)$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3ab^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3a^2b \operatorname{artanh}\left(\frac{c}{x}\right) + a^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="fricas")`

[Out] `integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x) + a^3)/x^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c/x))**3/x**2,x)`

[Out] `Integral((a + b*atanh(c/x))**3/x**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c/x) + a)^3/x^2, x)`

$$3.156 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3} dx$$

**Optimal.** Leaf size=139

$$\frac{3b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x}}\right)}{2c^2} + \frac{3b^2 \log\left(\frac{2}{1 - \frac{c}{x}}\right) \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2} - \frac{3b \left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} + \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2} - \frac{\left(a + b \coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}$$

[Out]  $(-3*b*(a + b*ArcCoth[x/c])^2)/(2*c^2) - (3*b*(a + b*ArcCoth[x/c])^2)/(2*c*x) + (a + b*ArcCoth[x/c])^3/(2*c^2) - (a + b*ArcCoth[x/c])^3/(2*x^2) + (3*b^2*(a + b*ArcCoth[x/c])*Log[2/(1 - c/x)])/c^2 + (3*b^3*PolyLog[2, 1 - 2/(1 - c/x)])/(2*c^2)$

**Rubi [F]** time = 2.10214, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c/x])^3/x^3, x]

[Out]  $(-3*b^3*(1 - c/x)^2)/(64*c^2) - (3*a*b^2*(1 + c/x)^2)/(16*c^2) + (3*b^3*(1 + c/x)^2)/(64*c^2) + (3*a^2*b)/(8*x^2) + (3*a*b^2)/(8*x^2) - (3*a^2*b)/(4*c*x) - (3*b^3)/(2*c*x) + (3*a*b^2*Log[1 - c/x])/(8*c^2) - (3*a*b^2*(1 - c/x)*Log[1 - c/x])/(4*c^2) - (3*b^3*(1 - c/x)*Log[1 - c/x])/(4*c^2) - (3*a*b^2*Log[1 - c/x])/(8*x^2) - (3*b^2*(1 - c/x)^2*(2*a - b*Log[1 - c/x]))/(32*c^2) + (3*b*(1 - c/x)*(2*a - b*Log[1 - c/x])^2)/(8*c^2) - (3*b*(1 - c/x)^2*(2*a - b*Log[1 - c/x])^2)/(32*c^2) + ((1 - c/x)*(2*a - b*Log[1 - c/x])^3)/(8*c^2) - ((1 - c/x)^2*(2*a - b*Log[1 - c/x])^3)/(16*c^2) + (3*a*b^2*Log[1 - c/x]*Log[1 + c/x])/(4*x^2) - (3*a*b^2*Log[1 + c/x]*Log[c - x])/(4*c^2) - (3*a*b^2*Log[c - x]*Log[x/c])/(4*c^2) - (3*a*b^2*Log[1 - c/x]*Log[c + x])/(4*c^2) + (3*a*b^2*Log[(c - x)/(2*c)]*Log[c + x])/(4*c^2) - (3*a*b^2*Log[-(x/c)]*Log[c + x])/(4*c^2) + (3*a*b^2*Log[c - x]*Log[(c + x)/(2*c)])/(4*c^2) + (3*a^2*b*Log[(c + x)/x])/(4*c^2) + (3*a*b^2*Log[(c + x)/x])/(8*c^2) - (9*a*b^2*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (3*b^3*(1 + c/x)*Log[(c + x)/x])/(4*c^2) + (3*a*b^2*(1 + c/x)^2*Log[(c + x)/x])/(8*c^2) - (3*b^3*(1 + c/x)^2*Log[(c + x)/x])/(32*c^2) - (3*a^2*b*Log[(c + x)/x])/(4*x^2) - (3*a*b^2*Log[(c + x)/x])/(4*c^2)$



$$\begin{aligned}
& x)/x]/(8*x^2) + (3*a*b^2*(1 + c/x)*\text{Log}[(c + x)/x]^2)/(4*c^2) - (3*b^3*(1 \\
& + c/x)*\text{Log}[(c + x)/x]^2)/(8*c^2) - (3*a*b^2*(1 + c/x)^2*\text{Log}[(c + x)/x]^2)/( \\
& 8*c^2) + (3*b^3*(1 + c/x)^2*\text{Log}[(c + x)/x]^2)/(32*c^2) + (b^3*(1 + c/x)*\text{Log} \\
& [(c + x)/x]^3)/(8*c^2) - (b^3*(1 + c/x)^2*\text{Log}[(c + x)/x]^3)/(16*c^2) + (3*a \\
& *b^2*\text{PolyLog}[2, (c - x)/(2*c)])/(4*c^2) + (3*a*b^2*\text{PolyLog}[2, -(c/x)])/(4*c \\
& ^2) + (3*a*b^2*\text{PolyLog}[2, c/x]/(4*c^2) + (3*a*b^2*\text{PolyLog}[2, (c + x)/(2*c) \\
& ])/(4*c^2) - (3*a*b^2*\text{PolyLog}[2, 1 - x/c]/(4*c^2) - (3*a*b^2*\text{PolyLog}[2, 1 \\
& + x/c]/(4*c^2) + (3*b^3*\text{Defer}[\text{Int}][(\text{Log}[1 - c/x]^2*\text{Log}[1 + c/x])/x^3, x])/ \\
& 8 - (3*b^3*\text{Defer}[\text{Int}][(\text{Log}[1 - c/x]*\text{Log}[1 + c/x]^2)/x^3, x])/8
\end{aligned}$$

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x}))^3}{x^3} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x}))^3}{8x^3} + \frac{3b(2a - b \log(1 - \frac{c}{x}))^2 \log(1 + \frac{c}{x})}{8x^3} + \frac{3b^2(2a - b \log(1 - \frac{c}{x})) \log^2(1 + \frac{c}{x})}{8x^3} + \frac{b^3 \log^3(1 + \frac{c}{x})}{8x^3} \right) dx \\
&= \frac{1}{8} \int \frac{(2a - b \log(1 - \frac{c}{x}))^3}{x^3} dx + \frac{1}{8}(3b) \int \frac{(2a - b \log(1 - \frac{c}{x}))^2 \log(1 + \frac{c}{x})}{x^3} dx + \frac{1}{8}(3b^2) \int \frac{(2a - b \log(1 - \frac{c}{x})) \log^2(1 + \frac{c}{x})}{x^3} dx + \frac{1}{8} \int \frac{b^3 \log^3(1 + \frac{c}{x})}{x^3} dx \\
&= -\left(\frac{1}{8} \text{Subst} \left( \int x(2a - b \log(1 - cx))^3 dx, x, \frac{1}{x} \right)\right) + \frac{1}{8}(3b) \int \left( \frac{4a^2 \log(1 + \frac{c}{x})}{x^3} - \frac{4ab \log(1 + \frac{c}{x})}{x^3} + \frac{3b^2 \log^2(1 + \frac{c}{x})}{x^3} - \frac{b^3 \log^3(1 + \frac{c}{x})}{x^3} \right) dx \\
&= -\left(\frac{1}{8} \text{Subst} \left( \int \left( \frac{(2a - b \log(1 - cx))^3}{c} - \frac{(1 - cx)(2a - b \log(1 - cx))^3}{c} \right) dx, x, \frac{1}{x} \right)\right) + \frac{1}{2} (3a^2b) \int \frac{\log(1 + \frac{c}{x})}{x^3} dx \\
&= \frac{3ab^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} - \frac{1}{2} (3a^2b) \text{Subst} \left( \int x \log(1 + cx) dx, x, \frac{1}{x} \right) - \frac{1}{4} (3ab^2) \text{Subst} \left( \int \frac{\log^2(1 + cx)}{x} dx, x, \frac{1}{x} \right) \\
&= \frac{3ab^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} - \frac{3a^2b \log(\frac{c+x}{x})}{4x^2} - \frac{1}{4} (3ab^2) \text{Subst} \left( \int \left( -\frac{\log^2(1 + cx)}{c} + \frac{(1 + cx) \log(1 + cx)}{c} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^3}{8c^2} - \frac{(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))^3}{16c^2} + \frac{3ab^2 \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4x^2} \\
&= \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} + \frac{3b(1 - \frac{c}{x})(2a - b \log(1 - \frac{c}{x}))^2}{8c^2} - \frac{3b(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))^2}{32c^2} + \frac{(1 - \frac{c}{x}) \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{4c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3ab^2}{2cx} - \frac{3b^3}{4cx} - \frac{3b^2(1 - \frac{c}{x})^2 (2a - b \log(1 - \frac{c}{x}))^2}{32c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} - \frac{3ab^2(1 + \frac{c}{x})^2}{16c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} - \frac{3b^3(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} - \frac{3ab^2(1 + \frac{c}{x})^2}{16c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} - \frac{3ab^2(1 - \frac{c}{x}) \log(1 - \frac{c}{x})}{4c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} - \frac{3ab^2(1 + \frac{c}{x})^2}{16c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ab^2 \log(1 - \frac{c}{x})}{8c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} - \frac{3ab^2(1 + \frac{c}{x})^2}{16c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ab^2 \log(1 - \frac{c}{x})}{8c^2} \\
&= -\frac{3b^3(1 - \frac{c}{x})^2}{64c^2} - \frac{3ab^2(1 + \frac{c}{x})^2}{16c^2} + \frac{3b^3(1 + \frac{c}{x})^2}{64c^2} + \frac{3a^2b}{8x^2} + \frac{3ab^2}{8x^2} - \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ab^2 \log(1 - \frac{c}{x})}{8c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.334353, size = 195, normalized size = 1.4

$$-6b^3x^2\text{PolyLog}\left(2, -e^{-2\text{tanh}^{-1}\left(\frac{c}{x}\right)}\right) + a\left(12b^2x^2\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right) - a\left(2ac^2 + 3bx^2\log\left(1 - \frac{c}{x}\right) - 3bx^2\log\left(\frac{c+x}{x}\right) + 6bcx\right)\right) + 6$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x])^3/x^3, x]

[Out] (6\*b^2\*(-c + x)\*(b\*x + a\*(c + x))\*ArcTanh[c/x]^2 + 2\*b^3\*(-c^2 + x^2)\*ArcTanh[c/x]^3 + 6\*b\*ArcTanh[c/x]\*(-(a\*c\*(a\*c + 2\*b\*x)) + 2\*b^2\*x^2\*Log[1 + E^(-2\*ArcTanh[c/x])]) + a\*(12\*b^2\*x^2\*Log[1/Sqrt[1 - c^2/x^2]] - a\*(2\*a\*c^2 + 6\*b\*c\*x + 3\*b\*x^2\*Log[1 - c/x] - 3\*b\*x^2\*Log[(c + x)/x])) - 6\*b^3\*x^2\*PolyLog[2, -E^(-2\*ArcTanh[c/x])])/(4\*c^2\*x^2)

---

**Maple [C]** time = 0.266, size = 6645, normalized size = 47.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x))^3/x^3, x)

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3/x^3, x, algorithm="maxima")

[Out] 3/4\*(c\*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2\*x)) - 2\*arctanh(c/x)/x^2)\*a^2\*b - 3/8\*(c^2\*((log(c + x))^2 - 2\*(log(c + x) - 2)\*log(-c + x) + log(-c + x)^2 + 4\*log(c + x))/c^4 - 8\*log(x)/c^4) - 4\*c\*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2\*x))\*arctanh(c/x)\*a\*b^2 + 1/64\*(32\*c^4\*integrate(-1/4\*log(

$x)^3/(c^4x^3 - c^2x^5), x) - 3c^3(\log(c + x)/c^5 - \log(-c + x)/c^5 - 2/(c^4x)) + 48c^3\int(-1/4x\log(x)^2/(c^4x^3 - c^2x^5), x) + 48c^3\int(-1/4x\log(x)/(c^4x^3 - c^2x^5), x) - 6c(2\log(-c + x)/c^3 - 2\log(x)/c^3 + (c + 2x)/(c^2x^2))\log(-c/x + 1)^2 + 21c^2(\log(c + x)/c^4 + \log(-c + x)/c^4 - 2\log(x)/c^4) - 32c^2\int(-1/4x^2\log(x)^3/(c^4x^3 - c^2x^5), x) + 48c^2\int(-1/4x^2\log(x)^2/(c^4x^3 - c^2x^5), x) - 384c^2\int(-1/4x^2\log(c + x)/(c^4x^3 - c^2x^5), x) + 144c^2\int(-1/4x^2\log(x)/(c^4x^3 - c^2x^5), x) - 18c(\log(c + x)/c^3 - \log(-c + x)/c^3) + c(6(2x^2\log(-c + x)^2 + 2x^2\log(x)^2 - 6x^2\log(x) + c^2 + 6cx - 2(2x^2\log(x) - 3x^2)\log(-c + x))\log(-c/x + 1)/(c^3x^2) - (4x^2\log(-c + x)^3 - 4x^2\log(x)^3 + 18x^2\log(x)^2 - 6(2x^2\log(x) - 3x^2)\log(-c + x)^2 - 42x^2\log(x) + 3c^2 + 42cx + 6(2x^2\log(x)^2 - 6x^2\log(x) + 7x^2)\log(-c + x))/(c^3x^2)) - 48c\int(-1/4x^3\log(x)^2/(c^4x^3 - c^2x^5), x) - 192c\int(-1/4x^3\log(c + x)/(c^4x^3 - c^2x^5), x) + 336c\int(-1/4x^3\log(x)/(c^4x^3 - c^2x^5), x) + 4\log(-c/x + 1)^3/x^2 - 2(12cx\log(c + x)^2 + 2(c^2 - x^2)\log(c + x)^3 - 3(c^2 - 2cx + x^2 - 2(c^2 - x^2)\log(c + x) + 2(c^2 - x^2)\log(x))\log(-c + x)^2 - 3(2(c^2 - x^2)\log(c + x)^2 - 2(c^2 - x^2)\log(x)^2 - c^2 - 6cx + 8(cx + x^2)\log(c + x) - 2(c^2 + 2cx + 5x^2)\log(x))\log(-c + x))/(c^2x^2) - 48\int(-1/4x^4\log(x)^2/(c^4x^3 - c^2x^5), x) - 192\int(-1/4x^4\log(c + x)/(c^4x^3 - c^2x^5), x) + 240\int(-1/4x^4\log(x)/(c^4x^3 - c^2x^5), x))b^3 - 3/2ab^2\operatorname{arctanh}(c/x)^2/x^2 - 1/2a^3/x^2$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \operatorname{artanh}\left(\frac{c}{x}\right)^3 + 3ab^2 \operatorname{artanh}\left(\frac{c}{x}\right)^2 + 3a^2b \operatorname{artanh}\left(\frac{c}{x}\right) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="fricas")`

[Out] `integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x) + a^3)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}\left(\frac{c}{x}\right))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x))\*\*3/x\*\*3,x)

[Out] Integral((a + b\*atanh(c/x))\*\*3/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x))^3/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x) + a)^3/x^3, x)

### 3.157 $\int x^7 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=54

$$\frac{1}{8}x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{8}bc^3x^2 - \frac{1}{8}bc^4 \tanh^{-1} \left( \frac{x^2}{c} \right) + \frac{1}{24}bcx^6$$

[Out]  $(b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcTanh[x^2/c])/8$

**Rubi [A]** time = 0.0417084, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 263, 275, 302, 207}

$$\frac{1}{8}x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{8}bc^3x^2 - \frac{1}{8}bc^4 \tanh^{-1} \left( \frac{x^2}{c} \right) + \frac{1}{24}bcx^6$$

Antiderivative was successfully verified.

[In] Int[x^7\*(a + b\*ArcTanh[c/x^2]),x]

[Out]  $(b*c^3*x^2)/8 + (b*c*x^6)/24 + (x^8*(a + b*ArcTanh[c/x^2]))/8 - (b*c^4*ArcTanh[x^2/c])/8$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^7 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4} (bc) \int \frac{x^5}{1 - \frac{c^2}{x^4}} dx \\
&= \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4} (bc) \int \frac{x^9}{-c^2 + x^4} dx \\
&= \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{8} (bc) \operatorname{Subst} \left( \int \frac{x^4}{-c^2 + x^2} dx, x, x^2 \right) \\
&= \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{8} (bc) \operatorname{Subst} \left( \int \left( c^2 + x^2 + \frac{c^4}{-c^2 + x^2} \right) dx, x, x^2 \right) \\
&= \frac{1}{8} bc^3 x^2 + \frac{1}{24} bcx^6 + \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{8} (bc^5) \operatorname{Subst} \left( \int \frac{1}{-c^2 + x^2} dx, x, x^2 \right) \\
&= \frac{1}{8} bc^3 x^2 + \frac{1}{24} bcx^6 + \frac{1}{8} x^8 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{8} bc^4 \tanh^{-1} \left( \frac{x^2}{c} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0140286, size = 73, normalized size = 1.35

$$\frac{ax^8}{8} + \frac{1}{8} bc^3 x^2 + \frac{1}{16} bc^4 \log(x^2 - c) - \frac{1}{16} bc^4 \log(c + x^2) + \frac{1}{24} bcx^6 + \frac{1}{8} bx^8 \tanh^{-1} \left( \frac{c}{x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c^3*x^2)/8 + (b*c*x^6)/24 + (a*x^8)/8 + (b*x^8*ArcTanh[c/x^2])/8 + (b*c^
4*Log[-c + x^2])/16 - (b*c^4*Log[c + x^2])/16
```

**Maple [A]** time = 0.016, size = 64, normalized size = 1.2

$$\frac{x^8 a}{8} + \frac{bx^8}{8} \operatorname{Arctanh}\left(\frac{c}{x^2}\right) - \frac{bc^4}{16} \ln\left(1 + \frac{c}{x^2}\right) + \frac{bcx^6}{24} + \frac{bc^3 x^2}{8} + \frac{bc^4}{16} \ln\left(\frac{c}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a+b*arctanh(c/x^2)),x)`

[Out] `1/8*x^8*a+1/8*b*x^8*arctanh(c/x^2)-1/16*b*c^4*ln(1+c/x^2)+1/24*b*c*x^6+1/8*b*c^3*x^2+1/16*b*c^4*ln(c/x^2-1)`

**Maxima [A]** time = 0.989646, size = 84, normalized size = 1.56

$$\frac{1}{8} ax^8 + \frac{1}{48} \left( 6x^8 \operatorname{artanh}\left(\frac{c}{x^2}\right) + (2x^6 + 6c^2x^2 - 3c^3 \log(x^2 + c) + 3c^3 \log(x^2 - c))c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="maxima")`

[Out] `1/8*a*x^8 + 1/48*(6*x^8*arctanh(c/x^2) + (2*x^6 + 6*c^2*x^2 - 3*c^3*log(x^2 + c) + 3*c^3*log(x^2 - c))*c)*b`

**Fricas [A]** time = 1.74963, size = 122, normalized size = 2.26

$$\frac{1}{8} ax^8 + \frac{1}{24} bcx^6 + \frac{1}{8} bc^3 x^2 + \frac{1}{16} (bx^8 - bc^4) \log\left(\frac{x^2 + c}{x^2 - c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="fricas")`

[Out] `1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*x^2 + 1/16*(b*x^8 - b*c^4)*log((x^2 + c)/(x^2 - c))`



**Sympy [A]** time = 18.4974, size = 51, normalized size = 0.94

$$\frac{ax^8}{8} - \frac{bc^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{8} + \frac{bc^3x^2}{8} + \frac{bcx^6}{24} + \frac{bx^8 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(a+b\*atanh(c/x\*\*2)),x)

[Out] a\*x\*\*8/8 - b\*c\*\*4\*atanh(c/x\*\*2)/8 + b\*c\*\*3\*x\*\*2/8 + b\*c\*x\*\*6/24 + b\*x\*\*8\*atanh(c/x\*\*2)/8

**Giac [A]** time = 1.33246, size = 96, normalized size = 1.78

$$\frac{1}{16}bx^8 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{8}ax^8 + \frac{1}{24}bcx^6 + \frac{1}{8}bc^3x^2 - \frac{1}{16}bc^4 \log(x^2+c) + \frac{1}{16}bc^4 \log(-x^2+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] 1/16\*b\*x^8\*log((x^2 + c)/(x^2 - c)) + 1/8\*a\*x^8 + 1/24\*b\*c\*x^6 + 1/8\*b\*c^3\*x^2 - 1/16\*b\*c^4\*log(x^2 + c) + 1/16\*b\*c^4\*log(-x^2 + c)

$$3.158 \quad \int x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$$

**Optimal.** Leaf size=45

$$\frac{1}{6}x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{12}bc^3 \log(c^2 - x^4) + \frac{1}{12}bcx^4$$

[Out] (b\*c\*x^4)/12 + (x^6\*(a + b\*ArcTanh[c/x^2]))/6 + (b\*c^3\*Log[c^2 - x^4])/12

**Rubi [A]** time = 0.0329161, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 266, 43}

$$\frac{1}{6}x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{12}bc^3 \log(c^2 - x^4) + \frac{1}{12}bcx^4$$

Antiderivative was successfully verified.

[In] Int[x^5\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (b\*c\*x^4)/12 + (x^6\*(a + b\*ArcTanh[c/x^2]))/6 + (b\*c^3\*Log[c^2 - x^4])/12

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{6} x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{3} (bc) \int \frac{x^3}{1 - \frac{c^2}{x^4}} dx \\
&= \frac{1}{6} x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{3} (bc) \int \frac{x^7}{-c^2 + x^4} dx \\
&= \frac{1}{6} x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{12} (bc) \operatorname{Subst} \left( \int \frac{x}{-c^2 + x} dx, x, x^4 \right) \\
&= \frac{1}{6} x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{12} (bc) \operatorname{Subst} \left( \int \left( 1 - \frac{c^2}{c^2 - x} \right) dx, x, x^4 \right) \\
&= \frac{1}{12} bcx^4 + \frac{1}{6} x^6 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{12} bc^3 \log(c^2 - x^4)
\end{aligned}$$

**Mathematica [A]** time = 0.0119029, size = 50, normalized size = 1.11

$$\frac{ax^6}{6} + \frac{1}{12}bc^3 \log(x^4 - c^2) + \frac{1}{12}bcx^4 + \frac{1}{6}bx^6 \tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTanh[c/x^2]),x]
```

```
[Out] (b*c*x^4)/12 + (a*x^6)/6 + (b*x^6*ArcTanh[c/x^2])/6 + (b*c^3*Log[-c^2 + x^4
])/12
```

**Maple [A]** time = 0.019, size = 65, normalized size = 1.4

$$\frac{x^6 a}{6} + \frac{bx^6}{6} \operatorname{Arctanh} \left( \frac{c}{x^2} \right) + \frac{bc^3}{12} \ln \left( 1 + \frac{c}{x^2} \right) + \frac{bcx^4}{12} - \frac{bc^3 \ln(x^{-1})}{3} + \frac{bc^3}{12} \ln \left( \frac{c}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctanh(c/x^2)),x)
```

[Out]  $\frac{1}{6}x^6a + \frac{1}{6}bx^6\operatorname{arctanh}(c/x^2) + \frac{1}{12}b^3c^3\ln(1+c/x^2) + \frac{1}{12}b^3c^3\ln(1/c/x^2-1) - \frac{1}{3}b^3c^3\ln(1/x) + \frac{1}{12}b^3c^3\ln(c/x^2-1)$

**Maxima [A]** time = 0.968106, size = 57, normalized size = 1.27

$$\frac{1}{6}ax^6 + \frac{1}{12}\left(2x^6\operatorname{artanh}\left(\frac{c}{x^2}\right) + (x^4 + c^2\log(x^4 - c^2))c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="maxima")`

[Out]  $\frac{1}{6}ax^6 + \frac{1}{12}(2x^6\operatorname{arctanh}(c/x^2) + (x^4 + c^2\log(x^4 - c^2))*c)*b$

**Fricas [A]** time = 1.71104, size = 124, normalized size = 2.76

$$\frac{1}{12}bx^6\log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{6}ax^6 + \frac{1}{12}bcx^4 + \frac{1}{12}bc^3\log(x^4 - c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="fricas")`

[Out]  $\frac{1}{12}bx^6\log((x^2+c)/(x^2-c)) + \frac{1}{6}ax^6 + \frac{1}{12}b^3c^3\log(x^4 - c^2) + \frac{1}{12}b^3c^3\log(x^4 - c^2)$

**Sympy [C]** time = 13.1943, size = 75, normalized size = 1.67

$$\frac{ax^6}{6} + \frac{bc^3\log(-i\sqrt{c}+x)}{6} + \frac{bc^3\log(i\sqrt{c}+x)}{6} - \frac{bc^3\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6} + \frac{bcx^4}{12} + \frac{bx^6\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*atanh(c/x**2)),x)`

[Out]  $a*x**6/6 + b*c**3*\log(-I*\sqrt{c} + x)/6 + b*c**3*\log(I*\sqrt{c} + x)/6 - b*c**3*\operatorname{atanh}(c/x**2)/6 + b*c*x**4/12 + b*x**6*\operatorname{atanh}(c/x**2)/6$

---

**Giac [A]** time = 1.24473, size = 70, normalized size = 1.56

$$\frac{1}{12}bx^6 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{6}ax^6 + \frac{1}{12}bcx^4 + \frac{1}{12}bc^3 \log(x^4 - c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] 1/12\*b\*x^6\*log((x^2 + c)/(x^2 - c)) + 1/6\*a\*x^6 + 1/12\*b\*c\*x^4 + 1/12\*b\*c^3\*log(x^4 - c^2)

### 3.159 $\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=43

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{4}bc^2 \tanh^{-1} \left( \frac{x^2}{c} \right) + \frac{1}{4}bcx^2$$

[Out] (b\*c\*x^2)/4 + (x^4\*(a + b\*ArcTanh[c/x^2]))/4 - (b\*c^2\*ArcTanh[x^2/c])/4

**Rubi [A]** time = 0.0297014, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 263, 275, 321, 207}

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{4}bc^2 \tanh^{-1} \left( \frac{x^2}{c} \right) + \frac{1}{4}bcx^2$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (b\*c\*x^2)/4 + (x^4\*(a + b\*ArcTanh[c/x^2]))/4 - (b\*c^2\*ArcTanh[x^2/c])/4

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

#### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{2} (bc) \int \frac{x}{1 - \frac{c^2}{x^4}} dx \\
&= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{2} (bc) \int \frac{x^5}{-c^2 + x^4} dx \\
&= \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4} (bc) \text{Subst} \left( \int \frac{x^2}{-c^2 + x^2} dx, x, x^2 \right) \\
&= \frac{1}{4} bcx^2 + \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4} (bc^3) \text{Subst} \left( \int \frac{1}{-c^2 + x^2} dx, x, x^2 \right) \\
&= \frac{1}{4} bcx^2 + \frac{1}{4} x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{4} bc^2 \tanh^{-1} \left( \frac{x^2}{c} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0111031, size = 62, normalized size = 1.44

$$\frac{ax^4}{4} + \frac{1}{8}bc^2 \log(x^2 - c) - \frac{1}{8}bc^2 \log(c + x^2) + \frac{1}{4}bcx^2 + \frac{1}{4}bx^4 \tanh^{-1} \left( \frac{c}{x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c/x^2]), x]
```

```
[Out] (b*c*x^2)/4 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x^2])/4 + (b*c^2*Log[-c + x^2])/
8 - (b*c^2*Log[c + x^2])/8
```

**Maple [A]** time = 0.006, size = 55, normalized size = 1.3

$$\frac{x^4 a}{4} + \frac{bx^4}{4} \operatorname{Artanh}\left(\frac{c}{x^2}\right) - \frac{bc^2}{8} \ln\left(1 + \frac{c}{x^2}\right) + \frac{bc^2}{8} \ln\left(\frac{c}{x^2} - 1\right) + \frac{bcx^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arctanh(c/x^2)),x)`

[Out] `1/4*x^4*a+1/4*arctanh(c/x^2)*b*x^4-1/8*b*c^2*ln(1+c/x^2)+1/8*b*c^2*ln(c/x^2-1)+1/4*b*c*x^2`

**Maxima [A]** time = 0.960415, size = 66, normalized size = 1.53

$$\frac{1}{4} ax^4 + \frac{1}{8} \left( 2x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right) + (2x^2 - c \log(x^2 + c) + c \log(x^2 - c))c \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="maxima")`

[Out] `1/4*a*x^4 + 1/8*(2*x^4*arctanh(c/x^2) + (2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c)*b`

**Fricas [A]** time = 1.585, size = 97, normalized size = 2.26

$$\frac{1}{4} ax^4 + \frac{1}{4} bcx^2 + \frac{1}{8} (bx^4 - bc^2) \log\left(\frac{x^2 + c}{x^2 - c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="fricas")`

[Out] `1/4*a*x^4 + 1/4*b*c*x^2 + 1/8*(b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c))`

**Sympy [A]** time = 15.3665, size = 41, normalized size = 0.95

$$\frac{ax^4}{4} - \frac{bc^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{4} + \frac{bcx^2}{4} + \frac{bx^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c/x\*\*2)),x)

[Out] a\*x\*\*4/4 - b\*c\*\*2\*atanh(c/x\*\*2)/4 + b\*c\*x\*\*2/4 + b\*x\*\*4\*atanh(c/x\*\*2)/4

**Giac [A]** time = 1.25419, size = 84, normalized size = 1.95

$$\frac{1}{8}bx^4 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{4}ax^4 + \frac{1}{4}bcx^2 - \frac{1}{8}bc^2 \log(x^2+c) + \frac{1}{8}bc^2 \log(-x^2+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] 1/8\*b\*x^4\*log((x^2 + c)/(x^2 - c)) + 1/4\*a\*x^4 + 1/4\*b\*c\*x^2 - 1/8\*b\*c^2\*log(x^2 + c) + 1/8\*b\*c^2\*log(-x^2 + c)

### 3.160 $\int x \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=34

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4}bc \log(c^2 - x^4)$$

[Out]  $(x^2*(a + b*\text{ArcTanh}[c/x^2]))/2 + (b*c*\text{Log}[c^2 - x^4])/4$

**Rubi [A]** time = 0.0180373, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 263, 260}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4}bc \log(c^2 - x^4)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{ArcTanh}[c/x^2]), x]$

[Out]  $(x^2*(a + b*\text{ArcTanh}[c/x^2]))/2 + (b*c*\text{Log}[c^2 - x^4])/4$

#### Rule 6097

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \\ \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /; \\ \text{FreeQ}\{a, b, c, d, m, n\}, x] \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 263

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[x^{(m+n*p)}*(b + a/x^n)^p, x] /; \text{FreeQ}\{a, b, m, n\}, x] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{NegQ}[n]$

#### Rule 260

$\text{Int}[(x_.)^{(m_.)}]/((a_.) + (b_.)*(x_.)^{(n_.)}), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

#### Rubi steps

$$\begin{aligned}
\int x \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + (bc) \int \frac{1}{\left( 1 - \frac{c^2}{x^4} \right) x} dx \\
&= \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + (bc) \int \frac{x^3}{-c^2 + x^4} dx \\
&= \frac{1}{2} x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{4} bc \log(c^2 - x^4)
\end{aligned}$$

**Mathematica [A]** time = 0.0068589, size = 39, normalized size = 1.15

$$\frac{ax^2}{2} + \frac{1}{4}bc \log(x^4 - c^2) + \frac{1}{2}bx^2 \tanh^{-1}\left(\frac{c}{x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (a\*x^2)/2 + (b\*x^2\*ArcTanh[c/x^2])/2 + (b\*c\*Log[-c^2 + x^4])/4

**Maple [A]** time = 0.016, size = 52, normalized size = 1.5

$$\frac{ax^2}{2} + \frac{bx^2}{2} \operatorname{Artanh}\left(\frac{c}{x^2}\right) + \frac{bc}{4} \ln\left(1 + \frac{c}{x^2}\right) - bc \ln(x^{-1}) + \frac{bc}{4} \ln\left(\frac{c}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c/x^2)),x)

[Out] 1/2\*a\*x^2+1/2\*b\*x^2\*arctanh(c/x^2)+1/4\*b\*c\*ln(1+c/x^2)-b\*c\*ln(1/x)+1/4\*b\*c\*ln(c/x^2-1)

**Maxima [A]** time = 0.981602, size = 46, normalized size = 1.35

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(2x^2 \operatorname{artanh}\left(\frac{c}{x^2}\right) + c \log(x^4 - c^2)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x^2)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/4\*(2\*x^2\*arctanh(c/x^2) + c\*log(x^4 - c^2))\*b

**Fricas [A]** time = 1.64731, size = 99, normalized size = 2.91

$$\frac{1}{4}bx^2 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{2}ax^2 + \frac{1}{4}bc \log(x^4 - c^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x^2)),x, algorithm="fricas")

[Out] 1/4\*b\*x^2\*log((x^2 + c)/(x^2 - c)) + 1/2\*a\*x^2 + 1/4\*b\*c\*log(x^4 - c^2)

**Sympy [C]** time = 10.1228, size = 61, normalized size = 1.79

$$\frac{ax^2}{2} + \frac{bc \log(-i\sqrt{c} + x)}{2} + \frac{bc \log(i\sqrt{c} + x)}{2} - \frac{bc \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{bx^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c/x\*\*2)),x)

[Out] a\*x\*\*2/2 + b\*c\*log(-I\*sqrt(c) + x)/2 + b\*c\*log(I\*sqrt(c) + x)/2 - b\*c\*atanh(c/x\*\*2)/2 + b\*x\*\*2\*atanh(c/x\*\*2)/2

**Giac [A]** time = 1.38631, size = 63, normalized size = 1.85

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(x^2 \log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right) + c \log(|x^4 - c^2|)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

```
[Out] 1/2*a*x^2 + 1/4*(x^2*log(-(c/x^2 + 1)/(c/x^2 - 1)) + c*log(abs(x^4 - c^2)))  
*b
```

$$3.161 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} dx$$

**Optimal.** Leaf size=30

$$\frac{1}{4}b\text{PolyLog}\left(2, -\frac{c}{x^2}\right) - \frac{1}{4}b\text{PolyLog}\left(2, \frac{c}{x^2}\right) + a \log(x)$$

[Out] a\*Log[x] + (b\*PolyLog[2, -(c/x^2)])/4 - (b\*PolyLog[2, c/x^2])/4

**Rubi [A]** time = 0.0327894, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6095, 5912}

$$\frac{1}{4}b\text{PolyLog}\left(2, -\frac{c}{x^2}\right) - \frac{1}{4}b\text{PolyLog}\left(2, \frac{c}{x^2}\right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])/x, x]

[Out] a\*Log[x] + (b\*PolyLog[2, -(c/x^2)])/4 - (b\*PolyLog[2, c/x^2])/4

### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, \frac{1}{x^2}\right)\right) \\ &= a \log(x) + \frac{1}{4}b\text{Li}_2\left(-\frac{c}{x^2}\right) - \frac{1}{4}b\text{Li}_2\left(\frac{c}{x^2}\right) \end{aligned}$$

**Mathematica [A]** time = 0.0139595, size = 28, normalized size = 0.93

$$\frac{1}{4}b \left( \text{PolyLog} \left( 2, -\frac{c}{x^2} \right) - \text{PolyLog} \left( 2, \frac{c}{x^2} \right) \right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x,x]

[Out] a\*Log[x] + (b\*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/4

**Maple [B]** time = 0.03, size = 154, normalized size = 5.1

$$-a \ln(x^{-1}) - b \ln(x^{-1}) \text{Arctanh}\left(\frac{c}{x^2}\right) + \frac{b \ln(x^{-1})}{2} \ln\left(1 + \frac{1}{x}\sqrt{-c}\right) + \frac{b \ln(x^{-1})}{2} \ln\left(1 - \frac{1}{x}\sqrt{-c}\right) + \frac{b}{2} \text{dilog}\left(1 + \frac{1}{x}\sqrt{-c}\right) + \frac{b}{2} \text{dilog}\left(1 - \frac{1}{x}\sqrt{-c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))/x,x)

[Out] -a\*ln(1/x)-b\*ln(1/x)\*arctanh(c/x^2)+1/2\*b\*ln(1/x)\*ln(1+(-c)^(1/2)/x)+1/2\*b\*ln(1/x)\*ln(1-(-c)^(1/2)/x)+1/2\*b\*dilog(1+(-c)^(1/2)/x)+1/2\*b\*dilog(1-(-c)^(1/2)/x)-1/2\*b\*ln(1/x)\*ln(1-1/x\*c^(1/2))-1/2\*b\*ln(1/x)\*ln(1+1/x\*c^(1/2))-1/2\*b\*dilog(1-1/x\*c^(1/2))-1/2\*b\*dilog(1+1/x\*c^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}b \int \frac{\log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x,x, algorithm="maxima")

[Out] 1/2\*b\*integrate((log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x) + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c/x^2) + a)/x, x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x,x)

[Out] Integral((a + b\*atanh(c/x\*\*2))/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)/x, x)



$$3.162 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3} dx$$

**Optimal.** Leaf size=37

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2} - \frac{b \log\left(1 - \frac{c^2}{x^4}\right)}{4c}$$

[Out]  $-(a + b \operatorname{ArcTanh}[c/x^2])/(2*x^2) - (b \operatorname{Log}[1 - c^2/x^4])/(4*c)$

**Rubi [A]** time = 0.0202372, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 260}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2} - \frac{b \log\left(1 - \frac{c^2}{x^4}\right)}{4c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{ArcTanh}[c/x^2])/x^3, x]$

[Out]  $-(a + b \operatorname{ArcTanh}[c/x^2])/(2*x^2) - (b \operatorname{Log}[1 - c^2/x^4])/(4*c)$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

#### Rubi steps

$$\int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^3} dx = -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2} - (bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^5} dx$$

$$= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2} - \frac{b \log\left(1 - \frac{c^2}{x^4}\right)}{4c}$$

**Mathematica [A]** time = 0.0089931, size = 42, normalized size = 1.14

$$-\frac{a}{2x^2} - \frac{b \log\left(1 - \frac{c^2}{x^4}\right)}{4c} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x^3,x]

[Out] -a/(2\*x^2) - (b\*ArcTanh[c/x^2])/(2\*x^2) - (b\*Log[1 - c^2/x^4])/(4\*c)

**Maple [A]** time = 0.004, size = 37, normalized size = 1.

$$-\frac{a}{2x^2} - \frac{b}{2x^2} \operatorname{Arctanh}\left(\frac{c}{x^2}\right) - \frac{b}{4c} \ln\left(1 - \frac{c^2}{x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))/x^3,x)

[Out] -1/2\*a/x^2-1/2\*b/x^2\*arctanh(c/x^2)-1/4\*b\*ln(1-c^2/x^4)/c

**Maxima [A]** time = 0.969016, size = 50, normalized size = 1.35

$$-\frac{b\left(\frac{2c \operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^2} + \log\left(-\frac{c^2}{x^4} + 1\right)\right)}{4c} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^3,x, algorithm="maxima")

[Out]  $-1/4*b*(2*c*arctanh(c/x^2)/x^2 + \log(-c^2/x^4 + 1))/c - 1/2*a/x^2$

**Fricas [A]** time = 1.77503, size = 126, normalized size = 3.41

$$\frac{bx^2 \log(x^4 - c^2) - 4bx^2 \log(x) + bc \log\left(\frac{x^2+c}{x^2-c}\right) + 2ac}{4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^3,x, algorithm="fricas")

[Out]  $-1/4*(b*x^2*\log(x^4 - c^2) - 4*b*x^2*\log(x) + b*c*\log((x^2 + c)/(x^2 - c)) + 2*a*c)/(c*x^2)$

**Sympy [A]** time = 18.5034, size = 76, normalized size = 2.05

$$\begin{cases} -\frac{a}{2x^2} - \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^2} + \frac{b \log(x)}{c} - \frac{b \log(-i\sqrt{c+x})}{2c} - \frac{b \log(i\sqrt{c+x})}{2c} + \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x\*\*3,x)

[Out] Piecewise((-a/(2\*x\*\*2) - b\*atanh(c/x\*\*2)/(2\*x\*\*2) + b\*log(x)/c - b\*log(-I\*sqrt(c) + x)/(2\*c) - b\*log(I\*sqrt(c) + x)/(2\*c) + b\*atanh(c/x\*\*2)/(2\*c), Ne(c, 0)), (-a/(2\*x\*\*2), True))

**Giac [A]** time = 1.27518, size = 70, normalized size = 1.89

$$-\frac{b \log(x^4 - c^2)}{4c} + \frac{b \log(x)}{c} - \frac{b \log\left(\frac{x^2+c}{x^2-c}\right)}{4x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*b*log(x^4 - c^2)/c + b*log(x)/c - 1/4*b*log((x^2 + c)/(x^2 - c))/x^2 -  
1/2*a/x^2
```

$$3.163 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5} dx$$

**Optimal.** Leaf size=45

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} + \frac{b \tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2} - \frac{b}{4cx^2}$$

[Out]  $-b/(4*c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2)$

**Rubi [A]** time = 0.0320788, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 263, 275, 325, 207}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} + \frac{b \tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2} - \frac{b}{4cx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])/x^5,x]

[Out]  $-b/(4*c*x^2) - (a + b*ArcTanh[c/x^2])/(4*x^4) + (b*ArcTanh[x^2/c])/(4*c^2)$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^5} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{1}{2}(bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^7} dx \\
&= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{1}{2}(bc) \int \frac{1}{x^3(-c^2 + x^4)} dx \\
&= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{1}{4}(bc) \operatorname{Subst}\left(\int \frac{1}{x^2(-c^2 + x^2)} dx, x, x^2\right) \\
&= -\frac{b}{4cx^2} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b \operatorname{Subst}\left(\int \frac{1}{-c^2 + x^2} dx, x, x^2\right)}{4c} \\
&= -\frac{b}{4cx^2} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4} + \frac{b \tanh^{-1}\left(\frac{x^2}{c}\right)}{4c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0113542, size = 64, normalized size = 1.42

$$-\frac{a}{4x^4} - \frac{b \log(x^2 - c)}{8c^2} + \frac{b \log(c + x^2)}{8c^2} - \frac{b}{4cx^2} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c/x^2])/x^5, x]
```

```
[Out] -a/(4*x^4) - b/(4*c*x^2) - (b*ArcTanh[c/x^2])/(4*x^4) - (b*Log[-c + x^2])/(8*c^2) + (b*Log[c + x^2])/(8*c^2)
```

---

**Maple [A]** time = 0.009, size = 57, normalized size = 1.3

$$-\frac{a}{4x^4} - \frac{b}{4x^4} \operatorname{Artanh}\left(\frac{c}{x^2}\right) - \frac{b}{4cx^2} - \frac{b}{8c^2} \ln\left(\frac{c}{x^2} - 1\right) + \frac{b}{8c^2} \ln\left(1 + \frac{c}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x^2))/x^5,x)`

[Out]  $-1/4*a/x^4 - 1/4*b/x^4*arctanh(c/x^2) - 1/4*b/c/x^2 - 1/8*b/c^2*\ln(c/x^2-1) + 1/8*b/c^2*\ln(1+c/x^2)$

---

**Maxima [A]** time = 0.967116, size = 76, normalized size = 1.69

$$\frac{1}{8} \left( c \left( \frac{\log(x^2 + c)}{c^3} - \frac{\log(x^2 - c)}{c^3} - \frac{2}{c^2 x^2} \right) - \frac{2 \operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^4} \right) b - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="maxima")`

[Out]  $1/8*(c*(\log(x^2 + c)/c^3 - \log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x^2)/x^4)*b - 1/4*a/x^4$

---

**Fricas [A]** time = 1.54632, size = 109, normalized size = 2.42

$$-\frac{2bcx^2 + 2ac^2 - (bx^4 - bc^2) \log\left(\frac{x^2+c}{x^2-c}\right)}{8c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="fricas")`

[Out]  $-1/8*(2*b*c*x^2 + 2*a*c^2 - (b*x^4 - b*c^2)*\log((x^2 + c)/(x^2 - c)))/(c^2*x^4)$

---

**Sympy [A]** time = 45.3849, size = 49, normalized size = 1.09

$$\begin{cases} -\frac{a}{4x^4} - \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b}{4cx^2} + \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{4c^2} & \text{for } c \neq 0 \\ -\frac{a}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x\*\*5,x)

[Out] Piecewise((-a/(4\*x\*\*4) - b\*atanh(c/x\*\*2)/(4\*x\*\*4) - b/(4\*c\*x\*\*2) + b\*atanh(c/x\*\*2)/(4\*c\*\*2), Ne(c, 0)), (-a/(4\*x\*\*4), True))

---

**Giac [A]** time = 1.2143, size = 89, normalized size = 1.98

$$\frac{b \log(x^2 + c)}{8c^2} - \frac{b \log(-x^2 + c)}{8c^2} - \frac{b \log\left(\frac{x^2+c}{x^2-c}\right)}{8x^4} - \frac{bx^2 + ac}{4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^5,x, algorithm="giac")

[Out] 1/8\*b\*log(x^2 + c)/c^2 - 1/8\*b\*log(-x^2 + c)/c^2 - 1/8\*b\*log((x^2 + c)/(x^2 - c))/x^4 - 1/4\*(b\*x^2 + a\*c)/(c\*x^4)



$$3.164 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7} dx$$

**Optimal.** Leaf size=57

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{b \log(c^2-x^4)}{12c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{12cx^4}$$

[Out]  $-\frac{b}{(12*c*x^4)} - (a + b*\text{ArcTanh}[c/x^2])/(6*x^6) + (b*\text{Log}[x])/(3*c^3) - (b*\text{Log}[c^2 - x^4])/(12*c^3)$

**Rubi [A]** time = 0.0400527, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 263, 266, 44}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{b \log(c^2-x^4)}{12c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{12cx^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c/x^2])/x^7, x]$

[Out]  $-\frac{b}{(12*c*x^4)} - (a + b*\text{ArcTanh}[c/x^2])/(6*x^6) + (b*\text{Log}[x])/(3*c^3) - (b*\text{Log}[c^2 - x^4])/(12*c^3)$

#### Rule 6097

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $\rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 263

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol]$   $\rightarrow \text{Int}[x^{(m+n*p)}*(b + a/x^n)^p, x] /;$   $\text{FreeQ}\{a, b, m, n\}, x\} \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{NegQ}[n]$

#### Rule 266

$\text{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol]$   $\rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)}*(a + b*x)^p, x], x, x^n], x] /;$   $\text{FreeQ}\{a, b$

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^7} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^9} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{3}(bc) \int \frac{1}{x^5(-c^2 + x^4)} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc) \operatorname{Subst}\left(\int \frac{1}{x^2(-c^2 + x)} dx, x, x^4\right) \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{1}{12}(bc) \operatorname{Subst}\left(\int \left(-\frac{1}{c^4(c^2 - x)} - \frac{1}{c^2x^2} - \frac{1}{c^4x}\right) dx, x, x^4\right) \\
 &= -\frac{b}{12cx^4} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6} + \frac{b \log(x)}{3c^3} - \frac{b \log(c^2 - x^4)}{12c^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.012913, size = 62, normalized size = 1.09

$$-\frac{a}{6x^6} - \frac{b \log(x^4 - c^2)}{12c^3} + \frac{b \log(x)}{3c^3} - \frac{b}{12cx^4} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x^7, x]

[Out] -a/(6\*x^6) - b/(12\*c\*x^4) - (b\*ArcTanh[c/x^2])/(6\*x^6) + (b\*Log[x])/(3\*c^3) - (b\*Log[-c^2 + x^4])/(12\*c^3)

**Maple [A]** time = 0.006, size = 45, normalized size = 0.8

$$-\frac{a}{6x^6} - \frac{b}{6x^6} \operatorname{Arctanh}\left(\frac{c}{x^2}\right) - \frac{b}{12cx^4} - \frac{b}{12c^3} \ln\left(\frac{c^2}{x^4} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x^2))/x^7,x)`

[Out] `-1/6*a/x^6-1/6*b/x^6*arctanh(c/x^2)-1/12*b/c/x^4-1/12*b/c^3*ln(c^2/x^4-1)`

**Maxima [A]** time = 0.979566, size = 74, normalized size = 1.3

$$-\frac{1}{12} \left( c \left( \frac{\log(x^4 - c^2)}{c^4} - \frac{\log(x^4)}{c^4} + \frac{1}{c^2 x^4} \right) + \frac{2 \operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^6} \right) b - \frac{a}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="maxima")`

[Out] `-1/12*(c*(log(x^4 - c^2)/c^4 - log(x^4)/c^4 + 1/(c^2*x^4)) + 2*arctanh(c/x^2)/x^6)*b - 1/6*a/x^6`

**Fricas [A]** time = 1.67821, size = 151, normalized size = 2.65

$$\frac{bx^6 \log(x^4 - c^2) - 4bx^6 \log(x) + bc^2x^2 + bc^3 \log\left(\frac{x^2+c}{x^2-c}\right) + 2ac^3}{12c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="fricas")`

[Out] `-1/12*(b*x^6*log(x^4 - c^2) - 4*b*x^6*log(x) + b*c^2*x^2 + b*c^3*log((x^2 + c)/(x^2 - c)) + 2*a*c^3)/(c^3*x^6)`

**Sympy [A]** time = 47.1008, size = 94, normalized size = 1.65

$$\begin{cases} \frac{a}{6x^6} - \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{6x^6} - \frac{b}{12cx^4} + \frac{b \log(x)}{3c^3} - \frac{b \log(-i\sqrt{c+x})}{6c^3} - \frac{b \log(i\sqrt{c+x})}{6c^3} + \frac{b \operatorname{atanh}\left(\frac{c}{x^2}\right)}{6c^3} & \text{for } c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x\*\*7,x)

[Out] Piecewise((-a/(6\*x\*\*6) - b\*atanh(c/x\*\*2)/(6\*x\*\*6) - b/(12\*c\*x\*\*4) + b\*log(x)/(3\*c\*\*3) - b\*log(-I\*sqrt(c) + x)/(6\*c\*\*3) - b\*log(I\*sqrt(c) + x)/(6\*c\*\*3) + b\*atanh(c/x\*\*2)/(6\*c\*\*3), Ne(c, 0)), (-a/(6\*x\*\*6), True))

**Giac [A]** time = 1.16046, size = 88, normalized size = 1.54

$$-\frac{b \log(x^4 - c^2)}{12c^3} + \frac{b \log(x)}{3c^3} - \frac{b \log\left(\frac{x^2+c}{x^2-c}\right)}{12x^6} - \frac{bx^2 + 2ac}{12cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^7,x, algorithm="giac")

[Out] -1/12\*b\*log(x^4 - c^2)/c^3 + 1/3\*b\*log(x)/c^3 - 1/12\*b\*log((x^2 + c)/(x^2 - c))/x^6 - 1/12\*(b\*x^2 + 2\*a\*c)/(c\*x^6)

### 3.165 $\int x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=63

$$\frac{1}{5}x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{5}bc^{5/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{1}{5}bc^{5/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{15}bcx^3$$

[Out] (2\*b\*c\*x^3)/15 + (b\*c^(5/2)\*ArcTan[x/Sqrt[c]])/5 + (x^5\*(a + b\*ArcTanh[c/x^2]))/5 - (b\*c^(5/2)\*ArcTanh[x/Sqrt[c]])/5

**Rubi [A]** time = 0.0347141, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6097, 263, 321, 298, 203, 206}

$$\frac{1}{5}x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{5}bc^{5/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{1}{5}bc^{5/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{15}bcx^3$$

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (2\*b\*c\*x^3)/15 + (b\*c^(5/2)\*ArcTan[x/Sqrt[c]])/5 + (x^5\*(a + b\*ArcTanh[c/x^2]))/5 - (b\*c^(5/2)\*ArcTanh[x/Sqrt[c]])/5

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x],

x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{5} x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{5} (2bc) \int \frac{x^2}{1 - \frac{c^2}{x^4}} dx \\
 &= \frac{1}{5} x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{5} (2bc) \int \frac{x^6}{-c^2 + x^4} dx \\
 &= \frac{2}{15} bcx^3 + \frac{1}{5} x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{5} (2bc^3) \int \frac{x^2}{-c^2 + x^4} dx \\
 &= \frac{2}{15} bcx^3 + \frac{1}{5} x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{5} (bc^3) \int \frac{1}{c - x^2} dx + \frac{1}{5} (bc^3) \int \frac{1}{c + x^2} dx \\
 &= \frac{2}{15} bcx^3 + \frac{1}{5} bc^{5/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{5} x^5 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{5} bc^{5/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0205633, size = 88, normalized size = 1.4

$$\frac{ax^5}{5} + \frac{1}{10} bc^{5/2} \log(\sqrt{c} - x) - \frac{1}{10} bc^{5/2} \log(\sqrt{c} + x) + \frac{1}{5} bc^{5/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{15} bcx^3 + \frac{1}{5} bx^5 \tanh^{-1} \left( \frac{c}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (2\*b\*c\*x^3)/15 + (a\*x^5)/5 + (b\*c^(5/2)\*ArcTan[x/Sqrt[c]])/5 + (b\*x^5\*ArcTanh[c/x^2])/5 + (b\*c^(5/2)\*Log[Sqrt[c] - x])/10 - (b\*c^(5/2)\*Log[Sqrt[c] + x])/10

**Maple [A]** time = 0.014, size = 53, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^5}{5} \operatorname{Artanh}\left(\frac{c}{x^2}\right) + \frac{b}{5} c^{\frac{5}{2}} \arctan\left(x \frac{1}{\sqrt{c}}\right) - \frac{b}{5} c^{\frac{5}{2}} \operatorname{Artanh}\left(\frac{1}{x} \sqrt{c}\right) + \frac{2bcx^3}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arctanh(c/x^2)),x)

[Out] 1/5\*a\*x^5+1/5\*b\*x^5\*arctanh(c/x^2)+1/5\*b\*c^(5/2)\*arctan(x/c^(1/2))-1/5\*b\*c^(5/2)\*arctanh(1/x\*c^(1/2))+2/15\*b\*c\*x^3

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c/x^2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.83608, size = 440, normalized size = 6.98

$$\left[ \frac{1}{10} bx^5 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{5} ax^5 + \frac{2}{15} bcx^3 + \frac{1}{5} bc^{\frac{5}{2}} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{10} bc^{\frac{5}{2}} \log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right), \frac{1}{10} bx^5 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c/x^2)),x, algorithm="fricas")

```
[Out] [1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^(5/2)*arctan(x/sqrt(c)) + 1/10*b*c^(5/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)), 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*sqrt(-c)*c^2*arctan(sqrt(-c)*x/c) + 1/10*b*sqrt(-c)*c^2*log((x^2 + 2*sqrt(-c)*x - c)/(x^2 + c))]
```

**Sympy [A]** time = 42.2058, size = 588, normalized size = 9.33

$$\left\{ \begin{array}{l} \frac{ax^5}{x^5(a-\infty b)} \\ \frac{x^5(a+\infty b)}{5} \\ -\frac{6ac^{69}x^5}{-30c^{69}+30c^{67}x^4} + \frac{6ac^{67}x^9}{-30c^{69}+30c^{67}x^4} - \frac{6bc^{\frac{143}{2}}\log(-\sqrt{c+x})}{-30c^{69}+30c^{67}x^4} + \frac{3bc^{\frac{143}{2}}\log(-i\sqrt{c+x})}{-30c^{69}+30c^{67}x^4} + \frac{3ibc^{\frac{143}{2}}\log(-i\sqrt{c+x})}{-30c^{69}+30c^{67}x^4} + \frac{3bc^{\frac{143}{2}}\log(i\sqrt{c+x})}{-30c^{69}+30c^{67}x^4} - \frac{3ibc^{\frac{143}{2}}\log(i\sqrt{c+x})}{-30c^{69}+30c^{67}x^4} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**5/5, Eq(c, 0)), (x**5*(a - oo*b)/5, Eq(c, -x**2)), (x**5*(a + oo*b)/5, Eq(c, x**2)), (-6*a*c**69*x**5/(-30*c**69 + 30*c**67*x**4) + 6*a*c**67*x**9/(-30*c**69 + 30*c**67*x**4) - 6*b*c**(143/2)*log(-sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) + 3*b*c**(143/2)*log(-I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) + 3*I*b*c**(143/2)*log(-I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) + 3*b*c**(143/2)*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*I*b*c**(143/2)*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 6*b*c**(143/2)*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4) + 6*b*c**(139/2)*x**4*log(-sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*b*c**(139/2)*x**4*log(-I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*I*b*c**(139/2)*x**4*log(-I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) - 3*b*c**(139/2)*x**4*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) + 3*I*b*c**(139/2)*x**4*log(I*sqrt(c) + x)/(-30*c**69 + 30*c**67*x**4) + 6*b*c**(139/2)*x**4*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4) - 4*b*c**70*x**3/(-30*c**69 + 30*c**67*x**4) - 6*b*c**69*x**5*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4) + 4*b*c**68*x**7/(-30*c**69 + 30*c**67*x**4) + 6*b*c**67*x**9*atanh(c/x**2)/(-30*c**69 + 30*c**67*x**4), True))
```



**Giac [A]** time = 1.30346, size = 90, normalized size = 1.43

$$\frac{1}{10} bx^5 \log\left(\frac{x^2 + c}{x^2 - c}\right) + \frac{1}{5} ax^5 + \frac{2}{15} bcx^3 + \frac{1}{5} bc^3 \left( \frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] 1/10\*b\*x^5\*log((x^2 + c)/(x^2 - c)) + 1/5\*a\*x^5 + 2/15\*b\*c\*x^3 + 1/5\*b\*c^3\*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c))

### 3.166 $\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=61

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{3}bc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{1}{3}bc^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2bcx}{3}$$

[Out] (2\*b\*c\*x)/3 - (b\*c^(3/2)\*ArcTan[x/Sqrt[c]])/3 + (x^3\*(a + b\*ArcTanh[c/x^2])/3 - (b\*c^(3/2)\*ArcTanh[x/Sqrt[c]])/3

**Rubi [A]** time = 0.0331548, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6097, 193, 321, 212, 206, 203}

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{3}bc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{1}{3}bc^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2bcx}{3}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (2\*b\*c\*x)/3 - (b\*c^(3/2)\*ArcTan[x/Sqrt[c]])/3 + (x^3\*(a + b\*ArcTanh[c/x^2])/3 - (b\*c^(3/2)\*ArcTanh[x/Sqrt[c]])/3

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

#### Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

$x]$  /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{3} (2bc) \int \frac{1}{1 - \frac{c^2}{x^4}} dx \\
 &= \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{3} (2bc) \int \frac{x^4}{-c^2 + x^4} dx \\
 &= \frac{2bcx}{3} + \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) + \frac{1}{3} (2bc^3) \int \frac{1}{-c^2 + x^4} dx \\
 &= \frac{2bcx}{3} + \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{3} (bc^2) \int \frac{1}{c - x^2} dx - \frac{1}{3} (bc^2) \int \frac{1}{c + x^2} dx \\
 &= \frac{2bcx}{3} - \frac{1}{3} bc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) - \frac{1}{3} bc^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0183367, size = 86, normalized size = 1.41

$$\frac{ax^3}{3} + \frac{1}{6} bc^{3/2} \log(\sqrt{c} - x) - \frac{1}{6} bc^{3/2} \log(\sqrt{c} + x) - \frac{1}{3} bc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} bx^3 \tanh^{-1} \left( \frac{c}{x^2} \right) + \frac{2bcx}{3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c/x^2]),x]

[Out] (2\*b\*c\*x)/3 + (a\*x^3)/3 - (b\*c^(3/2)\*ArcTan[x/Sqrt[c]])/3 + (b\*x^3\*ArcTanh[c/x^2])/3 + (b\*c^(3/2)\*Log[Sqrt[c] - x])/6 - (b\*c^(3/2)\*Log[Sqrt[c] + x])/6

**Maple [A]** time = 0.008, size = 51, normalized size = 0.8

$$\frac{x^3 a}{3} + \frac{b x^3}{3} \operatorname{Arctanh}\left(\frac{c}{x^2}\right) - \frac{b}{3} c^{\frac{3}{2}} \arctan\left(x \frac{1}{\sqrt{c}}\right) + \frac{2 x b c}{3} - \frac{b}{3} c^{\frac{3}{2}} \operatorname{Arctanh}\left(\frac{1}{x} \sqrt{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c/x^2)),x)

[Out] 1/3\*x^3\*a+1/3\*b\*x^3\*arctanh(c/x^2)-1/3\*b\*c^(3/2)\*arctan(x/c^(1/2))+2/3\*x\*b\*c-1/3\*b\*c^(3/2)\*arctanh(1/x\*c^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x^2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.77425, size = 421, normalized size = 6.9

$$\left[\frac{1}{6} b x^3 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{3} a x^3 - \frac{1}{3} b c^{\frac{3}{2}} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{6} b c^{\frac{3}{2}} \log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right) + \frac{2}{3} b c x, \frac{1}{6} b x^3 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{3} a x^3 + \frac{1}{3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x^2)),x, algorithm="fricas")

```
[Out] [1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 - 1/3*b*c^(3/2)*arctan(x/sqrt(c)) + 1/6*b*c^(3/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + 2/3*b*c*x, 1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 1/3*b*sqrt(-c)*c*arctan(sqrt(-c)*x/c) + 1/6*b*sqrt(-c)*c*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 2/3*b*c*x]
```

**Sympy [A]** time = 22.2281, size = 624, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atanh(c/x**2)),x)
```

```
[Out] Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - oo*b)/3, Eq(c, -x**2)), (x**3*(a + oo*b)/3, Eq(c, x**2)), (-2*a*c**(29/2)*x**3/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 2*a*c**(25/2)*x**7/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 4*b*c**(31/2)*x/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 2*b*c**(29/2)*x**3*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 4*b*c**(27/2)*x**5/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 2*b*c**(25/2)*x**7*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 2*b*c**16*log(-sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + b*c**16*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - I*b*c**16*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + b*c**16*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + I*b*c**16*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - 2*b*c**16*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 2*b*c**14*x**4*log(-sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4)*x**4 - b*c**14*x**4*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + I*b*c**14*x**4*log(-I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - b*c**14*x**4*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) - I*b*c**14*x**4*log(I*sqrt(c) + x)/(-6*c**(29/2) + 6*c**(25/2)*x**4) + 2*b*c**14*x**4*atanh(c/x**2)/(-6*c**(29/2) + 6*c**(25/2)*x**4), True))
```

**Giac [A]** time = 1.25818, size = 93, normalized size = 1.52

$$\frac{1}{3}bc^3 \left( \frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} \right) + \frac{1}{6}bx^3 \log\left(\frac{x^2+c}{x^2-c}\right) + \frac{1}{3}ax^3 + \frac{2}{3}bcx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

```
[Out] 1/3*b*c^3*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) + 1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 2/3*b*c*x
```

$$3.167 \quad \int \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$$

**Optimal.** Leaf size=44

$$ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) + b\sqrt{c} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - b\sqrt{c} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right)$$

[Out] a\*x + b\*Sqrt[c]\*ArcTan[x/Sqrt[c]] + b\*x\*ArcTanh[c/x^2] - b\*Sqrt[c]\*ArcTanh[x/Sqrt[c]]

**Rubi [A]** time = 0.0244142, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6091, 263, 298, 203, 206}

$$ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) + b\sqrt{c} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - b\sqrt{c} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c/x^2], x]

[Out] a\*x + b\*Sqrt[c]\*ArcTan[x/Sqrt[c]] + b\*x\*ArcTanh[c/x^2] - b\*Sqrt[c]\*ArcTanh[x/Sqrt[c]]

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

**Rule 203**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

**Rule 206**

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

**Rubi steps**

$$\begin{aligned}
 \int \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= ax + b \int \tanh^{-1} \left( \frac{c}{x^2} \right) dx \\
 &= ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) + (2bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right) x^2} dx \\
 &= ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) + (2bc) \int \frac{x^2}{-c^2 + x^4} dx \\
 &= ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) - (bc) \int \frac{1}{c - x^2} dx + (bc) \int \frac{1}{c + x^2} dx \\
 &= ax + b\sqrt{c} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + bx \tanh^{-1} \left( \frac{c}{x^2} \right) - b\sqrt{c} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.0158426, size = 54, normalized size = 1.23

$$ax + bx \tanh^{-1} \left( \frac{c}{x^2} \right) + \frac{1}{2} b \sqrt{c} \left( \log(\sqrt{c} - x) - \log(\sqrt{c} + x) + 2 \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTanh[c/x^2], x]
```

```
[Out] a*x + b*x*ArcTanh[c/x^2] + (b*Sqrt[c]*(2*ArcTan[x/Sqrt[c]] + Log[Sqrt[c] - x] - Log[Sqrt[c] + x]))/2
```

**Maple [A]** time = 0.01, size = 39, normalized size = 0.9

$$ax + bx \operatorname{Artanh} \left( \frac{c}{x^2} \right) - \operatorname{Artanh} \left( \frac{1}{x} \sqrt{c} \right) \sqrt{cb} + b \arctan \left( x \frac{1}{\sqrt{c}} \right) \sqrt{c}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arctanh(c/x^2),x)`

[Out] `a*x+b*x*arctanh(c/x^2)-arctanh(1/x*c^(1/2))*c^(1/2)*b+b*arctan(x/c^(1/2))*c^(1/2)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c/x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.76386, size = 351, normalized size = 7.98

$$\left[ \frac{1}{2} bx \log\left(\frac{x^2+c}{x^2-c}\right) + b\sqrt{c} \arctan\left(\frac{x}{\sqrt{c}}\right) + \frac{1}{2} b\sqrt{c} \log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right) + ax, \frac{1}{2} bx \log\left(\frac{x^2+c}{x^2-c}\right) + b\sqrt{-c} \arctan\left(\frac{\sqrt{-c}x}{c}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c/x^2),x, algorithm="fricas")`

[Out] `[1/2*b*x*log((x^2+c)/(x^2-c)) + b*sqrt(c)*arctan(x/sqrt(c)) + 1/2*b*sqrt(c)*log((x^2-2*sqrt(c)*x+c)/(x^2-c)) + a*x, 1/2*b*x*log((x^2+c)/(x^2-c)) + b*sqrt(-c)*arctan(sqrt(-c)*x/c) + 1/2*b*sqrt(-c)*log((x^2+2*sqrt(-c)*x-c)/(x^2+c)) + a*x]`

**Sympy [A]** time = 16.7275, size = 473, normalized size = 10.75

$$ax + b \begin{cases} 0 \\ -\infty x \\ \infty x \\ -\frac{2c^{\frac{21}{2}} x \operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} + \frac{2c^{\frac{17}{2}} x^5 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} - \frac{2c^{11} \log(-\sqrt{c}+x)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} + \frac{c^{11} \log(-i\sqrt{c}+x)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} + \frac{ic^{11} \log(-i\sqrt{c}+x)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} + \frac{c^{11} \log(i\sqrt{c}+x)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} - \frac{ic^{11} \log(i\sqrt{c}+x)}{-2c^{\frac{21}{2}} + 2c^{\frac{17}{2}} x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c/x\*\*2),x)

[Out] a\*x + b\*Piecewise((0, Eq(c, 0)), (-oo\*x, Eq(c, -x\*\*2)), (oo\*x, Eq(c, x\*\*2)), (-2\*c\*\*(21/2)\*x\*atanh(c/x\*\*2)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + 2\*c\*\*(17/2)\*x\*\*5\*atanh(c/x\*\*2)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - 2\*c\*\*11\*log(-sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + c\*\*11\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + I\*c\*\*11\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + c\*\*11\*log(I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - I\*c\*\*11\*log(I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - 2\*c\*\*11\*atanh(c/x\*\*2)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + 2\*c\*\*9\*x\*\*4\*log(-sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - c\*\*9\*x\*\*4\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - I\*c\*\*9\*x\*\*4\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) - c\*\*9\*x\*\*4\*log(I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + I\*c\*\*9\*x\*\*4\*log(I\*sqrt(c) + x)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4) + 2\*c\*\*9\*x\*\*4\*atanh(c/x\*\*2)/(-2\*c\*\*(21/2) + 2\*c\*\*(17/2)\*x\*\*4), True))

**Giac [A]** time = 1.21485, size = 77, normalized size = 1.75

$$\frac{1}{2} \left( 2c \left( \frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \right) + x \log\left(-\frac{\frac{c}{x^2} + 1}{\frac{c}{x^2} - 1}\right) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c/x^2),x, algorithm="giac")

[Out] 1/2\*(2\*c\*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c)) + x\*log(-(c/x^2 + 1)/(c/x^2 - 1)))\*b + a\*x

$$3.168 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} dx$$

**Optimal.** Leaf size=46

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

[Out] (b\*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b\*ArcTanh[c/x^2])/x + (b\*ArcTanh[x/Sqrt[c]])/Sqrt[c]

**Rubi [A]** time = 0.0309239, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6097, 263, 212, 206, 203}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])/x^2,x]

[Out] (b\*ArcTan[x/Sqrt[c]])/Sqrt[c] - (a + b\*ArcTanh[c/x^2])/x + (b\*ArcTanh[x/Sqrt[c]])/Sqrt[c]

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[x^(m+n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x],

$x] + \text{Dist}[r/(2*a), \text{Int}[1/(r + s*x^2), x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& !\text{GtQ}[a/b, 0]$

### Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 203

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^2} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - (2bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^4} dx \\ &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - (2bc) \int \frac{1}{-c^2 + x^4} dx \\ &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + b \int \frac{1}{c - x^2} dx + b \int \frac{1}{c + x^2} dx \\ &= \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \end{aligned}$$

**Mathematica [A]** time = 0.0180695, size = 72, normalized size = 1.57

$$-\frac{a}{x} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x} - \frac{b \log(\sqrt{c} - x)}{2\sqrt{c}} + \frac{b \log(\sqrt{c} + x)}{2\sqrt{c}} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x^2,x]

[Out] -(a/x) + (b\*ArcTan[x/Sqrt[c]])/Sqrt[c] - (b\*ArcTanh[c/x^2])/x - (b\*Log[Sqrt[c] - x])/(2\*Sqrt[c]) + (b\*Log[Sqrt[c] + x])/(2\*Sqrt[c])

---

**Maple [A]** time = 0.01, size = 44, normalized size = 1.

$$-\frac{a}{x} - \frac{b}{x} \operatorname{Artanh}\left(\frac{c}{x^2}\right) + b \arctan\left(x \frac{1}{\sqrt{c}}\right) \frac{1}{\sqrt{c}} + b \operatorname{Artanh}\left(\frac{1}{x} \sqrt{c}\right) \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))/x^2,x)

[Out] -a/x-b/x\*arctanh(c/x^2)+b\*arctan(x/c^(1/2))/c^(1/2)+b/c^(1/2)\*arctanh(1/x\*c^(1/2))

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.7739, size = 385, normalized size = 8.37

$$\left[ \frac{2b\sqrt{cx} \arctan\left(\frac{x}{\sqrt{c}}\right) + b\sqrt{cx} \log\left(\frac{x^2+2\sqrt{cx}+c}{x^2-c}\right) - bc \log\left(\frac{x^2+c}{x^2-c}\right) - 2ac}{2cx}, -\frac{2b\sqrt{-cx} \arctan\left(\frac{\sqrt{-cx}}{c}\right) + b\sqrt{-cx} \log\left(\frac{x^2-2\sqrt{-cx}-c}{x^2+c}\right)}{2cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^2,x, algorithm="fricas")

[Out] [1/2\*(2\*b\*sqrt(c)\*x\*arctan(x/sqrt(c)) + b\*sqrt(c)\*x\*log((x^2 + 2\*sqrt(c)\*x + c)/(x^2 - c)) - b\*c\*log((x^2 + c)/(x^2 - c)) - 2\*a\*c)/(c\*x), -1/2\*(2\*b\*sqrt(-c)\*x\*arctan(sqrt(-c)\*x/c) + b\*sqrt(-c)\*x\*log((x^2 - 2\*sqrt(-c)\*x - c)/(x^2 + c)) + b\*c\*log((x^2 + c)/(x^2 - c)) + 2\*a\*c)/(c\*x)]

---

**Sympy [A]** time = 13.8796, size = 593, normalized size = 12.89

$$\left\{ \begin{array}{l} -\frac{a}{x} \\ \frac{x}{a-\infty b} \\ \frac{x}{a+\infty b} \\ x \end{array} \right. - \frac{2ac^{\frac{39}{2}}}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} - \frac{2ac^{\frac{35}{2}}x^4}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} + \frac{2bc^{\frac{39}{2}}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} - \frac{2bc^{\frac{35}{2}}x^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} + \frac{2bc^{19}x\log(-\sqrt{c+x})}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} - \frac{bc^{19}x\log(-i\sqrt{c+x})}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5} + \frac{ibc^{19}x\log(-i\sqrt{c+x})}{-2c^{\frac{39}{2}}x+2c^{\frac{35}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x\*\*2,x)

[Out] Piecewise((-a/x, Eq(c, 0)), (-a - oo\*b)/x, Eq(c, -x\*\*2)), (-a + oo\*b)/x, Eq(c, x\*\*2)), (2\*a\*c\*\*(39/2)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - 2\*a\*c\*\*(35/2)\*x\*\*4/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + 2\*b\*c\*\*(39/2)\*atanh(c/x\*\*2)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - 2\*b\*c\*\*(35/2)\*x\*\*4\*atanh(c/x\*\*2)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + 2\*b\*c\*\*19\*x\*log(-sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - b\*c\*\*19\*x\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + I\*b\*c\*\*19\*x\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - b\*c\*\*19\*x\*log(I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - I\*b\*c\*\*19\*x\*log(I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + 2\*b\*c\*\*19\*x\*atanh(c/x\*\*2)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - 2\*b\*c\*\*17\*x\*\*5\*log(-sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + b\*c\*\*17\*x\*\*5\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - I\*b\*c\*\*17\*x\*\*5\*log(-I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + b\*c\*\*17\*x\*\*5\*log(I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) + I\*b\*c\*\*17\*x\*\*5\*log(I\*sqrt(c) + x)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5) - 2\*b\*c\*\*17\*x\*\*5\*atanh(c/x\*\*2)/(-2\*c\*\*(39/2)\*x + 2\*c\*\*(35/2)\*x\*\*5), True))

---

**Giac [A]** time = 1.23177, size = 84, normalized size = 1.83

$$-bc \left( \frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-cc}} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} \right) - \frac{b \log\left(\frac{x^2+c}{x^2-c}\right)}{2x} - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^2,x, algorithm="giac")

```
[Out] -b*c*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) - 1/2*b*  
log((x^2 + c)/(x^2 - c))/x - a/x
```

$$3.169 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4} dx$$

**Optimal.** Leaf size=65

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{2b}{3cx}$$

[Out]  $(-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))$

**Rubi [A]** time = 0.0371315, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6097, 263, 325, 298, 203, 206}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{2b}{3cx}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])/x^4,x]

[Out]  $(-2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (a + b*ArcTanh[c/x^2])/(3*x^3) + (b*ArcTanh[x/Sqrt[c]])/(3*c^(3/2))$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m+n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 325

Int[((c\_.)\*(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1))



+ 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 298

Int[(x\_)^2/((a\_) + (b\_.)\*(x\_)^4), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2\*b), Int[1/(r + s\*x^2), x], x] - Dist[s/(2\*b), Int[1/(r - s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^4} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{1}{3}(2bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^6} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{1}{3}(2bc) \int \frac{1}{x^2(-c^2 + x^4)} dx \\
 &= -\frac{2b}{3cx} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{(2b) \int \frac{x^2}{-c^2 + x^4} dx}{3c} \\
 &= -\frac{2b}{3cx} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} + \frac{b \int \frac{1}{c-x^2} dx}{3c} - \frac{b \int \frac{1}{c+x^2} dx}{3c} \\
 &= -\frac{2b}{3cx} - \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0305724, size = 90, normalized size = 1.38

$$-\frac{a}{3x^3} - \frac{b \log(\sqrt{c} - x)}{6c^{3/2}} + \frac{b \log(\sqrt{c} + x)}{6c^{3/2}} - \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{2b}{3cx}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x^4, x]

[Out] -a/(3\*x^3) - (2\*b)/(3\*c\*x) - (b\*ArcTan[x/Sqrt[c]])/(3\*c^(3/2)) - (b\*ArcTanh[c/x^2])/(3\*x^3) - (b\*Log[Sqrt[c] - x])/(6\*c^(3/2)) + (b\*Log[Sqrt[c] + x])/(6\*c^(3/2))

**Maple [A]** time = 0.012, size = 55, normalized size = 0.9

$$-\frac{a}{3x^3} - \frac{b}{3x^3} \operatorname{Artanh}\left(\frac{c}{x^2}\right) - \frac{2b}{3cx} - \frac{b}{3} \arctan\left(x \frac{1}{\sqrt{c}}\right) c^{-\frac{3}{2}} + \frac{b}{3} \operatorname{Artanh}\left(\frac{1}{x} \sqrt{c}\right) c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))/x^4, x)

[Out] -1/3\*a/x^3-1/3\*b/x^3\*arctanh(c/x^2)-2/3\*b/c/x-1/3\*b\*arctan(x/c^(1/2))/c^(3/2)+1/3\*b/c^(3/2)\*arctanh(1/x\*c^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.84796, size = 451, normalized size = 6.94

$$\left[ \frac{2b\sqrt{cx^3} \arctan\left(\frac{x}{\sqrt{c}}\right) - b\sqrt{cx^3} \log\left(\frac{x^2+2\sqrt{cx}+c}{x^2-c}\right) + 4bcx^2 + bc^2 \log\left(\frac{x^2+c}{x^2-c}\right) + 2ac^2}{6c^2x^3}, -\frac{2b\sqrt{-cx^3} \arctan\left(\frac{\sqrt{-cx}}{c}\right) + b\sqrt{-cx^3}}{6c^2x^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^4,x, algorithm="fricas")

[Out] [-1/6\*(2\*b\*sqrt(c)\*x^3\*arctan(x/sqrt(c)) - b\*sqrt(c)\*x^3\*log((x^2 + 2\*sqrt(c)\*x + c)/(x^2 - c)) + 4\*b\*c\*x^2 + b\*c^2\*log((x^2 + c)/(x^2 - c)) + 2\*a\*c^2)/(c^2\*x^3), -1/6\*(2\*b\*sqrt(-c)\*x^3\*arctan(sqrt(-c)\*x/c) + b\*sqrt(-c)\*x^3\*log((x^2 + 2\*sqrt(-c)\*x - c)/(x^2 + c)) + 4\*b\*c\*x^2 + b\*c^2\*log((x^2 + c)/(x^2 - c)) + 2\*a\*c^2)/(c^2\*x^3)]

**Sympy [A]** time = 25.6988, size = 706, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))/x\*\*4,x)

[Out] Piecewise((-a/(3\*x\*\*3), Eq(c, 0)), (-a - oo\*b)/(3\*x\*\*3), Eq(c, -x\*\*2)), (-a + oo\*b)/(3\*x\*\*3), Eq(c, x\*\*2)), (2\*a\*c\*\*(65/2)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*\*(61/2)\*x\*\*7) - 2\*a\*c\*\*(61/2)\*x\*\*4/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + 2\*b\*c\*\*(65/2)\*atanh(c/x\*\*2)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + 4\*b\*c\*\*\*(63/2)\*x\*\*2/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - 2\*b\*c\*\*(61/2)\*x\*\*4\*a\*tanh(c/x\*\*2)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - 4\*b\*c\*\*(59/2)\*x\*\*6/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + 2\*b\*c\*\*31\*x\*\*3\*log(-sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - b\*c\*\*31\*x\*\*3\*log(-I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - I\*b\*c\*\*31\*x\*\*3\*log(-I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - b\*c\*\*31\*x\*\*3\*log(I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + I\*b\*c\*\*31\*x\*\*3\*log(I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + 2\*b\*c\*\*31\*x\*\*3\*atanh(c/x\*\*2)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - 2\*b\*c\*\*29\*x\*\*7\*log(-sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + b\*c\*\*29\*x\*\*7\*log(-I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + I\*b\*c\*\*29\*x\*\*7\*log(-I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) + b\*c\*\*29\*x\*\*7\*log(I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7) - I\*b\*c\*\*29\*x\*\*7\*log(I\*sqrt(c) + x)/(-6\*c\*\*(65/2)\*x\*\*3 + 6\*c\*\*(61/2)\*x\*\*7)

```
*x**3 + 6*c**(61/2)*x**7) - 2*b*c**29*x**7*atanh(c/x**2)/(-6*c**(65/2)*x**3
+ 6*c**(61/2)*x**7), True))
```

**Giac [A]** time = 1.30144, size = 97, normalized size = 1.49

$$-\frac{1}{3}b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{6x^3} - \frac{2bx^2+ac}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c) + arctan(x/sqrt(c))/c^(3/2)) - 1/6*
b*log((x^2 + c)/(x^2 - c))/x^3 - 1/3*(2*b*x^2 + a*c)/(c*x^3)
```

$$3.170 \quad \int \frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6} dx$$

**Optimal.** Leaf size=65

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{2b}{15cx^3}$$

[Out]  $(-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))$

**Rubi [A]** time = 0.0375865, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6097, 263, 325, 212, 206, 203}

$$-\frac{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{2b}{15cx^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])/x^6,x]

[Out]  $(-2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (a + b*ArcTanh[c/x^2])/(5*x^5) + (b*ArcTanh[x/Sqrt[c]])/(5*c^(5/2))$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m+1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m+1)), x] - Dist[(b\*c\*n)/(d\*(m+1)), Int[(x^(n-1)\*(d\*x)^(m+1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Int[x^(m+n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 325

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] - Dist[(b\*(m+n\*(p+1))

+ 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{x^6} dx &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} - \frac{1}{5}(2bc) \int \frac{1}{\left(1 - \frac{c^2}{x^4}\right)x^8} dx \\
 &= -\frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} - \frac{1}{5}(2bc) \int \frac{1}{x^4(-c^2 + x^4)} dx \\
 &= -\frac{2b}{15cx^3} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} - \frac{(2b) \int \frac{1}{-c^2+x^4} dx}{5c} \\
 &= -\frac{2b}{15cx^3} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} + \frac{b \int \frac{1}{c-x^2} dx}{5c^2} + \frac{b \int \frac{1}{c+x^2} dx}{5c^2} \\
 &= -\frac{2b}{15cx^3} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{a + b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.023624, size = 90, normalized size = 1.38

$$-\frac{a}{5x^5} - \frac{b \log(\sqrt{c} - x)}{10c^{5/2}} + \frac{b \log(\sqrt{c} + x)}{10c^{5/2}} + \frac{b \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{2b}{15cx^3} - \frac{b \tanh^{-1}\left(\frac{c}{x^2}\right)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])/x^6,x]

[Out] -a/(5\*x^5) - (2\*b)/(15\*c\*x^3) + (b\*ArcTan[x/Sqrt[c]])/(5\*c^(5/2)) - (b\*ArcTanh[c/x^2])/(5\*x^5) - (b\*Log[Sqrt[c] - x])/(10\*c^(5/2)) + (b\*Log[Sqrt[c] + x])/(10\*c^(5/2))

**Maple [A]** time = 0.011, size = 55, normalized size = 0.9

$$-\frac{a}{5x^5} - \frac{b}{5x^5} \operatorname{Artanh}\left(\frac{c}{x^2}\right) - \frac{2b}{15cx^3} + \frac{b}{5} \arctan\left(x \frac{1}{\sqrt{c}}\right) c^{-\frac{5}{2}} + \frac{b}{5} \operatorname{Artanh}\left(\frac{1}{x} \sqrt{c}\right) c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))/x^6,x)

[Out] -1/5\*a/x^5-1/5/x^5\*b\*arctanh(c/x^2)-2/15\*b/c/x^3+1/5\*b\*arctan(x/c^(1/2))/c^(5/2)+1/5\*b/c^(5/2)\*arctanh(1/x\*c^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError





$8*x**6/(-30*c**71*x**5 + 30*c**69*x**9), True))$

**Giac [A]** time = 1.31428, size = 100, normalized size = 1.54

$$-\frac{1}{5}b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-cc^2}} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{10x^5} - \frac{2bx^2 + 3ac}{15cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))/x^6,x, algorithm="giac")

[Out]  $-1/5*b*(\arctan(x/\sqrt{-c})/(\sqrt{-c}*c^2) - \arctan(x/\sqrt{c})/c^{(5/2)}) - 1/10*b*\log((x^2 + c)/(x^2 - c))/x^5 - 1/15*(2*b*x^2 + 3*a*c)/(c*x^5)$

$$3.171 \quad \int x^3 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=94

$$-\frac{1}{4}c^2 \left( a + b \coth^{-1} \left( \frac{x^2}{c} \right) \right)^2 + \frac{1}{4}x^4 \left( a + b \coth^{-1} \left( \frac{x^2}{c} \right) \right)^2 + \frac{1}{2}bcx^2 \left( a + b \coth^{-1} \left( \frac{x^2}{c} \right) \right) + \frac{1}{4}b^2c^2 \log \left( 1 - \frac{c^2}{x^4} \right) + b^2c^2 \log(x)$$

[Out] (b\*c\*x^2\*(a + b\*ArcCoth[x^2/c]))/2 - (c^2\*(a + b\*ArcCoth[x^2/c])^2)/4 + (x^4\*(a + b\*ArcCoth[x^2/c])^2)/4 + (b^2\*c^2\*Log[1 - c^2/x^4])/4 + b^2\*c^2\*Log[x]

**Rubi [C]** time = 1.29574, antiderivative size = 599, normalized size of antiderivative = 6.37, number of steps used = 59, number of rules used = 33, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.063$ , Rules used = {6099, 2454, 2398, 2411, 2347, 2344, 2301, 2316, 2315, 2314, 31, 2455, 263, 266, 43, 193, 6742, 30, 2557, 12, 2466, 2448, 2462, 260, 2416, 2394, 2393, 2391, 2410, 2395, 36, 29, 2390}

$$-\frac{1}{8}b^2c^2 \text{PolyLog} \left( 2, -\frac{c}{x^2} \right) - \frac{1}{8}b^2c^2 \text{PolyLog} \left( 2, \frac{c}{x^2} \right) - \frac{1}{8}b^2c^2 \text{PolyLog} \left( 2, \frac{c-x^2}{2c} \right) - \frac{1}{8}b^2c^2 \text{PolyLog} \left( 2, \frac{c+x^2}{2c} \right) + \frac{1}{8}b^2c^2 \text{PolyLog} \left( 2, \frac{c+x^2}{2c} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[x^3\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] (a\*b\*c\*x^2)/4 - (b^2\*c\*x^2\*Log[1 - c/x^2])/8 + (b\*c\*(1 - c/x^2)\*x^2\*(2\*a - b\*Log[1 - c/x^2]))/8 - (c^2\*(2\*a - b\*Log[1 - c/x^2])^2)/16 + (x^4\*(2\*a - b\*Log[1 - c/x^2])^2)/16 + (b^2\*c^2\*Log[1 + c/x^2])/8 + (b^2\*c\*x^2\*Log[1 + c/x^2])/4 + (a\*b\*x^4\*Log[1 + c/x^2])/4 - (b^2\*x^4\*Log[1 - c/x^2]\*Log[1 + c/x^2])/8 - (b^2\*c^2\*Log[1 + c/x^2]^2)/16 + (b^2\*x^4\*Log[1 + c/x^2]^2)/16 + (a\*b\*c^2\*Log[x])/2 + (b^2\*c^2\*Log[x])/2 + (b^2\*c^2\*Log[c - x^2])/8 + (b^2\*c^2\*Log[1 + c/x^2]\*Log[c - x^2])/8 + (b^2\*c^2\*Log[x^2/c]\*Log[c - x^2])/8 - (a\*b\*c^2\*Log[c + x^2])/4 + (b^2\*c^2\*Log[c + x^2])/8 + (b^2\*c^2\*Log[1 - c/x^2]\*Log[c + x^2])/8 + (b^2\*c^2\*Log[-(x^2/c)]\*Log[c + x^2])/8 - (b^2\*c^2\*Log[(c - x^2)/(2\*c)]\*Log[c + x^2])/8 - (b^2\*c^2\*Log[c - x^2]\*Log[(c + x^2)/(2\*c)])/8 - (b^2\*c^2\*PolyLog[2, -(c/x^2)])/8 - (b^2\*c^2\*PolyLog[2, c/x^2])/8 - (b^2\*c^2\*PolyLog[2, (c - x^2)/(2\*c)])/8 - (b^2\*c^2\*PolyLog[2, (c + x^2)/(2\*c)])/8 + (b^2\*c^2\*PolyLog[2, (c + x^2)/c])/8 + (b^2\*c^2\*PolyLog[2, 1 - x^2/c])/8

**Rule 6099**

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 -

$c*x^n)/2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

#### Rule 2454

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*(x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*\text{Log}[c*(d + e*x)^p])^q}, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{GtQ}[(m+1)/n, 0] \parallel \text{IGtQ}[q, 0]) \&\& !(\text{EqQ}[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

#### Rule 2398

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_.)^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])^p/(g*(q+1)), x] - \text{Dist}[(b*e*n*p)/(g*(q+1)), \text{Int}[(f + g*x)^{(q+1)}*(a + b*\text{Log}[c*(d + e*x)^n])^{(p-1)}]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{GtQ}[p, 0] \&\& \text{NeQ}[q, -1] \&\& \text{IntegerQ}[2*p, 2*q] \&\& (!\text{IGtQ}[q, 0] \parallel (\text{EqQ}[p, 2] \&\& \text{NeQ}[q, 1]))$

#### Rule 2411

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})^{(p_.)}]*(b_.))^{(q_.)}*((f_.) + (g_.)*(x_.)^{(q_.)}*(h_.) + (i_.)*(x_.)^{(r_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x\} \&\& \text{EqQ}[e*f - d*g, 0] \&\& (\text{IGtQ}[p, 0] \parallel \text{IGtQ}[r, 0]) \&\& \text{IntegerQ}[2*r]$

#### Rule 2347

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}*((d_.) + (e_.)*(x_.)^{(q_.)})/(x_.), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(d + e*x)^{(q+1)}*(a + b*\text{Log}[c*x^n])^p]/x, x], x] - \text{Dist}[e/d, \text{Int}[(d + e*x)^q*(a + b*\text{Log}[c*x^n])^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1] \&\& \text{IntegerQ}[2*q]$

#### Rule 2344

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.))^{(p_.)}/((x_.)*((d_.) + (e_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[1/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/x, x], x] - \text{Dist}[e/d, \text{Int}[(a + b*\text{Log}[c*x^n])^p/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{IGtQ}[p, 0]$

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2316

Int[((a\_.) + Log[(c\_.)\*(x\_)\*(b\_.)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := Simp[((a + b\*Log[-((c\*d)/e)])\*Log[d + e\*x])/e, x] + Dist[b, Int[Log[-((e\*x)/d)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c\*d)/e), 0]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rule 2314

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))\*((d\_) + (e\_.)\*(x\_)^(r\_.))^(q\_), x\_Symbol] := Simp[(x\*(d + e\*x^r)^(q + 1)\*(a + b\*Log[c\*x^n]))/d, x] - Dist[(b\*n)/d, Int[(d + e\*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x] && EqQ[r\*(q + 1) + 1, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 193

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 30

Int[(x\_)^(m\_), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2557

Int[Log[v\_]\*Log[w\_]\*(u\_), x\_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v]\*Log[w], z, x] + (-Int[SimplifyIntegrand[(z\*Log[w]\*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z\*Log[v]\*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 2466

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_.) + (g\_.)\*(x\_))^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p])^q, x^m\*(f + g\*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]

Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d,

e, n, p}, x]

### Rule 2462

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[f + g\*x]\*(a + b\*Log[c\*(d + e\*x^n)^p])/g, x] - Dist[(b\*e\*n\*p)/g, Int[(x^(n - 1)\*Log[f + g\*x])/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 2416

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))\*((h\_.)\*(x\_)^(m\_.))\*((f\_) + (g\_.)\*(x\_)^(r\_.))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, (h\*x)^m\*(f + g\*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]^(p\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2410

Int[(Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]^(p\_.))\*(x\_)^(m\_)/((f\_) + (g\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[Log[c\*(d + e\*x)], x^m/(f + g\*x), x], x] /; FreeQ

$\{c, d, e, f, g\}, x \} \&\& \text{EqQ}[e*f - d*g, 0] \&\& \text{EqQ}[c*d, 1] \&\& \text{IntegerQ}[m]$

### Rule 2395

$\text{Int}[\{(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_))^{(n_.)}]*(b_.))*((f_.) + (g_.)*(x_))^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[\{(f + g*x)^{(q + 1)}*(a + b*\text{Log}[c*(d + e*x)^n]) / (g*(q + 1)), x] - \text{Dist}[(b*e*n)/(g*(q + 1)), \text{Int}[(f + g*x)^{(q + 1)} / (d + e*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, q\}, x \} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[q, -1]$

### Rule 36

$\text{Int}[1/\{(a_.) + (b_.)*(x_)*((c_.) + (d_.)*(x_))\}, x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   $\text{FreeQ}\{a, b, c, d\}, x \} \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 29

$\text{Int}[(x_)^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[x], x]$

### Rule 2390

$\text{Int}[\{(a_.) + \text{Log}[(c_.)*((d_) + (e_.)*(x_))^{(n_.)}]*(b_.))^{(p_.)}*((f_.) + (g_.)*(x_))^{(q_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[\{(f*x)/d\}^q*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x \} \&\& \text{EqQ}[e*f - d*g, 0]$

### Rubi steps





**Mathematica [A]** time = 0.0604428, size = 104, normalized size = 1.11

$$\frac{1}{4} \left( a^2 x^4 + bc^2(a+b) \log(x^2 - c) - abc^2 \log(c + x^2) + 2abcx^2 + 2bx^2 \tanh^{-1}\left(\frac{c}{x^2}\right) (ax^2 + bc) + b^2 c^2 \log(c + x^2) + b^2 (x^2 - c) \log(x^2 - c) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] (2\*a\*b\*c\*x^2 + a^2\*x^4 + 2\*b\*x^2\*(b\*c + a\*x^2)\*ArcTanh[c/x^2] + b^2\*(-c^2 + x^4)\*ArcTanh[c/x^2]^2 + b\*(a + b)\*c^2\*Log[-c + x^2] - a\*b\*c^2\*Log[c + x^2] + b^2\*c^2\*Log[c + x^2])/4

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x^3 \left( a + b \operatorname{Arctanh}\left(\frac{c}{x^2}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c/x^2))^2,x)

[Out] int(x^3\*(a+b\*arctanh(c/x^2))^2,x)

**Maxima [A]** time = 0.978379, size = 212, normalized size = 2.26

$$\frac{1}{4} b^2 x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{4} \left( 2x^4 \operatorname{artanh}\left(\frac{c}{x^2}\right) + (2x^2 - c \log(x^2 + c) + c \log(x^2 - c))c \right) ab + \frac{1}{16} \left( (\log(x^2 + c))^2 - 2(\log(x^2 + c) - 2) \log(x^2 - c) + \log(x^2 - c)^2 + 4 \log(x^2 + c) \right) c^2 + 4(2x^2 - c \log(x^2 + c) + c \log(x^2 - c)) c \operatorname{arctanh}\left(\frac{c}{x^2}\right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] 1/4\*b^2\*x^4\*arctanh(c/x^2)^2 + 1/4\*a^2\*x^4 + 1/4\*(2\*x^4\*arctanh(c/x^2) + (2\*x^2 - c\*log(x^2 + c) + c\*log(x^2 - c))\*c)\*a\*b + 1/16\*((log(x^2 + c))^2 - 2\*(log(x^2 + c) - 2)\*log(x^2 - c) + log(x^2 - c)^2 + 4\*log(x^2 + c))\*c^2 + 4\*(2\*x^2 - c\*log(x^2 + c) + c\*log(x^2 - c))\*c\*arctanh(c/x^2)\*b^2

**Fricas [A]** time = 1.81725, size = 278, normalized size = 2.96

$$\frac{1}{4}a^2x^4 + \frac{1}{2}abcx^2 - \frac{1}{4}(ab - b^2)c^2 \log(x^2 + c) + \frac{1}{4}(ab + b^2)c^2 \log(x^2 - c) + \frac{1}{16}(b^2x^4 - b^2c^2) \log\left(\frac{x^2 + c}{x^2 - c}\right)^2 + \frac{1}{4}(abx^4 + b^2cx^2) \log\left(\frac{x^2 + c}{x^2 - c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] 1/4\*a^2\*x^4 + 1/2\*a\*b\*c\*x^2 - 1/4\*(a\*b - b^2)\*c^2\*log(x^2 + c) + 1/4\*(a\*b + b^2)\*c^2\*log(x^2 - c) + 1/16\*(b^2\*x^4 - b^2\*c^2)\*log((x^2 + c)/(x^2 - c))^2 + 1/4\*(a\*b\*x^4 + b^2\*c\*x^2)\*log((x^2 + c)/(x^2 - c))

**Sympy [C]** time = 14.7646, size = 151, normalized size = 1.61

$$\frac{a^2x^4}{4} - \frac{abc^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{abcx^2}{2} + \frac{abx^4 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{b^2c^2 \log(-i\sqrt{c} + x)}{2} + \frac{b^2c^2 \log(i\sqrt{c} + x)}{2} - \frac{b^2c^2 \operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4} - \frac{b^2cx^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2} + \frac{b^2cx^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c/x\*\*2))\*\*2,x)

[Out] a\*\*2\*x\*\*4/4 - a\*b\*c\*\*2\*atanh(c/x\*\*2)/2 + a\*b\*c\*x\*\*2/2 + a\*b\*x\*\*4\*atanh(c/x\*\*2)/2 + b\*\*2\*c\*\*2\*log(-I\*sqrt(c) + x)/2 + b\*\*2\*c\*\*2\*log(I\*sqrt(c) + x)/2 - b\*\*2\*c\*\*2\*atanh(c/x\*\*2)\*\*2/4 - b\*\*2\*c\*\*2\*atanh(c/x\*\*2)/2 + b\*\*2\*c\*x\*\*2\*atanh(c/x\*\*2)/2 + b\*\*2\*x\*\*4\*atanh(c/x\*\*2)\*\*2/4

**Giac [A]** time = 1.2751, size = 180, normalized size = 1.91

$$\frac{1}{4}a^2x^4 + \frac{1}{2}abcx^2 + \frac{1}{16}(b^2x^4 - b^2c^2) \log\left(\frac{x^2 + c}{x^2 - c}\right)^2 - \frac{1}{4}(abc^2 - b^2c^2) \log(x^2 + c) + \frac{1}{4}(abc^2 + b^2c^2) \log(-x^2 + c) + \frac{1}{4}(abx^4 + b^2cx^2) \log\left(\frac{x^2 + c}{x^2 - c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c/x^2))^2,x, algorithm="giac")

[Out] 1/4\*a^2\*x^4 + 1/2\*a\*b\*c\*x^2 + 1/16\*(b^2\*x^4 - b^2\*c^2)\*log((x^2 + c)/(x^2 - c))^2 - 1/4\*(a\*b\*c^2 - b^2\*c^2)\*log(x^2 + c) + 1/4\*(a\*b\*c^2 + b^2\*c^2)\*log(-x^2 + c) + 1/4\*(a\*b\*x^4 + b^2\*c\*x^2)\*log((x^2 + c)/(x^2 - c))

### 3.172 $\int x \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$

**Optimal.** Leaf size=94

$$\frac{1}{2} b^2 c \operatorname{PolyLog} \left( 2, \frac{2}{\frac{c}{x^2} + 1} - 1 \right) + \frac{1}{2} x^2 \left( a + b \operatorname{coth}^{-1} \left( \frac{x^2}{c} \right) \right)^2 - \frac{1}{2} c \left( a + b \operatorname{coth}^{-1} \left( \frac{x^2}{c} \right) \right)^2 - bc \log \left( 2 - \frac{2}{\frac{c}{x^2} + 1} \right) \left( a + b \operatorname{coth}^{-1} \left( \frac{x^2}{c} \right) \right)$$

[Out]  $-(c*(a + b*\operatorname{ArcCoth}[x^2/c])^2)/2 + (x^2*(a + b*\operatorname{ArcCoth}[x^2/c])^2)/2 - b*c*(a + b*\operatorname{ArcCoth}[x^2/c])* \operatorname{Log}[2 - 2/(1 + c/x^2)] + (b^2*c*\operatorname{PolyLog}[2, -1 + 2/(1 + c/x^2)])/2$

**Rubi [B]** time = 0.696246, antiderivative size = 404, normalized size of antiderivative = 4.3, number of steps used = 34, number of rules used = 19, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.357$ , Rules used = {6099, 2454, 2397, 2392, 2391, 2455, 263, 260, 6715, 2448, 31, 6742, 2556, 12, 2462, 2416, 2394, 2315, 2393}

$$\frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, -\frac{c}{x^2} \right) - \frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, \frac{c}{x^2} \right) - \frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, \frac{c - x^2}{2c} \right) + \frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, \frac{c + x^2}{2c} \right) - \frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, \frac{c + x^2}{2c} \right) - \frac{1}{4} b^2 c \operatorname{PolyLog} \left( 2, \frac{c - x^2}{2c} \right)$$

Warning: Unable to verify antiderivative.

[In]  $\operatorname{Int}[x*(a + b*\operatorname{ArcTanh}[c/x^2])^2, x]$

[Out]  $((1 - c/x^2)*x^2*(2*a - b*\operatorname{Log}[1 - c/x^2])^2)/8 + (a*b*x^2*\operatorname{Log}[1 + c/x^2])/2 - (b^2*x^2*\operatorname{Log}[1 - c/x^2]*\operatorname{Log}[1 + c/x^2])/4 + (b^2*(1 + c/x^2)*x^2*\operatorname{Log}[1 + c/x^2]^2)/8 + a*b*c*\operatorname{Log}[x] - (b^2*c*\operatorname{Log}[1 - c/x^2]*\operatorname{Log}[-c - x^2])/4 - (b^2*c*\operatorname{Log}[-(x^2/c)]*\operatorname{Log}[-c - x^2])/4 + (b^2*c*\operatorname{Log}[-c - x^2]*\operatorname{Log}[(c - x^2)/(2*c)])/4 + (b^2*c*\operatorname{Log}[1 + c/x^2]*\operatorname{Log}[-c + x^2])/4 + (b^2*c*\operatorname{Log}[x^2/c]*\operatorname{Log}[-c + x^2])/4 + (a*b*c*\operatorname{Log}[c + x^2])/2 - (b^2*c*\operatorname{Log}[-c + x^2]*\operatorname{Log}[(c + x^2)/(2*c)])/4 + (b^2*c*\operatorname{PolyLog}[2, -(c/x^2)])/4 - (b^2*c*\operatorname{PolyLog}[2, c/x^2])/4 - (b^2*c*\operatorname{PolyLog}[2, (c - x^2)/(2*c)])/4 + (b^2*c*\operatorname{PolyLog}[2, (c + x^2)/(2*c)])/4 - (b^2*c*\operatorname{PolyLog}[2, (c + x^2)/c])/4 + (b^2*c*\operatorname{PolyLog}[2, 1 - x^2/c])/4$

#### Rule 6099

$\operatorname{Int}[(a + \operatorname{ArcTanh}[c*x^n])*(b*x^m)^p*(d*x^n)^m, x]$   
 Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.)^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Log[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2397

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))/(f_.) + (g_.)*(x_)^2, x_Symbol] := Simp[((d + e*x)*(a + b*Log[c*(d + e*x)^n])^p)/((e*f - d*g)*(f + g*x)), x] - Dist[(b*e*n*p)/(e*f - d*g), Int[(a + b*Log[c*(d + e*x)^n])^(p - 1)/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0] && GtQ[p, 0]
```

Rule 2392

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*d])*Log[x], x] + Dist[b, Int[Log[1 + (e*x)/d]/x, x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[c*d, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.)*((f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6715

```
Int[(u_)*(x_)^(m_), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]
```

### Rule 2448

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] := Simp[x*Log[v]*Log[w], x] + (-Int[SimplifyIntegrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2462

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_)^(n_))^(p_)])*(b_))/((f_) + (g_)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

### Rule 2416

```
Int[((a_) + Log[(c_)*((d_) + (e_)*(x_))^(n_)]*(b_))^(p_)*((h_)*(x_))^(m_)*((f_) + (g_)*(x_))^(r_))^(q_), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
```

, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rubi steps

$$\begin{aligned}
\int x \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx &= \int \left( \frac{1}{4} x \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 - \frac{1}{2} b x \left( -2a + b \log \left( 1 - \frac{c}{x^2} \right) \right) \log \left( 1 + \frac{c}{x^2} \right) + \frac{1}{4} b^2 x \log^2 \left( 1 + \frac{c}{x^2} \right) \right) dx \\
&= \frac{1}{4} \int x \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 dx - \frac{1}{2} b \int x \left( -2a + b \log \left( 1 - \frac{c}{x^2} \right) \right) \log \left( 1 + \frac{c}{x^2} \right) dx + \frac{1}{4} \int b^2 x \log^2 \left( 1 + \frac{c}{x^2} \right) dx \\
&= - \left( \frac{1}{8} \text{Subst} \left( \int \frac{(2a - b \log(1 - cx))^2}{x^2} dx, x, \frac{1}{x^2} \right) \right) - \frac{1}{4} b \text{Subst} \left( \int \left( -2a + b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) dx, x, \frac{1}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{8} b^2 \left( 1 + \frac{c}{x^2} \right) x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b \text{Subst} \left( \int \left( -2a + b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) dx, x, \frac{1}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{8} b^2 \left( 1 + \frac{c}{x^2} \right) x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) + abc \log(x) + \frac{1}{4} b \text{Subst} \left( \int \left( -2a + b \log \left( 1 - \frac{c}{x} \right) \right) \log \left( 1 + \frac{c}{x} \right) dx, x, \frac{1}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{1}{8} \left( 1 - \frac{c}{x^2} \right) x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{2} abx^2 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{4} b^2 x^2 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.13381, size = 107, normalized size = 1.14

$$\frac{1}{2} \left( b^2 c \text{PolyLog} \left( 2, e^{-2 \tanh^{-1} \left( \frac{c}{x^2} \right)} \right) + a \left( ax^2 + bc \log \left( 1 - \frac{c^2}{x^4} \right) - 2bc \log \left( \frac{c}{x^2} \right) \right) + 2b \tanh^{-1} \left( \frac{c}{x^2} \right) \left( ax^2 - bc \log \left( 1 - e^{-2 \tanh^{-1} \left( \frac{c}{x^2} \right)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcTanh[c/x^2])^2,x]

```
[Out] (b^2*(-c + x^2)*ArcTanh[c/x^2]^2 + 2*b*ArcTanh[c/x^2]*(a*x^2 - b*c*Log[1 - E^(-2*ArcTanh[c/x^2])]) + a*(a*x^2 + b*c*Log[1 - c^2/x^4] - 2*b*c*Log[c/x^2]) + b^2*c*PolyLog[2, E^(-2*ArcTanh[c/x^2])])/2
```

**Maple [F]** time = 180., size = 0, normalized size = 0.

$$\int x \left( a + b \operatorname{Arctanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c/x^2))^2,x)
```

```
[Out] int(x*(a+b*arctanh(c/x^2))^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} a^2 x^2 + \frac{1}{2} \left( 2 x^2 \operatorname{artanh} \left( \frac{c}{x^2} \right) + c \log(x^4 - c^2) \right) a b + \frac{1}{8} \left( x^2 \log(x^2 + c)^2 - 2(x^2 + c) \log(x^2 + c) \log(x^2 - c) + (x^2 - c) \log(x^2 - c)^2 \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*a*b + 1/8*(x^2*log(x^2 + c)^2 - 2*(x^2 + c)*log(x^2 + c)*log(x^2 - c) + (x^2 - c)*log(x^2 - c)^2 + 2*integrate(2*(3*c*x^3 + c^2*x)*log(x^2 + c)/(x^4 - c^2), x))*b^2
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( b^2 x \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 2 a b x \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^2 x, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```



[Out] `integral(b^2*x*arctanh(c/x^2)^2 + 2*a*b*x*arctanh(c/x^2) + a^2*x, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \left( a + b \operatorname{atanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c/x**2))**2,x)`

[Out] `Integral(x*(a + b*atanh(c/x**2))**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a \right)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c/x^2) + a)^2*x, x)`

$$3.173 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x} dx$$

**Optimal.** Leaf size=144

$$\frac{1}{2}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x^2}}\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x^2}{c}\right)\right) - \frac{1}{2}b \operatorname{PolyLog}\left(2, \frac{2}{1 - \frac{c}{x^2}} - 1\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x^2}{c}\right)\right) - \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x^2}}\right) + \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, \frac{2}{1 - \frac{c}{x^2}} - 1\right)$$

[Out] -((a + b\*ArcCoth[x^2/c])^2\*ArcTanh[1 - 2/(1 - c/x^2)]) + (b\*(a + b\*ArcCoth[x^2/c])\*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b\*(a + b\*ArcCoth[x^2/c])\*PolyLog[2, -1 + 2/(1 - c/x^2)])/2 - (b^2\*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2\*PolyLog[3, -1 + 2/(1 - c/x^2)])/4

**Rubi [A]** time = 0.32428, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$\frac{1}{2}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x^2}}\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x^2}{c}\right)\right) - \frac{1}{2}b \operatorname{PolyLog}\left(2, \frac{2}{1 - \frac{c}{x^2}} - 1\right) \left(a + b \operatorname{coth}^{-1}\left(\frac{x^2}{c}\right)\right) - \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - \frac{c}{x^2}}\right) + \frac{1}{4}b^2 \operatorname{PolyLog}\left(3, \frac{2}{1 - \frac{c}{x^2}} - 1\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])^2/x, x]

[Out] -((a + b\*ArcCoth[x^2/c])^2\*ArcTanh[1 - 2/(1 - c/x^2)]) + (b\*(a + b\*ArcCoth[x^2/c])\*PolyLog[2, 1 - 2/(1 - c/x^2)])/2 - (b\*(a + b\*ArcCoth[x^2/c])\*PolyLog[2, -1 + 2/(1 - c/x^2)])/2 - (b^2\*PolyLog[3, 1 - 2/(1 - c/x^2)])/4 + (b^2\*PolyLog[3, -1 + 2/(1 - c/x^2)])/4

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5914

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcTanh[c\*x])^p\*ArcTanh[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 - c\*x)])/(1 - c^2\*x^2), x], x] /; F

reeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 6052

Int[(ArcTanh[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^((p\_.))]/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[1 + u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^((p\_.))]/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6058

Int[(Log[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^((p\_.))]/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x} dx &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{\left(a + b \tanh^{-1}(cx)\right)^2}{x} dx, x, \frac{1}{x^2}\right)\right) \\
&= -\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) + (2bc) \operatorname{Subst}\left(\int \frac{\left(a + b \tanh^{-1}(cx)\right) \tanh^{-1}\left(\frac{2}{1-cx}\right)}{1 - c^2x^2} dx, x, \frac{1}{x^2}\right) \\
&= -\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) - (bc) \operatorname{Subst}\left(\int \frac{\left(a + b \tanh^{-1}(cx)\right) \log\left(\frac{2}{1-cx}\right)}{1 - c^2x^2} dx, x, \frac{1}{x^2}\right) \\
&= -\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) + \frac{1}{2}b\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right) \operatorname{Li}_2\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) - \frac{1}{2}bc \operatorname{Li}_2\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) \\
&= -\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 \tanh^{-1}\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) + \frac{1}{2}b\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right) \operatorname{Li}_2\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right) - \frac{1}{2}bc \operatorname{Li}_2\left(1 - \frac{2}{1 - \frac{c}{x^2}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0664234, size = 177, normalized size = 1.23

$$\frac{1}{2} \left( 4bc \left( \frac{1}{2} \left( \frac{b \operatorname{PolyLog}\left(3, \frac{\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right)}{4c} - \frac{\operatorname{PolyLog}\left(2, \frac{\frac{c}{x^2}-1}{\frac{c}{x^2}-1}\right) \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{2c} \right) \right) + \frac{1}{2} \left( \frac{\operatorname{PolyLog}\left(2, \frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right) \left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)}{2c} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x,x]

[Out] (-2\*ArcTanh[1 - 2/(1 - c/x^2)]\*(a + b\*ArcTanh[c/x^2])^2 + 4\*b\*c\*((-(a + b\*ArcTanh[c/x^2])\*PolyLog[2, (-1 - c/x^2)/(-1 + c/x^2)])/(2\*c) + (b\*PolyLog[3, (-1 - c/x^2)/(-1 + c/x^2)])/(4\*c))/2 + (((a + b\*ArcTanh[c/x^2])\*PolyLog[2, (1 + c/x^2)/(-1 + c/x^2)])/(2\*c) - (b\*PolyLog[3, (1 + c/x^2)/(-1 + c/x^2)])/(4\*c))/2)/2

**Maple [F]** time = 0.483, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(a + b \operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x^2))^2/x,x)`

[Out] `int((a+b*arctanh(c/x^2))^2/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2 \left( \log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right) \right)^2}{4x} + \frac{ab \left( \log\left(\frac{c}{x^2} + 1\right) - \log\left(-\frac{c}{x^2} + 1\right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="maxima")`

[Out] `a^2*log(x) + integrate(1/4*b^2*(log(c/x^2 + 1) - log(-c/x^2 + 1))^2/x + a*b*(log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c/x^2))^2 + 2*a*b*arctanh(c/x^2) + a^2)/x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c/x**2))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c/x**2))**2/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x, x)
```

$$3.174 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^3} dx$$

**Optimal.** Leaf size=99

$$\frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1 - \frac{c}{x^2}}\right)}{2c} - \frac{\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c} - \frac{\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2} + \frac{b \log\left(\frac{2}{1 - \frac{c}{x^2}}\right)\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)}{c}$$

[Out]  $-(a + b \cdot \text{ArcCoth}[x^2/c])^2/(2 \cdot c) - (a + b \cdot \text{ArcCoth}[x^2/c])^2/(2 \cdot x^2) + (b \cdot (a + b \cdot \text{ArcCoth}[x^2/c]) \cdot \text{Log}[2/(1 - c/x^2)])/c + (b^2 \cdot \text{PolyLog}[2, 1 - 2/(1 - c/x^2)])/(2 \cdot c)$

**Rubi [B]** time = 0.528957, antiderivative size = 207, normalized size of antiderivative = 2.09, number of steps used = 28, number of rules used = 12, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6099, 2454, 2389, 2296, 2295, 6715, 2430, 43, 2416, 2394, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}\left(1 - \frac{c}{x^2}\right)\right)}{4c} - \frac{b^2 \text{PolyLog}\left(2, \frac{1}{2}\left(\frac{c}{x^2} + 1\right)\right)}{4c} - \frac{b \log\left(\frac{1}{2}\left(\frac{c}{x^2} + 1\right)\right)\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{4c} - \frac{b \log\left(\frac{c}{x^2} + 1\right)\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{4x^2}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(a + b \cdot \text{ArcTanh}[c/x^2])^2/x^3, x]$

[Out]  $((1 - c/x^2) \cdot (2a - b \cdot \text{Log}[1 - c/x^2])^2)/(8 \cdot c) - (b \cdot (2a - b \cdot \text{Log}[1 - c/x^2]) \cdot \text{Log}[(1 + c/x^2)/2])/(4 \cdot c) - (b^2 \cdot \text{Log}[(1 - c/x^2)/2] \cdot \text{Log}[1 + c/x^2])/(4 \cdot c) - (b \cdot (2a - b \cdot \text{Log}[1 - c/x^2]) \cdot \text{Log}[1 + c/x^2])/(4 \cdot x^2) - (b^2 \cdot (1 + c/x^2) \cdot \text{Log}[1 + c/x^2]^2)/(8 \cdot c) + (b^2 \cdot \text{PolyLog}[2, (1 - c/x^2)/2])/(4 \cdot c) - (b^2 \cdot \text{PolyLog}[2, (1 + c/x^2)/2])/(4 \cdot c)$

### Rule 6099

$\text{Int}[(a + \text{ArcTanh}[c \cdot (x_1)^{n_1}] \cdot (b_1))^{p_1} \cdot ((d_1) \cdot (x_1))^{m_1}, x_{\text{Symbol}}] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d \cdot x)^m \cdot (a + (b \cdot \text{Log}[1 + c \cdot x^n])/2 - (b \cdot \text{Log}[1 - c \cdot x^n])/2)^p, x], x] /;$  FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

$\text{Int}[(a + \text{Log}[c \cdot ((d_1) + (e_1) \cdot (x_1)^{n_1})^{p_1}] \cdot (b_1))^{q_1} \cdot (x_1)^{m_1}, x_{\text{Symbol}}] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)} \cdot (a + b \cdot \text{Log}[c \cdot ((d_1) + (e_1) \cdot x^{n_1})^{p_1}] \cdot (b_1))^{q_1}, x], x]]$

$g[c*(d + e*x)^p]^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p, q\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]] \&\& (\text{GtQ}[(m + 1)/n, 0] \|\| \text{IGtQ}[q, 0]) \&\& !(\text{EqQ}[q, 1] \&\& \text{ILtQ}[n, 0] \&\& \text{IGtQ}[m, 0])$

### Rule 2389

$\text{Int}[(a + \text{Log}[c*(d + e*x)^n])^p, x\_Symbol] :> \text{Dist}[1/e, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x]$

### Rule 2296

$\text{Int}[(a + \text{Log}[c*x^n])^p, x\_Symbol] :> \text{Simp}[x*(a + b*\text{Log}[c*x^n])^p, x] - \text{Dist}[b*n*p, \text{Int}[(a + b*\text{Log}[c*x^n])^{p-1}, x], x] /; \text{FreeQ}\{a, b, c, n\}, x\} \&\& \text{GtQ}[p, 0] \&\& \text{IntegerQ}[2*p]$

### Rule 2295

$\text{Int}[\text{Log}[c*x^n], x\_Symbol] :> \text{Simp}[x*\text{Log}[c*x^n], x] - \text{Simp}[n*x, x] /; \text{FreeQ}\{c, n\}, x]$

### Rule 6715

$\text{Int}[u*(x^m), x\_Symbol] :> \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{m+1}, u, x], x, x^{m+1}], x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1] \&\& \text{Function0}[fQ[x^{m+1}, u, x]$

### Rule 2430

$\text{Int}[(a + \text{Log}[c*(d + e*x)^n])^p*(f + \text{Log}[h*(i + j*x)^m]), x\_Symbol] :> \text{Simp}[x*(a + b*\text{Log}[c*(d + e*x)^n])^p*(f + g*\text{Log}[h*(i + j*x)^m]), x] + (-\text{Dist}[g*j*m, \text{Int}[(x*(a + b*\text{Log}[c*(d + e*x)^n])^p]/(i + j*x), x], x] - \text{Dist}[b*e*n*p, \text{Int}[(x*(a + b*\text{Log}[c*(d + e*x)^n])^{p-1}*(f + g*\text{Log}[h*(i + j*x)^m])]/(d + e*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, m, n\}, x\} \&\& \text{IGtQ}[p, 0]$

### Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] \|\| (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \|\| \text{LtQ}[9*m + 5*(n + 1), 0] \|\| \text{GtQ}[m + n + 2, 0])$

### Rule 2416



```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n]]^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x^2}))^2}{x^3} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{4x^3} - \frac{b(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{2x^3} + \frac{b^2 \log^2(1 + \frac{c}{x^2})}{4x^3} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{x^3} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{x^3} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + \frac{c}{x^2})}{x^3} dx \\
&= -\left( \frac{1}{8} \text{Subst} \left( \int (2a - b \log(1 - cx))^2 dx, x, \frac{1}{x^2} \right) \right) + \frac{1}{4} b \text{Subst} \left( \int (-2a + b \log(1 - cx)) \log(1 + \frac{c}{x^2}) dx, x, \frac{1}{x^2} \right) \\
&= -\frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{4x^2} + \frac{\text{Subst} \left( \int (2a - b \log(x))^2 dx, x, 1 - \frac{c}{x^2} \right)}{8c} - \frac{b^2 \text{Subst} \left( \int \log^2(x) dx, x, 1 + \frac{c}{x^2} \right)}{8c} \\
&= \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c} - \frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{4x^2} - \frac{b^2(1 + \frac{c}{x^2}) \log^2(1 + \frac{c}{x^2})}{8c} \\
&= -\frac{ab}{2x^2} - \frac{b^2}{4x^2} + \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c} + \frac{b^2(1 + \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{4c} - \frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{4x^2} \\
&= -\frac{b^2}{2x^2} - \frac{b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{4c} + \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c} - \frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{4x^2} \\
&= -\frac{b^2}{4x^2} - \frac{b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{4c} + \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c} - \frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{4x^2} \\
&= \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c} - \frac{b(2a - b \log(1 - \frac{c}{x^2})) \log(\frac{1}{2}(1 + \frac{c}{x^2}))}{4c} - \frac{b^2 \log(\frac{1}{2}(1 - \frac{c}{x^2}))}{4c}
\end{aligned}$$

**Mathematica [A]** time = 0.0899924, size = 114, normalized size = 1.15

$$\frac{b^2 \left( \text{PolyLog} \left( 2, -e^{-2 \tanh^{-1}(\frac{c}{x^2})} \right) + \tanh^{-1} \left( \frac{c}{x^2} \right) \left( \frac{c \tanh^{-1}(\frac{c}{x^2})}{x^2} - \tanh^{-1} \left( \frac{c}{x^2} \right) - 2 \log \left( e^{-2 \tanh^{-1}(\frac{c}{x^2})} + 1 \right) \right) \right)}{2c} - \frac{a^2}{2x^2} - \frac{ab \left( \frac{c \tanh^{-1}(\frac{c}{x^2})}{x^2} - \tanh^{-1} \left( \frac{c}{x^2} \right) - 2 \log \left( e^{-2 \tanh^{-1}(\frac{c}{x^2})} + 1 \right) \right)}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^3,x]

[Out] -a^2/(2\*x^2) - (a\*b\*((c\*ArcTanh[c/x^2])/x^2 - Log[1/Sqrt[1 - c^2/x^4]]))/c - (b^2\*(ArcTanh[c/x^2]\*(-ArcTanh[c/x^2] + (c\*ArcTanh[c/x^2])/x^2 - 2\*Log[1 + E^(-2\*ArcTanh[c/x^2])]) + PolyLog[2, -E^(-2\*ArcTanh[c/x^2])]))/(2\*c)

---

**Maple [A]** time = 0.004, size = 144, normalized size = 1.5

$$-\frac{a^2}{2x^2} - \frac{b^2}{2x^2} \left( \operatorname{Arctanh}\left(\frac{c}{x^2}\right) \right)^2 - \frac{b^2}{2c} \left( \operatorname{Arctanh}\left(\frac{c}{x^2}\right) \right)^2 + \frac{b^2}{c} \operatorname{Arctanh}\left(\frac{c}{x^2}\right) \ln\left(\left(1 + \frac{c}{x^2}\right)^2 \left(1 - \frac{c^2}{x^4}\right)^{-1} + 1\right) + \frac{b^2}{2c} \operatorname{polylog}\left(2, \left(1 + \frac{c}{x^2}\right)^2 \left(1 - \frac{c^2}{x^4}\right)^{-1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c/x^2))^2/x^3,x)`

[Out] `-1/2*a^2/x^2-1/2*arctanh(c/x^2)^2/x^2*b^2-1/2/c*b^2*arctanh(c/x^2)^2+1/c*arctanh(c/x^2)*ln((1+c/x^2)^2/(1-c^2/x^4)+1)*b^2+1/2/c*polylog(2,-(1+c/x^2)^2/(1-c^2/x^4))*b^2-a*b/x^2*arctanh(c/x^2)-1/2/c*a*b*ln(1-c^2/x^4)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} \left( 8c^3 \int \frac{\log(x)^2}{cx^7 - c^3x^3} dx + c^2 \left( \frac{\log(x^2 + c)}{c^3} + \frac{\log(x^2 - c)}{c^3} - \frac{4 \log(x)}{c^3} \right) - 8c^2 \int \frac{x^2 \log(x^2 + c)}{cx^7 - c^3x^3} dx + 8c^2 \int \frac{x^2 \log(x)}{cx^7 - c^3x^3} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="maxima")`

[Out] `1/8*(8*c^3*integrate(log(x)^2/(c*x^7 - c^3*x^3), x) + c^2*(log(x^2 + c)/c^3 + log(x^2 - c)/c^3 - 4*log(x)/c^3) - 8*c^2*integrate(x^2*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 8*c^2*integrate(x^2*log(x)/(c*x^7 - c^3*x^3), x) + 2*c*(log(x^2 - c)/c^2 - log(x^2)/c^2 + 1/(c*x^2))*log(-c/x^2 + 1) - c*(log(x^2 + c)/c^2 - log(x^2 - c)/c^2) - 8*c*integrate(x^4*log(x)^2/(c*x^7 - c^3*x^3), x) - 4*c*integrate(x^4*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 16*c*integrate(x^4*log(x)/(c*x^7 - c^3*x^3), x) - log(-c/x^2 + 1)^2/x^2 - (x^2*log(x^2 - c)^2 + 4*x^2*log(x)^2 - 4*x^2*log(x) - 2*(2*x^2*log(x) - x^2)*log(x^2 - c) + 2*c)/(c*x^2) - (c*log(x^2 + c)^2 - 2*((x^2 + c)*log(x^2 + c) - 2*(x^2 + c)*log(x) - c)*log(x^2 - c))/(c*x^2) - 4*integrate(x^6*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 8*integrate(x^6*log(x)/(c*x^7 - c^3*x^3), x)*b^2 - 1/2*a*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a^2/x^2`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^3,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c/x^2)^2 + 2\*a\*b\*arctanh(c/x^2) + a^2)/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))\*\*2/x\*\*3,x)

[Out] Integral((a + b\*atanh(c/x\*\*2))\*\*2/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)^2/x^3, x)

$$3.175 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^5} dx$$

**Optimal.** Leaf size=97

$$\frac{\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2} - \frac{ab}{2cx^2} - \frac{\left(a + b \coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4} - \frac{b^2 \log\left(1 - \frac{c^2}{x^4}\right)}{4c^2} - \frac{b^2 \coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}$$

[Out]  $-(a*b)/(2*c*x^2) - (b^2*ArcCoth[x^2/c])/(2*c*x^2) + (a + b*ArcCoth[x^2/c])^2/(4*c^2) - (a + b*ArcCoth[x^2/c])^2/(4*x^4) - (b^2*Log[1 - c^2/x^4])/(4*c^2)$

**Rubi [C]** time = 1.5301, antiderivative size = 770, normalized size of antiderivative = 7.94, number of steps used = 67, number of rules used = 23, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$ , Rules used = {6099, 2454, 2401, 2389, 2296, 2295, 2390, 2305, 2304, 2395, 43, 6742, 30, 2557, 12, 2466, 2462, 260, 2416, 2394, 2315, 2393, 2391}

$$\frac{b^2 \text{PolyLog}\left(2, -\frac{c}{x^2}\right)}{8c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{c}{x^2}\right)}{8c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{c-x^2}{2c}\right)}{8c^2} + \frac{b^2 \text{PolyLog}\left(2, \frac{c+x^2}{2c}\right)}{8c^2} - \frac{b^2 \text{PolyLog}\left(2, \frac{c+x^2}{c}\right)}{8c^2} - b^2$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*ArcTanh[c/x^2])^2/x^5, x]

[Out]  $-(b^2*(1 - c/x^2)^2)/(32*c^2) - (b^2*(1 + c/x^2)^2)/(32*c^2) + (a*b)/(8*x^4) + b^2/(16*x^4) - (3*a*b)/(4*c*x^2) + (b^2*Log[1 - c/x^2])/(16*c^2) - (3*b^2*(1 - c/x^2)*Log[1 - c/x^2])/(8*c^2) - (b^2*Log[1 - c/x^2])/(16*x^4) - (b*(1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2]))/(16*c^2) + ((1 - c/x^2)*(2*a - b*Log[1 - c/x^2])^2)/(8*c^2) - ((1 - c/x^2)^2*(2*a - b*Log[1 - c/x^2])^2)/(16*c^2) - (b^2*(1 + c/x^2)*Log[1 + c/x^2])/(4*c^2) + (b^2*(1 + c/x^2)^2*Log[1 + c/x^2])/(16*c^2) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(8*x^4) + (b^2*(1 + c/x^2)*Log[1 + c/x^2]^2)/(8*c^2) - (b^2*(1 + c/x^2)^2*Log[1 + c/x^2]^2)/(16*c^2) - (b^2*Log[1 + c/x^2]*Log[c - x^2])/(8*c^2) - (b^2*Log[x^2/c]*Log[c - x^2])/(8*c^2) - (b^2*Log[1 - c/x^2]*Log[c + x^2])/(8*c^2) - (b^2*Log[-(x^2/c)]*Log[c + x^2])/(8*c^2) + (b^2*Log[(c - x^2)/(2*c)]*Log[c + x^2])/(8*c^2) + (b^2*Log[c - x^2]*Log[(c + x^2)/(2*c)])/(8*c^2) + (a*b*Log[(c + x^2)/x^2])/(4*c^2) + (b^2*Log[(c + x^2)/x^2])/(16*c^2) - (b^2*(1 + c/x^2)*Log[(c + x^2)/x^2])/(8*c^2) - (a*b*Log[(c + x^2)/x^2])/(4*x^4) - (b^2*Log[(c + x^2)/x^2])/(16*x^4) + (b^2*PolyLog[2, -(c/x^2)])/(8*c^2) + (b^2*PolyLog[2, c/x^2])/(8*c^2) + (b^2*PolyLog[2, (c - x^2)/(2*c)])/(8*c^2) + (b^2*PolyLog[2,$

$(c + x^2)/(2*c)]/(8*c^2) - (b^2*PolyLog[2, (c + x^2)/c])/(8*c^2) - (b^2*PolyLog[2, 1 - x^2/c])/(8*c^2)$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2454

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*Log[c\*(d + e\*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) && !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])

### Rule 2401

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.)]\*(b\_.))^(q\_.)\*(f\_.) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(f + g\*x)^q\*(a + b\*Log[c\*(d + e\*x)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[q, 0]

### Rule 2389

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]^(p\_.), x\_Symbol] := Dist[1/e, Subst[Int[(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, n, p}, x]

### Rule 2296

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*x^n])^p, x] - Dist[b\*n\*p, Int[(a + b\*Log[c\*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2\*p]

### Rule 2295

Int[Log[(c\_.)\*(x\_)^(n\_.)], x\_Symbol] := Simp[x\*Log[c\*x^n], x] - Simp[n\*x, x] /; FreeQ[{c, n}, x]

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

### Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :=
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^ (q_.), x_Symbol] := Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

### Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v_]*Log[w_], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2466

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && IntegerQ[m] && IntegerQ[r]
```

### Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(p_.)*((h_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2315



```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x^2}))^2}{x^5} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{4x^5} - \frac{b(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{2x^5} + \frac{b^2 \log^2(1 + \frac{c}{x^2})}{4x^5} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{x^5} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{x^5} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + \frac{c}{x^2})}{x^5} dx \\
&= -\left(\frac{1}{8} \text{Subst} \left( \int x(2a - b \log(1 - cx))^2 dx, x, \frac{1}{x^2} \right)\right) - \frac{1}{4} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{x^3} dx, x, \frac{1}{x^2} \right) \\
&= -\left(\frac{1}{8} \text{Subst} \left( \int \left( \frac{(2a - b \log(1 - cx))^2}{c} - \frac{(1 - cx)(2a - b \log(1 - cx))^2}{c} \right) dx, x, \frac{1}{x^2} \right)\right) - \frac{1}{4} b \text{Subst} \left( \int \frac{(-2a + b \log(1 - \frac{c}{x})) \log(1 + \frac{c}{x})}{x^3} dx, x, \frac{1}{x^2} \right) \\
&= \frac{1}{2} (ab) \text{Subst} \left( \int \frac{\log(1 + \frac{c}{x})}{x^3} dx, x, x^2 \right) - \frac{1}{4} b^2 \text{Subst} \left( \int \frac{\log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{x^3} dx, x, x^2 \right) \\
&= \frac{b^2 \log(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{8x^4} - \frac{1}{2} (ab) \text{Subst} \left( \int x \log(1 + cx) dx, x, \frac{1}{x^2} \right) + \frac{1}{4} b^2 \text{Subst} \left( \int \frac{c \log(1 - \frac{c}{x}) \log(1 + \frac{c}{x})}{2x^3} dx, x, \frac{1}{x^2} \right) \\
&= \frac{(1 - \frac{c}{x^2})(2a - b \log(1 - \frac{c}{x^2}))^2}{8c^2} - \frac{(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))^2}{16c^2} - \frac{ab \log(1 + \frac{c}{x^2})}{4x^4} + \frac{b^2 \log(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{8x^4} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} - \frac{ab}{2cx^2} + \frac{b^2}{4cx^2} - \frac{b(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))}{16c^2} + \frac{(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{4x^4} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{4c^2} - \frac{b(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))}{16c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{4c^2} - \frac{b(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))}{16c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{4c^2} - \frac{b^2 \log(1 - \frac{c}{x^2})}{16x^4} - \frac{b(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))}{16c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} - \frac{3ab}{4cx^2} - \frac{3b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{8c^2} - \frac{b^2 \log(1 - \frac{c}{x^2})}{16x^4} - \frac{b(1 - \frac{c}{x^2})^2 (2a - b \log(1 - \frac{c}{x^2}))}{16c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2 \log(1 - \frac{c}{x^2})}{16c^2} - \frac{3b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{8c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2 \log(1 - \frac{c}{x^2})}{16c^2} - \frac{3b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{8c^2} \\
&= -\frac{b^2(1 - \frac{c}{x^2})^2}{32c^2} - \frac{b^2(1 + \frac{c}{x^2})^2}{32c^2} + \frac{ab}{8x^4} + \frac{b^2}{16x^4} - \frac{3ab}{4cx^2} + \frac{b^2 \log(1 - \frac{c}{x^2})}{16c^2} - \frac{3b^2(1 - \frac{c}{x^2}) \log(1 - \frac{c}{x^2})}{8c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0809365, size = 131, normalized size = 1.35

$$\frac{a^2c^2 + 2abcx^2 + abx^4 \log(x^2 - c) - abx^4 \log(c + x^2) + 2bc \tanh^{-1}\left(\frac{c}{x^2}\right)(ac + bx^2) + b^2(c^2 - x^4) \tanh^{-1}\left(\frac{c}{x^2}\right)^2 + b^2x^4}{4c^2x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^5, x]

[Out]  $-(a^2c^2 + 2a*b*c*x^2 + 2*b*c*(a*c + b*x^2)*\text{ArcTanh}[c/x^2] + b^2*(c^2 - x^4)*\text{ArcTanh}[c/x^2]^2 - 4*b^2*x^4*\text{Log}[x] + a*b*x^4*\text{Log}[-c + x^2] + b^2*x^4*\text{Log}[-c + x^2] - a*b*x^4*\text{Log}[c + x^2] + b^2*x^4*\text{Log}[c + x^2])/(4*c^2*x^4)$

**Maple [B]** time = 88.027, size = 682002, normalized size = 7031.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))^2/x^5, x)

[Out] result too large to display

**Maxima [B]** time = 1.05709, size = 247, normalized size = 2.55

$$\frac{1}{4} \left( c \left( \frac{\log(x^2 + c)}{c^3} - \frac{\log(x^2 - c)}{c^3} - \frac{2}{c^2 x^2} \right) - \frac{2 \operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^4} \right) ab - \frac{1}{16} \left( c^2 \frac{\log(x^2 + c)^2 - 2(\log(x^2 + c) - 2)\log(x^2 - c)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^5, x, algorithm="maxima")

[Out]  $1/4*(c*(\log(x^2 + c)/c^3 - \log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*\operatorname{arctanh}(c/x^2)/x^4)*a*b - 1/16*(c^2*((\log(x^2 + c))^2 - 2*(\log(x^2 + c) - 2)*\log(x^2 - c) + \log(x^2 - c)^2 + 4*\log(x^2 + c))/c^4 - 16*\log(x)/c^4) - 4*c*(\log(x^2 + c)/c^3 - \log(x^2 - c)/c^3 - 2/(c^2*x^2))*\operatorname{arctanh}(c/x^2)*b^2 - 1/4*b^2*\operatorname{arctanh}(c/x^2)^2/x^4 - 1/4*a^2/x^4$

---

**Fricas [A]** time = 1.7995, size = 308, normalized size = 3.18

$$\frac{16b^2x^4 \log(x) + 4(ab - b^2)x^4 \log(x^2 + c) - 4(ab + b^2)x^4 \log(x^2 - c) - 8abcx^2 - 4a^2c^2 + (b^2x^4 - b^2c^2) \log\left(\frac{x^2+c}{x^2-c}\right)^2 - 4}{16c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^5,x, algorithm="fricas")

[Out] 1/16\*(16\*b^2\*x^4\*log(x) + 4\*(a\*b - b^2)\*x^4\*log(x^2 + c) - 4\*(a\*b + b^2)\*x^4\*log(x^2 - c) - 8\*a\*b\*c\*x^2 - 4\*a^2\*c^2 + (b^2\*x^4 - b^2\*c^2)\*log((x^2 + c)/(x^2 - c))^2 - 4\*(b^2\*c\*x^2 + a\*b\*c^2)\*log((x^2 + c)/(x^2 - c)))/(c^2\*x^4)

---

**Sympy [A]** time = 41.7701, size = 172, normalized size = 1.77

$$\begin{cases} -\frac{a^2}{4x^4} - \frac{ab \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^4} - \frac{ab}{2cx^2} + \frac{ab \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2} - \frac{b^2 \operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4x^4} - \frac{b^2 \operatorname{atanh}\left(\frac{c}{x^2}\right)}{2cx^2} + \frac{b^2 \log(x)}{c^2} - \frac{b^2 \log(-i\sqrt{c+x})}{2c^2} - \frac{b^2 \log(i\sqrt{c+x})}{2c^2} + \frac{b^2 \operatorname{atanh}^2}{4c^2} \\ -\frac{a^2}{4x^4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))\*\*2/x\*\*5,x)

[Out] Piecewise((-a\*\*2/(4\*x\*\*4) - a\*b\*atanh(c/x\*\*2)/(2\*x\*\*4) - a\*b/(2\*c\*x\*\*2) + a\*b\*atanh(c/x\*\*2)/(2\*c\*\*2) - b\*\*2\*atanh(c/x\*\*2)\*\*2/(4\*x\*\*4) - b\*\*2\*atanh(c/x\*\*2)/(2\*c\*x\*\*2) + b\*\*2\*log(x)/c\*\*2 - b\*\*2\*log(-I\*sqrt(c) + x)/(2\*c\*\*2) - b\*\*2\*log(I\*sqrt(c) + x)/(2\*c\*\*2) + b\*\*2\*atanh(c/x\*\*2)\*\*2/(4\*c\*\*2) + b\*\*2\*atanh(c/x\*\*2)/(2\*c\*\*2), Ne(c, 0)), (-a\*\*2/(4\*x\*\*4), True))

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^5, x)
```

$$3.176 \quad \int x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=1214

result too large to display

```
[Out] (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 -
(4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]
^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/5 -
(b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log
[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*ArcTanh
[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^2])^2)/
20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^
(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*
Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqr
t[c] - I*x)))/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x)
)/(Sqrt[c] - I*x)))/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/
(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c]
- x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTanh[x/S
qrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x)
])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)))/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
+ x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] -
(I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^
2*c^(5/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^
(5/2)*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^(5/2)*PolyLog[2, ((-I)*x)/S
qrt[c]] + (I/5)*b^2*c^(5/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(5/2)*PolyLo
g[2, x/Sqrt[c]])/5 + (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)
])/5 - (b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^
(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[
c] + x))])/10 - (b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sq
rt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^(5/2)*PolyLog[2, 1 - (
1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

---

**Rubi [A]** time = 2.71234, antiderivative size = 1214, normalized size of antiderivative = 1., number of steps used = 97, number of rules used = 33, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.063$ , Rules used = {6099, 2457, 2476, 2448, 263, 207, 2455, 193, 321, 2470, 12, 260, 6688, 5988, 5932, 2447, 302, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 204, 4928, 4848,

2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

[In] Int[x^4\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] 
$$\begin{aligned} & (8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^{(5/2)}*ArcTan[x/Sqrt[c]])/5 - \\ & (4*b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]] \\ & ^2 - (4*b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]] \\ & ]^2)/5 + (2*b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/5 - \\ & (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/5 + \\ & (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + \\ & (x^5*(2*a - b*Log[1 - c/x^2])^2)/20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 + (b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - \\ & (b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 + (b^2*x^5*Log[1 + c/x^2]^2)/20 - \\ & (2*b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/5 + (b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/5 - \\ & (2*b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/5 + (b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^{(5/2)}*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/5 + (2*b^2*c^{(5/2)}*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/5 + (I/5)*b^2*c^{(5/2)}*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/5)*b^2*c^{(5/2)}*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/10)*b^2*c^{(5/2)}*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^{(5/2)}*PolyLog[2, -(x/Sqrt[c])])/5 - (I/5)*b^2*c^{(5/2)}*PolyLog[2, ((-I)*x)/Sqrt[c]] + (I/5)*b^2*c^{(5/2)}*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^{(5/2)}*PolyLog[2, x/Sqrt[c]])/5 + (b^2*c^{(5/2)}*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^{(5/2)}*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/5 - (b^2*c^{(5/2)}*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/10 - (b^2*c^{(5/2)}*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/10 - (I/10)*b^2*c^{(5/2)}*PolyLog[2, 1 - (1 - I)*(Sqrt[c] + x)/(Sqrt[c] - I*x)] \end{aligned}$$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*((f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 193

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```



Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] := With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 5988

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Dist[1/
d, Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

Rule 5932

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] := Simp[(a + b*ArcTanh[c*x])^p*Log[2 - 2/(1 + (e*x)/d)]/d, x] -
Dist[(b*c*p)/d, Int[(a + b*ArcTanh[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]
/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^
2*d^2 - e^2, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (-Simp[(b*PolyLog[2, -(c*x)])]/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

### Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/((d_.) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_.) + (e_.)*(x_))]/((f_.) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 4928

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)*(x_)^(m_.)/((d_.) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[2/(1 - I*c*x)]]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)])/e, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

#### Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)), x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

#### Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_Symbol] := Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Dist[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]]/(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

#### Rubi steps



**Mathematica [F]** time = 5.64618, size = 0, normalized size = 0.

$$\int x^4 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^4\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] Integrate[x^4\*(a + b\*ArcTanh[c/x^2])^2, x]

---

**Maple [F]** time = 0.685, size = 0, normalized size = 0.

$$\int x^4 \left( a + b \operatorname{Artanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arctanh(c/x^2))^2,x)

[Out] int(x^4\*(a+b\*arctanh(c/x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( b^2 x^4 \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 2 a b x^4 \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^2 x^4, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctanh(c/x^2)^2 + 2*a*b*x^4*arctanh(c/x^2) + a^2*x^4, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \left( a + b \operatorname{atanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atanh(c/x**2))**2,x)
```

```
[Out] Integral(x**4*(a + b*atanh(c/x**2))**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a \right)^2 x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2*x^4, x)
```

$$3.177 \quad \int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=1172

result too large to display

```
[Out] (4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log[1 - c/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + (b^2*c^(3/2)*PolyLog[2, -(x/Sqrt[c])])/3 + (I/3)*b^2*c^(3/2)*PolyLog[2, ((-I)*x)/Sqrt[c]] - (I/3)*b^2*c^(3/2)*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^(3/2)*PolyLog[2, x/Sqrt[c]])/3 + (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/6 - (b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]
```

---

**Rubi [A]** time = 2.20796, antiderivative size = 1172, normalized size of antiderivative = 1., number of steps used = 79, number of rules used = 33, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.063$ , Rules used = {6099, 2457, 2471, 2448, 263, 207, 2470, 12, 260, 6688, 5988, 5932, 2447, 2455, 193, 321, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 2476, 204, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display



Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] 
$$\begin{aligned} & (4*a*b*c*x)/3 - (2*a*b*c^{(3/2)}*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]])/3 \\ & + (1/3)*b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]])/3 + (b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]^2)/3 \\ & - (2*b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*Log[1 - c/x^2])/3 \\ & + (b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 \\ & + (x^3*(2*a - b*Log[1 - c/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2])/3 - (b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 \\ & - (b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2])/6 + (b^2*x^3*Log[1 + c/x^2]^2)/12 \\ & + (2*b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/3 \\ & - (2*b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/3 + (b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 \\ & + (b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/3 - (b^2*c^{(3/2)}*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/3 \\ & + (2*b^2*c^{(3/2)}*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (1/3)*b^2*c^{(3/2)}*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] \\ & + (1/3)*b^2*c^{(3/2)}*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] + (1/6)*b^2*c^{(3/2)}*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] \\ & + (b^2*c^{(3/2)}*PolyLog[2, -(x/Sqrt[c])])/3 + (1/3)*b^2*c^{(3/2)}*PolyLog[2, ((-I)*x)/Sqrt[c]] - (1/3)*b^2*c^{(3/2)}*PolyLog[2, (I*x)/Sqrt[c]] - (b^2*c^{(3/2)}*PolyLog[2, x/Sqrt[c]])/3 \\ & + (b^2*c^{(3/2)}*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^2*c^{(3/2)}*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (b^2*c^{(3/2)}*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/6 \\ & - (b^2*c^{(3/2)}*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/6 + (1/6)*b^2*c^{(3/2)}*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] \end{aligned}$$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a +

$b \cdot \log[c \cdot (d + e \cdot x^n)^p]^{(q-1)} / (d + e \cdot x^n), x, x] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

### Rule 2471

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := With[{t = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p]]^q, (f + g\*x^s)^r, x]}, Int[t, x] /; SumQ[t]] /; FreeQ[{a, b, c, d, e, f, g, n, p, q, r, s}, x] && IntegerQ[n] && IGtQ[q, 0] && IntegerQ[r] && IntegerQ[s] && (EqQ[q, 1] || (GtQ[r, 0] && GtQ[s, 1]) || (LtQ[s, 0] && LtQ[r, 0]))

### Rule 2448

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)], x\_Symbol] := Simp[x\*Log[c\*(d + e\*x^n)^p], x] - Dist[e\*n\*p, Int[x^n/(d + e\*x^n), x], x] /; FreeQ[{c, d, e, n, p}, x]

### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/(f\_. + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n-1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/((1 - c^2\*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^n)]^(p\_.))\*(b\_.)\*((f\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rule 193

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]

Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x],

$x] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n - 1] \ \&\& \ \text{NeQ}[m + n * p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 6742

$\text{Int}[u_, x\_Symbol] \ :> \ \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \ \text{Int}[v, x] \ /; \ \text{SumQ}[v]$   
 $]$

### Rule 203

$\text{Int}[\{(a_) + (b_)*(x_)^2\}^{-1}, x\_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

### Rule 30

$\text{Int}[(x_)^{(m_.)}, x\_Symbol] \ :> \ \text{Simp}[x^{(m + 1)}/(m + 1), x] \ /; \ \text{FreeQ}[m, x] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 2557

$\text{Int}[\text{Log}[v_] * \text{Log}[w_] * (u_), x\_Symbol] \ :> \ \text{With}[\{z = \text{IntHide}[u, x]\}, \ \text{Dist}[\text{Log}[v] * \text{Log}[w], z, x] + (-\text{Int}[\text{SimplifyIntegrand}[(z * \text{Log}[w] * D[v, x])/v, x], x] - \text{Int}[\text{SimplifyIntegrand}[(z * \text{Log}[v] * D[w, x])/w, x], x]) \ /; \ \text{InverseFunctionFreeQ}[z, x] \ /; \ \text{InverseFunctionFreeQ}[v, x] \ \&\& \ \text{InverseFunctionFreeQ}[w, x]$

### Rule 5992

$\text{Int}[\{(a_) + \text{ArcTanh}[(c_)*(x_)] * (b_)\} * (x_)^{(m_.)} / \{(d_) + (e_)*(x_)^2\}, x\_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[a + b * \text{ArcTanh}[c * x], x^m / (d + e * x^2), x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{EqQ}[m, 1] \ \&\& \ \text{NeQ}[a, 0])$   
 $)$

### Rule 5912

$\text{Int}[\{(a_) + \text{ArcTanh}[(c_)*(x_)] * (b_)\} / (x_), x\_Symbol] \ :> \ \text{Simp}[a * \text{Log}[x], x] + (-\text{Simp}[(b * \text{PolyLog}[2, -(c * x)]) / 2, x] + \text{Simp}[(b * \text{PolyLog}[2, c * x]) / 2, x]) \ /; \ \text{FreeQ}[\{a, b, c\}, x]$

### Rule 5920

$\text{Int}[\{(a_) + \text{ArcTanh}[(c_)*(x_)] * (b_)\} / \{(d_) + (e_)*(x_)\}, x\_Symbol] \ :> \ -\text{Simp}[\{(a + b * \text{ArcTanh}[c * x]) * \text{Log}[2 / (1 + c * x)]\} / e, x] + (\text{Dist}[(b * c) / e, \text{Int}[\text{Log}[2 / (1 + c * x)] / (1 - c^2 * x^2), x], x] - \text{Dist}[(b * c) / e, \text{Int}[\text{Log}[(2 * c * (d + e * x)) / ((c * d + e) * (1 + c * x))] / (1 - c^2 * x^2), x], x] + \text{Simp}[\{(a + b * \text{ArcTanh}[c * x]) *$

$\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rule 2402

$\text{Int}[\text{Log}[(c\_)/((d\_ + (e\_)*(x\_)))/((f\_ + (g\_)*(x\_)^2)], x\_Symbol] \text{:>} -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c\_)*(x\_)]/((d\_ + (e\_)*(x\_))), x\_Symbol] \text{:>} -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

### Rule 2476

$\text{Int}[(a\_ + \text{Log}[(c\_)*((d\_ + (e\_)*(x\_)^{n\_})^{p\_})*(b\_)]^{q\_}*(x\_)^{m\_})*((f\_ + (g\_)*(x\_)^{s\_})^{r\_}), x\_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(a + b * \text{Log}[c*(d + e*x^n)^p]]^q, x^m*(f + g*x^s)^r, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x] \ \&\& \ \text{IGtQ}[q, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[r] \ \&\& \ \text{IntegerQ}[s]$

### Rule 204

$\text{Int}[(a\_ + (b\_)*(x_)^2)^{-1}, x\_Symbol] \text{:>} -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 4928

$\text{Int}[(a\_ + \text{ArcTan}[(c\_)*(x_)]*(b\_))*(x_)^{m\_}/((d\_ + (e\_)*(x_)^2), x\_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[a + b*\text{ArcTan}[c*x], x^m/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{EqQ}[m, 1] \ \&\& \ \text{NeQ}[a, 0])$

### Rule 4848

$\text{Int}[(a\_ + \text{ArcTan}[(c\_)*(x_)]*(b\_))/(x_), x\_Symbol] \text{:>} \text{Simp}[a*\text{Log}[x], x] + (\text{Dist}[(I*b)/2, \text{Int}[\text{Log}[1 - I*c*x]/x, x], x] - \text{Dist}[(I*b)/2, \text{Int}[\text{Log}[1 + I*c*x]/x, x], x]) /; \text{FreeQ}\{a, b, c\}, x]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_)*((d\_ + (e\_)*(x_)^{n\_}))]/(x_), x\_Symbol] \text{:>} -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[
((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log[
2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x)
)/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] := -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] := Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx &= \int \left( \frac{1}{4} x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 - \frac{1}{2} b x^2 \left( -2a + b \log \left( 1 - \frac{c}{x^2} \right) \right) \log \left( 1 + \frac{c}{x^2} \right) + \frac{1}{4} b^2 x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) \right) dx \\
&= \frac{1}{4} \int x^2 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 dx - \frac{1}{2} b \int x^2 \left( -2a + b \log \left( 1 - \frac{c}{x^2} \right) \right) \log \left( 1 + \frac{c}{x^2} \right) dx + \frac{1}{4} \int b^2 x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) dx \\
&= \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{12} b^2 x^3 \log^2 \left( 1 + \frac{c}{x^2} \right) - \frac{1}{2} b \int \left( -2a x^2 \log \left( 1 + \frac{c}{x^2} \right) + b x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) \right) dx \\
&= \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{12} b^2 x^3 \log^2 \left( 1 + \frac{c}{x^2} \right) + (ab) \int x^2 \log \left( 1 + \frac{c}{x^2} \right) dx - \frac{1}{2} b \int x^2 \log^2 \left( 1 + \frac{c}{x^2} \right) dx \\
&= \frac{2}{3} abc x + \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 + \frac{1}{3} ab x^3 \log \left( 1 + \frac{c}{x^2} \right) - \frac{1}{6} b^2 x^3 \log \left( 1 - \frac{c}{x^2} \right) \log \left( 1 + \frac{c}{x^2} \right) \\
&= \frac{2}{3} abc x - \frac{1}{3} b^2 c x \log \left( 1 - \frac{c}{x^2} \right) - \frac{1}{3} b c^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right) + \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 \\
&= \frac{4}{3} abc x - \frac{1}{3} b^2 c x \log \left( 1 - \frac{c}{x^2} \right) - \frac{1}{3} b c^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right) + \frac{1}{12} x^3 \left( 2a - b \log \left( 1 - \frac{c}{x^2} \right) \right)^2 \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{2}{3} b^2 c^{3/2} \tanh^{-1} \left( \frac{x}{\sqrt{c}} \right) - \frac{1}{3} b^2 c x \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{2}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{2}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{2}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{4}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{4}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{4}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{4}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{4}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{4}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right) \\
&= \frac{4}{3} abc x - \frac{2}{3} abc^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{4}{3} b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right) + \frac{1}{3} i b^2 c^{3/2} \tan^{-1} \left( \frac{x}{\sqrt{c}} \right)^2 - \frac{4}{3} b^2 c^{3/2} \log \left( 1 - \frac{c}{x^2} \right)
\end{aligned}$$

**Mathematica [F]** time = 2.97755, size = 0, normalized size = 0.

$$\int x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] Integrate[x^2\*(a + b\*ArcTanh[c/x^2])^2, x]

---

**Maple [F]** time = 0.641, size = 0, normalized size = 0.

$$\int x^2 \left( a + b \operatorname{Artanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c/x^2))^2,x)

[Out] int(x^2\*(a+b\*arctanh(c/x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( b^2 x^2 \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 2 a b x^2 \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^2 x^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x^2\*(a+b\*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*arctanh(c/x^2)^2 + 2\*a\*b\*x^2\*arctanh(c/x^2) + a^2\*x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \left( a + b \operatorname{atanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c/x\*\*2))\*\*2,x)

[Out] Integral(x\*\*2\*(a + b\*atanh(c/x\*\*2))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a \right)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c/x^2))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)^2\*x^2, x)

$$3.178 \quad \int \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=1549

result too large to display

```
[Out] a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]]
- a*b*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + (b^
2*x*Log[1 - c/x^2]^2)/4 + a*b*x*Log[1 + c/x^2] - b^2*Sqrt[c]*ArcTanh[x/Sqrt
[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^2])/2 + (b^2*x*Log[
1 + c/x^2]^2)/4 - (b^2*Sqrt[-c]*Log[1 + c/x^2]*Log[Sqrt[-c] - x])/2 + (b^2*
Sqrt[-c]*Log[Sqrt[-c] - x]^2)/4 - (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] -
x])/2 + (b^2*Sqrt[c]*Log[Sqrt[c] - x]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c
]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1
+ I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] - b^2*Sqrt[-c]*Log[Sqrt[-c] - x]*Log[
x/Sqrt[-c]] - b^2*Sqrt[c]*Log[Sqrt[c] - x]*Log[x/Sqrt[c]] + (b^2*Sqrt[-c]*L
og[1 + c/x^2]*Log[Sqrt[-c] + x])/2 - (b^2*Sqrt[-c]*Log[(Sqrt[-c] - x)/(2*Sq
rt[-c])])*Log[Sqrt[-c] + x])/2 + b^2*Sqrt[-c]*Log[-(x/Sqrt[-c])]*Log[Sqrt[-
c] + x] - (b^2*Sqrt[-c]*Log[Sqrt[-c] + x]^2)/4 + (b^2*Sqrt[-c]*Log[Sqrt[-c]
- x]*Log[(Sqrt[-c] + x)/(2*Sqrt[-c])])/2 - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]
*Log[(2*Sqrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqr
t[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*Ar
cTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt
[c] + x))] + (b^2*Sqrt[c]*Log[1 - c/x^2]*Log[Sqrt[c] + x])/2 - (b^2*Sqrt[c]
*Log[(Sqrt[c] - x)/(2*Sqrt[c])])*Log[Sqrt[c] + x])/2 + b^2*Sqrt[c]*Log[-(x/S
qrt[c])]*Log[Sqrt[c] + x] - (b^2*Sqrt[c]*Log[Sqrt[c] + x]^2)/4 + (b^2*Sqrt[
c]*Log[Sqrt[c] - x]*Log[(Sqrt[c] + x)/(2*Sqrt[c])])/2 + b^2*Sqrt[c]*ArcTan[
x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)] + I*b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 -
((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog[2, -(x/Sqrt
[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqrt[c]*PolyLog[
2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] - (b^2*Sqrt[c]*PolyLo
g[2, (Sqrt[c] + x)/(2*Sqrt[c])])/2 + (b^2*Sqrt[-c]*PolyLog[2, (1 - x/Sqrt[-
c])/2])/2 - b^2*Sqrt[-c]*PolyLog[2, 1 - x/Sqrt[-c]] + b^2*Sqrt[-c]*PolyLog[
2, 1 + x/Sqrt[-c]] - (b^2*Sqrt[-c]*PolyLog[2, (c - Sqrt[-c]*x)/(2*c)])/2 -
b^2*Sqrt[c]*PolyLog[2, 1 - x/Sqrt[c]] + (b^2*Sqrt[c]*PolyLog[2, 1/2 - x/(2*
Sqrt[c])])/2 + b^2*Sqrt[c]*PolyLog[2, 1 + x/Sqrt[c]] + b^2*Sqrt[c]*PolyLog[
2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)] - (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*
(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/2 - (b^2*Sqrt[c]*Pol
yLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x)
)])/2 - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] -
I*x)]
```

**Rubi [A]** time = 2.24891, antiderivative size = 1549, normalized size of antiderivative = 1., number of steps used = 99, number of rules used = 29, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 2.417$ , Rules used = {6093, 2448, 263, 207, 2450, 2476, 2462, 260, 2416, 2394, 2315, 2390, 2301, 2393, 2391, 203, 2556, 12, 2470, 6688, 5992, 5912, 5920, 2402, 2447, 204, 4928, 4848, 4856}

result too large to display

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])^2, x]

[Out]  $a^2x + 2ab\sqrt{c}\operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right] - 2ab\sqrt{c}\operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right] - abx\log\left[1 - \frac{c}{x^2}\right] - b^2\sqrt{c}\operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]\log\left[1 - \frac{c}{x^2}\right] + (b^2x\log\left[1 - \frac{c}{x^2}\right]^2)/4 + abx\log\left[1 + \frac{c}{x^2}\right] - b^2\sqrt{c}\operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]\log\left[1 + \frac{c}{x^2}\right] - (b^2x\log\left[1 - \frac{c}{x^2}\right]\log\left[1 + \frac{c}{x^2}\right])/2 + (b^2x\log\left[1 + \frac{c}{x^2}\right]^2)/4 - (b^2\sqrt{-c}\log\left[1 + \frac{c}{x^2}\right]\log\left[\sqrt{-c} - x\right])/2 + (b^2\sqrt{-c}\log\left[\sqrt{-c} - x\right]^2)/4 - (b^2\sqrt{c}\log\left[1 - \frac{c}{x^2}\right]\log\left[\sqrt{c} - x\right])/2 + (b^2\sqrt{c}\log\left[\sqrt{c} - x\right]^2)/4 - 2b^2\sqrt{c}\operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{2\sqrt{c}}{\sqrt{c} - Ix}\right] + b^2\sqrt{c}\operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{(1 + I)(\sqrt{c} - x)}{\sqrt{c} - Ix}\right] - b^2\sqrt{-c}\log\left[\sqrt{-c} - x\right]\log\left[\frac{x}{\sqrt{-c}}\right] - b^2\sqrt{c}\log\left[\sqrt{c} - x\right]\log\left[\frac{x}{\sqrt{c}}\right] + (b^2\sqrt{-c}\log\left[1 + \frac{c}{x^2}\right]\log\left[\sqrt{-c} + x\right])/2 - (b^2\sqrt{-c}\log\left[\sqrt{-c} - x\right]\log\left[\frac{2\sqrt{-c}}{\sqrt{-c}}\right])\log\left[\sqrt{-c} + x\right])/2 + b^2\sqrt{-c}\log\left[-\frac{x}{\sqrt{-c}}\right]\log\left[\sqrt{-c} + x\right] - (b^2\sqrt{-c}\log\left[\sqrt{-c} + x\right]^2)/4 + (b^2\sqrt{-c}\log\left[\sqrt{-c} - x\right]\log\left[\frac{\sqrt{-c} + x}{2\sqrt{-c}}\right])/2 - 2b^2\sqrt{c}\operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{2\sqrt{c}}{\sqrt{c} + x}\right] + b^2\sqrt{c}\operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{2\sqrt{c}(\sqrt{-c} - x)}{(\sqrt{-c} - \sqrt{c})(\sqrt{c} + x)}\right] + b^2\sqrt{c}\operatorname{ArcTanh}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{2\sqrt{c}(\sqrt{-c} + x)}{(\sqrt{-c} + \sqrt{c})(\sqrt{c} + x)}\right] + (b^2\sqrt{c}\log\left[1 - \frac{c}{x^2}\right]\log\left[\sqrt{c} + x\right])/2 - (b^2\sqrt{c}\log\left[\frac{\sqrt{c} - x}{2\sqrt{c}}\right]\log\left[\sqrt{c} + x\right])/2 + b^2\sqrt{c}\log\left[-\frac{x}{\sqrt{c}}\right]\log\left[\sqrt{c} + x\right] - (b^2\sqrt{c}\log\left[\sqrt{c} + x\right]^2)/4 + (b^2\sqrt{c}\log\left[\sqrt{c} - x\right]\log\left[\frac{\sqrt{c} + x}{2\sqrt{c}}\right])/2 + b^2\sqrt{c}\operatorname{ArcTan}\left[\frac{x}{\sqrt{c}}\right]\log\left[\frac{(1 - I)(\sqrt{c} + x)}{\sqrt{c} - Ix}\right] + I b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c} - Ix}\right] - \frac{I}{2} b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 - \frac{(1 + I)(\sqrt{c} - x)}{\sqrt{c} - Ix}\right] + b^2\sqrt{c}\operatorname{PolyLog}\left[2, -\frac{x}{\sqrt{c}}\right] - I b^2\sqrt{c}\operatorname{PolyLog}\left[2, \frac{(-I)x}{\sqrt{c}}\right] + I b^2\sqrt{c}\operatorname{PolyLog}\left[2, \frac{Ix}{\sqrt{c}}\right] - b^2\sqrt{c}\operatorname{PolyLog}\left[2, \frac{x}{\sqrt{c}}\right] - (b^2\sqrt{c}\operatorname{PolyLog}\left[2, \frac{\sqrt{c} + x}{2\sqrt{c}}\right])/2 + (b^2\sqrt{-c}\operatorname{PolyLog}\left[2, \frac{1 - x/\sqrt{-c}}{2}\right])/2 - b^2\sqrt{-c}\operatorname{PolyLog}\left[2, 1 - \frac{x}{\sqrt{-c}}\right] + b^2\sqrt{-c}\operatorname{PolyLog}\left[2, 1 + \frac{x}{\sqrt{-c}}\right] - (b^2\sqrt{-c}\operatorname{PolyLog}\left[2, \frac{c - \sqrt{-c}x}{2c}\right])/2 - b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 - \frac{x}{\sqrt{c}}\right] + (b^2\sqrt{c}\operatorname{PolyLog}\left[2, \frac{1}{2} - \frac{x}{2\sqrt{c}}\right])/2 + b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 + \frac{x}{\sqrt{c}}\right] + b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{\sqrt{c} + x}\right] - (b^2\sqrt{c}\operatorname{PolyLog}\left[2, 1 - \frac{2\sqrt{c}}{c}\right])$

$$\frac{(\sqrt{-c} - x)/((\sqrt{-c} - \sqrt{c})(\sqrt{c} + x))}{2} - (b^2 \sqrt{c} \text{PolyLog}[2, 1 - (2\sqrt{c})(\sqrt{-c} + x)/((\sqrt{-c} + \sqrt{c})(\sqrt{c} + x))]) / 2 - (I/2) * b^2 \sqrt{c} \text{PolyLog}[2, 1 - ((1 - I)(\sqrt{c} + x)/(\sqrt{c} - I * x))]$$
Rule 6093

$$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)(x_)^{(n_)}] * (b_.)^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + (b * \text{Log}[1 + c * x^n])/2 - (b * \text{Log}[1 - c * x^n])/2)^p, x], x] /; \text{FreeQ}\{a, b, c, n\}, x \&\& \text{IGtQ}[p, 0] \&\& \text{IntegerQ}[n]$$
Rule 2448

$$\text{Int}[\text{Log}[(c_.)((d_) + (e_.)(x_)^{(n_)})^{(p_.)}], x\_Symbol] \rightarrow \text{Simp}[x * \text{Log}[c * (d + e * x^n)^p], x] - \text{Dist}[e * n * p, \text{Int}[x^n / (d + e * x^n), x], x] /; \text{FreeQ}\{c, d, e, n, p\}, x]$$
Rule 263

$$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.)(x_)^{(n_)})^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[x^{(m + n * p)} * (b + a/x^n)^p, x] /; \text{FreeQ}\{a, b, m, n\}, x \&\& \text{IntegerQ}[p] \&\& \text{NegQ}[n]$$
Rule 207

$$\text{Int}[(a_) + (b_.)(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2] * x) / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] * \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{GtQ}[b, 0])$$
Rule 2450

$$\text{Int}[(a_.) + \text{Log}[(c_.)((d_) + (e_.)(x_)^{(n_)})^{(p_.)}] * (b_.)^{(q_.)}, x\_Symbol] \rightarrow \text{Simp}[x * (a + b * \text{Log}[c * (d + e * x^n)^p])^q, x] - \text{Dist}[b * e * n * p * q, \text{Int}[(x^n * (a + b * \text{Log}[c * (d + e * x^n)^p])^{(q - 1)}) / (d + e * x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x \&\& \text{IGtQ}[q, 0] \&\& (\text{EqQ}[q, 1] \parallel \text{IntegerQ}[n])$$
Rule 2476

$$\text{Int}[(a_.) + \text{Log}[(c_.)((d_) + (e_.)(x_)^{(n_)})^{(p_.)}] * (b_.)^{(q_.)} * (x_)^{(m_.)} * ((f_) + (g_.)(x_)^{(s_)})^{(r_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b * \text{Log}[c * (d + e * x^n)^p])^q, x^m * (f + g * x^s)^r, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n, p, q, r, s\}, x \&\& \text{IGtQ}[q, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[r] \&\& \text{IntegerQ}[s]$$
Rule 2462

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*(b_.))/((f_.) + (g_.
)*(x_)), x_Symbol] := Simp[(Log[f + g*x]*(a + b*Log[c*(d + e*x^n)^p])/g, x
] - Dist[(b*e*n*p)/g, Int[(x^(n - 1)*Log[f + g*x])/(d + e*x^n), x], x] /; F
reeQ[{a, b, c, d, e, f, g, n, p}, x] && RationalQ[n]
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((h_.)*(x_)
)^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a
+ b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.))*((f_) + (g_.
)*(x_)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]^(p_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
```

], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 2556

Int[Log[v\_] \* Log[w\_], x\_Symbol] := Simp[x \* Log[v] \* Log[w], x] + (-Int[SimplifyIntegrand[(x \* Log[w] \* D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x \* Log[v] \* D[w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p], x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rule 5992

Int[(((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))\*(x\_)^(m\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[a + b\*ArcTanh[c\*x], x^m/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])

Rule 5912

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (-Simp[(b*PolyLog[2, -(c*x)])/2, x] + Simp[(b*PolyLog[2, c*x])/2, x]) /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x]) /; FreeQ[{a, b, c, d, e},
x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_)])*(b_.))*(x_)^(m_.)/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
```

$I*c*x]/x, x], x]) /; \text{FreeQ}\{a, b, c\}, x]$

### Rule 4856

$\text{Int}[(a_.) + \text{ArcTan}[c_.](x_.)]*(b_.)/((d_.) + (e_.)(x_.)), x\_Symbol] :> -\text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[2/(1 - I*c*x)]/e, x] + (\text{Dist}[(b*c)/e, \text{Int}[\text{Log}[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - \text{Dist}[(b*c)/e, \text{Int}[\text{Log}[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]/(1 + c^2*x^2), x], x] + \text{Simp}[(a + b*\text{ArcTan}[c*x])*\text{Log}[(2*c*(d + e*x))/(c*d + I*e)*(1 - I*c*x)]/e, x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[c^2*d^2 + e^2, 0]$

### Rubi steps





**Mathematica [A]** time = 3.40588, size = 565, normalized size = 0.36

$$-\frac{1}{2}b^2x\sqrt{\frac{c}{x^2}}\left(-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}}-1\right)\right)\right)+$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2,x]

[Out] a^2\*x - 2\*a\*b\*Sqrt[c/x^2]\*x\*(ArcTan[Sqrt[c/x^2]] + ArcTanh[Sqrt[c/x^2]]) + 2\*a\*b\*x\*ArcTanh[c/x^2] - (b^2\*Sqrt[c/x^2]\*x\*((-2\*I)\*ArcTan[Sqrt[c/x^2]]^2 + 4\*ArcTan[Sqrt[c/x^2]]\*ArcTanh[c/x^2] - (2\*ArcTanh[c/x^2]^2)/Sqrt[c/x^2] + 2\*ArcTan[Sqrt[c/x^2]]\*Log[1 + E^((4\*I)\*ArcTan[Sqrt[c/x^2]])] - 2\*ArcTanh[c/x^2]\*Log[1 - Sqrt[c/x^2]] + Log[2]\*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]\*Log[(1/2 + I/2)\*(-I + Sqrt[c/x^2])]) + 2\*ArcTanh[c/x^2]\*Log[1 + Sqrt[c/x^2]] - Log[2]\*Log[1 + Sqrt[c/x^2]] - Log[((1 + I) - (1 - I)\*Sqrt[c/x^2])/2]\*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)\*(I + Sqrt[c/x^2])]\*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]\*Log[((1 + I) + (1 - I)\*Sqrt[c/x^2])/2] - (I/2)\*PolyLog[2, -E^((4\*I)\*ArcTan[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2 - I/2)\*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)\*(-1 + Sqrt[c/x^2])] + PolyLog[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)\*(1 + Sqrt[c/x^2])] - PolyLog[2, (1/2 + I/2)\*(1 + Sqrt[c/x^2])])]/2

**Maple [F]** time = 0.549, size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{Arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))^2,x)

[Out] int((a+b\*arctanh(c/x^2))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2,x, algorithm="fricas")`

[Out] `integral(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2, x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c/x**2))**2,x)`

[Out] `Integral((a + b*atanh(c/x**2))**2, x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c/x^2) + a)^2, x)`

$$3.179 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx$$

**Optimal.** Leaf size=1117

result too large to display

```
[Out] (2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/Sqrt[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/Sqrt[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^2*ArcCot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))/x + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Log[1 - c/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) + (2*b^2*ArcCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + (2*b^2*ArcCoth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt[c]]*Log[((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*PolyLog[2, 1 - 2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - ((1 + I)*(1 - Sqrt[c])/x)]/(1 - (I*Sqrt[c])/x))/Sqrt[c] - (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)])/Sqrt[c] + (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]/x)))/(2*Sqrt[c]) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x)))/(2*Sqrt[c]) + ((I/2)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (I*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]
```

---

**Rubi [A]** time = 2.15037, antiderivative size = 1117, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 29, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 1.812$ , Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6715, 2448, 321, 6742, 203, 2556, 5992, 5920, 2402, 2315, 4928, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])^2/x^2,x]

[Out] (2\*a\*b)/x - (2\*a\*b\*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2\*b^2\*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2\*b^2\*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2\*b^2\*ArcTan[x/Sqrt[c]])/Sqrt[c] - (I\*b^2\*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2\*b^2\*ArcTanh[x/Sqrt[c]])/Sqrt[c] - (b^2\*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2\*b^2\*ArcTan[x/Sqrt[c]]\*Log[2 - (2\*Sqrt[c])/(Sqrt[c] - I\*x)])/Sqrt[c] - (b^2\*Log[1 - c/x^2])/x + (b^2\*ArcCot[x/Sqrt[c]]\*Log[1 - c/x^2])/Sqrt[c] - (b\*(2\*a - b\*Log[1 - c/x^2])/x + (b\*ArcTanh[x/Sqrt[c]]\*(2\*a - b\*Log[1 - c/x^2]))/Sqrt[c] - (2\*a - b\*Log[1 - c/x^2])^2/(4\*x) - (a\*b\*Log[1 + c/x^2])/x + (b^2\*ArcCoth[x/Sqrt[c]]\*Log[1 + c/x^2])/Sqrt[c] + (b^2\*ArcTan[x/Sqrt[c]]\*Log[1 + c/x^2])/Sqrt[c] + (b^2\*Log[1 - c/x^2]\*Log[1 + c/x^2])/(2\*x) - (b^2\*Log[1 + c/x^2]^2)/(4\*x) + (2\*b^2\*ArcCot[x/Sqrt[c]]\*Log[2/(1 - (I\*Sqrt[c])/x)])/Sqrt[c] - (b^2\*ArcCot[x/Sqrt[c]]\*Log[((1 + I)\*(1 - Sqrt[c]/x))/(1 - (I\*Sqrt[c])/x)])/Sqrt[c] + (2\*b^2\*ArcCoth[x/Sqrt[c]]\*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2\*ArcCoth[x/Sqrt[c]]\*Log[(-2\*Sqrt[c]\*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])\*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2\*ArcCoth[x/Sqrt[c]]\*Log[(2\*Sqrt[c]\*(1 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])\*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2\*ArcCot[x/Sqrt[c]]\*Log[((1 - I)\*(1 + Sqrt[c]/x))/(1 - (I\*Sqrt[c])/x)])/Sqrt[c] - (2\*b^2\*ArcTanh[x/Sqrt[c]]\*Log[2 - (2\*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I\*b^2\*PolyLog[2, 1 - 2/(1 - (I\*Sqrt[c])/x)])/Sqrt[c] + ((I/2)\*b^2\*PolyLog[2, 1 - ((1 + I)\*(1 - Sqrt[c])/x)]/(1 - (I\*Sqrt[c])/x))/Sqrt[c] - (b^2\*PolyLog[2, 1 - 2/(1 + Sqrt[c]/x)])/Sqrt[c] + (b^2\*PolyLog[2, 1 + (2\*Sqrt[c]\*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])\*(1 + Sqrt[c]/x))]/(2\*Sqrt[c]) + (b^2\*PolyLog[2, 1 - (2\*Sqrt[c]\*(1 + Sqrt[-c]/x))/((Sqrt[-c] + Sqrt[c])\*(1 + Sqrt[c]/x))]/(2\*Sqrt[c]) + ((I/2)\*b^2\*PolyLog[2, 1 - ((1 - I)\*(1 + Sqrt[c]/x))/(1 - (I\*Sqrt[c])/x)])/Sqrt[c] - (I\*b^2\*PolyLog[2, -1 + (2\*Sqrt[c])/(Sqrt[c] - I\*x)])/Sqrt[c] + (b^2\*PolyLog[2, -1 + (2\*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c]

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(d\*x)^m\*(a + (b\*Log[1 + c\*x^n])/2 - (b\*Log[1 - c\*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]

### Rule 2457

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_))]^(p\_.)]\*(b\_.)^(q\_.)\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p])^q)/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p\*q)/(f^n\*(m + 1)), Int[((f\*x)^(m + n)\*(a + b\*Log[c\*(d + e\*x^n)^p])^(q - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]

Rule 2476

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))^(q\_.)\*(x\_)^(m\_.)\*((f\_) + (g\_.)\*(x\_)^(s\_))^(r\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*Log[c\*(d + e\*x^n)^p]]^q, x^m\*(f + g\*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]

Rule 2455

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))\*((f\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((f\*x)^(m + 1)\*(a + b\*Log[c\*(d + e\*x^n)^p]))/(f\*(m + 1)), x] - Dist[(b\*e\*n\*p)/(f\*(m + 1)), Int[(x^(n - 1)\*(f\*x)^(m + 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]

Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

Rule 325

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x]

, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)])/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

### Rule 2447

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

### Rule 6715

Int[(u\_)\*(x\_)^(m\_.), x\_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] := Simp[x*Log[c*(d
+ e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2556

```
Int[Log[v_]*Log[w_], x_Symbol] := Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[(x*Log[w]*D[v, x])/v, x], x] - Int[SimplifyIntegrand[(x*Log[v]*D[
w, x])/w, x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 5992

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] := -
Simp[(a + b*ArcTanh[c*x])*Log[2/(1 + c*x)]/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
```



$\text{Log}[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))]/e, x] /; \text{FreeQ}\{a, b, c, d, e, x\} \ \&\& \ \text{NeQ}[c^2*d^2 - e^2, 0]$

### Rule 2402

$\text{Int}[\text{Log}[(c\_)/((d\_ + (e\_)*(x\_)))/((f\_ + (g\_)*(x\_)^2)], x\_Symbol] :> -\text{Dist}[e/g, \text{Subst}[\text{Int}[\text{Log}[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; \text{FreeQ}\{c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[c, 2*d] \ \&\& \ \text{EqQ}[e^2*f + d^2*g, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c\_)*(x\_)]/((d\_ + (e\_)*(x\_))), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

### Rule 4928

$\text{Int}[(((a\_ + \text{ArcTan}[(c\_)*(x\_)]*(b\_))*(x\_)^{(m\_)})/((d\_ + (e\_)*(x\_)^2)), x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[a + b*\text{ArcTan}[c*x], x^m/(d + e*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{EqQ}[m, 1] \ \&\& \ \text{NeQ}[a, 0])$

### Rule 4856

$\text{Int}[((a\_ + \text{ArcTan}[(c\_)*(x\_)]*(b\_))/((d\_ + (e\_)*(x\_))), x\_Symbol] :> -\text{Simp}[(a + b*\text{ArcTan}[c*x])*Log[2/(1 - I*c*x)]/e, x] + (\text{Dist}[(b*c)/e, \text{Int}[\text{Log}[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - \text{Dist}[(b*c)/e, \text{Int}[\text{Log}[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + \text{Simp}[(a + b*\text{ArcTan}[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))]/e, x]) /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[c^2*d^2 + e^2, 0]$

### Rule 4924

$\text{Int}[((a\_ + \text{ArcTan}[(c\_)*(x\_)]*(b\_))^{(p\_)})/((x\_)*((d\_ + (e\_)*(x\_)^2))), x\_Symbol] :> -\text{Simp}[(I*(a + b*\text{ArcTan}[c*x])^{(p + 1)})/(b*d*(p + 1)), x] + \text{Dist}[I/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^p/(x*(I + c*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[e, c^2*d] \ \&\& \ \text{GtQ}[p, 0]$

### Rule 4868

$\text{Int}[((a\_ + \text{ArcTan}[(c\_)*(x\_)]*(b\_))^{(p\_)})/((x\_)*((d\_ + (e\_)*(x\_))), x\_Symbol] :> \text{Simp}[(a + b*\text{ArcTan}[c*x])^p*Log[2 - 2/(1 + (e*x)/d)]/d, x] - \text{Dist}[(b*c*p)/d, \text{Int}[(a + b*\text{ArcTan}[c*x])^{(p - 1)}*Log[2 - 2/(1 + (e*x)/d)]/(1 + c^2*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{EqQ}[c^2*d^2 + e^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx &= \int \left( \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x^2} - \frac{b\left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right)}{2x^2} + \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{4x^2} \right) dx \\
&= \frac{1}{4} \int \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{x^2} dx - \frac{1}{2} b \int \frac{\left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right)}{x^2} dx + \frac{1}{4} b^2 \int \frac{\log^2\left(1 + \frac{c}{x^2}\right)}{x^2} dx \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{4x} + \frac{1}{2} b \operatorname{Subst} \left( \int \left(-2a + b \log(1 - cx^2)\right) \log(1 + cx^2) dx \right) \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{4x} + \frac{1}{2} b \operatorname{Subst} \left( \int \left(-2a \log(1 + cx^2) + b \log(1 - cx^2)\right) dx \right) \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{4x} + b \int \frac{2a - b \log\left(1 - \frac{c}{x^2}\right)}{x^2} dx + b \int \frac{2a - b \log\left(1 - \frac{c}{x^2}\right)}{c - x^2} dx \\
&= -\frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x} \\
&= \frac{2ab}{x} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{\sqrt{c}} - \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x} \\
&= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right) \left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{4b^2}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{\sqrt{c}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} \\
&= \frac{2ab}{x} - \frac{2ab \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \cot^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \coth^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}
\end{aligned}$$

**Mathematica [A]** time = 3.03704, size = 568, normalized size = 0.51

$$b^2 \left( -\text{PolyLog}\left(2, \frac{1}{2}\left(1 - \sqrt{\frac{c}{x^2}}\right)\right) + \text{PolyLog}\left(2, \left(-\frac{1}{2} - \frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}} - 1\right)\right) + \text{PolyLog}\left(2, \left(-\frac{1}{2} + \frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}} - 1\right)\right) + \text{PolyLog}\left(2, \frac{1}{2}\left(\sqrt{\frac{c}{x^2}} + 1\right)\right) - \text{PolyLog}\left(2, \left(\frac{1}{2} - \frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}} + 1\right)\right) - \text{PolyLog}\left(2, \left(\frac{1}{2} + \frac{i}{2}\right)\left(\sqrt{\frac{c}{x^2}} + 1\right)\right) \right)$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^2, x]

[Out] 
$$\begin{aligned} & (-2*a^2 - (4*a*b*(\text{ArcTan}[\text{Sqrt}[c/x^2]] - \text{ArcTanh}[\text{Sqrt}[c/x^2]]))/\text{Sqrt}[c/x^2] \\ & - 4*a*b*\text{ArcTanh}[c/x^2] + (b^2*((2*I)*\text{ArcTan}[\text{Sqrt}[c/x^2]]^2 - 4*\text{ArcTan}[\text{Sqrt}[c/x^2]]*\text{ArcTanh}[c/x^2] \\ & - 2*\text{Sqrt}[c/x^2]*\text{ArcTanh}[c/x^2]^2 - 2*\text{ArcTan}[\text{Sqrt}[c/x^2]]*\text{Log}[1 + E^{((4*I)*\text{ArcTan}[\text{Sqrt}[c/x^2]])}] \\ & - 2*\text{ArcTanh}[c/x^2]*\text{Log}[1 - \text{Sqrt}[c/x^2]] + \text{Log}[2]*\text{Log}[1 - \text{Sqrt}[c/x^2]] - \text{Log}[1 - \text{Sqrt}[c/x^2]]^2/2 \\ & + \text{Log}[1 - \text{Sqrt}[c/x^2]]*\text{Log}[(1/2 + I/2)*(-I + \text{Sqrt}[c/x^2])] + 2*\text{ArcTanh}[c/x^2]*\text{Log}[1 \\ & + \text{Sqrt}[c/x^2]] - \text{Log}[2]*\text{Log}[1 + \text{Sqrt}[c/x^2]] - \text{Log}[(1 + I) - (1 - I)*\text{Sqrt}[c/x^2])/2 \\ & * \text{Log}[1 + \text{Sqrt}[c/x^2]] - \text{Log}[(-1/2 - I/2)*(I + \text{Sqrt}[c/x^2])] * \text{Log}[1 + \text{Sqrt}[c/x^2]] \\ & + \text{Log}[1 + \text{Sqrt}[c/x^2]]^2/2 + \text{Log}[1 - \text{Sqrt}[c/x^2]] * \text{Log}[(1 + I) + (1 - I)*\text{Sqrt}[c/x^2])/2 \\ & + (I/2)*\text{PolyLog}[2, -E^{((4*I)*\text{ArcTan}[\text{Sqrt}[c/x^2]])}] - \text{PolyLog}[2, (1 - \text{Sqrt}[c/x^2])/2] \\ & + \text{PolyLog}[2, (-1/2 - I/2)*(-1 + \text{Sqrt}[c/x^2])] + \text{PolyLog}[2, (-1/2 + I/2)*(-1 + \text{Sqrt}[c/x^2])] \\ & + \text{PolyLog}[2, (1 + \text{Sqrt}[c/x^2])/2] - \text{PolyLog}[2, (1/2 - I/2)*(1 + \text{Sqrt}[c/x^2])] - \text{PolyLog}[2, (1/2 + I/2)*(1 + \text{Sqrt}[c/x^2])]) \\ & )/\text{Sqrt}[c/x^2])/(2*x) \end{aligned}$$

---

**Maple [F]** time = 0.656, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \left( a + b \text{Arctanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))^2/x^2, x)

[Out] int((a+b\*arctanh(c/x^2))^2/x^2, x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="fricas")`

[Out] `integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c/x**2))**2/x**2,x)`

[Out] `Integral((a + b*atanh(c/x**2))**2/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^2/x^2, x)
```

$$3.180 \quad \int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4} dx$$

**Optimal.** Leaf size=1263

result too large to display

```
[Out] (2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) +
(4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(
(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)
/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)
])/ (3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x
) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1
- c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sq
rt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(
12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (
b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]
*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3)
- (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c]
)/(Sqrt[c] - I*x)])/ (3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt
[c] - x))/(Sqrt[c] - I*x)])/ (3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*
Sqrt[c])/(Sqrt[c] + x)])/ (3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/ (3*c^(3/2)) - (b^
2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(
Sqrt[c] + x))])/ (3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c]
+ x))/(Sqrt[c] - I*x)])/ (3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*
Sqrt[c])/(Sqrt[c] + x)])/ (3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c]
)/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[
c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sq
rt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/ (3*c^(3/2)) + ((I/3
)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/S
qrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/ (3*c^(3/2)) - (b^2*PolyLog[2
, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/ (3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqr
t[c])/(Sqrt[c] + x)])/ (3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c
] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/ (6*c^(3/2)) + (b^2*PolyLog[2
, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/ (6*
c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x
)])/c^(3/2)
```

---

**Rubi [A]** time = 2.58476, antiderivative size = 1263, normalized size of antiderivative = 1., number of steps used = 104, number of rules used = 30, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}}$

= 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])^2/x^4, x]

[Out]  $(2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(3*c*x) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*Log[1 - c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x^2])^2/(12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3*c*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(6*x^3) - (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(3/2) + ((I/3)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(3/2) + ((I/6)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c^(3/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(3*c^(3/2)) + ((I/3)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(3/2) - ((I/3)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(3/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(3*c^(3/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(6*c^(3/2)) + ((I/6)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(3/2)$

### Rule 6099

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)\*((d\_.)\*(x\_)^(m\_.), x\_Sy



```
mbol] := Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

### Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

### Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_.) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]
```

### Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol] := Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

### Rule 325

```
Int[((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 2470

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_))^(p\_.)]\*(b\_.))/((f\_) + (g\_.)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rule 5988

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)]\*(b\_.))^(p\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_))), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))*(x_)^(m_)/((d_) + (e_)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))/(x_), x_Symbol] := Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])]/2, x] + Simp[(b*PolyLog[2, c*x])/2, x] /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_.))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_.)/((d_) + (e_.)*(x_.)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x)
)/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)]/(1
+ c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(\frac{c}{x^2}))^2}{x^4} dx &= \int \left( \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{4x^4} - \frac{b(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{2x^4} + \frac{b^2 \log^2(1 + \frac{c}{x^2})}{4x^4} \right) dx \\
&= \frac{1}{4} \int \frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{x^4} dx - \frac{1}{2} b \int \frac{(-2a + b \log(1 - \frac{c}{x^2})) \log(1 + \frac{c}{x^2})}{x^4} dx + \frac{1}{4} b^2 \int \frac{\log^2(1 + \frac{c}{x^2})}{x^4} dx \\
&= -\frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{12x^3} - \frac{b^2 \log^2(1 + \frac{c}{x^2})}{12x^3} - \frac{1}{2} b \int \left( -\frac{2a \log(1 + \frac{c}{x^2})}{x^4} + \frac{b \log(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{x^4} \right) dx \\
&= -\frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{12x^3} - \frac{b^2 \log^2(1 + \frac{c}{x^2})}{12x^3} + (ab) \int \frac{\log(1 + \frac{c}{x^2})}{x^4} dx - \frac{1}{2} b^2 \int \frac{\log(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{x^4} dx \\
&= -\frac{(2a - b \log(1 - \frac{c}{x^2}))^2}{12x^3} - \frac{ab \log(1 + \frac{c}{x^2})}{3x^3} + \frac{b^2 \log(1 - \frac{c}{x^2}) \log(1 + \frac{c}{x^2})}{6x^3} - \frac{b^2 \log^2(1 + \frac{c}{x^2})}{12x^3} \\
&= -\frac{b(2a - b \log(1 - \frac{c}{x^2}))}{9x^3} - \frac{b(2a - b \log(1 - \frac{c}{x^2}))}{3cx} + \frac{b \tanh^{-1}(\frac{x}{\sqrt{c}})(2a - b \log(1 - \frac{c}{x^2}))}{3c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{3c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{b(2a - b \log(1 - \frac{c}{x^2}))}{9x^3} - \frac{b(2a - b \log(1 - \frac{c}{x^2}))}{3cx} + \frac{b \tanh^{-1}(\frac{x}{\sqrt{c}})(2a - b \log(1 - \frac{c}{x^2}))}{3c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{3c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{b(2a - b \log(1 - \frac{c}{x^2}))}{9x^3} - \frac{b(2a - b \log(1 - \frac{c}{x^2}))}{3cx} + \frac{b \tanh^{-1}(\frac{x}{\sqrt{c}})(2a - b \log(1 - \frac{c}{x^2}))}{3c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{3c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{2b^2 \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{ib^2 \tan^{-1}(\frac{x}{\sqrt{c}})^2}{3c^{3/2}} + \frac{2b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{3c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{4b^2}{27x^3} - \frac{2ab}{3cx} - \frac{2ab \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{8b^2 \tan^{-1}(\frac{x}{\sqrt{c}})}{9c^{3/2}} + \frac{ib^2 \tan^{-1}(\frac{x}{\sqrt{c}})^2}{3c^{3/2}} + \frac{8b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{9c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{8b^2 \tan^{-1}(\frac{x}{\sqrt{c}})}{9c^{3/2}} + \frac{ib^2 \tan^{-1}(\frac{x}{\sqrt{c}})^2}{3c^{3/2}} + \frac{8b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{9c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log^2(1 + \frac{c}{x^2})}{9c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{14b^2 \tan^{-1}(\frac{x}{\sqrt{c}})}{9c^{3/2}} + \frac{ib^2 \tan^{-1}(\frac{x}{\sqrt{c}})^2}{3c^{3/2}} + \frac{14b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{9c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log^2(1 + \frac{c}{x^2})}{9c^{3/2}} \\
&= \frac{2ab}{9x^3} - \frac{2ab}{3cx} - \frac{2ab \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{4b^2 \tan^{-1}(\frac{x}{\sqrt{c}})}{3c^{3/2}} + \frac{ib^2 \tan^{-1}(\frac{x}{\sqrt{c}})^2}{3c^{3/2}} + \frac{4b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log(1 + \frac{c}{x^2})}{3c^{3/2}} - \frac{b^2 \tanh^{-1}(\frac{x}{\sqrt{c}}) \log^2(1 + \frac{c}{x^2})}{3c^{3/2}}
\end{aligned}$$

**Mathematica [F]** time = 2.6617, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^4, x]

[Out] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^4, x]

**Maple [F]** time = 0.603, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \left(a + b \operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))^2/x^4, x)

[Out] int((a+b\*arctanh(c/x^2))^2/x^4, x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^4,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c/x^2)^2 + 2\*a\*b\*arctanh(c/x^2) + a^2)/x^4, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))\*\*2/x\*\*4,x)

[Out] Integral((a + b\*atanh(c/x\*\*2))\*\*2/x\*\*4, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)^2/x^4, x)



$$3.181 \quad \int \frac{\left(a+b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6} dx$$

**Optimal.** Leaf size=1337

result too large to display

```
[Out] (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*Log[1 - c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a - b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (a*b*Log[1 + c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b^2*Log[1 + c/x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/2) - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)])/c^(5/2) - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/c^(5/2) - (b^2*PolyLog[2, -(x/Sqrt[c])])/(5*c^(5/2)) - ((I/5)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]])/c^(5/2) + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]])/c^(5/2) + (b^2*PolyLog[2, x/Sqrt[c]])/(5*c^(5/2)) - (b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(10*c^(5/2)) - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/c^(5/2)
```

---

**Rubi [A]** time = 2.78992, antiderivative size = 1337, normalized size of antiderivative = 1., number of steps used = 129, number of rules used = 30, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}}$

= 1.875, Rules used = {6099, 2457, 2476, 2455, 263, 325, 207, 206, 2470, 12, 260, 6688, 5988, 5932, 2447, 6742, 203, 30, 2557, 5992, 5912, 5920, 2402, 2315, 4928, 4848, 2391, 4856, 4924, 4868}

result too large to display

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c/x^2])^2/x^6, x]

[Out] 
$$\begin{aligned} & (2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^2*x) \\ & + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(15*c^(5/2)) \\ & - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(15*c^(5/2)) \\ & - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) \\ & - (b^2*Log[1 - c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c/x^2])/(5*c^2*x) \\ & - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b*(2*a - b*Log[1 - c/x^2]))/(25*x^5) \\ & - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) \\ & + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) \\ & - (a*b*Log[1 + c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(5*c^(5/2)) \\ & + (b^2*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) \\ & - (b^2*Log[1 + c/x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) \\ & + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/(5*c^(5/2)) \\ & + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) \\ & - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) \\ & - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) \\ & + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/(5*c^(5/2)) \\ & - (2*b^2*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) \\ & + ((I/5)*b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)]) / c^(5/2) \\ & - ((I/5)*b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)]) / c^(5/2) \\ & - ((I/10)*b^2*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)]) / c^(5/2) \\ & - (b^2*PolyLog[2, -(x/Sqrt[c])]) / (5*c^(5/2)) - ((I/5)*b^2*PolyLog[2, ((-I)*x)/Sqrt[c]]) / c^(5/2) \\ & + ((I/5)*b^2*PolyLog[2, (I*x)/Sqrt[c]]) / c^(5/2) + (b^2*PolyLog[2, x/Sqrt[c]]) / (5*c^(5/2)) \\ & - (b^2*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] + x)]) / (5*c^(5/2)) + (b^2*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] + x)]) / (5*c^(5/2)) \\ & + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))]) / (10*c^(5/2)) \\ & + (b^2*PolyLog[2, 1 - (2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] + x))]) / (10*c^(5/2)) \\ & - ((I/10)*b^2*PolyLog[2, 1 - ((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)]) / c^(5/2) \end{aligned}$$

Rule 6099

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(d*x)^m*(a + (b*Log[1 + c*x^n])/2 - (b*Log[1 - c*x^n])/2)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] && IntegerQ[m] && IntegerQ[n]
```

### Rule 2457

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_.)*((f_.)*(x_)^(m_.), x_Symbol]
:> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p])^q)/(f*(m + 1)), x] - Dist[(b*e*n*p*q)/(f^n*(m + 1)), Int[((f*x)^(m + n)*(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

### Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))^(q_.)*(x_)^(m_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] && IntegerQ[s]
```

### Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]^(p_.)]*(b_.))*((f_.)*(x_)^(m_.), x_Symbol]
:> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

### Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Int[x^(m + n*p)*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

### Rule 325

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

, 0] || GtQ[b, 0])

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 2470

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(p\_)]\*(b\_))/((f\_) + (g\_)\*(x\_)^2), x\_Symbol] := With[{u = IntHide[1/(f + g\*x^2), x]}, Simp[u\*(a + b\*Log[c\*(d + e\*x^n)^p]), x] - Dist[b\*e\*n\*p, Int[(u\*x^(n - 1))/(d + e\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

### Rule 5988

Int[((a\_) + ArcTanh[(c\_)\*(x\_)^(n\_)]\*(b\_))^(p\_)/((x\_)\*((d\_) + (e\_)\*(x\_)^2)), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*d\*(p + 1)), x] + Dist[1/d, Int[(a + b\*ArcTanh[c\*x])^p/(x\*(1 + c\*x)), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2\*d + e, 0] && GtQ[p, 0]

### Rule 5932

Int[((a\_) + ArcTanh[(c\_)\*(x\_)^(n\_)]\*(b\_))^(p\_)/((x\_)\*((d\_) + (e\_)\*(x\_)^2)), x\_Symbol] := Simp[((a + b\*ArcTanh[c\*x])^p\*Log[2 - 2/(1 + (e\*x)/d)])/d, x] - Dist[(b\*c\*p)/d, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*Log[2 - 2/(1 + (e\*x)/d)]]/(1 - c^2\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d^2 - e^2, 0]

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2557

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] := With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[(z*Log[w]*D[v, x])/v, x], x] - In
t[SimplifyIntegrand[(z*Log[v]*D[w, x])/w, x], x]) /; InverseFunctionFreeQ[z
, x] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 5992

```
Int[(((a_) + ArcTanh[(c_)*(x_)])*(b_))*(x_)^(m_)/((d_) + (e_)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 5912

```
Int[((a_) + ArcTanh[(c_)*(x_)])*(b_)/(x_), x_Symbol] := Simp[a*Log[x], x
] + (-Simp[(b*PolyLog[2, -(c*x)])]/2, x] + Simp[(b*PolyLog[2, c*x])/2, x] /
; FreeQ[{a, b, c}, x]
```

Rule 5920

```
Int[((a_.) + ArcTanh[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -
Simp[((a + b*ArcTanh[c*x])*Log[2/(1 + c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x))
/((c*d + e)*(1 + c*x))]/(1 - c^2*x^2), x], x] + Simp[((a + b*ArcTanh[c*x])*
Log[(2*c*(d + e*x))/((c*d + e)*(1 + c*x))])/e, x]) /; FreeQ[{a, b, c, d, e}
, x] && NeQ[c^2*d^2 - e^2, 0]
```

### Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_.))]/((f_) + (g_.)*(x_)^2), x_Symbol] := -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

### Rule 2315

```
Int[Log[(c_.)*(x_.)/((d_) + (e_.)*(x_.)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 4928

```
Int[(((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
/; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

### Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))/(x_), x_Symbol] := Simp[a*Log[x], x]
+ (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))/((d_) + (e_.)*(x_.)), x_Symbol] := -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x)
)/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 4924

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*d*(p + 1)), x] + Dist
[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 4868

```
Int[((a_.) + ArcTan[(c_.)*(x_.)]*(b_.))^(p_.)/((x_.)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[((a + b*ArcTan[c*x])^p*Log[2 - 2/(1 + (e*x)/d)])/d, x] - Di
st[(b*c*p)/d, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2 - 2/(1 + (e*x)/d)])/
(1 + c^2*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6} dx &= \int \left( \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{4x^6} - \frac{b\left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right)}{2x^6} + \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{4x^6} \right) dx \\
&= \frac{1}{4} \int \frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{x^6} dx - \frac{1}{2} b \int \frac{\left(-2a + b \log\left(1 - \frac{c}{x^2}\right)\right) \log\left(1 + \frac{c}{x^2}\right)}{x^6} dx + \frac{1}{4} b^2 \int \frac{\log^2\left(1 + \frac{c}{x^2}\right)}{x^6} dx \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{20x^5} - \frac{1}{2} b \int \left( -\frac{2a \log\left(1 + \frac{c}{x^2}\right)}{x^6} + \frac{b \log\left(1 - \frac{c}{x^2}\right) \log\left(1 + \frac{c}{x^2}\right)}{x^6} \right) dx \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{20x^5} + (ab) \int \frac{\log\left(1 + \frac{c}{x^2}\right)}{x^6} dx - \frac{1}{2} b^2 \int \frac{\log\left(1 - \frac{c}{x^2}\right) \log\left(1 + \frac{c}{x^2}\right)}{x^6} dx \\
&= -\frac{\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)^2}{20x^5} - \frac{ab \log\left(1 + \frac{c}{x^2}\right)}{5x^5} + \frac{b^2 \log\left(1 - \frac{c}{x^2}\right) \log\left(1 + \frac{c}{x^2}\right)}{10x^5} - \frac{b^2 \log^2\left(1 + \frac{c}{x^2}\right)}{20x^5} \\
&= -\frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{5c^2x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{5c^2x} + \frac{b \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} - \frac{4b^2}{5c^2x} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{25x^5} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{15cx^3} - \frac{b\left(2a - b \log\left(1 - \frac{c}{x^2}\right)\right)}{5c^2x} \\
&= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{16b^2}{15c^2x} - \frac{2b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{4b^2}{125x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{92b^2}{75c^2x} + \frac{2ab \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{8b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{15c^{5/2}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{32b^2}{75c^2x} + \frac{2ab \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{46b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{52b^2}{75c^2x} + \frac{2ab \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{16b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} \\
&= \frac{2ab}{25x^5} - \frac{2ab}{15cx^3} + \frac{2ab}{5c^2x} - \frac{8b^2}{15c^2x} + \frac{2ab \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}} - \frac{26b^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)}{75c^{5/2}} - \frac{ib^2 \tan^{-1}\left(\frac{x}{\sqrt{c}}\right)^2}{5c^{5/2}} + \frac{2b^2 \tanh^{-1}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}
\end{aligned}$$



**Mathematica [F]** time = 2.60961, size = 0, normalized size = 0.

$$\int \frac{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}{x^6} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^6, x]

[Out] Integrate[(a + b\*ArcTanh[c/x^2])^2/x^6, x]

**Maple [F]** time = 0.665, size = 0, normalized size = 0.

$$\int \frac{1}{x^6} \left(a + b \operatorname{Artanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c/x^2))^2/x^6, x)

[Out] int((a+b\*arctanh(c/x^2))^2/x^6, x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}\left(\frac{c}{x^2}\right)^2 + 2ab \operatorname{artanh}\left(\frac{c}{x^2}\right) + a^2}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^6,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c/x^2)^2 + 2\*a\*b\*arctanh(c/x^2) + a^2)/x^6, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c/x\*\*2))\*\*2/x\*\*6,x)

[Out] Integral((a + b\*atanh(c/x\*\*2))\*\*2/x\*\*6, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c/x^2))^2/x^6,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)^2/x^6, x)

$$3.182 \quad \int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^3, x]

**Rubi [A]** time = 0.0248424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^3, x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3 dx = \int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3 dx$$

**Mathematica [A]** time = 2.43724, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^3, x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^3, x]

---

**Maple [A]** time = 0.664, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{Arctanh} \left( \frac{c}{x^2} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c/x^2))^3,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c/x^2))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \left( b^3 \operatorname{artanh} \left( \frac{c}{x^2} \right)^3 + 3ab^2 \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 3a^2b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^3 \right) (dx)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2))^3,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c/x^2)^3 + 3\*a\*b^2\*arctanh(c/x^2)^2 + 3\*a^2\*b\*arctanh(c/x^2) + a^3)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2))**3,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a \right)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c/x^2) + a)^3*(d*x)^m, x)
```

$$\mathbf{3.183} \quad \int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^2, x]

**Rubi [A]** time = 0.0247062, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx = \int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

**Mathematica [A]** time = 1.62497, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c/x^2])^2, x]

---

**Maple [A]** time = 0.38, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{Arctanh} \left( \frac{c}{x^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c/x^2))^2,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c/x^2))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \left( b^2 \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 2ab \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^2 \right) (dx)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c/x^2)^2 + 2\*a\*b\*arctanh(c/x^2) + a^2)\*(d\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*atanh(c/x\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a \right)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)^2\*(d\*x)^m, x)



### 3.184 $\int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx$

**Optimal.** Leaf size=75

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(m+1)} - \frac{2bcd(dx)^{m-1} \text{Hypergeometric2F1} \left( 1, \frac{1-m}{4}, \frac{5-m}{4}, \frac{c^2}{x^4} \right)}{1-m^2}$$

[Out]  $((d*x)^{(1+m)*(a+b*\text{ArcTanh}[c/x^2]))/(d*(1+m)) - (2*b*c*d*(d*x)^{-1+m})*\text{Hypergeometric2F1}[1, (1-m)/4, (5-m)/4, c^2/x^4])/(1-m^2)$

**Rubi [A]** time = 0.0557798, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 16, 339, 364}

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(m+1)} - \frac{2bcd(dx)^{m-1} {}_2F_1 \left( 1, \frac{1-m}{4}; \frac{5-m}{4}; \frac{c^2}{x^4} \right)}{1-m^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d*x)^m*(a + b*\text{ArcTanh}[c/x^2]), x]$

[Out]  $((d*x)^{(1+m)*(a+b*\text{ArcTanh}[c/x^2]))/(d*(1+m)) - (2*b*c*d*(d*x)^{-1+m})*\text{Hypergeometric2F1}[1, (1-m)/4, (5-m)/4, c^2/x^4])/(1-m^2)$

#### Rule 6097

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)*(a+b*\text{ArcTanh}[c*x^n])}/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1-c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 16

$\text{Int}[(u_.)*(v_.)^{(m_.)}*((b_.)*(v_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$   
 $\text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

#### Rule 339

$\text{Int}[(c_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow -\text{Dist}[(c*x)^{(m+1)}*(1/x)^{(m+1)}/c, \text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x]$

], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]

### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])/((c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rubi steps

$$\begin{aligned} \int (dx)^m \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) dx &= \frac{(dx)^{1+m} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(1+m)} + \frac{(2bc) \int \frac{(dx)^{1+m}}{\left( 1 - \frac{c^2}{x^4} \right) x^3} dx}{d(1+m)} \\ &= \frac{(dx)^{1+m} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(1+m)} + \frac{(2bcd^2) \int \frac{(dx)^{-2+m}}{1 - \frac{c^2}{x^4}} dx}{1+m} \\ &= \frac{(dx)^{1+m} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(1+m)} - \frac{\left( 2bcd \left( \frac{1}{x} \right)^{-1+m} (dx)^{-1+m} \right) \text{Subst} \left( \int \frac{x^{-m}}{1 - c^2 x^4} dx, x, \frac{1}{x} \right)}{1+m} \\ &= \frac{(dx)^{1+m} \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right)}{d(1+m)} - \frac{2bcd(dx)^{-1+m} {}_2F_1 \left( 1, \frac{1-m}{4}; \frac{5-m}{4}; \frac{c^2}{x^4} \right)}{1-m^2} \end{aligned}$$

**Mathematica [A]** time = 0.0673567, size = 68, normalized size = 0.91

$$\frac{(dx)^m \left( 2bc \text{Hypergeometric2F1} \left( 1, \frac{1}{4} - \frac{m}{4}, \frac{5}{4} - \frac{m}{4}, \frac{c^2}{x^4} \right) + (m-1)x^2 \left( a + b \tanh^{-1} \left( \frac{c}{x^2} \right) \right) \right)}{(m-1)(m+1)x}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c/x^2]), x]

[Out] ((d\*x)^m\*((-1 + m)\*x^2\*(a + b\*ArcTanh[c/x^2]) + 2\*b\*c\*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, c^2/x^4]))/((-1 + m)\*(1 + m)\*x)

**Maple [F]** time = 0.294, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \text{Artanh} \left( \frac{c}{x^2} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c/x**2)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a \right) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] integrate((b\*arctanh(c/x^2) + a)\*(d\*x)^m, x)

$$3.185 \quad \int \frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c/x^2]), x]

**Rubi [A]** time = 0.027983, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c/x^2]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c/x^2]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)} dx = \int \frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)} dx$$

**Mathematica [A]** time = 0.402949, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \tanh^{-1}\left(\frac{c}{x^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c/x^2]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c/x^2]), x]

---

**Maple [A]** time = 0.185, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{Arctanh} \left( \frac{c}{x^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c/x^2)),x)

[Out] int((d\*x)^m/(a+b\*arctanh(c/x^2)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c/x^2)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arctanh(c/x^2) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(dx)^m}{b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c/x^2)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arctanh(c/x^2) + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*atanh(c/x\*\*2)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c/x^2)),x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arctanh(c/x^2) + a), x)

$$3.186 \quad \int \frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c/x^2])^2, x]

**Rubi [A]** time = 0.0263933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c/x^2])^2,x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c/x^2])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.389505, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}\left(\frac{c}{x^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c/x^2])^2,x]



[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c/x^2])^2, x]

**Maple [A]** time = 0.146, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \operatorname{Arctanh} \left( \frac{c}{x^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c/x^2))^2,x)

[Out] int((d\*x)^m/(a+b\*arctanh(c/x^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(d^m x^4 - c^2 d^m) x^m}{b^2 c x \log(x^2 + c) - b^2 c x \log(x^2 - c) + 2 a b c x} + \int -\frac{(d^m (m + 3) x^4 - c^2 d^m (m - 1)) x^m}{b^2 c x^2 \log(x^2 + c) - b^2 c x^2 \log(x^2 - c) + 2 a b c x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c/x^2))^2,x, algorithm="maxima")

[Out] (d^m\*x^4 - c^2\*d^m)\*x^m/(b^2\*c\*x\*log(x^2 + c) - b^2\*c\*x\*log(x^2 - c) + 2\*a\*b\*c\*x) + integrate(-(d^m\*(m + 3)\*x^4 - c^2\*d^m\*(m - 1))\*x^m/(b^2\*c\*x^2\*log(x^2 + c) - b^2\*c\*x^2\*log(x^2 - c) + 2\*a\*b\*c\*x^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(dx)^m}{b^2 \operatorname{artanh} \left( \frac{c}{x^2} \right)^2 + 2 a b \operatorname{artanh} \left( \frac{c}{x^2} \right) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c/x^2))^2,x, algorithm="fricas")

[Out] `integral((d*x)^m/(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m/(a+b*atanh(c/x**2))**2,x)`

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="giac")`

[Out] `integrate((d*x)^m/(b*arctanh(c/x^2) + a)^2, x)`

### 3.187 $\int x^3 (a + b \tanh^{-1}(c\sqrt{x})) dx$

**Optimal.** Leaf size=88

$$\frac{1}{4}x^4(a + b \tanh^{-1}(c\sqrt{x})) + \frac{bx^{5/2}}{20c^3} + \frac{bx^{3/2}}{12c^5} + \frac{b\sqrt{x}}{4c^7} - \frac{b \tanh^{-1}(c\sqrt{x})}{4c^8} + \frac{bx^{7/2}}{28c}$$

[Out] (b\*Sqrt[x])/(4\*c^7) + (b\*x^(3/2))/(12\*c^5) + (b\*x^(5/2))/(20\*c^3) + (b\*x^(7/2))/(28\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(4\*c^8) + (x^4\*(a + b\*ArcTanh[c\*Sqrt[x]]))/4

**Rubi [A]** time = 0.0415064, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 50, 63, 206}

$$\frac{1}{4}x^4(a + b \tanh^{-1}(c\sqrt{x})) + \frac{bx^{5/2}}{20c^3} + \frac{bx^{3/2}}{12c^5} + \frac{b\sqrt{x}}{4c^7} - \frac{b \tanh^{-1}(c\sqrt{x})}{4c^8} + \frac{bx^{7/2}}{28c}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(a + b\*ArcTanh[c\*Sqrt[x]]),x]

[Out] (b\*Sqrt[x])/(4\*c^7) + (b\*x^(3/2))/(12\*c^5) + (b\*x^(5/2))/(20\*c^3) + (b\*x^(7/2))/(28\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(4\*c^8) + (x^4\*(a + b\*ArcTanh[c\*Sqrt[x]]))/4

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x^n))/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \tanh^{-1}(c\sqrt{x})) dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{1}{8}(bc) \int \frac{x^{7/2}}{1-c^2x} dx \\
&= \frac{bx^{7/2}}{28c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{x^{5/2}}{1-c^2x} dx}{8c} \\
&= \frac{bx^{5/2}}{20c^3} + \frac{bx^{7/2}}{28c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{x^{3/2}}{1-c^2x} dx}{8c^3} \\
&= \frac{bx^{3/2}}{12c^5} + \frac{bx^{5/2}}{20c^3} + \frac{bx^{7/2}}{28c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{\sqrt{x}}{1-c^2x} dx}{8c^5} \\
&= \frac{b\sqrt{x}}{4c^7} + \frac{bx^{3/2}}{12c^5} + \frac{bx^{5/2}}{20c^3} + \frac{bx^{7/2}}{28c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{1}{\sqrt{x}(1-c^2x)} dx}{8c^7} \\
&= \frac{b\sqrt{x}}{4c^7} + \frac{bx^{3/2}}{12c^5} + \frac{bx^{5/2}}{20c^3} + \frac{bx^{7/2}}{28c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \text{Subst}\left(\int \frac{1}{1-c^2x^2} dx, x, \sqrt{x}\right)}{4c^7} \\
&= \frac{b\sqrt{x}}{4c^7} + \frac{bx^{3/2}}{12c^5} + \frac{bx^{5/2}}{20c^3} + \frac{bx^{7/2}}{28c} - \frac{b \tanh^{-1}(c\sqrt{x})}{4c^8} + \frac{1}{4}x^4 (a + b \tanh^{-1}(c\sqrt{x}))
\end{aligned}$$

**Mathematica [A]** time = 0.031173, size = 114, normalized size = 1.3

$$\frac{ax^4}{4} + \frac{bx^{5/2}}{20c^3} + \frac{bx^{3/2}}{12c^5} + \frac{b\sqrt{x}}{4c^7} + \frac{b \log(1 - c\sqrt{x})}{8c^8} - \frac{b \log(c\sqrt{x} + 1)}{8c^8} + \frac{bx^{7/2}}{28c} + \frac{1}{4}bx^4 \tanh^{-1}(c\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

[Out]  $(b\sqrt{x})/(4c^7) + (bx^{3/2})/(12c^5) + (bx^{5/2})/(20c^3) + (bx^{7/2})/(28c) + (ax^4)/4 + (bx^4 \operatorname{ArcTanh}[c\sqrt{x}])/4 + (b\log[1 - c\sqrt{x}])/8c^8 - (b\log[1 + c\sqrt{x}])/8c^8$

**Maple [A]** time = 0.026, size = 84, normalized size = 1.

$$\frac{x^4 a}{4} + \frac{bx^4}{4} \operatorname{Artanh}(c\sqrt{x}) + \frac{b}{28c} x^{\frac{7}{2}} + \frac{b}{20c^3} x^{\frac{5}{2}} + \frac{b}{12c^5} x^{\frac{3}{2}} + \frac{b}{4c^7} \sqrt{x} + \frac{b}{8c^8} \ln(c\sqrt{x} - 1) - \frac{b}{8c^8} \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arctanh(c*x^(1/2))),x)`

[Out]  $1/4*x^4*a + 1/4*b*x^4*\operatorname{arctanh}(c*x^{(1/2)}) + 1/28*b*x^{(7/2)}/c + 1/20*b*x^{(5/2)}/c^3 + 1/12*b*x^{(3/2)}/c^5 + 1/4*b*x^{(1/2)}/c^7 + 1/8/c^8*b*\ln(c*x^{(1/2)}-1) - 1/8/c^8*b*\ln(1+c*x^{(1/2)})$

**Maxima [A]** time = 1.023, size = 116, normalized size = 1.32

$$\frac{1}{4} ax^4 + \frac{1}{840} \left( 210x^4 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2 \left( 15c^6x^{\frac{7}{2}} + 21c^4x^{\frac{5}{2}} + 35c^2x^{\frac{3}{2}} + 105\sqrt{x} \right)}{c^8} - \frac{105 \log(c\sqrt{x} + 1)}{c^9} + \frac{105 \log(c\sqrt{x} - 1)}{c^9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")`

[Out]  $1/4*a*x^4 + 1/840*(210*x^4*\operatorname{arctanh}(c*\sqrt{x}) + c*(2*(15*c^6*x^{(7/2)} + 21*c^4*x^{(5/2)} + 35*c^2*x^{(3/2)} + 105*\sqrt{x})/c^8 - 105*\log(c*\sqrt{x} + 1)/c^9 + 105*\log(c*\sqrt{x} - 1)/c^9))*b$

**Fricas [A]** time = 1.78655, size = 213, normalized size = 2.42

$$\frac{210ac^8x^4 + 105(bc^8x^4 - b) \log\left(-\frac{c^2x + 2c\sqrt{x} + 1}{c^2x - 1}\right) + 2(15bc^7x^3 + 21bc^5x^2 + 35bc^3x + 105bc)\sqrt{x}}{840c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2))),x, algorithm="fricas")

[Out] 1/840\*(210\*a\*c^8\*x^4 + 105\*(b\*c^8\*x^4 - b)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1)) + 2\*(15\*b\*c^7\*x^3 + 21\*b\*c^5\*x^2 + 35\*b\*c^3\*x + 105\*b\*c)\*sqrt(x))/c^8

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atanh}(c\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*(1/2))),x)

[Out] Integral(x\*\*3\*(a + b\*atanh(c\*sqrt(x))), x)

**Giac [A]** time = 1.20434, size = 142, normalized size = 1.61

$$\frac{1}{4}ax^4 + \frac{1}{840} \left( 105x^4 \log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right) - c \left( \frac{105 \log(|c\sqrt{x}+1|)}{c^9} - \frac{105 \log(|c\sqrt{x}-1|)}{c^9} - \frac{2(15c^{12}x^{\frac{7}{2}} + 21c^{10}x^{\frac{5}{2}} + 35c^8x^{\frac{3}{2}})}{c^{14}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2))),x, algorithm="giac")

[Out] 1/4\*a\*x^4 + 1/840\*(105\*x^4\*log(-(c\*sqrt(x) + 1)/(c\*sqrt(x) - 1)) - c\*(105\*log(abs(c\*sqrt(x) + 1))/c^9 - 105\*log(abs(c\*sqrt(x) - 1))/c^9 - 2\*(15\*c^12\*x^(7/2) + 21\*c^10\*x^(5/2) + 35\*c^8\*x^(3/2) + 105\*c^6\*sqrt(x))/c^14))\*b

### 3.188 $\int x^2 (a + b \tanh^{-1}(c\sqrt{x})) dx$

**Optimal.** Leaf size=75

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) + \frac{bx^{3/2}}{9c^3} + \frac{b\sqrt{x}}{3c^5} - \frac{b \tanh^{-1}(c\sqrt{x})}{3c^6} + \frac{bx^{5/2}}{15c}$$

[Out] (b\*Sqrt[x])/(3\*c^5) + (b\*x^(3/2))/(9\*c^3) + (b\*x^(5/2))/(15\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(3\*c^6) + (x^3\*(a + b\*ArcTanh[c\*Sqrt[x]]))/3

**Rubi [A]** time = 0.0345661, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 50, 63, 206}

$$\frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) + \frac{bx^{3/2}}{9c^3} + \frac{b\sqrt{x}}{3c^5} - \frac{b \tanh^{-1}(c\sqrt{x})}{3c^6} + \frac{bx^{5/2}}{15c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*Sqrt[x]]),x]

[Out] (b\*Sqrt[x])/(3\*c^5) + (b\*x^(3/2))/(9\*c^3) + (b\*x^(5/2))/(15\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(3\*c^6) + (x^3\*(a + b\*ArcTanh[c\*Sqrt[x]]))/3

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(c\sqrt{x})) dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{1}{6}(bc) \int \frac{x^{5/2}}{1-c^2x} dx \\
&= \frac{bx^{5/2}}{15c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{x^{3/2}}{1-c^2x} dx}{6c} \\
&= \frac{bx^{3/2}}{9c^3} + \frac{bx^{5/2}}{15c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{\sqrt{x}}{1-c^2x} dx}{6c^3} \\
&= \frac{b\sqrt{x}}{3c^5} + \frac{bx^{3/2}}{9c^3} + \frac{bx^{5/2}}{15c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{1}{\sqrt{x}(1-c^2x)} dx}{6c^5} \\
&= \frac{b\sqrt{x}}{3c^5} + \frac{bx^{3/2}}{9c^3} + \frac{bx^{5/2}}{15c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \operatorname{Subst}\left(\int \frac{1}{1-c^2x^2} dx, x, \sqrt{x}\right)}{3c^5} \\
&= \frac{b\sqrt{x}}{3c^5} + \frac{bx^{3/2}}{9c^3} + \frac{bx^{5/2}}{15c} - \frac{b \tanh^{-1}(c\sqrt{x})}{3c^6} + \frac{1}{3}x^3 (a + b \tanh^{-1}(c\sqrt{x}))
\end{aligned}$$

**Mathematica [A]** time = 0.0236325, size = 101, normalized size = 1.35

$$\frac{ax^3}{3} + \frac{bx^{3/2}}{9c^3} + \frac{b\sqrt{x}}{3c^5} + \frac{b \log(1 - c\sqrt{x})}{6c^6} - \frac{b \log(c\sqrt{x} + 1)}{6c^6} + \frac{bx^{5/2}}{15c} + \frac{1}{3}bx^3 \tanh^{-1}(c\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

```
[Out] (b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) + (a*x^3)/3
+ (b*x^3*ArcTanh[c*Sqrt[x]])/3 + (b*Log[1 - c*Sqrt[x]])/(6*c^6) - (b*Log[1
```



+ c\*Sqrt[x]])/(6\*c^6)

**Maple [A]** time = 0.026, size = 75, normalized size = 1.

$$\frac{x^3 a}{3} + \frac{bx^3}{3} \operatorname{Artanh}(c\sqrt{x}) + \frac{b}{15c} x^{\frac{5}{2}} + \frac{b}{9c^3} x^{\frac{3}{2}} + \frac{b}{3c^5} \sqrt{x} + \frac{b}{6c^6} \ln(c\sqrt{x}-1) - \frac{b}{6c^6} \ln(1+c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^(1/2))),x)

[Out] 1/3\*x^3\*a+1/3\*b\*x^3\*arctanh(c\*x^(1/2))+1/15\*b\*x^(5/2)/c+1/9\*b\*x^(3/2)/c^3+1/3\*b\*x^(1/2)/c^5+1/6/c^6\*b\*ln(c\*x^(1/2)-1)-1/6/c^6\*b\*ln(1+c\*x^(1/2))

**Maxima [A]** time = 0.958278, size = 105, normalized size = 1.4

$$\frac{1}{3} ax^3 + \frac{1}{90} \left( 30x^3 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2 \left( 3c^4 x^{\frac{5}{2}} + 5c^2 x^{\frac{3}{2}} + 15\sqrt{x} \right)}{c^6} - \frac{15 \log(c\sqrt{x}+1)}{c^7} + \frac{15 \log(c\sqrt{x}-1)}{c^7} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(1/2))),x, algorithm="maxima")

[Out] 1/3\*a\*x^3 + 1/90\*(30\*x^3\*arctanh(c\*sqrt(x)) + c\*(2\*(3\*c^4\*x^(5/2) + 5\*c^2\*x^(3/2) + 15\*sqrt(x))/c^6 - 15\*log(c\*sqrt(x) + 1)/c^7 + 15\*log(c\*sqrt(x) - 1)/c^7))\*b

**Fricas [A]** time = 1.77937, size = 185, normalized size = 2.47

$$\frac{30ac^6x^3 + 15(bc^6x^3 - b) \log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right) + 2(3bc^5x^2 + 5bc^3x + 15bc)\sqrt{x}}{90c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(1/2))),x, algorithm="fricas")

[Out]  $\frac{1}{90}(30ac^6x^3 + 15(bc^6x^3 - b)\log(-(c^2x + 2c\sqrt{x} + 1)/(c^2x - 1)) + 2(3b^2c^5x^2 + 5b^2c^3x + 15b^2c)\sqrt{x})/c^6$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(c\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c*x**(1/2))),x)`

[Out] `Integral(x**2*(a + b*atanh(c*sqrt(x))), x)`

**Giac [A]** time = 1.17049, size = 131, normalized size = 1.75

$$\frac{1}{3}ax^3 + \frac{1}{90} \left( 15x^3 \log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right) - c \left( \frac{15 \log(|c\sqrt{x}+1|)}{c^7} - \frac{15 \log(|c\sqrt{x}-1|)}{c^7} - \frac{2(3c^8x^{\frac{5}{2}} + 5c^6x^{\frac{3}{2}} + 15c^4\sqrt{x})}{c^{10}} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")`

[Out]  $\frac{1}{3}ax^3 + \frac{1}{90}(15x^3\log(-(c\sqrt{x} + 1)/(c\sqrt{x} - 1)) - c(15\log(\operatorname{abs}(c\sqrt{x} + 1))/c^7 - 15\log(\operatorname{abs}(c\sqrt{x} - 1))/c^7 - 2(3c^8x^{(5/2)} + 5c^6x^{(3/2)} + 15c^4\sqrt{x})/c^{10}))b$

### 3.189 $\int x \left( a + b \tanh^{-1} \left( c\sqrt{x} \right) \right) dx$

**Optimal.** Leaf size=62

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( c\sqrt{x} \right) \right) + \frac{b\sqrt{x}}{2c^3} - \frac{b \tanh^{-1} \left( c\sqrt{x} \right)}{2c^4} + \frac{bx^{3/2}}{6c}$$

[Out] (b\*Sqrt[x])/(2\*c^3) + (b\*x^(3/2))/(6\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(2\*c^4) + (x^2\*(a + b\*ArcTanh[c\*Sqrt[x]]))/2

**Rubi [A]** time = 0.0238849, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6097, 50, 63, 206}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( c\sqrt{x} \right) \right) + \frac{b\sqrt{x}}{2c^3} - \frac{b \tanh^{-1} \left( c\sqrt{x} \right)}{2c^4} + \frac{bx^{3/2}}{6c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcTanh[c\*Sqrt[x]]),x]

[Out] (b\*Sqrt[x])/(2\*c^3) + (b\*x^(3/2))/(6\*c) - (b\*ArcTanh[c\*Sqrt[x]])/(2\*c^4) + (x^2\*(a + b\*ArcTanh[c\*Sqrt[x]]))/2

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(
b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int x(a + b \tanh^{-1}(c\sqrt{x})) dx &= \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x})) - \frac{1}{4}(bc) \int \frac{x^{3/2}}{1 - c^2x} dx \\
&= \frac{bx^{3/2}}{6c} + \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{\sqrt{x}}{1 - c^2x} dx}{4c} \\
&= \frac{b\sqrt{x}}{2c^3} + \frac{bx^{3/2}}{6c} + \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \int \frac{1}{\sqrt{x}(1 - c^2x)} dx}{4c^3} \\
&= \frac{b\sqrt{x}}{2c^3} + \frac{bx^{3/2}}{6c} + \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x})) - \frac{b \operatorname{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, \sqrt{x}\right)}{2c^3} \\
&= \frac{b\sqrt{x}}{2c^3} + \frac{bx^{3/2}}{6c} - \frac{b \tanh^{-1}(c\sqrt{x})}{2c^4} + \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x}))
\end{aligned}$$

**Mathematica [A]** time = 0.0208938, size = 88, normalized size = 1.42

$$\frac{ax^2}{2} + \frac{b\sqrt{x}}{2c^3} + \frac{b \log(1 - c\sqrt{x})}{4c^4} - \frac{b \log(c\sqrt{x} + 1)}{4c^4} + \frac{bx^{3/2}}{6c} + \frac{1}{2}bx^2 \tanh^{-1}(c\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]]), x]
```

```
[Out] (b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*Sqrt
[x]])/2 + (b*Log[1 - c*Sqrt[x]])/(4*c^4) - (b*Log[1 + c*Sqrt[x]])/(4*c^4)
```

**Maple [A]** time = 0.027, size = 66, normalized size = 1.1

$$\frac{ax^2}{2} + \frac{bx^2}{2} \operatorname{Artanh}(c\sqrt{x}) + \frac{b}{6c} x^{\frac{3}{2}} + \frac{b}{2c^3} \sqrt{x} + \frac{b}{4c^4} \ln(c\sqrt{x}-1) - \frac{b}{4c^4} \ln(1+c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^(1/2))),x)`

[Out]  $\frac{1}{2}ax^2 + \frac{1}{2}bx^2 \operatorname{arctanh}(c\sqrt{x}) + \frac{1}{6}bx^{\frac{3}{2}}/c + \frac{1}{2}bx^{\frac{1}{2}}/c^3 + 1/4/c^4 * b * \ln(c\sqrt{x}-1) - 1/4/c^4 * b * \ln(1+c\sqrt{x})$

**Maxima [A]** time = 1.00239, size = 93, normalized size = 1.5

$$\frac{1}{2}ax^2 + \frac{1}{12} \left( 6x^2 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2(c^2x^{\frac{3}{2}} + 3\sqrt{x})}{c^4} - \frac{3 \log(c\sqrt{x}+1)}{c^5} + \frac{3 \log(c\sqrt{x}-1)}{c^5} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")`

[Out]  $\frac{1}{2}ax^2 + \frac{1}{12}(6x^2 \operatorname{arctanh}(c\sqrt{x}) + c(2(c^2x^{\frac{3}{2}} + 3\sqrt{x})/c^4 - 3\log(c\sqrt{x}+1)/c^5 + 3\log(c\sqrt{x}-1)/c^5)) * b$

**Fricas [A]** time = 1.76095, size = 159, normalized size = 2.56

$$\frac{6ac^4x^2 + 3(bc^4x^2 - b) \log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right) + 2(bc^3x + 3bc)\sqrt{x}}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")`

[Out]  $\frac{1}{12}(6ac^4x^2 + 3(bc^4x^2 - b) \log(-(c^2x + 2c\sqrt{x} + 1)/(c^2x - 1)) + 2(bc^3x + 3bc)\sqrt{x})/c^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{atanh}(c\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x\*\*(1/2))),x)

[Out] Integral(x\*(a + b\*atanh(c\*sqrt(x))), x)

**Giac [A]** time = 1.19396, size = 119, normalized size = 1.92

$$\frac{1}{2} ax^2 + \frac{1}{12} \left( 3x^2 \log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right) - c \left( \frac{3 \log(|c\sqrt{x}+1|)}{c^5} - \frac{3 \log(|c\sqrt{x}-1|)}{c^5} - \frac{2(c^4x^{\frac{3}{2}} + 3c^2\sqrt{x})}{c^6} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2))),x, algorithm="giac")

[Out] 1/2\*a\*x^2 + 1/12\*(3\*x^2\*log(-(c\*sqrt(x) + 1)/(c\*sqrt(x) - 1)) - c\*(3\*log(abs(c\*sqrt(x) + 1))/c^5 - 3\*log(abs(c\*sqrt(x) - 1))/c^5 - 2\*(c^4\*x^(3/2) + 3\*c^2\*sqrt(x))/c^6))\*b

### 3.190 $\int (a + b \tanh^{-1}(c\sqrt{x})) dx$

**Optimal.** Leaf size=39

$$ax - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2} + \frac{b\sqrt{x}}{c} + bx \tanh^{-1}(c\sqrt{x})$$

[Out] (b\*Sqrt[x])/c + a\*x - (b\*ArcTanh[c\*Sqrt[x]])/c^2 + b\*x\*ArcTanh[c\*Sqrt[x]]

**Rubi [A]** time = 0.0204042, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6091, 50, 63, 206}

$$ax - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2} + \frac{b\sqrt{x}}{c} + bx \tanh^{-1}(c\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*Sqrt[x]], x]

[Out] (b\*Sqrt[x])/c + a\*x - (b\*ArcTanh[c\*Sqrt[x]])/c^2 + b\*x\*ArcTanh[c\*Sqrt[x]]

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int (a + b \tanh^{-1}(c\sqrt{x})) dx &= ax + b \int \tanh^{-1}(c\sqrt{x}) dx \\
 &= ax + bx \tanh^{-1}(c\sqrt{x}) - \frac{1}{2}(bc) \int \frac{\sqrt{x}}{1-c^2x} dx \\
 &= \frac{b\sqrt{x}}{c} + ax + bx \tanh^{-1}(c\sqrt{x}) - \frac{b \int \frac{1}{\sqrt{x}(1-c^2x)} dx}{2c} \\
 &= \frac{b\sqrt{x}}{c} + ax + bx \tanh^{-1}(c\sqrt{x}) - \frac{b \text{Subst}\left(\int \frac{1}{1-c^2x^2} dx, x, \sqrt{x}\right)}{c} \\
 &= \frac{b\sqrt{x}}{c} + ax - \frac{b \tanh^{-1}(c\sqrt{x})}{c^2} + bx \tanh^{-1}(c\sqrt{x})
 \end{aligned}$$

**Mathematica [A]** time = 0.0244588, size = 42, normalized size = 1.08

$$ax - bc \left( \frac{\tanh^{-1}(c\sqrt{x})}{c^3} - \frac{\sqrt{x}}{c^2} \right) + bx \tanh^{-1}(c\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*Sqrt[x]], x]

[Out] a\*x + b\*x\*ArcTanh[c\*Sqrt[x]] - b\*c\*(-(Sqrt[x]/c^2) + ArcTanh[c\*Sqrt[x]]/c^3)

**Maple [A]** time = 0.024, size = 50, normalized size = 1.3

$$ax + bx \text{Artanh}(c\sqrt{x}) + \frac{b}{c}\sqrt{x} + \frac{b}{2c^2} \ln(c\sqrt{x} - 1) - \frac{b}{2c^2} \ln(1 + c\sqrt{x})$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arctanh(c*x^(1/2)),x)`

[Out]  $a*x+b*x*arctanh(c*x^{(1/2)})+b*x^{(1/2)}/c+1/2*b/c^2*\ln(c*x^{(1/2)}-1)-1/2*b/c^2*\ln(1+c*x^{(1/2)})$

**Maxima [A]** time = 0.971143, size = 72, normalized size = 1.85

$$\frac{1}{2} \left( c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(c\sqrt{x}+1)}{c^3} + \frac{\log(c\sqrt{x}-1)}{c^3} \right) + 2x \operatorname{artanh}(c\sqrt{x}) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="maxima")`

[Out]  $1/2*(c*(2*\sqrt{x})/c^2 - \log(c*\sqrt{x} + 1)/c^3 + \log(c*\sqrt{x} - 1)/c^3) + 2*x*arctanh(c*\sqrt{x}))*b + a*x$

**Fricas [A]** time = 1.71946, size = 131, normalized size = 3.36

$$\frac{2ac^2x + 2bc\sqrt{x} + (bc^2x - b) \log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="fricas")`

[Out]  $1/2*(2*a*c^2*x + 2*b*c*\sqrt{x} + (b*c^2*x - b)*\log(-(c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1)))/c^2$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(c\sqrt{x})) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c\*x\*\*(1/2)),x)

[Out] Integral(a + b\*atanh(c\*sqrt(x)), x)

**Giac [A]** time = 1.22042, size = 90, normalized size = 2.31

$$\frac{1}{2} \left( c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(|c\sqrt{x}+1|)}{c^3} + \frac{\log(|c\sqrt{x}-1|)}{c^3} \right) + x \log \left( -\frac{c\sqrt{x}+1}{c\sqrt{x}-1} \right) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^(1/2)),x, algorithm="giac")

[Out] 1/2\*(c\*(2\*sqrt(x)/c^2 - log(abs(c\*sqrt(x) + 1))/c^3 + log(abs(c\*sqrt(x) - 1))/c^3) + x\*log(-(c\*sqrt(x) + 1)/(c\*sqrt(x) - 1)))\*b + a\*x

$$3.191 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x} dx$$

**Optimal.** Leaf size=29

$$-b\text{PolyLog}(2, -c\sqrt{x}) + b\text{PolyLog}(2, c\sqrt{x}) + a \log(x)$$

[Out] a\*Log[x] - b\*PolyLog[2, -(c\*Sqrt[x])] + b\*PolyLog[2, c\*Sqrt[x]]

**Rubi [A]** time = 0.0315208, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {6095, 5912}

$$-b\text{PolyLog}(2, -c\sqrt{x}) + b\text{PolyLog}(2, c\sqrt{x}) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*Sqrt[x]])/x, x]

[Out] a\*Log[x] - b\*PolyLog[2, -(c\*Sqrt[x])] + b\*PolyLog[2, c\*Sqrt[x]]

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x} dx &= 2 \text{Subst} \left( \int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, \sqrt{x} \right) \\ &= a \log(x) - b\text{Li}_2(-c\sqrt{x}) + b\text{Li}_2(c\sqrt{x}) \end{aligned}$$

**Mathematica [A]** time = 0.0114324, size = 29, normalized size = 1.

$$-b\text{PolyLog}(2, -c\sqrt{x}) + b\text{PolyLog}(2, c\sqrt{x}) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])/x,x]

[Out] a\*Log[x] - b\*PolyLog[2, -(c\*Sqrt[x])] + b\*PolyLog[2, c\*Sqrt[x]]

**Maple [B]** time = 0.036, size = 63, normalized size = 2.2

$$2a \ln(c\sqrt{x}) + 2b \ln(c\sqrt{x}) \text{Artanh}(c\sqrt{x}) - b \text{dilog}(c\sqrt{x}) - b \text{dilog}(1 + c\sqrt{x}) - b \ln(c\sqrt{x}) \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))/x,x)

[Out] 2\*a\*ln(c\*x^(1/2))+2\*b\*ln(c\*x^(1/2))\*arctanh(c\*x^(1/2))-b\*dilog(c\*x^(1/2))-b\*dilog(1+c\*x^(1/2))-b\*ln(c\*x^(1/2))\*ln(1+c\*x^(1/2))

**Maxima [B]** time = 1.49312, size = 82, normalized size = 2.83

$$-(\log(c\sqrt{x}) \log(-c\sqrt{x} + 1) + \text{Li}_2(-c\sqrt{x} + 1))b + (\log(c\sqrt{x} + 1) \log(-c\sqrt{x}) + \text{Li}_2(c\sqrt{x} + 1))b + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x,x, algorithm="maxima")

[Out] -(log(c\*sqrt(x))\*log(-c\*sqrt(x) + 1) + dilog(-c\*sqrt(x) + 1))\*b + (log(c\*sqrt(x) + 1)\*log(-c\*sqrt(x)) + dilog(c\*sqrt(x) + 1))\*b + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \text{artanh}(c\sqrt{x}) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*sqrt(x)) + a)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(c\sqrt{x})}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))/x,x)

[Out] Integral((a + b\*atanh(c\*sqrt(x)))/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(c\sqrt{x}) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)/x, x)

$$3.192 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^2} dx$$

**Optimal.** Leaf size=40

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{x} + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{\sqrt{x}}$$

[Out]  $-\left(\frac{bc}{\sqrt{x}}\right) + bc^2 \operatorname{ArcTanh}[c\sqrt{x}] - \left(a + b \operatorname{ArcTanh}[c\sqrt{x}]\right)/x$

**Rubi [A]** time = 0.0229893, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 51, 63, 206}

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{x} + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{ArcTanh}[c\sqrt{x}])/x^2, x]$

[Out]  $-\left(\frac{bc}{\sqrt{x}}\right) + bc^2 \operatorname{ArcTanh}[c\sqrt{x}] - \left(a + b \operatorname{ArcTanh}[c\sqrt{x}]\right)/x$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^2} dx &= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + \frac{1}{2}(bc) \int \frac{1}{x^{3/2}(1 - c^2x)} dx \\
&= -\frac{bc}{\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + \frac{1}{2}(bc^3) \int \frac{1}{\sqrt{x}(1 - c^2x)} dx \\
&= -\frac{bc}{\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{x} + (bc^3) \text{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{\sqrt{x}} + bc^2 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0241474, size = 67, normalized size = 1.68

$$-\frac{a}{x} - \frac{1}{2}bc^2 \log(1 - c\sqrt{x}) + \frac{1}{2}bc^2 \log(c\sqrt{x} + 1) - \frac{bc}{\sqrt{x}} - \frac{b \tanh^{-1}(c\sqrt{x})}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^2, x]
```

```
[Out] -(a/x) - (b*c)/Sqrt[x] - (b*ArcTanh[c*Sqrt[x]])/x - (b*c^2*Log[1 - c*Sqrt[x]
]])/2 + (b*c^2*Log[1 + c*Sqrt[x]])/2
```

**Maple [A]** time = 0.033, size = 55, normalized size = 1.4

$$-\frac{a}{x} - \frac{b}{x} \operatorname{Arctanh}(c\sqrt{x}) - bc \frac{1}{\sqrt{x}} - \frac{c^2 b}{2} \ln(c\sqrt{x} - 1) + \frac{c^2 b}{2} \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))/x^2,x)

[Out] -a/x-b/x\*arctanh(c\*x^(1/2))-b\*c/x^(1/2)-1/2\*c^2\*b\*ln(c\*x^(1/2)-1)+1/2\*c^2\*b\*ln(1+c\*x^(1/2))

**Maxima [A]** time = 0.961342, size = 69, normalized size = 1.72

$$\frac{1}{2} \left( \left( c \log(c\sqrt{x} + 1) - c \log(c\sqrt{x} - 1) - \frac{2}{\sqrt{x}} \right) c - \frac{2 \operatorname{artanh}(c\sqrt{x})}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] 1/2\*((c\*log(c\*sqrt(x) + 1) - c\*log(c\*sqrt(x) - 1) - 2/sqrt(x))\*c - 2\*arctanh(c\*sqrt(x))/x)\*b - a/x

**Fricas [A]** time = 1.74626, size = 122, normalized size = 3.05

$$\frac{2bc\sqrt{x} - (bc^2x - b) \log\left(-\frac{c^2x + 2c\sqrt{x} + 1}{c^2x - 1}\right) + 2a}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x^2,x, algorithm="fricas")

[Out] -1/2\*(2\*b\*c\*sqrt(x) - (b\*c^2\*x - b)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1)) + 2\*a)/x



**Sympy [A]** time = 40.8538, size = 231, normalized size = 5.78

$$\begin{cases} -\frac{a}{x} + \frac{b \operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c = -\sqrt{\frac{1}{x}} \\ -\frac{a}{x} - \frac{b \operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c = \sqrt{\frac{1}{x}} \\ -\frac{ac^4x^{\frac{5}{2}}}{c^2x^2-x^2} + \frac{a\sqrt{x}}{c^2x^2-x^2} + \frac{bc^4x^{\frac{5}{2}} \operatorname{atanh}(c\sqrt{x})}{c^2x^2-x^2} - \frac{bc^3x^2}{c^2x^2-x^2} - \frac{2bc^2x^{\frac{3}{2}} \operatorname{atanh}(c\sqrt{x})}{c^2x^2-x^2} + \frac{bcx}{c^2x^2-x^2} + \frac{b\sqrt{x} \operatorname{atanh}(c\sqrt{x})}{c^2x^2-x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))/x\*\*2,x)

[Out] Piecewise((-a/x + b\*atanh(sqrt(x)\*sqrt(1/x))/x, Eq(c, -sqrt(1/x))), (-a/x - b\*atanh(sqrt(x)\*sqrt(1/x))/x, Eq(c, sqrt(1/x))), (-a\*c\*\*4\*x\*\*(5/2)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + a\*sqrt(x)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + b\*c\*\*4\*x\*\*(5/2)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - b\*c\*\*3\*x\*\*2/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 2\*b\*c\*\*2\*x\*\*(3/2)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + b\*c\*x/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + b\*sqrt(x)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)), True))

**Giac [A]** time = 1.33223, size = 90, normalized size = 2.25

$$\frac{1}{2}bc^2 \log(c\sqrt{x} + 1) - \frac{1}{2}bc^2 \log(c\sqrt{x} - 1) - \frac{b \log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{2x} - \frac{bc\sqrt{x} + a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x^2,x, algorithm="giac")

[Out] 1/2\*b\*c^2\*log(c\*sqrt(x) + 1) - 1/2\*b\*c^2\*log(c\*sqrt(x) - 1) - 1/2\*b\*log(-(c\*sqrt(x) + 1)/(c\*sqrt(x) - 1))/x - (b\*c\*sqrt(x) + a)/x

$$3.193 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^3} dx$$

**Optimal.** Leaf size=60

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{bc^3}{2\sqrt{x}} + \frac{1}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{6x^{3/2}}$$

[Out]  $-(b*c)/(6*x^(3/2)) - (b*c^3)/(2*sqrt[x]) + (b*c^4*ArcTanh[c*sqrt[x]])/2 - (a + b*ArcTanh[c*sqrt[x]])/(2*x^2)$

**Rubi [A]** time = 0.0263707, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 51, 63, 206}

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{2x^2} - \frac{bc^3}{2\sqrt{x}} + \frac{1}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{6x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*sqrt[x]])/x^3, x]

[Out]  $-(b*c)/(6*x^(3/2)) - (b*c^3)/(2*sqrt[x]) + (b*c^4*ArcTanh[c*sqrt[x]])/2 - (a + b*ArcTanh[c*sqrt[x]])/(2*x^2)$

### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^3} dx &= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} + \frac{1}{4}(bc) \int \frac{1}{x^{5/2}(1 - c^2x)} dx \\
&= -\frac{bc}{6x^{3/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} + \frac{1}{4}(bc^3) \int \frac{1}{x^{3/2}(1 - c^2x)} dx \\
&= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} + \frac{1}{4}(bc^5) \int \frac{1}{\sqrt{x}(1 - c^2x)} dx \\
&= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2} + \frac{1}{2}(bc^5) \text{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} + \frac{1}{2}bc^4 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0258193, size = 86, normalized size = 1.43

$$-\frac{a}{2x^2} - \frac{bc^3}{2\sqrt{x}} - \frac{1}{4}bc^4 \log(1 - c\sqrt{x}) + \frac{1}{4}bc^4 \log(c\sqrt{x} + 1) - \frac{bc}{6x^{3/2}} - \frac{b \tanh^{-1}(c\sqrt{x})}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])/x^3, x]

[Out] -a/(2\*x^2) - (b\*c)/(6\*x^(3/2)) - (b\*c^3)/(2\*Sqrt[x]) - (b\*ArcTanh[c\*Sqrt[x]])/(2\*x^2) - (b\*c^4\*Log[1 - c\*Sqrt[x]])/4 + (b\*c^4\*Log[1 + c\*Sqrt[x]])/4

---

**Maple [A]** time = 0.033, size = 64, normalized size = 1.1

$$-\frac{a}{2x^2} - \frac{b}{2x^2} \operatorname{Arctanh}(c\sqrt{x}) - \frac{c^4 b}{4} \ln(c\sqrt{x} - 1) - \frac{bc}{6} x^{-\frac{3}{2}} - \frac{bc^3}{2} \frac{1}{\sqrt{x}} + \frac{c^4 b}{4} \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/x^3,x)`

[Out] `-1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(1/2))-1/4*c^4*b*ln(c*x^(1/2)-1)-1/6*b*c/x^(3/2)-1/2*b*c^3/x^(1/2)+1/4*c^4*b*ln(1+c*x^(1/2))`

---

**Maxima [A]** time = 0.956247, size = 86, normalized size = 1.43

$$\frac{1}{12} \left( \left( 3c^3 \log(c\sqrt{x} + 1) - 3c^3 \log(c\sqrt{x} - 1) - \frac{2(3c^2x + 1)}{x^{\frac{3}{2}}} \right) c - \frac{6 \operatorname{artanh}(c\sqrt{x})}{x^2} \right) b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="maxima")`

[Out] `1/12*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*b - 1/2*a/x^2`

---

**Fricas [A]** time = 1.65234, size = 149, normalized size = 2.48

$$\frac{3(bc^4x^2 - b) \log\left(-\frac{c^2x + 2c\sqrt{x} + 1}{c^2x - 1}\right) - 2(3bc^3x + bc)\sqrt{x} - 6a}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="fricas")`

[Out] `1/12*(3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(3*b*c^3*x + b*c)*sqrt(x) - 6*a)/x^2`

---

**Sympy [A]** time = 179.21, size = 342, normalized size = 5.7

$$\left( \begin{array}{l} -\frac{a}{2x^2} + \frac{b \operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ \frac{a}{2x^2} - \frac{b \operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{3ac^2x^{\frac{3}{2}}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{3a\sqrt{x}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{3bc^6x^{\frac{7}{2}} \operatorname{atanh}(c\sqrt{x})}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{3bc^5x^3}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{3bc^4x^{\frac{5}{2}} \operatorname{atanh}(c\sqrt{x})}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{2bc^3x^2}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} - \frac{3bc^2x^{\frac{3}{2}} \operatorname{atanh}(c\sqrt{x})}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} + \frac{bcx}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))/x\*\*3,x)

[Out] Piecewise((-a/(2\*x\*\*2) + b\*atanh(sqrt(x)\*sqrt(1/x))/(2\*x\*\*2), Eq(c, -sqrt(1/x))), (-a/(2\*x\*\*2) - b\*atanh(sqrt(x)\*sqrt(1/x))/(2\*x\*\*2), Eq(c, sqrt(1/x))), (-3\*a\*c\*\*2\*x\*\*(3/2)/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 3\*a\*sqrt(x)/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 3\*b\*c\*\*6\*x\*\*(7/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*b\*c\*\*5\*x\*\*3/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*b\*c\*\*4\*x\*\*(5/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 2\*b\*c\*\*3\*x\*\*2/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 3\*b\*c\*\*2\*x\*\*(3/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + b\*c\*x/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 3\*b\*sqrt(x)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)), True))

---

**Giac [A]** time = 1.32737, size = 105, normalized size = 1.75

$$\frac{1}{4}bc^4 \log(c\sqrt{x} + 1) - \frac{1}{4}bc^4 \log(c\sqrt{x} - 1) - \frac{b \log\left(\frac{-c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{4x^2} - \frac{3bc^3x^{\frac{3}{2}} + bc\sqrt{x} + 3a}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))/x^3,x, algorithm="giac")

[Out] 1/4\*b\*c^4\*log(c\*sqrt(x) + 1) - 1/4\*b\*c^4\*log(c\*sqrt(x) - 1) - 1/4\*b\*log(-(c\*sqrt(x) + 1)/(c\*sqrt(x) - 1))/x^2 - 1/6\*(3\*b\*c^3\*x^(3/2) + b\*c\*sqrt(x) + 3\*a)/x^2

$$3.194 \quad \int \frac{a+b \tanh^{-1}(c\sqrt{x})}{x^4} dx$$

**Optimal.** Leaf size=73

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} + \frac{1}{3}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{15x^{5/2}}$$

[Out]  $-(b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*sqrt[x]) + (b*c^6*ArcTan[c*sqrt[x]])/3 - (a + b*ArcTan[c*sqrt[x]])/(3*x^3)$

**Rubi [A]** time = 0.0335061, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {6097, 51, 63, 206}

$$-\frac{a+b \tanh^{-1}(c\sqrt{x})}{3x^3} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} + \frac{1}{3}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{bc}{15x^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTan[c\*sqrt[x]])/x^4, x]

[Out]  $-(b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*sqrt[x]) + (b*c^6*ArcTan[c*sqrt[x]])/3 - (a + b*ArcTan[c*sqrt[x]])/(3*x^3)$

### Rule 6097

```
Int[((a_.) + ArcTan[c_.*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(c\sqrt{x})}{x^4} dx &= -\frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{6}(bc) \int \frac{1}{x^{7/2}(1-c^2x)} dx \\
&= -\frac{bc}{15x^{5/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{6}(bc^3) \int \frac{1}{x^{5/2}(1-c^2x)} dx \\
&= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{6}(bc^5) \int \frac{1}{x^{3/2}(1-c^2x)} dx \\
&= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{6}(bc^7) \int \frac{1}{\sqrt{x}(1-c^2x)} dx \\
&= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3} + \frac{1}{3}(bc^7) \text{Subst}\left(\int \frac{1}{1-c^2x^2} dx, x, \sqrt{x}\right) \\
&= -\frac{bc}{15x^{5/2}} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} + \frac{1}{3}bc^6 \tanh^{-1}(c\sqrt{x}) - \frac{a + b \tanh^{-1}(c\sqrt{x})}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0302242, size = 99, normalized size = 1.36

$$-\frac{a}{3x^3} - \frac{bc^3}{9x^{3/2}} - \frac{bc^5}{3\sqrt{x}} - \frac{1}{6}bc^6 \log(1 - c\sqrt{x}) + \frac{1}{6}bc^6 \log(c\sqrt{x} + 1) - \frac{bc}{15x^{5/2}} - \frac{b \tanh^{-1}(c\sqrt{x})}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^4, x]
```

[Out]  $-a/(3x^3) - (b*c)/(15*x^{(5/2)}) - (b*c^3)/(9*x^{(3/2)}) - (b*c^5)/(3*\text{Sqrt}[x])$   
 $- (b*\text{ArcTanh}[c*\text{Sqrt}[x]])/(3*x^3) - (b*c^6*\text{Log}[1 - c*\text{Sqrt}[x]])/6 + (b*c^6*\text{Log}[1 + c*\text{Sqrt}[x]])/6$

**Maple [A]** time = 0.037, size = 73, normalized size = 1.

$$-\frac{a}{3x^3} - \frac{b}{3x^3} \text{Artanh}(c\sqrt{x}) - \frac{c^6 b}{6} \ln(c\sqrt{x} - 1) - \frac{bc}{15} x^{-\frac{5}{2}} - \frac{bc^3}{9} x^{-\frac{3}{2}} - \frac{bc^5}{3} \frac{1}{\sqrt{x}} + \frac{c^6 b}{6} \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(1/2)))/x^4,x)`

[Out]  $-1/3*a/x^3 - 1/3*b/x^3*\text{arctanh}(c*x^{(1/2)}) - 1/6*c^6*b*\ln(c*x^{(1/2)} - 1) - 1/15*b*c/x^{(5/2)}$   
 $- 1/9*b*c^3/x^{(3/2)} - 1/3*b*c^5/x^{(1/2)} + 1/6*c^6*b*\ln(1+c*x^{(1/2)})$

**Maxima [A]** time = 0.957319, size = 97, normalized size = 1.33

$$\frac{1}{90} \left( \left( 15c^5 \log(c\sqrt{x} + 1) - 15c^5 \log(c\sqrt{x} - 1) - \frac{2(15c^4x^2 + 5c^2x + 3)}{x^{\frac{5}{2}}} \right) c - \frac{30 \text{artanh}(c\sqrt{x})}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="maxima")`

[Out]  $1/90*((15*c^5*\log(c*\text{sqrt}(x) + 1) - 15*c^5*\log(c*\text{sqrt}(x) - 1) - 2*(15*c^4*x^2 + 5*c^2*x + 3)/x^{(5/2)})*c - 30*\text{arctanh}(c*\text{sqrt}(x))/x^3)*b - 1/3*a/x^3$

**Fricas [A]** time = 1.62725, size = 174, normalized size = 2.38

$$\frac{15(bc^6x^3 - b) \log\left(-\frac{c^2x + 2c\sqrt{x} + 1}{c^2x - 1}\right) - 2(15bc^5x^2 + 5bc^3x + 3bc)\sqrt{x} - 30a}{90x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="fricas")`



[Out]  $\frac{1}{90}*(15*(b*c^6*x^3 - b)*\log(-(c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1)) - 2*(15*b*c^5*x^2 + 5*b*c^3*x + 3*b*c)*\sqrt{x} - 30*a)/x^3$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**(1/2)))/x**4,x)`

[Out] Timed out

**Giac [A]** time = 1.39889, size = 119, normalized size = 1.63

$$\frac{1}{6}bc^6 \log(c\sqrt{x} + 1) - \frac{1}{6}bc^6 \log(c\sqrt{x} - 1) - \frac{b \log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{6x^3} - \frac{15bc^5x^{\frac{5}{2}} + 5bc^3x^{\frac{3}{2}} + 3bc\sqrt{x} + 15a}{45x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="giac")`

[Out]  $\frac{1}{6}*b*c^6*\log(c*\sqrt{x} + 1) - \frac{1}{6}*b*c^6*\log(c*\sqrt{x} - 1) - \frac{1}{6}*b*\log(-(c*\sqrt{x} + 1)/(c*\sqrt{x} - 1))/x^3 - \frac{1}{45}*(15*b*c^5*x^{(5/2)} + 5*b*c^3*x^{(3/2)} + 3*b*c*\sqrt{x} + 15*a)/x^3$

### 3.195 $\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$

**Optimal.** Leaf size=211

$$\frac{bx^{5/2}(a + b \tanh^{-1}(c\sqrt{x}))}{10c^3} + \frac{bx^{3/2}(a + b \tanh^{-1}(c\sqrt{x}))}{6c^5} + \frac{ab\sqrt{x}}{2c^7} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{4c^8} + \frac{1}{4}x^4(a + b \tanh^{-1}(c\sqrt{x}))^2$$

[Out] (a\*b\*Sqrt[x])/(2\*c^7) + (71\*b^2\*x)/(420\*c^6) + (3\*b^2\*x^2)/(70\*c^4) + (b^2\*x^3)/(84\*c^2) + (b^2\*Sqrt[x]\*ArcTanh[c\*Sqrt[x]])/(2\*c^7) + (b\*x^(3/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(6\*c^5) + (b\*x^(5/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(10\*c^3) + (b\*x^(7/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(14\*c) - (a + b\*ArcTanh[c\*Sqrt[x]])^2/(4\*c^8) + (x^4\*(a + b\*ArcTanh[c\*Sqrt[x]])^2)/4 + (44\*b^2\*Log[1 - c^2\*x])/(105\*c^8)

**Rubi [F]** time = 0.0251889, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

[Out] Defer[Int][x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

Rubi steps

$$\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^2 dx = \int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$$

**Mathematica [A]** time = 0.112294, size = 224, normalized size = 1.06

$$\frac{105a^2c^8x^4 + 30abc^7x^{7/2} + 42abc^5x^{5/2} + 70abc^3x^{3/2} + 2bc\sqrt{x} \tanh^{-1}(c\sqrt{x}) (105ac^7x^{7/2} + b(15c^6x^3 + 21c^4x^2 + 35c^2x + 10))}{105c^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

[Out] (210\*a\*b\*c\*Sqrt[x] + 71\*b^2\*c^2\*x + 70\*a\*b\*c^3\*x^(3/2) + 18\*b^2\*c^4\*x^2 + 4\*2\*a\*b\*c^5\*x^(5/2) + 5\*b^2\*c^6\*x^3 + 30\*a\*b\*c^7\*x^(7/2) + 105\*a^2\*c^8\*x^4 + 2\*b\*c\*Sqrt[x]\*(105\*a\*c^7\*x^(7/2) + b\*(105 + 35\*c^2\*x + 21\*c^4\*x^2 + 15\*c^6\*x^3))\*ArcTanh[c\*Sqrt[x]] + 105\*b^2\*(-1 + c^8\*x^4)\*ArcTanh[c\*Sqrt[x]]^2 + b\*(105\*a + 176\*b)\*Log[1 - c\*Sqrt[x]] - 105\*a\*b\*Log[1 + c\*Sqrt[x]] + 176\*b^2\*Log[1 + c\*Sqrt[x]])/(420\*c^8)

**Maple [B]** time = 0.056, size = 396, normalized size = 1.9

$$\frac{b^2}{2c^7} \operatorname{Arctanh}(c\sqrt{x}) \sqrt{x} + \frac{ab}{10c^3} x^{\frac{5}{2}} + \frac{ab}{14c} x^{\frac{7}{2}} + \frac{b^2}{14c} \operatorname{Arctanh}(c\sqrt{x}) x^{\frac{7}{2}} + \frac{abx^4}{2} \operatorname{Arctanh}(c\sqrt{x}) - \frac{b^2}{4c^8} \operatorname{Arctanh}(c\sqrt{x}) \ln(1 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x^(1/2)))^2,x)

[Out] 1/14/c\*b^2\*arctanh(c\*x^(1/2))\*x^(7/2)+1/2\*a\*b\*x^4\*arctanh(c\*x^(1/2))-1/4/c^8\*b^2\*arctanh(c\*x^(1/2))\*ln(1+c\*x^(1/2))+1/10/c^3\*a\*b\*x^(5/2)+1/14/c\*x^(7/2)\*a\*b+1/10/c^3\*b^2\*arctanh(c\*x^(1/2))\*x^(5/2)+1/6/c^5\*b^2\*arctanh(c\*x^(1/2))\*x^(3/2)+1/6/c^5\*a\*b\*x^(3/2)-1/8/c^8\*b^2\*ln(c\*x^(1/2)-1)\*ln(1/2+1/2\*c\*x^(1/2))+1/8/c^8\*b^2\*ln(-1/2\*c\*x^(1/2)+1/2)\*ln(1/2+1/2\*c\*x^(1/2))+1/4/c^8\*b^2\*arctanh(c\*x^(1/2))\*ln(c\*x^(1/2)-1)-1/8/c^8\*b^2\*ln(-1/2\*c\*x^(1/2)+1/2)\*ln(1+c\*x^(1/2))+1/4/c^8\*a\*b\*ln(c\*x^(1/2)-1)-1/4/c^8\*a\*b\*ln(1+c\*x^(1/2))+71/420\*b^2\*x/c^6+1/84\*b^2\*x^3/c^2+1/4\*b^2\*x^4\*arctanh(c\*x^(1/2))^2+1/16/c^8\*b^2\*ln(1+c\*x^(1/2))^2+44/105/c^8\*b^2\*ln(c\*x^(1/2)-1)+44/105/c^8\*b^2\*ln(1+c\*x^(1/2))+1/16/c^8\*b^2\*ln(c\*x^(1/2)-1)^2+3/70\*b^2\*x^2/c^4+1/4\*a^2\*x^4+1/2\*a\*b\*x^(1/2)/c^7+1/2\*b^2\*arctanh(c\*x^(1/2))\*x^(1/2)/c^7

**Maxima [A]** time = 0.987937, size = 358, normalized size = 1.7

$$\frac{1}{4} b^2 x^4 \operatorname{artanh}(c\sqrt{x})^2 + \frac{1}{4} a^2 x^4 + \frac{1}{420} \left( 210 x^4 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2 \left( 15 c^6 x^{\frac{7}{2}} + 21 c^4 x^{\frac{5}{2}} + 35 c^2 x^{\frac{3}{2}} + 105 \sqrt{x} \right)}{c^8} - \frac{105 \log}{c^8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="maxima")

```
[Out] 1/4*b^2*x^4*arctanh(c*sqrt(x))^2 + 1/4*a^2*x^4 + 1/420*(210*x^4*arctanh(c*sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9))*a*b + 1/1680*(4*c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9)*arctanh(c*sqrt(x)) + (20*c^6*x^3 + 72*c^4*x^2 + 284*c^2*x - 2*(105*log(c*sqrt(x) - 1) - 352)*log(c*sqrt(x) + 1) + 105*log(c*sqrt(x) + 1)^2 + 105*log(c*sqrt(x) - 1)^2 + 704*log(c*sqrt(x) - 1))/c^8)*b^2
```

**Fricas [A]** time = 2.12589, size = 662, normalized size = 3.14

$$420 a^2 c^8 x^4 + 20 b^2 c^6 x^3 + 72 b^2 c^4 x^2 + 284 b^2 c^2 x + 105 (b^2 c^8 x^4 - b^2) \log\left(-\frac{c^2 x + 2c\sqrt{x} + 1}{c^2 x - 1}\right)^2 + 4 (105 abc^8 - 105 ab + 176 b^2) \log(c\sqrt{x} + 1) - 4 (105 abc^8 - 105 ab - 176 b^2) \log(c\sqrt{x} - 1) + 4 (105 a^2 b c^8 x^4 - 105 a^2 b c^8 + (15 b^2 c^7 x^3 + 21 b^2 c^5 x^2 + 35 b^2 c^3 x + 105 b^2 c) \sqrt{x}) \log\left(-\frac{c^2 x + 2c\sqrt{x} + 1}{c^2 x - 1}\right) + 8 (15 a^2 b c^7 x^3 + 21 a^2 b c^5 x^2 + 35 a^2 b c^3 x + 105 a^2 b c) \sqrt{x} / c^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] 1/1680*(420*a^2*c^8*x^4 + 20*b^2*c^6*x^3 + 72*b^2*c^4*x^2 + 284*b^2*c^2*x + 105*(b^2*c^8*x^4 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(105*a*b*c^8 - 105*a*b + 176*b^2)*log(c*sqrt(x) + 1) - 4*(105*a*b*c^8 - 105*a*b - 176*b^2)*log(c*sqrt(x) - 1) + 4*(105*a*b*c^8*x^4 - 105*a*b*c^8 + (15*b^2*c^7*x^3 + 21*b^2*c^5*x^2 + 35*b^2*c^3*x + 105*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(15*a*b*c^7*x^3 + 21*a*b*c^5*x^2 + 35*a*b*c^3*x + 105*a*b*c)*sqrt(x))/c^8
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{atanh}(c\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atanh(c*x**(1/2)))**2,x)
```

```
[Out] Integral(x**3*(a + b*atanh(c*sqrt(x)))**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^3, x)
```

### 3.196 $\int x^2 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$

**Optimal.** Leaf size=173

$$\frac{2bx^{3/2}(a + b \tanh^{-1}(c\sqrt{x}))}{9c^3} + \frac{2ab\sqrt{x}}{3c^5} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{3c^6} + \frac{1}{3}x^3(a + b \tanh^{-1}(c\sqrt{x}))^2 + \frac{2bx^{5/2}(a + b \tanh^{-1}(c\sqrt{x}))}{15c}$$

[Out] (2\*a\*b\*Sqrt[x])/(3\*c^5) + (8\*b^2\*x)/(45\*c^4) + (b^2\*x^2)/(30\*c^2) + (2\*b^2\*Sqrt[x]\*ArcTanh[c\*Sqrt[x]])/(3\*c^5) + (2\*b\*x^(3/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(9\*c^3) + (2\*b\*x^(5/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(15\*c) - (a + b\*ArcTanh[c\*Sqrt[x]])^2/(3\*c^6) + (x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^2)/3 + (23\*b^2\*Log[1 - c^2\*x])/(45\*c^6)

**Rubi [F]** time = 0.023916, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

[Out] Defer[Int][x^2\*(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

Rubi steps

$$\int x^2 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx = \int x^2 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$$

**Mathematica [A]** time = 0.103839, size = 194, normalized size = 1.12

$$\frac{30a^2c^6x^3 + 12abc^5x^{5/2} + 20abc^3x^{3/2} + 4bc\sqrt{x} \tanh^{-1}(c\sqrt{x}) (15ac^5x^{5/2} + b(3c^4x^2 + 5c^2x + 15)) + 60abc\sqrt{x} + 2b(15a + 2b^2)}{90c^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

```
[Out] (60*a*b*c*Sqrt[x] + 16*b^2*c^2*x + 20*a*b*c^3*x^(3/2) + 3*b^2*c^4*x^2 + 12*
a*b*c^5*x^(5/2) + 30*a^2*c^6*x^3 + 4*b*c*Sqrt[x]*(15*a*c^5*x^(5/2) + b*(15
+ 5*c^2*x + 3*c^4*x^2))*ArcTanh[c*Sqrt[x]] + 30*b^2*(-1 + c^6*x^3)*ArcTanh[
c*Sqrt[x]]^2 + 2*b*(15*a + 23*b)*Log[1 - c*Sqrt[x]] - 30*a*b*Log[1 + c*Sqrt
[x]] + 46*b^2*Log[1 + c*Sqrt[x]])/(90*c^6)
```

**Maple [B]** time = 0.049, size = 358, normalized size = 2.1

$$\frac{x^3 a^2}{3} + \frac{x^3 b^2}{3} \left( \operatorname{Arctanh}(c\sqrt{x}) \right)^2 + \frac{2b^2}{15c} \operatorname{Arctanh}(c\sqrt{x}) x^{\frac{5}{2}} + \frac{2b^2}{9c^3} \operatorname{Arctanh}(c\sqrt{x}) x^{\frac{3}{2}} + \frac{2b^2}{3c^5} \operatorname{Arctanh}(c\sqrt{x}) \sqrt{x} + \frac{b^2}{3c^6} \operatorname{Arctanh}(c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] 1/3*x^3*a^2+1/3*b^2*x^3*arctanh(c*x^(1/2))^2+2/15/c*b^2*arctanh(c*x^(1/2))*
x^(5/2)+2/9/c^3*b^2*arctanh(c*x^(1/2))*x^(3/2)+2/3*b^2*arctanh(c*x^(1/2))*x
^(1/2)/c^5+1/3/c^6*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/3/c^6*b^2*arcta
nh(c*x^(1/2))*ln(1+c*x^(1/2))-1/6/c^6*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1
/2))+1/12/c^6*b^2*ln(c*x^(1/2)-1)^2+1/6/c^6*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1
/2+1/2*c*x^(1/2))-1/6/c^6*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/12/c
^6*b^2*ln(1+c*x^(1/2))^2+1/30*b^2*x^2/c^2+8/45*b^2*x/c^4+23/45/c^6*b^2*ln(c
*x^(1/2)-1)+23/45/c^6*b^2*ln(1+c*x^(1/2))+2/3*a*b*x^3*arctanh(c*x^(1/2))+2/
15/c*a*b*x^(5/2)+2/9/c^3*a*b*x^(3/2)+2/3*a*b*x^(1/2)/c^5+1/3/c^6*a*b*ln(c*x
^(1/2)-1)-1/3/c^6*a*b*ln(1+c*x^(1/2))
```

**Maxima [A]** time = 1.00531, size = 325, normalized size = 1.88

$$\frac{1}{3} b^2 x^3 \operatorname{artanh}(c\sqrt{x})^2 + \frac{1}{3} a^2 x^3 + \frac{1}{45} \left( 30 x^3 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2 \left( 3 c^4 x^{\frac{5}{2}} + 5 c^2 x^{\frac{3}{2}} + 15 \sqrt{x} \right)}{c^6} - \frac{15 \log(c\sqrt{x} + 1)}{c^7} + \frac{15 \log(c\sqrt{x} - 1)}{c^7} \right) \right) a b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arctanh(c*sqrt(x))^2 + 1/3*a^2*x^3 + 1/45*(30*x^3*arctanh(c*sq
rt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sq
rt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1)/c^7))*a*b + 1/180*(4*c*(2*(3*c^4*x^(
```

$5/2) + 5*c^2*x^{(3/2)} + 15*\sqrt{x})/c^6 - 15*\log(c*\sqrt{x} + 1)/c^7 + 15*\log$   
 $(c*\sqrt{x} - 1)/c^7)*\operatorname{arctanh}(c*\sqrt{x}) + (6*c^4*x^2 + 32*c^2*x - 2*(15*\log$   
 $(c*\sqrt{x} - 1) - 46)*\log(c*\sqrt{x} + 1) + 15*\log(c*\sqrt{x} + 1)^2 + 15*\log$   
 $(c*\sqrt{x} - 1)^2 + 92*\log(c*\sqrt{x} - 1))/c^6)*b^2$

**Fricas [A]** time = 2.18744, size = 567, normalized size = 3.28

$60 a^2 c^6 x^3 + 6 b^2 c^4 x^2 + 32 b^2 c^2 x + 15 (b^2 c^6 x^3 - b^2) \log\left(-\frac{c^2 x + 2 c \sqrt{x} + 1}{c^2 x - 1}\right)^2 + 4 (15 a b c^6 - 15 a b + 23 b^2) \log(c \sqrt{x} + 1) - 4 (15$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="fricas")

[Out]  $1/180*(60*a^2*c^6*x^3 + 6*b^2*c^4*x^2 + 32*b^2*c^2*x + 15*(b^2*c^6*x^3 - b^2)*\log(-c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1))^2 + 4*(15*a*b*c^6 - 15*a*b + 23*b^2)*\log(c*\sqrt{x} + 1) - 4*(15*a*b*c^6 - 15*a*b - 23*b^2)*\log(c*\sqrt{x} - 1) + 4*(15*a*b*c^6*x^3 - 15*a*b*c^6 + (3*b^2*c^5*x^2 + 5*b^2*c^3*x + 15*b^2*c)*\sqrt{x})*\log(-c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1) + 8*(3*a*b*c^5*x^2 + 5*a*b*c^3*x + 15*a*b*c)*\sqrt{x})/c^6$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(c\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*\*2\*(a + b\*atanh(c\*sqrt(x)))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2*x^2, x)
```

### 3.197 $\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$

**Optimal.** Leaf size=129

$$\frac{ab\sqrt{x}}{c^3} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{2c^4} + \frac{bx^{3/2}(a + b \tanh^{-1}(c\sqrt{x}))}{3c} + \frac{1}{2}x^2(a + b \tanh^{-1}(c\sqrt{x}))^2 + \frac{b^2x}{6c^2} + \frac{2b^2 \log(1 - c^2x)}{3c^4} +$$

[Out] (a\*b\*Sqrt[x])/c^3 + (b^2\*x)/(6\*c^2) + (b^2\*Sqrt[x]\*ArcTanh[c\*Sqrt[x]])/c^3 + (b\*x^(3/2)\*(a + b\*ArcTanh[c\*Sqrt[x]]))/(3\*c) - (a + b\*ArcTanh[c\*Sqrt[x]])^2/(2\*c^4) + (x^2\*(a + b\*ArcTanh[c\*Sqrt[x]])^2)/2 + (2\*b^2\*Log[1 - c^2\*x])/(3\*c^4)

**Rubi [F]** time = 0.013963, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[x\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

[Out] Defer[Int][x\*(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

Rubi steps

$$\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx = \int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^2 dx$$

**Mathematica [A]** time = 0.0859147, size = 160, normalized size = 1.24

$$\frac{3a^2c^4x^2 + 2abc^3x^{3/2} + 2bc\sqrt{x} \tanh^{-1}(c\sqrt{x})(3ac^3x^{3/2} + b(c^2x + 3)) + 6abc\sqrt{x} + b(3a + 4b) \log(1 - c\sqrt{x}) - 3ab \log(c\sqrt{x})}{6c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*Sqrt[x]])^2,x]

[Out]  $(6ab\sqrt{x} + b^2c^2x + 2ab\sqrt{x} + 3a^2c^4x^2 + 2b\sqrt{x} + 3ac^3x^{3/2} + b(3 + c^2x))\operatorname{ArcTanh}[c\sqrt{x}] + 3b^2(-1 + c^4x^2)\operatorname{ArcTanh}[c\sqrt{x}]^2 + b(3a + 4b)\operatorname{Log}[1 - c\sqrt{x}] - 3ab\operatorname{Log}[1 + c\sqrt{x}] + 4b^2\operatorname{Log}[1 + c\sqrt{x}])/(6c^4)$

**Maple [B]** time = 0.051, size = 317, normalized size = 2.5

$$\frac{a^2x^2}{2} + \frac{b^2x^2}{2} (\operatorname{Arctanh}(c\sqrt{x}))^2 + \frac{b^2}{3c} \operatorname{Arctanh}(c\sqrt{x})x^{\frac{3}{2}} + \frac{b^2}{c^3} \operatorname{Arctanh}(c\sqrt{x})\sqrt{x} + \frac{b^2}{2c^4} \operatorname{Arctanh}(c\sqrt{x}) \ln(c\sqrt{x} - 1) - \frac{b^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^(1/2)))^2,x)`

[Out]  $\frac{1}{2}a^2x^2 + \frac{1}{2}b^2x^2\operatorname{arctanh}(c\sqrt{x})^2 + \frac{1}{3}cb^2\operatorname{arctanh}(c\sqrt{x})x^{\frac{3}{2}} + \frac{1}{c^3}b^2\operatorname{arctanh}(c\sqrt{x})\sqrt{x} + \frac{1}{2c^4}b^2\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x} - 1) - \frac{1}{2c^4}b^2\operatorname{arctanh}(c\sqrt{x})\ln(1 + c\sqrt{x}) - \frac{1}{4c^4}b^2\ln(c\sqrt{x} - 1)\ln(1/2 + 1/2c\sqrt{x}) + \frac{1}{8c^4}b^2\ln(c\sqrt{x} - 1)^2 + \frac{1}{4c^4}b^2\ln(-1/2c\sqrt{x} + 1/2)\ln(1/2 + 1/2c\sqrt{x}) - \frac{1}{4c^4}b^2\ln(-1/2c\sqrt{x} + 1/2)^2 + \frac{1}{2}abx^2\operatorname{arctanh}(c\sqrt{x}) + \frac{1}{3}abx^{\frac{3}{2}}/c + abx/c^3 + \frac{1}{2}c^4ab\ln(c\sqrt{x} - 1) - \frac{1}{2}c^4ab\ln(1 + c\sqrt{x})$

**Maxima [B]** time = 1.02672, size = 290, normalized size = 2.25

$$\frac{1}{2}b^2x^2 \operatorname{artanh}(c\sqrt{x})^2 + \frac{1}{2}a^2x^2 + \frac{1}{6} \left( 6x^2 \operatorname{artanh}(c\sqrt{x}) + c \left( \frac{2(c^2x^{\frac{3}{2}} + 3\sqrt{x})}{c^4} - \frac{3 \log(c\sqrt{x} + 1)}{c^5} + \frac{3 \log(c\sqrt{x} - 1)}{c^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2}b^2x^2\operatorname{arctanh}(c\sqrt{x})^2 + \frac{1}{2}a^2x^2 + \frac{1}{6}(6x^2\operatorname{arctanh}(c\sqrt{x}) + c(2(c^2x^{3/2} + 3\sqrt{x})/c^4 - 3\log(c\sqrt{x} + 1)/c^5 + 3\log(c\sqrt{x} - 1)/c^5))ab + \frac{1}{24}(4c(2(c^2x^{3/2} + 3\sqrt{x})/c^4 - 3\log(c\sqrt{x} + 1)/c^5 + 3\log(c\sqrt{x} - 1)/c^5))\operatorname{arctanh}(c\sqrt{x}) + (4c^2x - 2(3\log(c\sqrt{x} - 1) - 8)\log(c\sqrt{x} + 1) + 3\log(c\sqrt{x} + 1) - 3\log(c\sqrt{x} - 1))ab$

$$1)^2 + 3 \log(c \sqrt{x} - 1)^2 + 16 \log(c \sqrt{x} - 1) / c^4) * b^2$$

**Fricas [A]** time = 2.20403, size = 479, normalized size = 3.71

$$\frac{12 a^2 c^4 x^2 + 4 b^2 c^2 x + 3 (b^2 c^4 x^2 - b^2) \log\left(-\frac{c^2 x + 2 c \sqrt{x} + 1}{c^2 x - 1}\right)^2 + 4 (3 a b c^4 - 3 a b + 4 b^2) \log(c \sqrt{x} + 1) - 4 (3 a b c^4 - 3 a b - 4 b^2)}{24 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="fricas")

[Out] 1/24\*(12\*a^2\*c^4\*x^2 + 4\*b^2\*c^2\*x + 3\*(b^2\*c^4\*x^2 - b^2)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1))^2 + 4\*(3\*a\*b\*c^4 - 3\*a\*b + 4\*b^2)\*log(c\*sqrt(x) + 1) - 4\*(3\*a\*b\*c^4 - 3\*a\*b - 4\*b^2)\*log(c\*sqrt(x) - 1) + 4\*(3\*a\*b\*c^4\*x^2 - 3\*a\*b\*c^4 + (b^2\*c^3\*x + 3\*b^2\*c)\*sqrt(x))\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1)) + 8\*(a\*b\*c^3\*x + 3\*a\*b\*c)\*sqrt(x))/c^4

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{atanh}(c \sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x\*\*(1/2)))\*\*2,x)

[Out] Integral(x\*(a + b\*atanh(c\*sqrt(x)))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c \sqrt{x}) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^2\*x, x)

$$3.198 \quad \int (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$$

**Optimal.** Leaf size=85

$$-\frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{c^2} + \frac{2ab\sqrt{x}}{c} + x(a + b \tanh^{-1}(c\sqrt{x}))^2 + \frac{b^2 \log(1 - c^2x)}{c^2} + \frac{2b^2\sqrt{x} \tanh^{-1}(c\sqrt{x})}{c}$$

[Out] (2\*a\*b\*Sqrt[x])/c + (2\*b^2\*Sqrt[x]\*ArcTanh[c\*Sqrt[x]])/c - (a + b\*ArcTanh[c\*Sqrt[x]])^2/c^2 + x\*(a + b\*ArcTanh[c\*Sqrt[x]])^2 + (b^2\*Log[1 - c^2\*x])/c^2

**Rubi [F]** time = 0.0063505, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

[Out] Defer[Int][(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

Rubi steps

$$\int (a + b \tanh^{-1}(c\sqrt{x}))^2 dx = \int (a + b \tanh^{-1}(c\sqrt{x}))^2 dx$$

**Mathematica [A]** time = 0.0614956, size = 115, normalized size = 1.35

$$\frac{a^2c^2x + 2abc\sqrt{x} + b(a + b) \log(1 - c\sqrt{x}) - ab \log(c\sqrt{x} + 1) + 2bc\sqrt{x} \tanh^{-1}(c\sqrt{x})(ac\sqrt{x} + b) + b^2(c^2x - 1) \tanh^{-1}(c\sqrt{x})}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])^2, x]

```
[Out] (2*a*b*c*Sqrt[x] + a^2*c^2*x + 2*b*c*(b + a*c*Sqrt[x])*Sqrt[x]*ArcTanh[c*Sqrt[x]] + b^2*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*Log[1 - c*Sqrt[x]] - a*b*Log[1 + c*Sqrt[x]] + b^2*Log[1 + c*Sqrt[x]])/c^2
```

**Maple [B]** time = 0.05, size = 272, normalized size = 3.2

$$a^2x + b^2x(\operatorname{Arctanh}(c\sqrt{x}))^2 + 2\frac{b^2\operatorname{Arctanh}(c\sqrt{x})\sqrt{x}}{c} + \frac{b^2}{c^2}\operatorname{Arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1) - \frac{b^2}{c^2}\operatorname{Arctanh}(c\sqrt{x})\ln(1+c\sqrt{x}) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^2,x)
```

```
[Out] a^2*x+b^2*x*arctanh(c*x^(1/2))^2+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c+1/c^2*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/c^2*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/2/c^2*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/4/c^2*b^2*ln(c*x^(1/2)-1)^2+1/c^2*b^2*ln(c*x^(1/2)-1)+1/c^2*b^2*ln(1+c*x^(1/2))-1/2/c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+1/2/c^2*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+1/4/c^2*b^2*ln(1+c*x^(1/2))^2+2*a*b*x*arctanh(c*x^(1/2))+2*a*b*x^(1/2)/c+1/c^2*a*b*ln(c*x^(1/2)-1)-1/c^2*a*b*ln(1+c*x^(1/2))
```

**Maxima [B]** time = 1.01408, size = 236, normalized size = 2.78

$$\left( c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(c\sqrt{x}+1)}{c^3} + \frac{\log(c\sqrt{x}-1)}{c^3} \right) + 2x \operatorname{artanh}(c\sqrt{x}) \right) ab + \frac{1}{4} \left( 4c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(c\sqrt{x}+1)}{c^3} + \frac{\log(c\sqrt{x}-1)}{c^3} \right) \operatorname{artanh}(c\sqrt{x}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] (c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x*arctanh(c*sqrt(x)))*a*b + 1/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*b^2 + a^2*x
```

**Fricas [B]** time = 1.99055, size = 382, normalized size = 4.49

$$\frac{4a^2c^2x + 8abc\sqrt{x} + (b^2c^2x - b^2)\log\left(-\frac{c^2x+2c\sqrt{x}+1}{c^2x-1}\right)^2 + 4(abc^2 - ab + b^2)\log(c\sqrt{x} + 1) - 4(abc^2 - ab - b^2)\log(c\sqrt{x} - 1)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="fricas")

[Out] 1/4\*(4\*a^2\*c^2\*x + 8\*a\*b\*c\*sqrt(x) + (b^2\*c^2\*x - b^2)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1))^2 + 4\*(a\*b\*c^2 - a\*b + b^2)\*log(c\*sqrt(x) + 1) - 4\*(a\*b\*c^2 - a\*b - b^2)\*log(c\*sqrt(x) - 1) + 4\*(a\*b\*c^2\*x - a\*b\*c^2 + b^2\*c\*sqrt(x))\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1)))/c^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(c\sqrt{x}))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*2,x)

[Out] Integral((a + b\*atanh(c\*sqrt(x)))\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^2, x)

$$3.199 \quad \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x} dx$$

**Optimal.** Leaf size=145

$$-2b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) + 2b \operatorname{PolyLog}\left(2, \frac{2}{1 - c\sqrt{x}} - 1\right) (a + b \tanh^{-1}(c\sqrt{x})) + b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) - b^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))$$

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

---

**Rubi [A]** time = 0.317312, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-2b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) + 2b \operatorname{PolyLog}\left(2, \frac{2}{1 - c\sqrt{x}} - 1\right) (a + b \tanh^{-1}(c\sqrt{x})) + b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) - b^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x, x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 2*b*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + b^2*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - b^2*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])]
```

#### Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

#### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_.)/(x_), x_Symbol] := Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```



Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{x} dx &= 2 \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 - (8bc) \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \tanh^{-1}(cx)}{1 - c^2x^2} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + (4bc) \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx)) \log \left( \frac{1 + c\sqrt{x}}{1 - c\sqrt{x}} \right)}{1 - c^2x^2} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 - 2b(a + b \tanh^{-1}(c\sqrt{x})) \operatorname{Li}_2 \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 - 2b(a + b \tanh^{-1}(c\sqrt{x})) \operatorname{Li}_2 \left( 1 - \frac{2}{1 - c\sqrt{x}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.0829929, size = 164, normalized size = 1.13

$$4 \tanh^{-1} \left( \frac{2}{c\sqrt{x} - 1} + 1 \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 - b \left( -2 \operatorname{PolyLog} \left( 2, \frac{c\sqrt{x} + 1}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x})) + 2 \operatorname{PolyLog} \left( 2, \frac{c\sqrt{x}}{c\sqrt{x} - 1} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])^2/x,x]

[Out] 4\*ArcTanh[1 + 2/(-1 + c\*Sqrt[x])]\*(a + b\*ArcTanh[c\*Sqrt[x]])^2 - b\*(-2\*(a + b\*ArcTanh[c\*Sqrt[x]])\*PolyLog[2, (1 + c\*Sqrt[x])/(1 - c\*Sqrt[x])] + 2\*(a + b\*ArcTanh[c\*Sqrt[x]])\*PolyLog[2, (1 + c\*Sqrt[x])/(-1 + c\*Sqrt[x])] + b\*(PolyLog[3, (1 + c\*Sqrt[x])/(1 - c\*Sqrt[x])] - PolyLog[3, (1 + c\*Sqrt[x])/(-1 + c\*Sqrt[x])]))

---

**Maple [C]** time = 0.316, size = 742, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))^2/x,x)

```
[Out] 2*a^2*ln(c*x^(1/2))+2*b^2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+b^2*polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1))-2*b^2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+2*b^2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*b^2*arctanh(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*b^2*polylog(3,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*b^2*arctanh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*b^2*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-I*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2-I*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+I*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*arctanh(c*x^(1/2))^2+I*b^2*Pi*csgn(I*((1+c*x^(1/2))^2/(-c^2*x+1)-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2-2*a*b*ln(c*x^(1/2))*ln(1+c*x^(1/2))+4*a*b*ln(c*x^(1/2))*arctanh(c*x^(1/2))-2*a*b*dilog(1+c*x^(1/2))-2*a*b*dilog(c*x^(1/2))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2 \int \frac{\log(c\sqrt{x}+1)^2}{x} dx - \frac{1}{2}b^2 \int \frac{\log(c\sqrt{x}+1)\log(-c\sqrt{x}+1)}{x} dx + \frac{1}{4}b^2 \int \frac{\log(-c\sqrt{x}+1)^2}{x} dx + ab \int \frac{\log(c\sqrt{x}+1)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 1/4*b^2*integrate(log(-c*sqrt(x) + 1)^2/x, x) + a*b*integrate(log(c*sqrt(x) + 1)/x, x) - a*b*integrate(log(-c*sqrt(x) + 1)/x, x) + a^2*log(x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 \operatorname{artanh}(c\sqrt{x})^2 + 2ab \operatorname{artanh}(c\sqrt{x}) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*sqrt(x))^2 + 2*a*b*arctanh(c*sqrt(x)) + a^2)/x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(c\sqrt{x}))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(1/2)))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*sqrt(x)))**2/x, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x, x)
```

$$3.200 \quad \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^2} dx$$

**Optimal.** Leaf size=85

$$c^2 (a+b \tanh^{-1}(c\sqrt{x}))^2 - \frac{2bc(a+b \tanh^{-1}(c\sqrt{x}))}{\sqrt{x}} - \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x} + b^2 c^2 \log(x) - b^2 c^2 \log(1-c^2 x)$$

[Out]  $(-2*b*c*(a + b*ArcTanh[c*Sqrt[x]]))/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]])^2 - (a + b*ArcTanh[c*Sqrt[x]])^2/x + b^2*c^2*Log[x] - b^2*c^2*Log[1 - c^2*x]$

**Rubi [F]** time = 0.0238263, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2, x]$

[Out]  $\text{Defer[Int]}[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2, x]$

Rubi steps

$$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^2} dx = \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^2}{x^2} dx$$

**Mathematica [A]** time = 0.110467, size = 129, normalized size = 1.52

$$\frac{a^2 - abc^2 x \log(c\sqrt{x} + 1) + bc^2 x(a + b) \log(1 - c\sqrt{x}) + 2abc\sqrt{x} + 2b \tanh^{-1}(c\sqrt{x})(a + bc\sqrt{x}) + b^2 c^2 x \log(c\sqrt{x} + 1)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])^2/x^2,x]

[Out] -((a^2 + 2\*a\*b\*c\*Sqrt[x] + 2\*b\*(a + b\*c\*Sqrt[x])\*ArcTanh[c\*Sqrt[x]] - b^2\*(-1 + c^2\*x)\*ArcTanh[c\*Sqrt[x]]^2 + b\*(a + b)\*c^2\*x\*Log[1 - c\*Sqrt[x]] - a\*b\*c^2\*x\*Log[1 + c\*Sqrt[x]] + b^2\*c^2\*x\*Log[1 + c\*Sqrt[x]] - b^2\*c^2\*x\*Log[x])/x)

**Maple [B]** time = 0.054, size = 292, normalized size = 3.4

$$-\frac{a^2}{x} - \frac{b^2}{x} \left( \operatorname{Artanh}(c\sqrt{x}) \right)^2 - 2 \frac{b^2 c \operatorname{Artanh}(c\sqrt{x})}{\sqrt{x}} - c^2 b^2 \operatorname{Artanh}(c\sqrt{x}) \ln(c\sqrt{x} - 1) + c^2 b^2 \operatorname{Artanh}(c\sqrt{x}) \ln(1 + c\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))^2/x^2,x)

[Out] -a^2/x-b^2/x\*arctanh(c\*x^(1/2))^2-2\*c\*b^2/x^(1/2)\*arctanh(c\*x^(1/2))-c^2\*b^2\*arctanh(c\*x^(1/2))\*ln(c\*x^(1/2)-1)+c^2\*b^2\*arctanh(c\*x^(1/2))\*ln(1+c\*x^(1/2))+1/2\*c^2\*b^2\*ln(c\*x^(1/2)-1)\*ln(1/2+1/2\*c\*x^(1/2))-1/4\*c^2\*b^2\*ln(c\*x^(1/2)-1)^2-c^2\*b^2\*ln(c\*x^(1/2)-1)+2\*c^2\*b^2\*ln(c\*x^(1/2))-c^2\*b^2\*ln(1+c\*x^(1/2))+1/2\*c^2\*b^2\*ln(-1/2\*c\*x^(1/2)+1/2)\*ln(1+c\*x^(1/2))-1/2\*c^2\*b^2\*ln(-1/2\*c\*x^(1/2)+1/2)\*ln(1/2+1/2\*c\*x^(1/2))-1/4\*c^2\*b^2\*ln(1+c\*x^(1/2))^2-2\*a\*b/x\*arctanh(c\*x^(1/2))-2\*c\*a\*b/x^(1/2)-c^2\*a\*b\*ln(c\*x^(1/2)-1)+c^2\*a\*b\*ln(1+c\*x^(1/2))

**Maxima [B]** time = 1.01967, size = 235, normalized size = 2.76

$$\left( \left( c \log(c\sqrt{x} + 1) - c \log(c\sqrt{x} - 1) - \frac{2}{\sqrt{x}} \right) c - \frac{2 \operatorname{artanh}(c\sqrt{x})}{x} \right) ab + \frac{1}{4} \left( \left( 2(\log(c\sqrt{x} - 1) - 2) \log(c\sqrt{x} + 1) - \log(c\sqrt{x} + 1)^2 - \log(c\sqrt{x} - 1)^2 - 4\log(c\sqrt{x} - 1) + 4\log(x) \right) c^2 + 4(c \log(c\sqrt{x} + 1) - c \log(c\sqrt{x} - 1) - 2/\sqrt{x}) c \operatorname{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2/x^2,x, algorithm="maxima")

[Out] ((c\*log(c\*sqrt(x) + 1) - c\*log(c\*sqrt(x) - 1) - 2/sqrt(x))\*c - 2\*arctanh(c\*sqrt(x))/x)\*a\*b + 1/4\*((2\*(log(c\*sqrt(x) - 1) - 2)\*log(c\*sqrt(x) + 1) - log(c\*sqrt(x) + 1)^2 - log(c\*sqrt(x) - 1)^2 - 4\*log(c\*sqrt(x) - 1) + 4\*log(x))\*c^2 + 4\*(c\*log(c\*sqrt(x) + 1) - c\*log(c\*sqrt(x) - 1) - 2/sqrt(x))\*c\*arctan

$$h(c\sqrt{x}) * b^2 - b^2 * \operatorname{arctanh}(c\sqrt{x})^2/x - a^2/x$$

**Fricas [B]** time = 1.83377, size = 375, normalized size = 4.41

$$\frac{8b^2c^2x \log(\sqrt{x}) + 4(ab - b^2)c^2x \log(c\sqrt{x} + 1) - 4(ab + b^2)c^2x \log(c\sqrt{x} - 1) - 8abc\sqrt{x} + (b^2c^2x - b^2) \log\left(-\frac{c^2x + 2c\sqrt{x}}{c^2x - 1}\right)}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2/x^2,x, algorithm="fricas")

[Out] 1/4\*(8\*b^2\*c^2\*x\*log(sqrt(x)) + 4\*(a\*b - b^2)\*c^2\*x\*log(c\*sqrt(x) + 1) - 4\*(a\*b + b^2)\*c^2\*x\*log(c\*sqrt(x) - 1) - 8\*a\*b\*c\*sqrt(x) + (b^2\*c^2\*x - b^2)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1))^2 - 4\*a^2 - 4\*(b^2\*c\*sqrt(x) + a\*b)\*log(-(c^2\*x + 2\*c\*sqrt(x) + 1)/(c^2\*x - 1)))/x

**Sympy [A]** time = 42.9305, size = 680, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*2/x\*\*2,x)

[Out] Piecewise((-a\*\*2/x + 2\*a\*b\*atanh(sqrt(x)\*sqrt(1/x))/x - b\*\*2\*atanh(sqrt(x)\*sqrt(1/x))\*\*2/x, Eq(c, -sqrt(1/x))), (-a\*\*2/x - 2\*a\*b\*atanh(sqrt(x)\*sqrt(1/x))/x - b\*\*2\*atanh(sqrt(x)\*sqrt(1/x))\*\*2/x, Eq(c, sqrt(1/x))), (-a\*\*2/x, Eq(c, 0)), (-a\*\*2\*c\*\*4\*x\*\*(5/2)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + a\*\*2\*sqrt(x)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + 2\*a\*b\*c\*\*4\*x\*\*(5/2)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 2\*a\*b\*c\*\*3\*x\*\*2/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 4\*a\*b\*c\*\*2\*x\*(3/2)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + 2\*a\*b\*c\*x/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + 2\*a\*b\*sqrt(x)\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + b\*\*2\*c\*\*4\*x\*\*(5/2)\*log(x)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 2\*b\*\*2\*c\*\*4\*x\*\*(5/2)\*log(sqrt(x) - 1/c)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + b\*\*2\*c\*\*4\*x\*\*(5/2)\*atanh(c\*sqrt(x))\*\*2/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 2\*b\*\*2\*c\*\*3\*x\*\*2\*atanh(c\*sqrt(x))/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - b\*\*2\*c\*\*2\*x\*\*(3/2)\*log(x)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + 2\*b\*\*2\*c\*\*2\*x\*\*(3/2)\*log(sqrt(x) - 1/c)/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) - 2\*b\*\*2\*c\*\*2\*x\*\*(3/2)\*atanh(c\*sqrt(x))\*\*2/(c\*\*2\*x\*\*(5/2) - x\*\*(3/2)) + 2\*b\*\*2\*

```
c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*b**2*c*x*atan
h(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + b**2*sqrt(x)*atanh(c*sqrt(x))**2/
(c**2*x**(5/2) - x**(3/2)), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*sqrt(x)) + a)^2/x^2, x)
```



$$3.201 \quad \int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{x^3} dx$$

**Optimal.** Leaf size=133

$$\frac{1}{2}c^4(a + b \tanh^{-1}(c\sqrt{x}))^2 - \frac{bc^3(a + b \tanh^{-1}(c\sqrt{x}))}{\sqrt{x}} - \frac{bc(a + b \tanh^{-1}(c\sqrt{x}))}{3x^{3/2}} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{2x^2} - \frac{b^2c^2}{6x} + \frac{2}{3}$$

[Out]  $-(b^2c^2)/(6x) - (bc^3(a + b \operatorname{ArcTanh}[c\sqrt{x}]))/(3x^{3/2}) - (bc^3(a + b \operatorname{ArcTanh}[c\sqrt{x}]))/\sqrt{x} + (c^4(a + b \operatorname{ArcTanh}[c\sqrt{x}])^2)/2 - (a + b \operatorname{ArcTanh}[c\sqrt{x}])^2/(2x^2) + (2b^2c^4 \operatorname{Log}[x])/3 - (2b^2c^4 \operatorname{Log}[1 - c^2x])/3$

**Rubi [F]** time = 0.0240127, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*Sqrt[x]])^2/x^3, x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*Sqrt[x]])^2/x^3, x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{x^3} dx = \int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^2}{x^3} dx$$

**Mathematica [A]** time = 0.127832, size = 178, normalized size = 1.34

$$\frac{3a^2 + 6abc^3x^{3/2} + bc^4x^2(3a + 4b) \log(1 - c\sqrt{x}) - 3abc^4x^2 \log(c\sqrt{x} + 1) + 2b \tanh^{-1}(c\sqrt{x})(3a + bc\sqrt{x}(3c^2x + 1))}{6x^2} + \frac{2}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])^2/x^3,x]

[Out]  $-(3a^2 + 2ab\sqrt{x} + b^2c^2x + 6ab^2c^3x^{3/2} + 2b(3a + b\sqrt{x}(1 + 3c^2x))\operatorname{ArcTanh}[c\sqrt{x}] - 3b^2(-1 + c^4x^2)\operatorname{ArcTanh}[c\sqrt{x}]^2 + b(3a + 4b)c^4x^2\log[1 - c\sqrt{x}] - 3ab^2c^4x^2\log[1 + c\sqrt{x}] + 4b^2c^4x^2\log[1 + c\sqrt{x}] - 4b^2c^4x^2\log[x])/(6x^2)$

**Maple [B]** time = 0.062, size = 332, normalized size = 2.5

$$-\frac{a^2}{2x^2} - \frac{b^2}{2x^2} (\operatorname{Arctanh}(c\sqrt{x}))^2 - \frac{c^4b^2}{2} \operatorname{Arctanh}(c\sqrt{x}) \ln(c\sqrt{x} - 1) - \frac{b^2c}{3} \operatorname{Arctanh}(c\sqrt{x}) x^{-\frac{3}{2}} - c^3b^2 \operatorname{Arctanh}(c\sqrt{x}) \frac{1}{\sqrt{x}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))^2/x^3,x)

[Out]  $-1/2a^2/x^2 - 1/2b^2/x^2 \operatorname{arctanh}(c\sqrt{x})^2 - 1/2c^4b^2 \operatorname{arctanh}(c\sqrt{x}) \ln(c\sqrt{x} - 1) - 1/3c^4b^2/x^{3/2} \operatorname{arctanh}(c\sqrt{x}) - c^3b^2/x^{1/2} \operatorname{arctanh}(c\sqrt{x}) + 1/2c^4b^2 \operatorname{arctanh}(c\sqrt{x}) \ln(1 + c\sqrt{x}) - 1/8c^4b^2 \ln(c\sqrt{x} - 1)^2 + 1/4c^4b^2 \ln(c\sqrt{x} - 1) \ln(1/2 + 1/2c\sqrt{x}) - 1/4c^4b^2 \ln(-1/2c\sqrt{x} + 1/2) \ln(1/2 + 1/2c\sqrt{x}) + 1/4c^4b^2 \ln(-1/2c\sqrt{x} + 1/2) \ln(1 + c\sqrt{x}) - 1/8c^4b^2 \ln(1 + c\sqrt{x})^2 - 2/3c^4b^2 \ln(c\sqrt{x} - 1) - 1/6b^2c^2/x + 4/3c^4b^2 \ln(c\sqrt{x}) - 2/3c^4b^2 \ln(1 + c\sqrt{x}) - ab/x^2 \operatorname{arctanh}(c\sqrt{x}) - 1/2c^4ab \ln(c\sqrt{x} - 1) - 1/3c^4ab/x^{3/2} - c^3ab/x^{1/2} + 1/2c^4ab \ln(1 + c\sqrt{x})$

**Maxima [B]** time = 0.984568, size = 316, normalized size = 2.38

$$\frac{1}{6} \left( \left( 3c^3 \log(c\sqrt{x} + 1) - 3c^3 \log(c\sqrt{x} - 1) - \frac{2(3c^2x + 1)}{x^{\frac{3}{2}}} \right) c - \frac{6 \operatorname{arctanh}(c\sqrt{x})}{x^2} \right) ab + \frac{1}{24} \left( \left( 16c^2 \log(x) - \frac{3c^2x \log(c\sqrt{x})}{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2/x^3,x, algorithm="maxima")

[Out]  $1/6*((3c^3\log(c\sqrt{x}) + 1) - 3c^3\log(c\sqrt{x} - 1) - 2*(3c^2x + 1)/x^{3/2})*c - 6*\operatorname{arctanh}(c\sqrt{x})/x^2)*ab + 1/24*((16c^2*\log(x) - (3c^2$

$$*x*\log(c*\sqrt{x} + 1)^2 + 3*c^2*x*\log(c*\sqrt{x} - 1)^2 + 16*c^2*x*\log(c*\sqrt{x} - 1) - 2*(3*c^2*x*\log(c*\sqrt{x} - 1) - 8*c^2*x*\log(c*\sqrt{x} + 1) + 4)/x)*c^2 + 4*(3*c^3*\log(c*\sqrt{x} + 1) - 3*c^3*\log(c*\sqrt{x} - 1) - 2*(3*c^2*x + 1)/x^{(3/2)})*c*\operatorname{arctanh}(c*\sqrt{x}))*b^2 - 1/2*b^2*\operatorname{arctanh}(c*\sqrt{x})^2/x^2 - 1/2*a^2/x^2$$

**Fricas [A]** time = 1.92422, size = 471, normalized size = 3.54

$$\frac{32b^2c^4x^2 \log(\sqrt{x}) + 4(3ab - 4b^2)c^4x^2 \log(c\sqrt{x} + 1) - 4(3ab + 4b^2)c^4x^2 \log(c\sqrt{x} - 1) - 4b^2c^2x + 3(b^2c^4x^2 - b^2) \log(\sqrt{x})}{24x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2/x^3,x, algorithm="fricas")

[Out]  $\frac{1}{24}*(32*b^2*c^4*x^2*\log(\sqrt{x}) + 4*(3*a*b - 4*b^2)*c^4*x^2*\log(c*\sqrt{x} + 1) - 4*(3*a*b + 4*b^2)*c^4*x^2*\log(c*\sqrt{x} - 1) - 4*b^2*c^2*x + 3*(b^2*c^4*x^2 - b^2)*\log(-c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1))^2 - 12*a^2 - 4*(3*a*b + (3*b^2*c^3*x + b^2*c)*\sqrt{x})*\log(-c^2*x + 2*c*\sqrt{x} + 1)/(c^2*x - 1) - 8*(3*a*b*c^3*x + a*b*c)*\sqrt{x})/x^2$

**Sympy [A]** time = 179.052, size = 972, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*2/x\*\*3,x)

[Out] Piecewise((-a\*\*2/(2\*x\*\*2) + a\*b\*atanh(sqrt(x)\*sqrt(1/x))/x\*\*2 - b\*\*2\*atanh(sqrt(x)\*sqrt(1/x))\*\*2/(2\*x\*\*2), Eq(c, -sqrt(1/x))), (-a\*\*2/(2\*x\*\*2) - a\*b\*atanh(sqrt(x)\*sqrt(1/x))/x\*\*2 - b\*\*2\*atanh(sqrt(x)\*sqrt(1/x))\*\*2/(2\*x\*\*2), Eq(c, sqrt(1/x))), (-a\*\*2/(2\*x\*\*2), Eq(c, 0)), (-3\*a\*\*2\*c\*\*2\*x\*\*(3/2)/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 3\*a\*\*2\*sqrt(x)/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 6\*a\*b\*c\*\*6\*x\*\*(7/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 6\*a\*b\*c\*\*5\*x\*\*3/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 6\*a\*b\*c\*\*4\*x\*\*(5/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 4\*a\*b\*c\*\*3\*x\*\*2/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) - 6\*a\*b\*c\*\*2\*x\*\*(3/2)\*atanh(c\*sqrt(x))/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 2\*a\*b\*c\*x/(6\*c\*\*2\*x\*\*(7/2) - 6\*x\*\*(5/2)) + 6\*a\*b\*sqrt(x)\*atanh(c\*sqrt(x))

```

rt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**6*x**(7/2)*log(x)/(6*c**2
*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*log(sqrt(x) - 1/c)/(6*c**2*x
**(7/2) - 6*x**(5/2)) + 3*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))**2/(6*c**2*x*
*(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/
2) - 6*x**(5/2)) - b**2*c**6*x**(7/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*b*
*2*c**5*x**3*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) - 4*b**2*c**4*
x**(5/2)*log(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2*c**4*x**(5/2)*log(s
qrt(x) - 1/c)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**4*x**(5/2)*atanh(c
*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)) + 8*b**2*c**4*x**(5/2)*atanh(c*
sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**3*x**2*atanh(c*sqrt(x))
/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/
(6*c**2*x**(7/2) - 6*x**(5/2)) + b**2*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x*
*(5/2)) + 2*b**2*c*x*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*
*2*sqrt(x)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*x**(5/2)), True))

```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^2/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^2/x^3, x)

### 3.202 $\int x^3 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx$

**Optimal.** Leaf size=374

$$-\frac{44b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-c\sqrt{x}}\right)}{35c^8} + \frac{b^2 x^3 (a + b \tanh^{-1}(c\sqrt{x}))}{28c^2} + \frac{9b^2 x^2 (a + b \tanh^{-1}(c\sqrt{x}))}{70c^4} + \frac{71b^2 x (a + b \tanh^{-1}(c\sqrt{x}))}{140c^6}$$

[Out]  $(47*b^3*\text{Sqrt}[x])/(70*c^7) + (23*b^3*x^{(3/2)})/(420*c^5) + (b^3*x^{(5/2)})/(140*c^3) - (47*b^3*\text{ArcTanh}[c*\text{Sqrt}[x]])/(70*c^8) + (71*b^2*x*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/(140*c^6) + (9*b^2*x^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/(70*c^4) + (b^2*x^3*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]]))/(28*c^2) + (44*b*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/(35*c^8) + (3*b*\text{Sqrt}[x]*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/(4*c^7) + (b*x^{(3/2)}*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/(4*c^5) + (3*b*x^{(5/2)}*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/(20*c^3) + (3*b*x^{(7/2)}*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^2)/(28*c) - (a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^3/(4*c^8) + (x^4*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])^3)/4 - (88*b^2*(a + b*\text{ArcTanh}[c*\text{Sqrt}[x]])*\text{Log}[2/(1 - c*\text{Sqrt}[x])])/(35*c^8) - (44*b^3*\text{PolyLog}[2, 1 - 2/(1 - c*\text{Sqrt}[x])])/(35*c^8)$

**Rubi [F]** time = 0.0233636, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

Verification is Not applicable to the result.

[In] Int[x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^3,x]

[Out] Defer[Int][x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^3, x]

Rubi steps

$$\int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx = \int x^3 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

**Mathematica [A]** time = 1.1936, size = 418, normalized size = 1.12

$1056b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + 6b \tanh^{-1}(c\sqrt{x}) \left(105a^2 c^8 x^4 + 2abc\sqrt{x} (15c^6 x^3 + 21c^4 x^2 + 35c^2 x + 105) + b^2 (5\right.$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*Sqrt[x]])^3,x]

[Out] (-564\*a\*b^2 + 630\*a^2\*b\*c\*Sqrt[x] + 564\*b^3\*c\*Sqrt[x] + 426\*a\*b^2\*c^2\*x + 210\*a^2\*b\*c^3\*x^(3/2) + 46\*b^3\*c^3\*x^(3/2) + 108\*a\*b^2\*c^4\*x^2 + 126\*a^2\*b\*c^5\*x^(5/2) + 6\*b^3\*c^5\*x^(5/2) + 30\*a\*b^2\*c^6\*x^3 + 90\*a^2\*b\*c^7\*x^(7/2) + 210\*a^3\*c^8\*x^4 + 6\*b^2\*(b\*(-176 + 105\*c\*Sqrt[x] + 35\*c^3\*x^(3/2) + 21\*c^5\*x^(5/2) + 15\*c^7\*x^(7/2)) + 105\*a\*(-1 + c^8\*x^4))\*ArcTanh[c\*Sqrt[x]]^2 + 210\*b^3\*(-1 + c^8\*x^4)\*ArcTanh[c\*Sqrt[x]]^3 + 6\*b\*ArcTanh[c\*Sqrt[x]]\*(105\*a^2\*c^8\*x^4 + b^2\*(-94 + 71\*c^2\*x + 18\*c^4\*x^2 + 5\*c^6\*x^3) + 2\*a\*b\*c\*Sqrt[x]\*(105 + 35\*c^2\*x + 21\*c^4\*x^2 + 15\*c^6\*x^3) - 352\*b^2\*Log[1 + E^(-2\*ArcTanh[c\*Sqrt[x]])]) + 315\*a^2\*b\*Log[1 - c\*Sqrt[x]] - 315\*a^2\*b\*Log[1 + c\*Sqrt[x]] + 1056\*a\*b^2\*Log[1 - c^2\*x] + 1056\*b^3\*PolyLog[2, -E^(-2\*ArcTanh[c\*Sqrt[x]])])/(840\*c^8)

**Maple [C]** time = 1.179, size = 1518, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x^(1/2)))^3,x)

[Out] -1/4/c^8\*b^3\*arctanh(c\*x^(1/2))^3+1/4\*b^3\*x^4\*arctanh(c\*x^(1/2))^3-11/15/c^8\*b^3-47/70\*b^3\*arctanh(c\*x^(1/2))/c^8+47/70\*b^3\*x^(1/2)/c^7+23/420\*b^3\*x^(3/2)/c^5+1/140\*b^3\*x^(5/2)/c^3+1/28\*a\*b^2\*x^3/c^2+9/70/c^4\*x^2\*a\*b^2+71/140/c^6\*b^2\*x\*a+3/4/c^7\*a^2\*b\*x^(1/2)+3/20/c^3\*a^2\*b\*x^(5/2)+1/4/c^5\*a^2\*b\*x^(3/2)+3/16/c^8\*a\*b^2\*ln(c\*x^(1/2)-1)^2+3/16/c^8\*a\*b^2\*ln(1+c\*x^(1/2))^2+44/35/c^8\*a\*b^2\*ln(c\*x^(1/2)-1)+44/35/c^8\*a\*b^2\*ln(1+c\*x^(1/2))+3/8/c^8\*a^2\*b\*ln(c\*x^(1/2)-1)-3/8/c^8\*a^2\*b\*ln(1+c\*x^(1/2))-88/35/c^8\*b^3\*arctanh(c\*x^(1/2))\*ln(1+I\*(1+c\*x^(1/2))/(-c^2\*x+1)^(1/2))-88/35/c^8\*b^3\*arctanh(c\*x^(1/2))\*ln(1-I\*(1+c\*x^(1/2))/(-c^2\*x+1)^(1/2))+3/8/c^8\*b^3\*arctanh(c\*x^(1/2))^2\*ln(c\*x^(1/2)-1)-3/8/c^8\*b^3\*arctanh(c\*x^(1/2))^2\*ln(1+c\*x^(1/2))+3/4/c^8\*b^3\*arctanh(c\*x^(1/2))^2\*ln((1+c\*x^(1/2))/(-c^2\*x+1)^(1/2))+3/28/c\*x^(7/2)\*a^2\*b+9/70/c^4\*b^3\*arctanh(c\*x^(1/2))\*x^2+71/140/c^6\*b^3\*arctanh(c\*x^(1/2))\*x+1/28/c^2\*b^3\*arctanh(c\*x^(1/2))\*x^3+3/4/c^7\*b^3\*arctanh(c\*x^(1/2))^2\*x^(1/2)+3/28/c\*b^3\*arctanh(c\*x^(1/2))^2\*x^(7/2)+3/20/c^3\*b^3\*arctanh(c\*x^(1/2))^2\*x^(5/2)+1/4/c^5\*b^3\*arctanh(c\*x^(1/2))^2\*x^(3/2)+3/4\*a\*b^2\*x^4\*arctanh(c\*x^(1/2))^2+3/4\*a^2\*b\*x^4\*arctanh(c\*x^(1/2))+1/4\*x^4\*a^3+3/8/c^8\*a\*b^2\*ln(-1/2\*c\*x^(1/2)+1/2)\*ln(1/2+1/2\*c\*x^(1/2))+3/4/c^8\*a\*b^2\*arctanh(c\*x^(1/2))\*ln(c\*x^(1/2)-1)-3/4/c^8\*a\*b^2\*arctanh(c\*x^(1/2))\*ln(1+c\*x^(1/2))-3/8/c^8\*a\*b^2\*1

$$\begin{aligned} & n(c*x^{(1/2)}-1)*\ln(1/2+1/2*c*x^{(1/2)})+3/14/c*a*b^2*\operatorname{arctanh}(c*x^{(1/2)})*x^{(7/2)} \\ & )+3/10/c^3*a*b^2*\operatorname{arctanh}(c*x^{(1/2)})*x^{(5/2)}+1/2/c^5*a*b^2*\operatorname{arctanh}(c*x^{(1/2)}) \\ & )*x^{(3/2)}+3/2/c^7*a*b^2*x^{(1/2)}*\operatorname{arctanh}(c*x^{(1/2)})-3/8/c^8*a*b^2*\ln(-1/2*c* \\ & x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})-3/8*I/c^8*b^3*Pi*\operatorname{arctanh}(c*x^{(1/2)})^2-3/16*I/c \\ & ^8*b^3*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^{(1/2)})^2/(c^ \\ & 2*x-1))*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*ar \\ & ctanh(c*x^{(1/2)})^2+44/35/c^8*b^3*\operatorname{arctanh}(c*x^{(1/2)})^2-88/35/c^8*b^3*dilog(1 \\ & -I*(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})-88/35/c^8*b^3*dilog(1+I*(1+c*x^{(1/2)})/(- \\ & c^2*x+1)^{(1/2)})+3/16*I/c^8*b^3*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*cs \\ & gsn(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\operatorname{arctanh}(c* \\ & x^{(1/2)})^2-3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1))*csgn(I*(1+c* \\ & x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\operatorname{arctanh}(c*x^{(1/2)})^2 \\ & +3/8*I/c^8*b^3*Pi*csgn(I*(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})*csgn(I*(1+c*x^{(1/2)} \\ & ))^2/(c^2*x-1))^2*\operatorname{arctanh}(c*x^{(1/2)})^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^{(1/2)} \\ & ))/(-c^2*x+1)^{(1/2)})^2*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1))*\operatorname{arctanh}(c*x^{(1/2)}) \\ & ^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1))^3*\operatorname{arctanh}(c*x^{(1/2)}) \\ & ^2+3/8*I/c^8*b^3*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\operatorname{arctanh}(c*x^{(1 \\ & /2)})^2-3/8*I/c^8*b^3*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^3*\operatorname{arctanh}(c* \\ & x^{(1/2)})^2+3/16*I/c^8*b^3*Pi*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)} \\ & ))^2/(-c^2*x+1)+1))^3*\operatorname{arctanh}(c*x^{(1/2)})^2 \end{aligned}$$

**Maxima [B]** time = 4.15179, size = 2662, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}a^3x^4 - \frac{1}{26880}ab^2c((315c^7x^4 + 500c^5x^3 + 1002c^3x^2 + 3684cx - 12(105c^7x^4 + 120c^6x^{(7/2)} + 140c^5x^3 + 168c^4x^{(5/2)} + 210c^3x^2 + 280c^2x^{(3/2)} + 420cx + 840\sqrt{x})*\log(c\sqrt{x} + 1))/c^8 - 6396\log(c\sqrt{x} + 1)/c^9 - 6396\log(c\sqrt{x} - 1)/c^9 - \frac{1}{2240}(840x^4\log(c\sqrt{x} + 1) - c((105c^7x^4 - 120c^6x^{(7/2)} + 140c^5x^3 - 168c^4x^{(5/2)} + 210c^3x^2 - 280c^2x^{(3/2)} + 420cx - 840\sqrt{x}))/c^8 + 840\log(c\sqrt{x} + 1)/c^9))a^2b^2\log(-c\sqrt{x} + 1) + \frac{1}{2240}(840x^4\log(c\sqrt{x} + 1) - c((105c^7x^4 - 120c^6x^{(7/2)} + 140c^5x^3 - 168c^4x^{(5/2)} + 210c^3x^2 - 280c^2x^{(3/2)} + 420cx - 840\sqrt{x}))/c^8 + 840\log(c\sqrt{x} + 1)/c^9))a^2b - \frac{1}{2240}(840x^4\log(-c\sqrt{x} + 1) - c((105c^7x^4 + 120c^6x^{(7/2)} + 140c^5x^3 + 168c^4x^{(5/2)} + 210c^3x^2 + 280c^2x^{(3/2)} + 420cx + 840\sqrt{x}))/c^8 + 840\log(c\sqrt{x} - 1)/c^9))a^2b + \frac{1}{1881600}(11025(32\log(-c\sqrt{x} + 1))^2 - 8*10$

$$\begin{aligned}
&g(-c\sqrt{x} + 1) + 1)(c\sqrt{x} - 1)^8 + 57600*(49*\log(-c\sqrt{x} + 1)^2 \\
&- 14*\log(-c\sqrt{x} + 1) + 2)*(c\sqrt{x} - 1)^7 + 548800*(18*\log(-c\sqrt{x} \\
&+ 1)^2 - 6*\log(-c\sqrt{x} + 1) + 1)*(c\sqrt{x} - 1)^6 + 790272*(25*\log(-c\sqrt{x} \\
&\sqrt{x} + 1)^2 - 10*\log(-c\sqrt{x} + 1) + 2)*(c\sqrt{x} - 1)^5 + 3087000*(8 \\
&*\log(-c\sqrt{x} + 1)^2 - 4*\log(-c\sqrt{x} + 1) + 1)*(c\sqrt{x} - 1)^4 + 219 \\
&5200*(9*\log(-c\sqrt{x} + 1)^2 - 6*\log(-c\sqrt{x} + 1) + 2)*(c\sqrt{x} - 1)^3 \\
&+ 4939200*(2*\log(-c\sqrt{x} + 1)^2 - 2*\log(-c\sqrt{x} + 1) + 1)*(c\sqrt{x} \\
&- 1)^2 + 2822400*(\log(-c\sqrt{x} + 1)^2 - 2*\log(-c\sqrt{x} + 1) + 2)*(c\sqrt{x} \\
&- 1))*a*b^2/c^8 - 1/3161088000*(385875*(256*\log(-c\sqrt{x} + 1)^3 - \\
&96*\log(-c\sqrt{x} + 1)^2 + 24*\log(-c\sqrt{x} + 1) - 3)*(c\sqrt{x} - 1)^8 + \\
&2304000*(343*\log(-c\sqrt{x} + 1)^3 - 147*\log(-c\sqrt{x} + 1)^2 + 42*\log(-c\sqrt{x} \\
&\sqrt{x} + 1) - 6)*(c\sqrt{x} - 1)^7 + 76832000*(36*\log(-c\sqrt{x} + 1)^3 - \\
&18*\log(-c\sqrt{x} + 1)^2 + 6*\log(-c\sqrt{x} + 1) - 1)*(c\sqrt{x} - 1)^6 + 4 \\
&4255232*(125*\log(-c\sqrt{x} + 1)^3 - 75*\log(-c\sqrt{x} + 1)^2 + 30*\log(-c\sqrt{x} \\
&\sqrt{x} + 1) - 6)*(c\sqrt{x} - 1)^5 + 216090000*(32*\log(-c\sqrt{x} + 1)^3 - \\
&24*\log(-c\sqrt{x} + 1)^2 + 12*\log(-c\sqrt{x} + 1) - 3)*(c\sqrt{x} - 1)^4 + \\
&614656000*(9*\log(-c\sqrt{x} + 1)^3 - 9*\log(-c\sqrt{x} + 1)^2 + 6*\log(-c\sqrt{x} \\
&\sqrt{x} + 1) - 2)*(c\sqrt{x} - 1)^3 + 691488000*(4*\log(-c\sqrt{x} + 1)^3 - 6* \\
&\log(-c\sqrt{x} + 1)^2 + 6*\log(-c\sqrt{x} + 1) - 3)*(c\sqrt{x} - 1)^2 + 79027 \\
&2000*(\log(-c\sqrt{x} + 1)^3 - 3*\log(-c\sqrt{x} + 1)^2 + 6*\log(-c\sqrt{x} + \\
&1) - 6)*(c\sqrt{x} - 1))*b^3/c^8 + 44/35*(\log(c\sqrt{x} + 1)*\log(-1/2*c\sqrt{x} \\
&\sqrt{x} + 1/2) + \operatorname{dilog}(1/2*c\sqrt{x} + 1/2))*b^3/c^8 - 1881559/3763200*b^3*\log \\
&(c\sqrt{x} - 1)/c^8 + 1/2240*(2283*a*b^2 - 752*b^3)*\log(c\sqrt{x} + 1)/c^8 \\
&+ 1/3161088000*(1157625*(16*a*b^2*c^8 - b^3*c^8)*x^4 - 27000*(1680*a*b^2*c^ \\
&7 + 169*b^3*c^7)*x^(7/2) + 3500*(24528*a*b^2*c^6 - 3565*b^3*c^6)*x^3 + 9878 \\
&4000*(b^3*c^8*x^4 - b^3)*\log(c\sqrt{x} + 1)^3 - 168*(895440*a*b^2*c^5 + 442 \\
&69*b^3*c^5)*x^(5/2) + 210*(1248240*a*b^2*c^4 - 334699*b^3*c^4)*x^2 + 564480 \\
&0*(105*a*b^2*c^8*x^4 + 15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3 \\
&*x^(3/2) + 105*b^3*c*\sqrt{x} - 105*a*b^2 + 176*b^3)*\log(c\sqrt{x} + 1)^2 - \\
&352800*(105*b^3*c^8*x^4 - 120*b^3*c^7*x^(7/2) + 140*b^3*c^6*x^3 - 168*b^3*c \\
&^5*x^(5/2) + 210*b^3*c^4*x^2 - 280*b^3*c^3*x^(3/2) + 420*b^3*c^2*x - 840*b^ \\
&3*c*\sqrt{x} + 533*b^3 - 840*(b^3*c^8*x^4 - b^3)*\log(c\sqrt{x} + 1))*\log(-c\sqrt{x} \\
&\sqrt{x} + 1)^2 - 280*(1718640*a*b^2*c^3 + 2899*b^3*c^3)*x^(3/2) + 420*(2424 \\
&240*a*b^2*c^2 - 1227199*b^3*c^2)*x - 1411200*(105*a*b^2*c^8*x^4 - 120*a*b^2 \\
&*c^7*x^(7/2) - 168*a*b^2*c^5*x^(5/2) - 280*a*b^2*c^3*x^(3/2) - 840*a*b^2*c \\
&\sqrt{x} + 20*(7*a*b^2*c^6 - 2*b^3*c^6)*x^3 + 6*(35*a*b^2*c^4 - 24*b^3*c^4)* \\
&x^2 + 4*(105*a*b^2*c^2 - 142*b^3*c^2)*x)*\log(c\sqrt{x} + 1) + 840*(11025*b^ \\
&3*c^8*x^4 + 27000*b^3*c^7*x^(7/2) - 16100*b^3*c^6*x^3 + 89544*b^3*c^5*x^(5/ \\
&2) - 85890*b^3*c^4*x^2 + 286440*b^3*c^3*x^(3/2) - 348180*b^3*c^2*x + 191772 \\
&0*b^3*c*\sqrt{x} - 352800*(b^3*c^8*x^4 - b^3)*\log(c\sqrt{x} + 1)^2 - 13440*( \\
&15*b^3*c^7*x^(7/2) + 21*b^3*c^5*x^(5/2) + 35*b^3*c^3*x^(3/2) + 105*b^3*c*\sqrt{x} \\
&\sqrt{x} + 176*b^3)*\log(c\sqrt{x} + 1))*\log(-c\sqrt{x} + 1) - 840*(3835440*a*b \\
&^2*c + 618199*b^3*c)*\sqrt{x})/c^8
\end{aligned}$$



---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x^3 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2x^3 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2bx^3 \operatorname{artanh}(c\sqrt{x}) + a^3x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^3\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*x^3\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*x^3\*arctanh(c\*sqrt(x)) + a^3\*x^3, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*(1/2)))\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3\*x^3, x)

### 3.203 $\int x^2 \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx$

**Optimal.** Leaf size=304

$$\frac{23b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-c\sqrt{x}}\right)}{15c^6} + \frac{b^2 x^2 (a + b \tanh^{-1}(c\sqrt{x}))}{10c^2} + \frac{8b^2 x (a + b \tanh^{-1}(c\sqrt{x}))}{15c^4} - \frac{46b^2 \log\left(\frac{2}{1-c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))}{15c^6}$$

[Out]  $(19b^3 \sqrt{x}) / (30c^5) + (b^3 x^{3/2}) / (30c^3) - (19b^3 \text{ArcTanh}[c\sqrt{x}]) / (30c^6) + (8b^2 x (a + b \text{ArcTanh}[c\sqrt{x}])) / (15c^4) + (b^2 x^2 (a + b \text{ArcTanh}[c\sqrt{x}])) / (10c^2) + (23b^3 (a + b \text{ArcTanh}[c\sqrt{x}])^2) / (15c^6) + (b \sqrt{x} (a + b \text{ArcTanh}[c\sqrt{x}])^2) / c^5 + (b x^{3/2} (a + b \text{ArcTanh}[c\sqrt{x}])^2) / (3c^3) + (b x^{5/2} (a + b \text{ArcTanh}[c\sqrt{x}])^2) / (5c) - (a + b \text{ArcTanh}[c\sqrt{x}])^3 / (3c^6) + (x^3 (a + b \text{ArcTanh}[c\sqrt{x}])^3) / 3 - (46b^2 (a + b \text{ArcTanh}[c\sqrt{x}]) \text{Log}[2 / (1 - c\sqrt{x})]) / (15c^6) - (23b^3 \text{PolyLog}[2, 1 - 2 / (1 - c\sqrt{x})]) / (15c^6)$

**Rubi [F]** time = 0.0237346, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[x^2 (a + b \text{ArcTanh}[c\sqrt{x}])^3, x]$

[Out]  $\text{Defer}[\text{Int}[x^2 (a + b \text{ArcTanh}[c\sqrt{x}])^3, x]$

Rubi steps

$$\int x^2 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx = \int x^2 (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

**Mathematica [A]** time = 0.789276, size = 351, normalized size = 1.15

$$46b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + b \tanh^{-1}(c\sqrt{x}) \left(30a^2 c^6 x^3 + 4abc\sqrt{x} (3c^4 x^2 + 5c^2 x + 15) + b^2 (3c^4 x^2 + 16c^2 x - 19)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*Sqrt[x]])^3,x]

[Out]  $(-19ab^2 + 30a^2bc\sqrt{x} + 19b^3c\sqrt{x} + 16a^2b^2c^2x + 10a^2b^2c^3x^{3/2} + b^3c^3x^{3/2} + 3a^2b^2c^4x^2 + 6a^2b^2c^5x^{5/2} + 10a^3c^6x^3 + 2b^2(b(-23 + 15c\sqrt{x} + 5c^3x^{3/2} + 3c^5x^{5/2})) + 15a(-1 + c^6x^3)\text{ArcTanh}[c\sqrt{x}]^2 + 10b^3(-1 + c^6x^3)\text{ArcTanh}[c\sqrt{x}]^3 + b\text{ArcTanh}[c\sqrt{x}](30a^2c^6x^3 + 4a^2bc\sqrt{x})(15 + 5c^2x + 3c^4x^2) + b^2(-19 + 16c^2x + 3c^4x^2) - 92b^2\text{Log}[1 + E^{-2\text{ArcTanh}[c\sqrt{x}]})] + 15a^2b\text{Log}[1 - c\sqrt{x}] - 15a^2b\text{Log}[1 + c\sqrt{x}] + 46a^2b^2\text{Log}[1 - c^2x] + 46b^3\text{PolyLog}[2, -E^{-2\text{ArcTanh}[c\sqrt{x}]})])/(30c^6)$

**Maple [C]** time = 0.326, size = 1423, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^(1/2)))^3,x)

[Out]  $-1/4I/c^6b^3\text{Pisgn}(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))*\text{csgn}(I*(1+c*x^{1/2})^2/(c^2*x-1))*\text{csgn}(I*(1+c*x^{1/2})^2/(c^2*x-1)/((1+c*x^{1/2})^2/(-c^2*x+1)+1))*\text{arctanh}(c*x^{1/2})^2 - 19/30b^3\text{arctanh}(c*x^{1/2})/c^6 + 19/30b^3x^{1/2}/c^5 + 1/3b^3x^3\text{arctanh}(c*x^{1/2})^3 + 23/15/c^6b^3\text{arctanh}(c*x^{1/2})^2 - 46/15/c^6b^3\text{dilog}(1-I*(1+c*x^{1/2})/(-c^2*x+1)^{1/2}) - 46/15/c^6b^3\text{dilog}(1+I*(1+c*x^{1/2})/(-c^2*x+1)^{1/2}) - 1/3/c^6b^3\text{arctanh}(c*x^{1/2})^3 + 1/3x^3a^3 + 1/30b^3x^{3/2}/c^3 + 1/2I/c^6b^3\text{Pisgn}(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{1/2})^2 + 1/4I/c^6b^3\text{Pisgn}(I*(1+c*x^{1/2})^2/(c^2*x-1))^3*\text{arctanh}(c*x^{1/2})^2 + 1/4I/c^6b^3\text{Pisgn}(I*(1+c*x^{1/2})^2/(c^2*x-1))^3*\text{arctanh}(c*x^{1/2})^2 - 1/2I/c^6b^3\text{Pisgn}(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))^3*\text{arctanh}(c*x^{1/2})^2 + a^2b^2x^3*\text{arctanh}(c*x^{1/2}) + 1/10/c^2b^3*\text{arctanh}(c*x^{1/2}) * x^2 + 8/15/c^4b^3*\text{arctanh}(c*x^{1/2}) * x + 1/c^5b^3*\text{arctanh}(c*x^{1/2})^2 * x^{1/2} + 1/5/c^6b^3*\text{arctanh}(c*x^{1/2})^2 * x^{5/2} + 1/3/c^3b^3*\text{arctanh}(c*x^{1/2})^2 * x^{3/2} + 1/c^5a^2b^2x^{1/2} + 1/5/c^5a^2b^2x^{5/2} + 1/3/c^3a^2b^2x^{3/2} - 46/15/c^6b^3*\text{arctanh}(c*x^{1/2}) * \ln(1+I*(1+c*x^{1/2})/(-c^2*x+1)^{1/2}) - 46/15/c^6b^3*\text{arctanh}(c*x^{1/2}) * \ln(1-I*(1+c*x^{1/2})/(-c^2*x+1)^{1/2}) + 1/2/c^6b^3*\text{arctanh}(c*x^{1/2})^2 * \ln(c*x^{1/2}-1) - 1/2/c^6b^3*\text{arctanh}(c*x^{1/2})^2 * \ln(1+c*x^{1/2}) + 1/c^6b^3*\text{arctanh}(c*x^{1/2})^2 * \ln((1+c*x^{1/2})/(-c^2*x+1)^{1/2}) + 1/4/c^6a^2b^2 * \ln(c*x^{1/2}-1)^2 + 1/4/c^6a^2b^2 * \ln(1+c*x^{1/2})^2 + 23$

$$\begin{aligned} & /15/c^6*a*b^2*\ln(c*x^{(1/2)}-1)+23/15/c^6*a*b^2*\ln(1+c*x^{(1/2)})+1/2/c^6*a^2*b \\ & * \ln(c*x^{(1/2)}-1)-1/2/c^6*a^2*b*\ln(1+c*x^{(1/2)})+8/15*a*b^2*x/c^4-1/4*I/c^6*b \\ & ^3*Pi*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1))*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/(( \\ & 1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\operatorname{arctanh}(c*x^{(1/2)})^2-2/3/c^6*b^3+1/4*I/c^6* \\ & b^3*Pi*csgn(I*(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})^2*csgn(I*(1+c*x^{(1/2)})^2/(c^2 \\ & *x-1))*\operatorname{arctanh}(c*x^{(1/2)})^2+1/4*I/c^6*b^3*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x \\ & +1)+1))*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2 \\ & *\operatorname{arctanh}(c*x^{(1/2)})^2+1/2*I/c^6*b^3*Pi*csgn(I*(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2) \\ & }))*csgn(I*(1+c*x^{(1/2)})^2/(c^2*x-1))^2*\operatorname{arctanh}(c*x^{(1/2)})^2-1/2/c^6*a*b^2*1 \\ & n(-1/2*c*x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})+1/2/c^6*a*b^2*\ln(-1/2*c*x^{(1/2)}+1/2)* \\ & \ln(1/2+1/2*c*x^{(1/2)})+1/c^6*a*b^2*\operatorname{arctanh}(c*x^{(1/2)})*\ln(c*x^{(1/2)}-1)-1/c^6* \\ & a*b^2*\operatorname{arctanh}(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})-1/2/c^6*a*b^2*\ln(c*x^{(1/2)}-1)*\ln(1 \\ & /2+1/2*c*x^{(1/2)})+2/5/c*a*b^2*\operatorname{arctanh}(c*x^{(1/2)})*x^{(5/2)}+2/3/c^3*a*b^2*\operatorname{arct} \\ & \operatorname{anh}(c*x^{(1/2)})*x^{(3/2)}+2/c^5*a*b^2*x^{(1/2)}*\operatorname{arctanh}(c*x^{(1/2)})-1/2*I/c^6*b^3 \\ & *Pi*\operatorname{arctanh}(c*x^{(1/2)})^2+1/10/c^2*x^2*a*b^2 \end{aligned}$$

**Maxima [B]** time = 3.73239, size = 2132, normalized size = 7.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^3x^3 - \frac{1}{720}ab^2c((20c^5x^3 + 39c^3x^2 + 138cx - 6(10c^5x^3 + 12c^4x^{5/2} + 15c^3x^2 + 20c^2x^{3/2} + 30cx + 60\sqrt{x}))\log(c\sqrt{x} + 1))/c^6 - 222\log(c\sqrt{x} + 1)/c^7 - 222\log(c\sqrt{x} - 1)/c^7 - 1/120(60x^3\log(c\sqrt{x} + 1) - c((10c^5x^3 - 12c^4x^{5/2}) + 15c^3x^2 - 20c^2x^{3/2} + 30cx - 60\sqrt{x}))/c^6 + 60\log(c\sqrt{x} + 1)/c^7))a^2b - 1/120(60x^3\log(-c\sqrt{x} + 1) - c((10c^5x^3 - 12c^4x^{5/2}) + 15c^3x^2 - 20c^2x^{3/2} + 30cx - 60\sqrt{x}))/c^6 + 60\log(c\sqrt{x} + 1)/c^7))a^2b - 1/120(60x^3\log(-c\sqrt{x} + 1) - c((10c^5x^3 + 12c^4x^{5/2} + 15c^3x^2 + 20c^2x^{3/2}) + 30cx + 60\sqrt{x}))/c^6 + 60\log(c\sqrt{x} - 1)/c^7))a^2b + 1/7200(100(18\log(-c\sqrt{x} + 1)^2 - 6\log(-c\sqrt{x} + 1) + 1)(c\sqrt{x} - 1)^6 + 432(25\log(-c\sqrt{x} + 1)^2 - 10\log(-c\sqrt{x} + 1) + 2)(c\sqrt{x} - 1)^5 + 3375(8\log(-c\sqrt{x} + 1)^2 - 4\log(-c\sqrt{x} + 1) + 1)(c\sqrt{x} - 1)^4 + 4000(9\log(-c\sqrt{x} + 1)^2 - 6\log(-c\sqrt{x} + 1) + 2)(c\sqrt{x} - 1)^3 + 13500(2\log(-c\sqrt{x} + 1)^2 - 2\log(-c\sqrt{x} + 1) + 1)(c\sqrt{x} - 1)^2 + 10800(\log(-c\sqrt{x} + 1)^2 - 2\log(-c\sqrt{x} + 1) + 2)(c\sqrt{x} - 1))a^2b^2/c^6 - 1/864000(1000(36\log(-c\sqrt{x} + 1)^3 - 18\log(-c\sqrt{x} + 1)^2 + 6\log(-c\sqrt{x} + 1) - 1)(c\sqrt{x} - 1)^6 +$

$$\begin{aligned}
& 1728*(125*\log(-c*\sqrt{x} + 1)^3 - 75*\log(-c*\sqrt{x} + 1)^2 + 30*\log(-c*\sqrt{x} + 1) - 6)*(c*\sqrt{x} - 1)^5 + 16875*(32*\log(-c*\sqrt{x} + 1)^3 - 24*\log(-c*\sqrt{x} + 1)^2 + 12*\log(-c*\sqrt{x} + 1) - 3)*(c*\sqrt{x} - 1)^4 + 80000*(9*\log(-c*\sqrt{x} + 1)^3 - 9*\log(-c*\sqrt{x} + 1)^2 + 6*\log(-c*\sqrt{x} + 1) - 2)*(c*\sqrt{x} - 1)^3 + 135000*(4*\log(-c*\sqrt{x} + 1)^3 - 6*\log(-c*\sqrt{x} + 1)^2 + 6*\log(-c*\sqrt{x} + 1) - 3)*(c*\sqrt{x} - 1)^2 + 216000*(\log(-c*\sqrt{x} + 1)^3 - 3*\log(-c*\sqrt{x} + 1)^2 + 6*\log(-c*\sqrt{x} + 1) - 6)*(c*\sqrt{x} - 1) + b^3/c^6 + 23/15*(\log(c*\sqrt{x} + 1)*\log(-1/2*c*\sqrt{x} + 1/2) + \operatorname{dilog}(1/2*c*\sqrt{x} + 1/2))*b^3/c^6 - 8929/14400*b^3*\log(c*\sqrt{x} - 1)/c^6 + 1/120*(147*a*b^2 - 38*b^3)*\log(c*\sqrt{x} + 1)/c^6 + 1/864000*(1000*(12*a*b^2*c^6 - b^3*c^6)*x^3 + 36000*(b^3*c^6*x^3 - b^3)*\log(c*\sqrt{x} + 1)^3 - 48*(660*a*b^2*c^5 + 91*b^3*c^5)*x^{5/2} + 15*(4440*a*b^2*c^4 - 919*b^3*c^4)*x^2 + 14400*(15*a*b^2*c^6*x^3 + 3*b^3*c^5*x^{5/2} + 5*b^3*c^3*x^{3/2} + 15*b^3*c*\sqrt{x} - 15*a*b^2 + 23*b^3)*\log(c*\sqrt{x} + 1)^2 - 1800*(10*b^3*c^6*x^3 - 12*b^3*c^5*x^{5/2} + 15*b^3*c^4*x^2 - 20*b^3*c^3*x^{3/2} + 30*b^3*c^2*x - 60*b^3*c*\sqrt{x} + 37*b^3 - 60*(b^3*c^6*x^3 - b^3)*\log(c*\sqrt{x} + 1))*\log(-c*\sqrt{x} + 1)^2 - 20*(6840*a*b^2*c^3 + 619*b^3*c^3)*x^{3/2} + 870*(360*a*b^2*c^2 - 161*b^3*c^2)*x - 7200*(10*a*b^2*c^6*x^3 - 12*a*b^2*c^5*x^{5/2}) - 20*a*b^2*c^3*x^{3/2} - 60*a*b^2*c*\sqrt{x} + 3*(5*a*b^2*c^4 - 2*b^3*c^4)*x^2 + 2*(15*a*b^2*c^2 - 16*b^3*c^2)*x*\log(c*\sqrt{x} + 1) + 60*(100*b^3*c^6*x^3 + 264*b^3*c^5*x^{5/2} - 165*b^3*c^4*x^2 + 1140*b^3*c^3*x^{3/2} - 1230*b^3*c^2*x + 8820*b^3*c*\sqrt{x} - 1800*(b^3*c^6*x^3 - b^3)*\log(c*\sqrt{x} + 1)^2 - 480*(3*b^3*c^5*x^{5/2} + 5*b^3*c^3*x^{3/2} + 15*b^3*c*\sqrt{x} + 23*b^3)*\log(c*\sqrt{x} + 1))*\log(-c*\sqrt{x} + 1) - 60*(17640*a*b^2*c + 4369*b^3*c)*\sqrt{x})/c^6
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(b^3x^2 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2x^2 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2bx^2 \operatorname{artanh}(c\sqrt{x}) + a^3x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")`

[Out] `integral(b^3*x^2*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^2*arctanh(c*sqrt(x))^2 + 3*a^2*b*x^2*arctanh(c*sqrt(x)) + a^3*x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{atanh}(c\sqrt{x}))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x\*\*(1/2)))\*3,x)

[Out] Integral(x\*\*2\*(a + b\*atanh(c\*sqrt(x)))\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3\*x^2, x)

### 3.204 $\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx$

**Optimal.** Leaf size=234

$$-\frac{2b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-c\sqrt{x}}\right)}{c^4} + \frac{b^2 x (a + b \tanh^{-1}(c\sqrt{x}))}{2c^2} - \frac{4b^2 \log\left(\frac{2}{1-c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x}))}{c^4} + \frac{3b\sqrt{x} (a + b \tanh^{-1}(c\sqrt{x}))}{2c^3}$$

[Out]  $(b^3 \sqrt{x}) / (2c^3) - (b^3 \text{ArcTanh}[c\sqrt{x}]) / (2c^4) + (b^2 x (a + b \text{ArcTanh}[c\sqrt{x}])) / (2c^2) + (2b^2 (a + b \text{ArcTanh}[c\sqrt{x}])^2) / c^4 + (3b^2 \sqrt{x} (a + b \text{ArcTanh}[c\sqrt{x}])^2) / (2c^3) + (b^2 x^{3/2} (a + b \text{ArcTanh}[c\sqrt{x}])^2) / (2c) - (a + b \text{ArcTanh}[c\sqrt{x}])^3 / (2c^4) + (x^2 (a + b \text{ArcTanh}[c\sqrt{x}])^3) / 2 - (4b^2 (a + b \text{ArcTanh}[c\sqrt{x}]) \text{Log}[2 / (1 - c\sqrt{x})]) / c^4 - (2b^3 \text{PolyLog}[2, 1 - 2 / (1 - c\sqrt{x})]) / c^4$

**Rubi [F]** time = 0.0142464, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[x\*(a + b\*ArcTanh[c\*Sqrt[x]])^3, x]

[Out] Defer[Int][x\*(a + b\*ArcTanh[c\*Sqrt[x]])^3, x]

Rubi steps

$$\int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx = \int x \left( a + b \tanh^{-1} (c\sqrt{x}) \right)^3 dx$$

**Mathematica [A]** time = 0.527349, size = 285, normalized size = 1.22

$$8b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + 2b \tanh^{-1}(c\sqrt{x}) \left(3a^2 c^4 x^2 + 2abc\sqrt{x}(c^2 x + 3) + b^2(c^2 x - 1)\right) - 8b^2 \log\left(e^{-2 \tanh^{-1}(c\sqrt{x})}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

```
[Out] (-2*a*b^2 + 6*a^2*b*c*Sqrt[x] + 2*b^3*c*Sqrt[x] + 2*a*b^2*c^2*x + 2*a^2*b*c^3*x^(3/2) + 2*a^3*c^4*x^2 + 2*b^2*(b*(-4 + 3*c*Sqrt[x] + c^3*x^(3/2)) + 3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2*c^4*x^2 + b^2*(-1 + c^2*x) + 2*a*b*c*Sqrt[x]*(3 + c^2*x) - 8*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*a^2*b*Log[1 - c*Sqrt[x]] - 3*a^2*b*Log[1 + c*Sqrt[x]] + 8*a*b^2*Log[1 - c^2*x] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(4*c^4)
```

**Maple [C]** time = 0.286, size = 1339, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] 1/2*b^3*x^2*arctanh(c*x^(1/2))^3+2/c^4*b^3*arctanh(c*x^(1/2))^2-4/c^4*b^3*dilog(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-1/2/c^4*b^3*arctanh(c*x^(1/2))^3-3/8*I/c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))*arctanh(c*x^(1/2))^2-1/2*b^3*arctanh(c*x^(1/2))/c^4+1/2*b^3*x^(1/2)/c^3-3/4*I/c^4*b^3*Pi*arctanh(c*x^(1/2))^2+1/c*a*b^2*arctanh(c*x^(1/2))*x^(3/2)+3/c^3*a*b^2*x^(1/2)*arctanh(c*x^(1/2))-3/4/c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1+c*x^(1/2))+3/4/c^4*a*b^2*ln(-1/2*c*x^(1/2)+1/2)*ln(1/2+1/2*c*x^(1/2))+3/2/c^4*a*b^2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-3/2/c^4*a*b^2*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-3/4/c^4*a*b^2*ln(c*x^(1/2)-1)*ln(1/2+1/2*c*x^(1/2))+1/2*a*b^2*x/c^2-3/4*I/c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2+3/4*I/c^4*b^3*Pi*csgn(I/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^2*arctanh(c*x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^(1/2))^2+3/8*I/c^4*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/((1+c*x^(1/2))^2/(-c^2*x+1)+1))^3*arctanh(c*x^(1/2))^2-4/c^4*b^3*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4/c^4*b^3*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4/c^4*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-3/4/c^4*b^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))+3/2/c^4*b^3*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/2*a*b^2*x^2*arctanh(c*x^(1/2))^2+3/2*a^2*b*x^2*arctanh(c*x^(1/2))+2/c^4*a*b^2*ln(1+c*x^(1/2))+3/4/c^4*a^2*b*ln(c*x^(1/2)-1)-3/4/c^4*a^2*b*ln(1+c*x^(1/2))+3/8/c^4*a*b^2*ln(c*x^(1/2)-1)^2+3/8/c^4*a*b^2*ln(1+c*x^(1/2))^2+2/c^4*a*b^2*ln(c*x^(1/2)-1)+1/2/c^2*b^3*arctanh(c*x^(1/2))*x+3/2/c^3*b^3*arctanh(c*x^(1/2))^2*x^(1/2)+1/2/c*b^3*arctanh(c
```



$$\begin{aligned}
& *x^{(1/2)} \wedge 2 * x^{(3/2)} + 3/2 / c^3 * a^2 * b * x^{(1/2)} + 1/2 / c * a^2 * b * x^{(3/2)} - 3/8 * I / c^4 * b^3 \\
& * \text{Pi} * \text{csgn}(I * (1 + c * x^{(1/2)}) \wedge 2 / (c^2 * x - 1)) * \text{csgn}(I * (1 + c * x^{(1/2)}) \wedge 2 / (c^2 * x - 1) / ((1 + \\
& c * x^{(1/2)}) \wedge 2 / (-c^2 * x + 1) + 1)) \wedge 2 * \text{arctanh}(c * x^{(1/2)}) \wedge 2 + 3/4 * I / c^4 * b^3 * \text{Pi} * \text{csgn}(I * \\
& (1 + c * x^{(1/2)}) / (-c^2 * x + 1) \wedge (1/2)) * \text{csgn}(I * (1 + c * x^{(1/2)}) \wedge 2 / (c^2 * x - 1)) \wedge 2 * \text{arctanh} \\
& (c * x^{(1/2)}) \wedge 2 + 3/8 * I / c^4 * b^3 * \text{Pi} * \text{csgn}(I * (1 + c * x^{(1/2)}) / (-c^2 * x + 1) \wedge (1/2)) \wedge 2 * \text{csg} \\
& \text{n}(I * (1 + c * x^{(1/2)}) \wedge 2 / (c^2 * x - 1)) * \text{arctanh}(c * x^{(1/2)}) \wedge 2 + 3/8 * I / c^4 * b^3 * \text{Pi} * \text{csgn}(I \\
& / ((1 + c * x^{(1/2)}) \wedge 2 / (-c^2 * x + 1) + 1)) * \text{csgn}(I * (1 + c * x^{(1/2)}) \wedge 2 / (c^2 * x - 1) / ((1 + c * x^{(1/2)}) \\
& \wedge 2 / (-c^2 * x + 1) + 1)) \wedge 2 * \text{arctanh}(c * x^{(1/2)}) \wedge 2 + 1/2 * x^2 * a^3 - 1/2 / c^4 * b^3
\end{aligned}$$

**Maxima [B]** time = 3.39122, size = 1598, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="maxima")

[Out]  $1/2 * a^3 * x^2 - 1/32 * a * b^2 * c * ((3 * c^3 * x^2 + 10 * c * x - 2 * (3 * c^3 * x^2 + 4 * c^2 * x^{(3/2)} + 6 * c * x + 12 * \text{sqrt}(x)) * \log(c * \text{sqrt}(x) + 1)) / c^4 - 14 * \log(c * \text{sqrt}(x) + 1) / c^5 - 14 * \log(c * \text{sqrt}(x) - 1) / c^5) - 1/16 * (12 * x^2 * \log(c * \text{sqrt}(x) + 1) - c * ((3 * c^3 * x^2 - 4 * c^2 * x^{(3/2)} + 6 * c * x - 12 * \text{sqrt}(x)) / c^4 + 12 * \log(c * \text{sqrt}(x) + 1) / c^5)) * a * b^2 * \log(-c * \text{sqrt}(x) + 1) + 1/16 * (12 * x^2 * \log(c * \text{sqrt}(x) + 1) - c * ((3 * c^3 * x^2 - 4 * c^2 * x^{(3/2)} + 6 * c * x - 12 * \text{sqrt}(x)) / c^4 + 12 * \log(c * \text{sqrt}(x) + 1) / c^5)) * a^2 * b - 1/16 * (12 * x^2 * \log(-c * \text{sqrt}(x) + 1) - c * ((3 * c^3 * x^2 + 4 * c^2 * x^{(3/2)} + 6 * c * x + 12 * \text{sqrt}(x)) / c^4 + 12 * \log(c * \text{sqrt}(x) - 1) / c^5)) * a^2 * b + 1/192 * (9 * (8 * \log(-c * \text{sqrt}(x) + 1))^2 - 4 * \log(-c * \text{sqrt}(x) + 1) + 1) * (c * \text{sqrt}(x) - 1)^4 + 32 * (9 * \log(-c * \text{sqrt}(x) + 1))^2 - 6 * \log(-c * \text{sqrt}(x) + 1) + 2) * (c * \text{sqrt}(x) - 1)^3 + 2 * 16 * (2 * \log(-c * \text{sqrt}(x) + 1))^2 - 2 * \log(-c * \text{sqrt}(x) + 1) + 1) * (c * \text{sqrt}(x) - 1)^2 + 288 * (\log(-c * \text{sqrt}(x) + 1))^2 - 2 * \log(-c * \text{sqrt}(x) + 1) + 2) * (c * \text{sqrt}(x) - 1) * a * b^2 / c^4 - 1/4608 * (9 * (32 * \log(-c * \text{sqrt}(x) + 1))^3 - 24 * \log(-c * \text{sqrt}(x) + 1))^2 + 12 * \log(-c * \text{sqrt}(x) + 1) - 3) * (c * \text{sqrt}(x) - 1)^4 + 128 * (9 * \log(-c * \text{sqrt}(x) + 1))^3 - 9 * \log(-c * \text{sqrt}(x) + 1))^2 + 6 * \log(-c * \text{sqrt}(x) + 1) - 2) * (c * \text{sqrt}(x) - 1)^3 + 432 * (4 * \log(-c * \text{sqrt}(x) + 1))^3 - 6 * \log(-c * \text{sqrt}(x) + 1))^2 + 6 * \log(-c * \text{sqrt}(x) + 1) - 3) * (c * \text{sqrt}(x) - 1)^2 + 1152 * (\log(-c * \text{sqrt}(x) + 1))^3 - 3 * \log(-c * \text{sqrt}(x) + 1))^2 + 6 * \log(-c * \text{sqrt}(x) + 1) - 6) * (c * \text{sqrt}(x) - 1)) * b^3 / c^4 + 2 * (\log(c * \text{sqrt}(x) + 1) * \log(-1/2 * c * \text{sqrt}(x) + 1/2) + \text{dilog}(1/2 * c * \text{sqrt}(x) + 1/2)) * b^3 / c^4 - 319/384 * b^3 * \log(c * \text{sqrt}(x) - 1) / c^4 + 1/16 * (25 * a * b^2 - 4 * b^3) * \log(c * \text{sqrt}(x) + 1) / c^4 + 1/4608 * (288 * (b^3 * c^4 * x^2 - b^3) * \log(c * \text{sqrt}(x) + 1))^3 + 27 * (8 * a * b^2 * c^4 - b^3 * c^4) * x^2 + 576 * (3 * a * b^2 * c^4 * x^2 + b^3 * c^3 * x^{(3/2)} + 3 * b^3 * c * \text{sqrt}(x) - 3 * a * b^2 + 4 * b^3) * \log(c * \text{sqrt}(x) + 1))^2 - 72 * (3 * b^3 * c^4 * x^2 - 4 * b^3 * c^3 * x^{(3/2)} + 6 * b^3 * c^2 * x - 12 * b^3 * c * \text{sqrt}(x) + 7 * b^3 - 12 * (b^3 * c^4 * x^2 - b^3) * \log(c * \text{sqrt}(x) + 1)) * \log(-c * \text{sqrt}(x) + 1))^2 - 4 * (168 * a * b^2 * c^3 + 37 * b$

$$\begin{aligned} &^3c^3)x^{(3/2)} + 6*(312*a*b^2*c^2 - 115*b^3*c^2)*x - 288*(3*a*b^2*c^4*x^2 \\ &- 4*a*b^2*c^3*x^{(3/2)} - 12*a*b^2*c*\sqrt{x} + 2*(3*a*b^2*c^2 - 2*b^3*c^2)*x) \\ &*\log(c*\sqrt{x} + 1) + 12*(9*b^3*c^4*x^2 + 28*b^3*c^3*x^{(3/2)} - 18*b^3*c^2*x \\ &+ 300*b^3*c*\sqrt{x} - 72*(b^3*c^4*x^2 - b^3)*\log(c*\sqrt{x} + 1)^2 - 96*(b^ \\ &3*c^3*x^{(3/2)} + 3*b^3*c*\sqrt{x} + 4*b^3)*\log(c*\sqrt{x} + 1))*\log(-c*\sqrt{x} \\ &+ 1) - 12*(600*a*b^2*c + 223*b^3*c)*\sqrt{x})/c^4 \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3x \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2x \operatorname{artanh}(c\sqrt{x})^2 + 3a^2bx \operatorname{artanh}(c\sqrt{x}) + a^3x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="fricas")

[Out] integral(b^3\*x\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*x\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*x\*arctanh(c\*sqrt(x)) + a^3\*x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \left( a + b \operatorname{atanh}(c\sqrt{x}) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x\*\*(1/2)))\*\*3,x)

[Out] Integral(x\*(a + b\*atanh(c\*sqrt(x)))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3\*x, x)

### 3.205 $\int (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$

**Optimal.** Leaf size=142

$$\frac{3b^3 \text{PolyLog}\left(2, 1 - \frac{2}{1-c\sqrt{x}}\right)}{c^2} - \frac{6b^2 \log\left(\frac{2}{1-c\sqrt{x}}\right)(a + b \tanh^{-1}(c\sqrt{x}))}{c^2} + \frac{3b(a + b \tanh^{-1}(c\sqrt{x}))^2}{c^2} - \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{c^2}$$

```
[Out] (3*b*(a + b*ArcTanh[c*Sqrt[x]])^2)/c^2 + (3*b*Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2)/c - (a + b*ArcTanh[c*Sqrt[x]])^3/c^2 + x*(a + b*ArcTanh[c*Sqrt[x]])^3 - (6*b^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c^2 - (3*b^3*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/c^2
```

**Rubi [F]** time = 0.0063135, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

Rubi steps

$$\int (a + b \tanh^{-1}(c\sqrt{x}))^3 dx = \int (a + b \tanh^{-1}(c\sqrt{x}))^3 dx$$

**Mathematica [A]** time = 0.276754, size = 201, normalized size = 1.42

$$\frac{6b^3 \text{PolyLog}\left(2, -e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + a(2a^2c^2x + 6abc\sqrt{x} + 3ab \log(1 - c\sqrt{x}) - 3ab \log(c\sqrt{x} + 1) + 6b^2 \log(1 - c^2x))}{c^2} + \dots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3, x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + b + a*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(2*a*b*c*Sqrt[x] + a^2*c^2*x - 2*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(6*a*b*c*Sqrt[x] + 2*a^2*c^2*x + 3*a*b*Log[1 - c*Sqrt[x]] - 3*a*b*Log[1 + c*Sqrt[x]] + 6*b^2*Log[1 - c^2*x]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(2*c^2)
```

**Maple [C]** time = 0.388, size = 6235, normalized size = 43.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} \left( c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(c\sqrt{x}+1)}{c^3} + \frac{\log(c\sqrt{x}-1)}{c^3} \right) + 2x \operatorname{artanh}(c\sqrt{x}) \right) a^2 b + \frac{3}{4} \left( 4c \left( \frac{2\sqrt{x}}{c^2} - \frac{\log(c\sqrt{x}+1)}{c^3} + \frac{\log(c\sqrt{x}-1)}{c^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

```
[Out] 3/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x*arctanh(c*sqrt(x)))*a^2*b + 3/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*a*b^2 + a^3*x - 1/32*b^3*((4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 8*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))/c^2 - 4*integrate(log(c*sqrt(x) + 1)^3 - 3*log(c*sqrt(x) + 1)^2*log(-c*sqrt(x) + 1) + 3*log(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)^2, x))
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(b^3 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2b \operatorname{artanh}(c\sqrt{x}) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="fricas")

[Out] integral(b^3\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*arctanh(c\*sqrt(x)) + a^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(c\sqrt{x}))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*3,x)

[Out] Integral((a + b\*atanh(c\*sqrt(x)))\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(c\sqrt{x}) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3, x)

$$3.206 \quad \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x} dx$$

**Optimal.** Leaf size=224

$$3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) - 3b^2 \text{PolyLog}\left(3, \frac{2}{1 - c\sqrt{x}} - 1\right) (a + b \tanh^{-1}(c\sqrt{x})) - 3b \text{PolyLog}$$

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sqrt[x])]) + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```

**Rubi [A]** time = 0.511527, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6062, 6610}

$$3b^2 \text{PolyLog}\left(3, 1 - \frac{2}{1 - c\sqrt{x}}\right) (a + b \tanh^{-1}(c\sqrt{x})) - 3b^2 \text{PolyLog}\left(3, \frac{2}{1 - c\sqrt{x}} - 1\right) (a + b \tanh^{-1}(c\sqrt{x})) - 3b \text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x, x]
```

```
[Out] 4*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 3*b*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])] + 3*b*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])] + 3*b^2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])] - 3*b^2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])] - (3*b^3*PolyLog[4, 1 - 2/(1 - c*Sqrt[x])]) + (3*b^3*PolyLog[4, -1 + 2/(1 - c*Sqrt[x])])/2
```

### Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] := Simp[2*(a +
b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[((a + b
*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x))]/(1 - c^2*x^2), x], x] /; F
reeQ[{a, b, c}, x] && IGtQ[p, 1]
```

### Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d +
e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0
] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

### Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d),
x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d
+ e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6062

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] := Simp[((a + b*ArcTanh[c*x])^p*PolyLog[k + 1, u])/(2
*c*d), x] - Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[k + 1,
u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && Eq
Q[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{x} dx &= 2 \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^3}{x} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^3 - (12bc) \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \operatorname{tanh}^{-1}(cx)}{1 - c^2x} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^3 + (6bc) \operatorname{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2 \log \left( \frac{1 + c\sqrt{x}}{1 - c\sqrt{x}} \right)}{1 - c^2x^2} dx, x, \sqrt{x} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^3 - 3b (a + b \tanh^{-1}(c\sqrt{x}))^2 \operatorname{Li}_2 \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^3 - 3b (a + b \tanh^{-1}(c\sqrt{x}))^2 \operatorname{Li}_2 \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) \\
&= 4 \tanh^{-1} \left( 1 - \frac{2}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^3 - 3b (a + b \tanh^{-1}(c\sqrt{x}))^2 \operatorname{Li}_2 \left( 1 - \frac{2}{1 - c\sqrt{x}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.199004, size = 248, normalized size = 1.11

$$\frac{3}{2}b \left( 2 \operatorname{PolyLog} \left( 2, \frac{c\sqrt{x} + 1}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 - 2 \operatorname{PolyLog} \left( 2, \frac{c\sqrt{x} + 1}{c\sqrt{x} - 1} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + b \left( -2 \operatorname{PolyLog} \left( 3, \frac{1 + c\sqrt{x}}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + 2 \operatorname{PolyLog} \left( 3, \frac{1 + c\sqrt{x}}{-1 + c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + 2 \operatorname{PolyLog} \left( 3, \frac{1 + c\sqrt{x}}{1 - c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + 2 \operatorname{PolyLog} \left( 3, \frac{1 + c\sqrt{x}}{-1 + c\sqrt{x}} \right) (a + b \tanh^{-1}(c\sqrt{x}))^2 + b \left( \operatorname{PolyLog} \left( 4, \frac{1 + c\sqrt{x}}{1 - c\sqrt{x}} \right) - \operatorname{PolyLog} \left( 4, \frac{1 + c\sqrt{x}}{-1 + c\sqrt{x}} \right) \right) \right) \right) / 2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

```
[Out] 4*ArcTanh[1 + 2/(-1 + c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 + (3*b*(2*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - 2*(a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])] + b*(-2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] + 2*(a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])]) + b*(PolyLog[4, (1 + c*Sqrt[x])/(1 - c*Sqrt[x])] - PolyLog[4, (1 + c*Sqrt[x])/(-1 + c*Sqrt[x])])))/2
```

---

**Maple [C]** time = 0.173, size = 1542, normalized size = 6.9

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b*\text{arctanh}(c*x^{(1/2)}))^3/x,x)$

[Out]  $3*I*a*b^2*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1))*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*\text{arctanh}(c*x^{(1/2)})^2+2*b^3*\text{arctanh}(c*x^{(1/2)})^3*\ln(1+(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+6*b^3*\text{arctanh}(c*x^{(1/2)})^2*\text{polylog}(2,-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})-12*b^3*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(3,-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+2*b^3*\ln(c*x^{(1/2)})*\text{arctanh}(c*x^{(1/2)})^3-2*b^3*\text{arctanh}(c*x^{(1/2)})^3*\ln((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)-3*b^3*\text{arctanh}(c*x^{(1/2)})^2*\text{polylog}(2,-(1+c*x^{(1/2)})^2/(-c^2*x+1))+3*b^3*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(3,-(1+c*x^{(1/2)})^2/(-c^2*x+1))-3*a^2*b*\text{dilog}(c*x^{(1/2)})-3*a^2*b*\text{dilog}(1+c*x^{(1/2)})+3*a*b^2*\text{polylog}(3,-(1+c*x^{(1/2)})^2/(-c^2*x+1))-12*a*b^2*\text{polylog}(3,(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})-12*a*b^2*\text{polylog}(3,-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+2*b^3*\text{arctanh}(c*x^{(1/2)})^3*\ln(1-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+6*b^3*\text{arctanh}(c*x^{(1/2)})^2*\text{polylog}(2,(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})-12*b^3*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(3,(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})-6*a*b^2*\text{arctanh}(c*x^{(1/2)})^2*\ln((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)+6*a*b^2*\text{arctanh}(c*x^{(1/2)})^2*\ln(1-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+12*a*b^2*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(2,(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+6*a*b^2*\text{arctanh}(c*x^{(1/2)})^2*\ln(1+(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+12*a*b^2*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(2,-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+6*a*b^2*\ln(c*x^{(1/2)})*\text{arctanh}(c*x^{(1/2)})^2-6*a*b^2*\text{arctanh}(c*x^{(1/2)})*\text{polylog}(2,-(1+c*x^{(1/2)})^2/(-c^2*x+1))-I*b^3*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{(1/2)})^3-I*b^3*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{(1/2)})^3+3*I*a*b^2*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^3*\text{arctanh}(c*x^{(1/2)})^2+6*a^2*b*\ln(c*x^{(1/2)})*\text{arctanh}(c*x^{(1/2)})-3*a^2*b*\ln(c*x^{(1/2)})*\ln(1+c*x^{(1/2)})+I*b^3*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1))*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*\text{arctanh}(c*x^{(1/2)})^3-3*I*a*b^2*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{(1/2)})^2-3*I*a*b^2*Pi*csgn(I/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{(1/2)})^2+2*a^3*\ln(c*x^{(1/2)})-3/2*b^3*\text{polylog}(4,-(1+c*x^{(1/2)})^2/(-c^2*x+1))+12*b^3*\text{polylog}(4,(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+12*b^3*\text{polylog}(4,-(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})+I*b^3*Pi*csgn(I*((1+c*x^{(1/2)})^2/(-c^2*x+1)-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^3*\text{arctanh}(c*x^{(1/2)})^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8}b^3 \int \frac{\log(c\sqrt{x}+1)^3}{x} dx - \frac{3}{8}b^3 \int \frac{\log(c\sqrt{x}+1)^2 \log(-c\sqrt{x}+1)}{x} dx + \frac{3}{8}b^3 \int \frac{\log(c\sqrt{x}+1) \log(-c\sqrt{x}+1)^2}{x} dx - \frac{1}{8}b^3 \int \frac{\log(-c\sqrt{x}+1)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x,x, algorithm="maxima")

[Out] 1/8\*b^3\*integrate(log(c\*sqrt(x) + 1)^3/x, x) - 3/8\*b^3\*integrate(log(c\*sqrt(x) + 1)^2\*log(-c\*sqrt(x) + 1)/x, x) + 3/8\*b^3\*integrate(log(c\*sqrt(x) + 1)\*log(-c\*sqrt(x) + 1)^2/x, x) - 1/8\*b^3\*integrate(log(-c\*sqrt(x) + 1)^3/x, x) + 3/4\*a\*b^2\*integrate(log(c\*sqrt(x) + 1)^2/x, x) - 3/2\*a\*b^2\*integrate(log(c\*sqrt(x) + 1)\*log(-c\*sqrt(x) + 1)/x, x) + 3/4\*a\*b^2\*integrate(log(-c\*sqrt(x) + 1)^2/x, x) + 3/2\*a^2\*b\*integrate(log(c\*sqrt(x) + 1)/x, x) - 3/2\*a^2\*b\*integrate(log(-c\*sqrt(x) + 1)/x, x) + a^3\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2b \operatorname{artanh}(c\sqrt{x}) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*arctanh(c\*sqrt(x)) + a^3)/x, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(c\sqrt{x}))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*3/x,x)

[Out] Integral((a + b\*atanh(c\*sqrt(x)))\*\*3/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3/x, x)

$$3.207 \quad \int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{x^2} dx$$

**Optimal.** Leaf size=142

$$-3b^3c^2 \text{PolyLog}\left(2, \frac{2}{c\sqrt{x}+1} - 1\right) + 6b^2c^2 \log\left(2 - \frac{2}{c\sqrt{x}+1}\right) (a + b \tanh^{-1}(c\sqrt{x})) + 3bc^2 (a + b \tanh^{-1}(c\sqrt{x}))^2 + c^2 (a + b \tanh^{-1}(c\sqrt{x}))^3$$

```
[Out] 3*b*c^2*(a + b*ArcTanh[c*Sqrt[x]])^2 - (3*b*c*(a + b*ArcTanh[c*Sqrt[x]])^2)
/Sqrt[x] + c^2*(a + b*ArcTanh[c*Sqrt[x]])^3 - (a + b*ArcTanh[c*Sqrt[x]])^3/
x + 6*b^2*c^2*(a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - 3*b^3
*c^2*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])]
```

**Rubi [F]** time = 0.0222653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{x^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2, x]
```

```
[Out] Defer[Int][(a + b*ArcTanh[c*Sqrt[x]])^3/x^2, x]
```

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{x^2} dx = \int \frac{(a + b \tanh^{-1}(c\sqrt{x}))^3}{x^2} dx$$

**Mathematica [A]** time = 0.307345, size = 230, normalized size = 1.62

$$-6b^3c^2x \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + a\left(-2a^2 - 3abc^2x \log(1 - c\sqrt{x}) + 3abc^2x \log(c\sqrt{x} + 1) - 6abc\sqrt{x} + 12b^2c^2x \log\left(\frac{1 + c\sqrt{x}}{1 - c\sqrt{x}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

```
[Out] (6*b^2*(-1 + c*Sqrt[x])*(a + a*c*Sqrt[x] + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 - 6*b*ArcTanh[c*Sqrt[x]]*(a^2 + 2*a*b*c*Sqrt[x] - 2*b^2*c^2*x*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(-2*a^2 - 6*a*b*c*Sqrt[x] - 3*a*b*c^2*x*Log[1 - c*Sqrt[x]] + 3*a*b*c^2*x*Log[1 + c*Sqrt[x]] + 12*b^2*c^2*x*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]]) - 6*b^3*c^2*x*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(2*x)
```

**Maple [C]** time = 0.412, size = 5199, normalized size = 36.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctanh(c*x^(1/2)))^3/x^2,x)
```

```
[Out] result too large to display
```

**Maxima [B]** time = 5.86528, size = 713, normalized size = 5.02

$$-3 \left( \log(c\sqrt{x} + 1) \log\left(-\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right) + \text{Li}_2\left(\frac{1}{2}c\sqrt{x} + \frac{1}{2}\right) \right) b^3 c^2 - 3 \left( \log(c\sqrt{x}) \log(-c\sqrt{x} + 1) + \text{Li}_2(-c\sqrt{x} + 1) \right) b^3 c^2 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1/2))*b^3*c^2 - 3*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b^3*c^2 + 3*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b^3*c^2 - 3*a*b^2*c^2*log(c*sqrt(x) - 1) - 3/4*((2*c*log(c*sqrt(x) - 1) - c*log(x) + 2/sqrt(x))*c - 2*log(-c*sqrt(x) + 1)/x)*a^2*b - a^3/x + 3/2*(a^2*b*c^2 - 2*a*b^2*c^2)*log(c*sqrt(x) + 1) - 3/4*(a^2*b*c^2 - 4*a*b^2*c^2)*log(x) - 1/8*(12*a^2*b*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^3 + (b^3*c^2*x - b^3)*log(-c*sqrt(x) + 1)^3 + 6*(b^3*c*sqrt(x) + a*b^2 - (a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1)^2 + 3*(2*b^3*c*sqrt(x) + 2*a*b^2 - 2*(a*b^2*c^2 + b^3*c^2)*x - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x))
```

$$+ 1)^2 + 12*(2*a*b^2*c*\sqrt{x} + a^2*b)*\log(c*\sqrt{x} + 1) - 3*(8*a*b^2*c*\sqrt{x} - (b^3*c^2*x - b^3)*\log(c*\sqrt{x} + 1)^2 + 4*(b^3*c*\sqrt{x} + a*b^2 - (a*b^2*c^2 - b^3*c^2)*x)*\log(c*\sqrt{x} + 1))*\log(-c*\sqrt{x} + 1)/x$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2b \operatorname{artanh}(c\sqrt{x}) + a^3}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x^2,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*arctanh(c\*sqrt(x)) + a^3)/x^2, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(c\sqrt{x}))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*3/x\*\*2,x)

[Out] Integral((a + b\*atanh(c\*sqrt(x)))\*\*3/x\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3/x^2, x)

$$3.208 \quad \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^3} dx$$

**Optimal.** Leaf size=234

$$-2b^3c^4 \text{PolyLog}\left(2, \frac{2}{c\sqrt{x}+1} - 1\right) - \frac{b^2c^2(a+b \tanh^{-1}(c\sqrt{x}))}{2x} + 4b^2c^4 \log\left(2 - \frac{2}{c\sqrt{x}+1}\right)(a+b \tanh^{-1}(c\sqrt{x})) + \frac{1}{2}c^4$$

[Out]  $-(b^3c^3)/(2\sqrt{x}) + (b^3c^4 \text{ArcTanh}[c\sqrt{x}])/2 - (b^2c^2(a + b \text{ArcTanh}[c\sqrt{x}]))/(2x) + 2b^2c^4(a + b \text{ArcTanh}[c\sqrt{x}])^2 - (b^2c^2(a + b \text{ArcTanh}[c\sqrt{x}])^2)/(2x^{3/2}) - (3b^2c^3(a + b \text{ArcTanh}[c\sqrt{x}])^2)/(2\sqrt{x}) + (c^4(a + b \text{ArcTanh}[c\sqrt{x}])^3)/2 - (a + b \text{ArcTanh}[c\sqrt{x}])^3/(2x^2) + 4b^2c^4(a + b \text{ArcTanh}[c\sqrt{x}]) \log[2 - 2/(1 + c\sqrt{x})] - 2b^3c^4 \text{PolyLog}[2, -1 + 2/(1 + c\sqrt{x})]$

**Rubi [F]** time = 0.0232819, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*sqrt[x]])^3/x^3, x]

[Out] Defer[Int][(a + b\*ArcTanh[c\*sqrt[x]])^3/x^3, x]

Rubi steps

$$\int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^3} dx = \int \frac{(a+b \tanh^{-1}(c\sqrt{x}))^3}{x^3} dx$$

**Mathematica [A]** time = 0.692146, size = 333, normalized size = 1.42

$$8b^3c^4x^2 \text{PolyLog}\left(2, e^{-2 \tanh^{-1}(c\sqrt{x})}\right) + 2b \tanh^{-1}(c\sqrt{x}) \left(3a^2 + 2abc\sqrt{x}(3c^2x + 1) - 8b^2c^4x^2 \log\left(1 - e^{-2 \tanh^{-1}(c\sqrt{x})}\right)\right) +$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcTanh[c\*Sqrt[x]])^3/x^3,x]

[Out]  $-(2*a^3 + 2*a^2*b*c*\text{Sqrt}[x] + 2*a*b^2*c^2*x + 6*a^2*b*c^3*x^{3/2} + 2*b^3*c^3*x^{3/2} - 2*a*b^2*c^4*x^2 - 2*b^2*(b*c*\text{Sqrt}[x]*(-1 - 3*c^2*x + 4*c^3*x^{3/2})) + 3*a*(-1 + c^4*x^2))*\text{ArcTanh}[c*\text{Sqrt}[x]]^2 - 2*b^3*(-1 + c^4*x^2)*\text{ArcTanh}[c*\text{Sqrt}[x]]^3 + 2*b*\text{ArcTanh}[c*\text{Sqrt}[x]]*(3*a^2 + b^2*c^2*x*(1 - c^2*x) + 2*a*b*c*\text{Sqrt}[x]*(1 + 3*c^2*x) - 8*b^2*c^4*x^2*\text{Log}[1 - E^{(-2*\text{ArcTanh}[c*\text{Sqrt}[x]])}])) + 3*a^2*b*c^4*x^2*\text{Log}[1 - c*\text{Sqrt}[x]] - 3*a^2*b*c^4*x^2*\text{Log}[1 + c*\text{Sqrt}[x]] - 16*a*b^2*c^4*x^2*\text{Log}[(c*\text{Sqrt}[x])/ \text{Sqrt}[1 - c^2*x]] + 8*b^3*c^4*x^2*\text{PolyLog}[2, E^{(-2*\text{ArcTanh}[c*\text{Sqrt}[x])}]])/(4*x^2)$

**Maple [C]** time = 0.31, size = 1365, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(1/2)))^3/x^3,x)

[Out]  $-3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^{1/2})^2/(c^2*x-1)/((1+c*x^{1/2})^2/(-c^2*x+1)+1))^3*arctanh(c*x^{1/2})^2-3/4*I*c^4*b^3*Pi*csgn(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))^2*arctanh(c*x^{1/2})^2-3/8*I*c^4*b^3*Pi*csgn(I*(1+c*x^{1/2})^2/(c^2*x-1))^3*arctanh(c*x^{1/2})^2+3/4*I*c^4*b^3*Pi*csgn(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))^3*arctanh(c*x^{1/2})^2-1/2*a^3/x^2+3/4*c^4*b^3*arctanh(c*x^{1/2})^2*\ln(1+c*x^{1/2})-3/2*c^4*b^3*arctanh(c*x^{1/2})^2*\ln((1+c*x^{1/2})/(-c^2*x+1)^{1/2})-3/2*a*b^2/x^2*arctanh(c*x^{1/2})^2-3/2*a^2*b/x^2*arctanh(c*x^{1/2})-1/2*c^2*b^3*arctanh(c*x^{1/2})/x-1/2*c*b^3*arctanh(c*x^{1/2})^2/x^{3/2}-3/2*c^3*b^3*arctanh(c*x^{1/2})^2/x^{1/2}-3/2*c^3*a^2*b/x^{1/2}-1/2*c*a^2*b/x^{3/2}+4*c^4*a*b^2*\ln(c*x^{1/2})-3/8*c^4*a*b^2*\ln(c*x^{1/2}-1)^2-3/8*c^4*a*b^2*\ln(1+c*x^{1/2})^2-2*c^4*a*b^2*\ln(c*x^{1/2}-1)-2*c^4*a*b^2*\ln(1+c*x^{1/2})-3/4*c^4*a^2*b*\ln(c*x^{1/2}-1)+3/4*c^4*a^2*b*\ln(1+c*x^{1/2})+4*c^4*b^3*arctanh(c*x^{1/2})*\ln(1+(1+c*x^{1/2})/(-c^2*x+1)^{1/2})-1/2*c^4*b^3/(c*x^{1/2}+1-(-c^2*x+1)^{1/2})*(-c^2*x+1)^{1/2}+1/2*c^4*b^3/((-c^2*x+1)^{1/2}+c*x^{1/2}+1)*(-c^2*x+1)^{1/2}-3/4*c^4*b^3*arctanh(c*x^{1/2})^2*\ln(c*x^{1/2}-1)+3/8*I*c^4*b^3*Pi*csgn(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1))*csgn(I*(1+c*x^{1/2})^2/(c^2*x-1))*csgn(I*(1+c*x^{1/2})^2/(c^2*x-1)/((1+c*x^{1/2})^2/(-c^2*x+1)+1))*arctanh(c*x^{1/2})^2-1/2*b^3/x^2*arctanh(c*x^{1/2})^3+4*c^4*b^3*dilog(1+(1+c*x^{1/2})/(-c^2*x+1)^{1/2})-4*c^4*b^3*dilog((1+c*x^{1/2})/(-c^2*x+1)^{1/2})-2*c^4*b^3*arctanh(c*x^{1/2})^2+1/2*c^4*b^3*arctanh(c*x^{1/2})^3+1/2*b^3*c^4*arctanh(c*x^{1/2})-3/8*I*c^4*b^3*Pi*csgn(I/((1+c*x^{1/2})^2/(-c^2*x+1)+1)))$



$$\begin{aligned}
& -c^2*x+1)+1))*\text{csgn}(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/((1+c*x^{(1/2)})^2/(-c^2*x+1)+ \\
& 1))^2*\text{arctanh}(c*x^{(1/2)})^2-3/8*I*c^4*b^3*Pi*\text{csgn}(I*(1+c*x^{(1/2)})/(-c^2*x+1) \\
& ^{(1/2)})^2*\text{csgn}(I*(1+c*x^{(1/2)})^2/(c^2*x-1))*\text{arctanh}(c*x^{(1/2)})^2+3/8*I*c^4* \\
& b^3*Pi*\text{csgn}(I*(1+c*x^{(1/2)})^2/(c^2*x-1))*\text{csgn}(I*(1+c*x^{(1/2)})^2/(c^2*x-1)/ \\
& (1+c*x^{(1/2)})^2/(-c^2*x+1)+1))^2*\text{arctanh}(c*x^{(1/2)})^2-3/4*I*c^4*b^3*Pi*\text{csgn} \\
& (I*(1+c*x^{(1/2)})/(-c^2*x+1)^{(1/2)})*\text{csgn}(I*(1+c*x^{(1/2)})^2/(c^2*x-1))^2*\text{arct} \\
& \text{anh}(c*x^{(1/2)})^2+3/4*c^4*a*b^2*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1+c*x^{(1/2)})-3/4*c \\
& ^4*a*b^2*\ln(-1/2*c*x^{(1/2)}+1/2)*\ln(1/2+1/2*c*x^{(1/2)})-3/2*c^4*a*b^2*\text{arctanh} \\
& (c*x^{(1/2)})*\ln(c*x^{(1/2)}-1)+3/2*c^4*a*b^2*\text{arctanh}(c*x^{(1/2)})*\ln(1+c*x^{(1/2)} \\
& )+3/4*c^4*a*b^2*\ln(c*x^{(1/2)}-1)*\ln(1/2+1/2*c*x^{(1/2)})-c*a*b^2*\text{arctanh}(c*x^{( \\
& 1/2)))/x^{(3/2)}-3*c^3*a*b^2/x^{(1/2)}*\text{arctanh}(c*x^{(1/2)})+3/4*I*c^4*b^3*Pi*\text{arcta} \\
& \text{nh}(c*x^{(1/2)})^2-1/2*c^2*a*b^2/x
\end{aligned}$$

**Maxima [B]** time = 6.64872, size = 949, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x^3,x, algorithm="maxima")

$$\begin{aligned}
& [\text{Out}] -2*(\log(c*\text{sqrt}(x) + 1)*\log(-1/2*c*\text{sqrt}(x) + 1/2) + \text{dilog}(1/2*c*\text{sqrt}(x) + 1/ \\
& 2))*b^3*c^4 - 2*(\log(c*\text{sqrt}(x))*\log(-c*\text{sqrt}(x) + 1) + \text{dilog}(-c*\text{sqrt}(x) + 1) \\
& )*b^3*c^4 + 2*(\log(c*\text{sqrt}(x) + 1)*\log(-c*\text{sqrt}(x)) + \text{dilog}(c*\text{sqrt}(x) + 1))*b \\
& ^3*c^4 - 1/8*((6*c^3*\log(c*\text{sqrt}(x) - 1) - 3*c^3*\log(x) + (6*c^2*x + 3*c*\text{sqrt} \\
& \text{t}(x) + 2)/x^{(3/2)})*c - 6*\log(-c*\text{sqrt}(x) + 1)/x^2)*a^2*b + 1/4*(3*a^2*b*c^4 \\
& - 8*a*b^2*c^4 + b^3*c^4)*\log(c*\text{sqrt}(x) + 1) - 1/4*(8*a*b^2*c^4 + b^3*c^4)*\log \\
& (c*\text{sqrt}(x) - 1) - 1/8*(3*a^2*b*c^4 - 16*a*b^2*c^4)*\log(x) - 1/2*a^3/x^2 - \\
& 1/16*(4*a^2*b*c*\text{sqrt}(x) - (b^3*c^4*x^2 - b^3)*\log(c*\text{sqrt}(x) + 1))^3 + (b^3*c \\
& ^4*x^2 - b^3)*\log(-c*\text{sqrt}(x) + 1)^3 + 2*(3*b^3*c^3*x^{(3/2)} + b^3*c*\text{sqrt}(x) \\
& + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*\log(c*\text{sqrt}(x) + 1)^2 + (6*b^3*c \\
& ^3*x^{(3/2)} + 2*b^3*c*\text{sqrt}(x) + 6*a*b^2 - 2*(3*a*b^2*c^4 + 4*b^3*c^4)*x^2 - \\
& 3*(b^3*c^4*x^2 - b^3)*\log(c*\text{sqrt}(x) + 1))*\log(-c*\text{sqrt}(x) + 1)^2 + 4*(3*a^2* \\
& b*c^3 + 2*b^3*c^3)*x^{(3/2)} - 2*(3*a^2*b*c^2 - 4*a*b^2*c^2)*x + 4*(6*a*b^2*c \\
& ^3*x^{(3/2)} + b^3*c^2*x + 2*a*b^2*c*\text{sqrt}(x) + 3*a^2*b)*\log(c*\text{sqrt}(x) + 1) - \\
& (24*a*b^2*c^3*x^{(3/2)} + 4*b^3*c^2*x + 8*a*b^2*c*\text{sqrt}(x) - 3*(b^3*c^4*x^2 - \\
& b^3)*\log(c*\text{sqrt}(x) + 1)^2 + 4*(3*b^3*c^3*x^{(3/2)} + b^3*c*\text{sqrt}(x) + 3*a*b^2 \\
& - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*\log(c*\text{sqrt}(x) + 1))*\log(-c*\text{sqrt}(x) + 1))/x \\
& ^2
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3 \operatorname{artanh}(c\sqrt{x})^3 + 3ab^2 \operatorname{artanh}(c\sqrt{x})^2 + 3a^2b \operatorname{artanh}(c\sqrt{x}) + a^3}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x^3,x, algorithm="fricas")

[Out] integral((b^3\*arctanh(c\*sqrt(x))^3 + 3\*a\*b^2\*arctanh(c\*sqrt(x))^2 + 3\*a^2\*b\*arctanh(c\*sqrt(x)) + a^3)/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(1/2)))\*\*3/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(c\sqrt{x}) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(1/2)))^3/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*sqrt(x)) + a)^3/x^3, x)

### 3.209 $\int x^{3/2} \tanh^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=38

$$\frac{x^2}{10} + \frac{2}{5}x^{5/2} \tanh^{-1}(\sqrt{x}) + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

[Out]  $x/5 + x^2/10 + (2*x^{(5/2)*ArcTanh[Sqrt[x]])/5 + \text{Log}[1 - x]/5$

**Rubi [A]** time = 0.0162326, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6097, 43}

$$\frac{x^2}{10} + \frac{2}{5}x^{5/2} \tanh^{-1}(\sqrt{x}) + \frac{x}{5} + \frac{1}{5} \log(1-x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3/2)*ArcTanh[Sqrt[x]]}, x]$

[Out]  $x/5 + x^2/10 + (2*x^{(5/2)*ArcTanh[Sqrt[x]])/5 + \text{Log}[1 - x]/5$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
 \int x^{3/2} \tanh^{-1}(\sqrt{x}) \, dx &= \frac{2}{5} x^{5/2} \tanh^{-1}(\sqrt{x}) - \frac{1}{5} \int \frac{x^2}{1-x} \, dx \\
 &= \frac{2}{5} x^{5/2} \tanh^{-1}(\sqrt{x}) - \frac{1}{5} \int \left( -1 + \frac{1}{1-x} - x \right) dx \\
 &= \frac{x}{5} + \frac{x^2}{10} + \frac{2}{5} x^{5/2} \tanh^{-1}(\sqrt{x}) + \frac{1}{5} \log(1-x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0159567, size = 31, normalized size = 0.82

$$\frac{1}{10} \left( 4x^{5/2} \tanh^{-1}(\sqrt{x}) + (x+2)x + 2 \log(1-x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2)\*ArcTanh[Sqrt[x]],x]

[Out] (x\*(2 + x) + 4\*x^(5/2)\*ArcTanh[Sqrt[x]] + 2\*Log[1 - x])/10

**Maple [A]** time = 0.024, size = 35, normalized size = 0.9

$$\frac{2}{5} x^{\frac{5}{2}} \operatorname{Artanh}(\sqrt{x}) + \frac{x^2}{10} + \frac{x}{5} + \frac{1}{5} \ln(-1 + \sqrt{x}) + \frac{1}{5} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2)\*arctanh(x^(1/2)),x)

[Out] 2/5\*x^(5/2)\*arctanh(x^(1/2))+1/10\*x^2+1/5\*x+1/5\*ln(-1+x^(1/2))+1/5\*ln(1+x^(1/2))

**Maxima [A]** time = 0.970501, size = 32, normalized size = 0.84

$$\frac{2}{5} x^{\frac{5}{2}} \operatorname{artanh}(\sqrt{x}) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arctanh(x^(1/2)),x, algorithm="maxima")

[Out] 2/5\*x^(5/2)\*arctanh(sqrt(x)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

**Fricas [A]** time = 1.65311, size = 112, normalized size = 2.95

$$\frac{1}{5} x^{\frac{5}{2}} \log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)\*arctanh(x^(1/2)),x, algorithm="fricas")

[Out] 1/5\*x^(5/2)\*log(-(x + 2\*sqrt(x) + 1)/(x - 1)) + 1/10\*x^2 + 1/5\*x + 1/5\*log(x - 1)

**Sympy [B]** time = 18.1094, size = 128, normalized size = 3.37

$$\frac{4x^{\frac{7}{2}} \operatorname{atanh}(\sqrt{x})}{10x-10} - \frac{4x^{\frac{5}{2}} \operatorname{atanh}(\sqrt{x})}{10x-10} + \frac{x^3}{10x-10} + \frac{x^2}{10x-10} + \frac{4x \log(\sqrt{x}+1)}{10x-10} - \frac{4x \operatorname{atanh}(\sqrt{x})}{10x-10} - \frac{x}{10x-10} - \frac{4 \log(\sqrt{x}+1)}{10x-10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3/2)\*atanh(x\*\*(1/2)),x)

[Out] 4\*x\*\*(7/2)\*atanh(sqrt(x))/(10\*x - 10) - 4\*x\*\*(5/2)\*atanh(sqrt(x))/(10\*x - 10) + x\*\*3/(10\*x - 10) + x\*\*2/(10\*x - 10) + 4\*x\*log(sqrt(x) + 1)/(10\*x - 10) - 4\*x\*atanh(sqrt(x))/(10\*x - 10) - x/(10\*x - 10) - 4\*log(sqrt(x) + 1)/(10\*x - 10) + 4\*atanh(sqrt(x))/(10\*x - 10) - 1/(10\*x - 10)

**Giac [A]** time = 1.17898, size = 49, normalized size = 1.29

$$\frac{1}{5} x^{\frac{5}{2}} \log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \frac{1}{10} x^2 + \frac{1}{5} x + \frac{1}{5} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*x^(5/2)*log(-(sqrt(x) + 1)/(sqrt(x) - 1)) + 1/10*x^2 + 1/5*x + 1/5*log(
abs(x - 1))
```

### 3.210 $\int \sqrt{x} \tanh^{-1}(\sqrt{x}) dx$

**Optimal.** Leaf size=31

$$\frac{2}{3}x^{3/2} \tanh^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

[Out]  $x/3 + (2*x^{(3/2)}*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3$

**Rubi [A]** time = 0.0129167, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6097, 43}

$$\frac{2}{3}x^{3/2} \tanh^{-1}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \log(1-x)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]*ArcTanh[Sqrt[x]],x]`

[Out]  $x/3 + (2*x^{(3/2)}*ArcTanh[Sqrt[x]])/3 + Log[1 - x]/3$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
 \int \sqrt{x} \tanh^{-1}(\sqrt{x}) dx &= \frac{2}{3} x^{3/2} \tanh^{-1}(\sqrt{x}) - \frac{1}{3} \int \frac{x}{1-x} dx \\
 &= \frac{2}{3} x^{3/2} \tanh^{-1}(\sqrt{x}) - \frac{1}{3} \int \left(-1 + \frac{1}{1-x}\right) dx \\
 &= \frac{x}{3} + \frac{2}{3} x^{3/2} \tanh^{-1}(\sqrt{x}) + \frac{1}{3} \log(1-x)
 \end{aligned}$$

**Mathematica [A]** time = 0.0111748, size = 25, normalized size = 0.81

$$\frac{1}{3} (2x^{3/2} \tanh^{-1}(\sqrt{x}) + x + \log(1-x))$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]\*ArcTanh[Sqrt[x]],x]

[Out] (x + 2\*x^(3/2)\*ArcTanh[Sqrt[x]] + Log[1 - x])/3

**Maple [A]** time = 0.024, size = 30, normalized size = 1.

$$\frac{2}{3} x^{\frac{3}{2}} \operatorname{Artanh}(\sqrt{x}) + \frac{x}{3} + \frac{1}{3} \ln(-1 + \sqrt{x}) + \frac{1}{3} \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(x^(1/2))\*x^(1/2),x)

[Out] 2/3\*x^(3/2)\*arctanh(x^(1/2))+1/3\*x+1/3\*ln(-1+x^(1/2))+1/3\*ln(1+x^(1/2))

**Maxima [A]** time = 0.97717, size = 26, normalized size = 0.84

$$\frac{2}{3} x^{\frac{3}{2}} \operatorname{artanh}(\sqrt{x}) + \frac{1}{3} x + \frac{1}{3} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))\*x^(1/2),x, algorithm="maxima")



[Out]  $2/3*x^{(3/2)}*\operatorname{arctanh}(\operatorname{sqrt}(x)) + 1/3*x + 1/3*\log(x - 1)$

---

**Fricas [A]** time = 1.70567, size = 97, normalized size = 3.13

$$\frac{1}{3}x^{\frac{3}{2}}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right) + \frac{1}{3}x + \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="fricas")`

[Out]  $1/3*x^{(3/2)}*\log(-(x + 2*\operatorname{sqrt}(x) + 1)/(x - 1)) + 1/3*x + 1/3*\log(x - 1)$

---

**Sympy [A]** time = 1.71664, size = 39, normalized size = 1.26

$$\frac{2x^{\frac{3}{2}}\operatorname{atanh}(\sqrt{x})}{3} + \frac{x}{3} + \frac{2\log(\sqrt{x}+1)}{3} - \frac{2\operatorname{atanh}(\sqrt{x})}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(atanh(x**(1/2))*x**(1/2),x)`

[Out]  $2*x^{(3/2)}*\operatorname{atanh}(\operatorname{sqrt}(x))/3 + x/3 + 2*\log(\operatorname{sqrt}(x) + 1)/3 - 2*\operatorname{atanh}(\operatorname{sqrt}(x))/3$

---

**Giac [A]** time = 1.16795, size = 42, normalized size = 1.35

$$\frac{1}{3}x^{\frac{3}{2}}\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \frac{1}{3}x + \frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="giac")`

[Out]  $1/3*x^{(3/2)}*\log(-(\operatorname{sqrt}(x) + 1)/(\operatorname{sqrt}(x) - 1)) + 1/3*x + 1/3*\log(\operatorname{abs}(x - 1))$

$$3.211 \quad \int \frac{\tanh^{-1}(\sqrt{x})}{\sqrt{x}} dx$$

**Optimal.** Leaf size=20

$$\log(1-x) + 2\sqrt{x} \tanh^{-1}(\sqrt{x})$$

[Out] 2\*Sqrt[x]\*ArcTanh[Sqrt[x]] + Log[1 - x]

**Rubi [A]** time = 0.0091329, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6097, 31}

$$\log(1-x) + 2\sqrt{x} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[Sqrt[x]]/Sqrt[x], x]

[Out] 2\*Sqrt[x]\*ArcTanh[Sqrt[x]] + Log[1 - x]

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\sqrt{x})}{\sqrt{x}} dx &= 2\sqrt{x} \tanh^{-1}(\sqrt{x}) - \int \frac{1}{1-x} dx \\ &= 2\sqrt{x} \tanh^{-1}(\sqrt{x}) + \log(1-x) \end{aligned}$$

**Mathematica [A]** time = 0.0084416, size = 20, normalized size = 1.

$$\log(1-x) + 2\sqrt{x} \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[Sqrt[x]]/Sqrt[x],x]

[Out] 2\*Sqrt[x]\*ArcTanh[Sqrt[x]] + Log[1 - x]

**Maple [A]** time = 0.025, size = 17, normalized size = 0.9

$$\ln(1-x) + 2 \operatorname{Artanh}(\sqrt{x}) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(x^(1/2))/x^(1/2),x)

[Out] ln(1-x)+2\*arctanh(x^(1/2))\*x^(1/2)

**Maxima [A]** time = 0.954426, size = 22, normalized size = 1.1

$$2\sqrt{x} \operatorname{artanh}(\sqrt{x}) + \log(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="maxima")

[Out] 2\*sqrt(x)\*arctanh(sqrt(x)) + log(-x + 1)

**Fricas [A]** time = 1.70806, size = 76, normalized size = 3.8

$$\sqrt{x} \log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="fricas")

[Out] sqrt(x)\*log(-(x + 2\*sqrt(x) + 1)/(x - 1)) + log(x - 1)

**Sympy [B]** time = 0.892376, size = 87, normalized size = 4.35

$$\frac{2x^{\frac{3}{2}} \operatorname{atanh}(\sqrt{x})}{x-1} - \frac{2\sqrt{x} \operatorname{atanh}(\sqrt{x})}{x-1} + \frac{2x \log(\sqrt{x}+1)}{x-1} - \frac{2x \operatorname{atanh}(\sqrt{x})}{x-1} - \frac{2 \log(\sqrt{x}+1)}{x-1} + \frac{2 \operatorname{atanh}(\sqrt{x})}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(x\*\*(1/2))/x\*\*(1/2),x)

[Out] 2\*x\*\*(3/2)\*atanh(sqrt(x))/(x - 1) - 2\*sqrt(x)\*atanh(sqrt(x))/(x - 1) + 2\*x\*log(sqrt(x) + 1)/(x - 1) - 2\*x\*atanh(sqrt(x))/(x - 1) - 2\*log(sqrt(x) + 1)/(x - 1) + 2\*atanh(sqrt(x))/(x - 1)

**Giac [A]** time = 1.18187, size = 34, normalized size = 1.7

$$\sqrt{x} \log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="giac")

[Out] sqrt(x)\*log(-(sqrt(x) + 1)/(sqrt(x) - 1)) + log(abs(x - 1))

$$3.212 \quad \int \frac{\tanh^{-1}(\sqrt{x})}{x^{3/2}} dx$$

**Optimal.** Leaf size=24

$$-\log(1-x) + \log(x) - \frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}}$$

[Out]  $(-2*\text{ArcTanh}[\text{Sqrt}[x]])/\text{Sqrt}[x] - \text{Log}[1 - x] + \text{Log}[x]$

**Rubi [A]** time = 0.0102569, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6097, 36, 31, 29}

$$-\log(1-x) + \log(x) - \frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{ArcTanh}[\text{Sqrt}[x]]/x^{(3/2)}, x]$

[Out]  $(-2*\text{ArcTanh}[\text{Sqrt}[x]])/\text{Sqrt}[x] - \text{Log}[1 - x] + \text{Log}[x]$

#### Rule 6097

$\text{Int}[(a_.) + \text{ArcTanh}[(c_.)*(x_.)^{(n_.)}]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x\_Symbol]$   
 $:\> \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 36

$\text{Int}[1/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.)), x\_Symbol] :\> \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_.))^{(-1)}, x\_Symbol] :\> \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /;$   
 $\text{FreeQ}\{a, b\}, x]$

Rule 29

`Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]`

Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(\sqrt{x})}{x^{3/2}} dx &= -\frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{(1-x)x} dx \\ &= -\frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}} + \int \frac{1}{1-x} dx + \int \frac{1}{x} dx \\ &= -\frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x) \end{aligned}$$

**Mathematica [A]** time = 0.0176779, size = 24, normalized size = 1.

$$-\log(1-x) + \log(x) - \frac{2 \tanh^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] `Integrate[ArcTanh[Sqrt[x]]/x^(3/2), x]`

[Out] `(-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]`

**Maple [A]** time = 0.03, size = 29, normalized size = 1.2

$$-2 \frac{\operatorname{Artanh}(\sqrt{x})}{\sqrt{x}} - \ln(-1 + \sqrt{x}) + \ln(x) - \ln(1 + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(arctanh(x^(1/2))/x^(3/2), x)`

[Out] `-2*arctanh(x^(1/2))/x^(1/2)-ln(-1+x^(1/2))+ln(x)-ln(1+x^(1/2))`

**Maxima [A]** time = 0.958226, size = 24, normalized size = 1.

$$-\frac{2 \operatorname{artanh}(\sqrt{x})}{\sqrt{x}} - \log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="maxima")

[Out] -2\*arctanh(sqrt(x))/sqrt(x) - log(x - 1) + log(x)

**Fricas [A]** time = 1.80816, size = 100, normalized size = 4.17

$$\frac{x \log(x-1) - x \log(x) + \sqrt{x} \log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="fricas")

[Out] -(x\*log(x - 1) - x\*log(x) + sqrt(x)\*log(-(x + 2\*sqrt(x) + 1)/(x - 1)))/x

**Sympy [B]** time = 2.41666, size = 126, normalized size = 5.25

$$-\frac{2x^{\frac{3}{2}} \operatorname{atanh}(\sqrt{x})}{x^2 - x} + \frac{2\sqrt{x} \operatorname{atanh}(\sqrt{x})}{x^2 - x} + \frac{x^2 \log(x)}{x^2 - x} - \frac{2x^2 \log(\sqrt{x} + 1)}{x^2 - x} + \frac{2x^2 \operatorname{atanh}(\sqrt{x})}{x^2 - x} - \frac{x \log(x)}{x^2 - x} + \frac{2x \log(\sqrt{x} + 1)}{x^2 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(x\*\*(1/2))/x\*\*(3/2),x)

[Out] -2\*x\*\*(3/2)\*atanh(sqrt(x))/(x\*\*2 - x) + 2\*sqrt(x)\*atanh(sqrt(x))/(x\*\*2 - x) + x\*\*2\*log(x)/(x\*\*2 - x) - 2\*x\*\*2\*log(sqrt(x) + 1)/(x\*\*2 - x) + 2\*x\*\*2\*atanh(sqrt(x))/(x\*\*2 - x) - x\*log(x)/(x\*\*2 - x) + 2\*x\*log(sqrt(x) + 1)/(x\*\*2 - x) - 2\*x\*atanh(sqrt(x))/(x\*\*2 - x)

**Giac [A]** time = 1.16425, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\sqrt{x}} + \log(x) - \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] -log(-(sqrt(x) + 1)/(sqrt(x) - 1))/sqrt(x) + log(x) - log(abs(x - 1))
```



### 3.213 $\int x^3 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) dx$

**Optimal.** Leaf size=190

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) + \frac{b \log \left( c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1 \right)}{16c^{8/3}} - \frac{b \log \left( c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1 \right)}{16c^{8/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{8c^{8/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{8c^{8/3}}$$

[Out]  $(3bx^{5/2})/(20c) - (\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2c^{1/3}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/(8c^{8/3}) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2c^{1/3}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/(8c^{8/3}) - (b*\text{ArcTanh}[c^{1/3}*\text{Sqrt}[x]])/(4c^{8/3}) + (x^4*(a + b*\text{ArcTanh}[c*x^{3/2}]))/4 + (b*\text{Log}[1 - c^{1/3}*\text{Sqrt}[x] + c^{2/3}*x])/(16c^{8/3}) - (b*\text{Log}[1 + c^{1/3}*\text{Sqrt}[x] + c^{2/3}*x])/(16c^{8/3})$

**Rubi [A]** time = 0.300586, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6097, 321, 329, 296, 634, 618, 204, 628, 206}

$$\frac{1}{4}x^4 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) + \frac{b \log \left( c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1 \right)}{16c^{8/3}} - \frac{b \log \left( c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1 \right)}{16c^{8/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{8c^{8/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{8c^{8/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(a + b*\text{ArcTanh}[c*x^{3/2}]), x]$

[Out]  $(3bx^{5/2})/(20c) - (\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2c^{1/3}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/(8c^{8/3}) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2c^{1/3}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/(8c^{8/3}) - (b*\text{ArcTanh}[c^{1/3}*\text{Sqrt}[x]])/(4c^{8/3}) + (x^4*(a + b*\text{ArcTanh}[c*x^{3/2}]))/4 + (b*\text{Log}[1 - c^{1/3}*\text{Sqrt}[x] + c^{2/3}*x])/(16c^{8/3}) - (b*\text{Log}[1 + c^{1/3}*\text{Sqrt}[x] + c^{2/3}*x])/(16c^{8/3})$

#### Rule 6097

$\text{Int}[\left( (a_{.}) + \text{ArcTanh}[(c_{.})*(x_{.})^{(n_{.})}]* (b_{.}) \right) * \left( (d_{.})*(x_{.})^{(m_{.})} \right), x\_Symbol]$   
 $:= \text{Simp}[\left( (d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n]) \right) / (d*(m+1)), x] - \text{Dist}[\left( (b*c^n) / (d*(m+1)) \right), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)}) / (1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 321

$\text{Int}[\left( (c_{.})*(x_{.})^{(m_{.})} * \left( (a_{.}) + (b_{.})*(x_{.})^{(n_{.})} \right)^{(p_{.})} \right), x\_Symbol] := \text{Simp}[\left( c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)} \right) / (b*(m+n*p+1)), x] - \text{Dist}[\left( (b*c^n) / (d*(m+1)) \right), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)}) / (1 - c^2*x^{(2*n)}), x], x] /;$

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n)^p, x], x, (c*x)^(1/k), x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 296

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},
```

e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int x^3 (a + b \tanh^{-1}(cx^{3/2})) dx &= \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) - \frac{1}{8}(3bc) \int \frac{x^{9/2}}{1 - c^2x^3} dx \\
 &= \frac{3bx^{5/2}}{20c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) - \frac{(3b) \int \frac{x^{3/2}}{1 - c^2x^3} dx}{8c} \\
 &= \frac{3bx^{5/2}}{20c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) - \frac{(3b) \text{Subst}\left(\int \frac{x^4}{1 - c^2x^6} dx, x, \sqrt{x}\right)}{4c} \\
 &= \frac{3bx^{5/2}}{20c} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) - \frac{b \text{Subst}\left(\int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x}\right)}{4c^{7/3}} - \frac{b \text{Subst}\left(\int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x}\right)}{4c^{7/3}} \\
 &= \frac{3bx^{5/2}}{20c} - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{4c^{8/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) + \frac{b \text{Subst}\left(\int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{c}x + c^{2/3}x^2} dx, x, \sqrt{x}\right)}{16c^{8/3}} \\
 &= \frac{3bx^{5/2}}{20c} - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{4c^{8/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2})) + \frac{b \log(1 - \sqrt[3]{c}\sqrt{x} + c^{2/3}x)}{16c^{8/3}} \\
 &= \frac{3bx^{5/2}}{20c} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}} - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{4c^{8/3}} + \frac{1}{4}x^4 (a + b \tanh^{-1}(cx^{3/2}))
 \end{aligned}$$

**Mathematica [A]** time = 0.0534598, size = 222, normalized size = 1.17

$$\frac{ax^4}{4} + \frac{b \log(1 - \sqrt[3]{c}\sqrt{x})}{8c^{8/3}} - \frac{b \log(\sqrt[3]{c}\sqrt{x} + 1)}{8c^{8/3}} + \frac{b \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1)}{16c^{8/3}} - \frac{b \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1)}{16c^{8/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcTanh[c\*x^(3/2)]), x]

[Out] (3\*b\*x^(5/2))/(20\*c) + (a\*x^4)/4 + (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(8\*c^(8/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(8\*c^(8/3))

$$\begin{aligned} & ])/(8*c^{(8/3)}) + (b*x^4*ArcTanh[c*x^{(3/2)}])/4 + (b*Log[1 - c^{(1/3)*Sqrt[x]}] \\ & ])/(8*c^{(8/3)}) - (b*Log[1 + c^{(1/3)*Sqrt[x]}])/ (8*c^{(8/3)}) + (b*Log[1 - c^{(1/3)*Sqrt[x]} + c^{(2/3)*x}])/ (16*c^{(8/3)}) - (b*Log[1 + c^{(1/3)*Sqrt[x]} + c^{(2/3)*x}])/ (16*c^{(8/3)}) \end{aligned}$$

**Maple [A]** time = 0.036, size = 194, normalized size = 1.

$$\frac{x^4 a}{4} + \frac{x^4 b}{4} \operatorname{Arctanh}\left(cx^{\frac{3}{2}}\right) + \frac{3b}{20c} x^{\frac{5}{2}} + \frac{b}{8c^3} \ln\left(\sqrt{x} - \sqrt[3]{c^{-1}}\right) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{b}{16c^3} \ln\left(x + \sqrt[3]{c^{-1}}\sqrt{x} + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{b\sqrt{3}}{8c^3} \arctan\left(\frac{\sqrt{3}\sqrt{x}}{1 + \sqrt[3]{c^{-1}}\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arctanh(c\*x^(3/2))),x)

[Out]  $\frac{1}{4}x^4a + \frac{1}{4}x^4b \operatorname{arctanh}(cx^{3/2}) + \frac{3}{20}bx^{5/2}/c + \frac{1}{8}b/c^3/(1/c)^{1/3} \ln(x^{1/2} - (1/c)^{1/3}) - \frac{1}{16}b/c^3/(1/c)^{1/3} \ln(x + (1/c)^{1/3}x^{1/2} + (1/c)^{2/3}) + \frac{1}{8}b/c^3 \frac{3^{1/2}}{(1/c)^{1/3}} \operatorname{arctan}\left(\frac{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} + 1}{1 + (1/c)^{1/3}}\right) - \frac{1}{8}b/c^3/(1/c)^{1/3} \ln(x^{1/2} + (1/c)^{1/3}) + \frac{1}{16}b/c^3/(1/c)^{1/3} \ln(x - (1/c)^{1/3}x^{1/2} + (1/c)^{2/3}) + \frac{1}{8}b/c^3 \frac{3^{1/2}}{(1/c)^{1/3}} \operatorname{arctan}\left(\frac{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} - 1}{1 + (1/c)^{1/3}}\right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(3/2))),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 10.9902, size = 4629, normalized size = 24.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(3/2))),x, algorithm="fricas")

[Out]  $\frac{1}{160} \cdot (40 \cdot a \cdot c \cdot x^4 + 24 \cdot b \cdot x^{5/2} - 20 \cdot \sqrt{3} \cdot \sqrt{((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 - 4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b + 4 \cdot b^2) \cdot c \cdot \arctan(1/24 \cdot (4 \cdot \sqrt{3} \cdot \sqrt{((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 \cdot b^2 \cdot c^5 \cdot \sqrt{x} + 4 \cdot b^4 \cdot c^5 \cdot \sqrt{x} + 4 \cdot b^4 \cdot c^2 + 4 \cdot b^4 \cdot x - 2 \cdot (2 \cdot b^3 \cdot c^5 \cdot \sqrt{x} + b^3 \cdot c^2) \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)) \cdot \sqrt{((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 - 4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b + 4 \cdot b^2) \cdot c^3 - \sqrt{3} \cdot (((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 \cdot c^8 - 4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b \cdot c^8 + 4 \cdot b^2 \cdot c^8 + 8 \cdot b^2 \cdot c^3 \cdot \sqrt{x})) \cdot \sqrt{((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 - 4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b + 4 \cdot b^2) / b^3 - 10 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot c \cdot \log(-1/4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 \cdot c^5 + ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b \cdot c^5 - b^2 \cdot c^5 + b^2 \cdot \sqrt{x}) - 20 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot c \cdot \log((4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot c^5 + 2 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b \cdot c^5 + b^2 \cdot c^5 + b^2 \cdot \sqrt{x}) - 40 \cdot \sqrt{3} \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 + 6 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b + 3 \cdot b^2) \cdot c \cdot \arctan(1/3 \cdot ((4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot c^8 + 2 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b \cdot c^8 + b^2 \cdot c^8 - 2 \cdot b^2 \cdot c^3 \cdot \sqrt{x} + \sqrt{-4 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot b^2 \cdot c^5 \cdot \sqrt{x} - 4 \cdot b^4 \cdot c^5 \cdot \sqrt{x} + 4 \cdot b^4 \cdot c^2 + 4 \cdot b^4 \cdot x - 4 \cdot (2 \cdot b^3 \cdot c^5 \cdot \sqrt{x} - b^3 \cdot c^2) \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)) \cdot c^3) \cdot \sqrt{3 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 + 6 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b + 3 \cdot b^2) / b^3} + 5 \cdot (((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot c - 6 \cdot b \cdot c) \cdot \log(((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 \cdot b^2 \cdot c^5 \cdot \sqrt{x} + 4 \cdot b^4 \cdot c^5 \cdot \sqrt{x} + 4 \cdot b^4 \cdot c^2 + 4 \cdot b^4 \cdot x - 2 \cdot (2 \cdot b^3 \cdot c^5 \cdot \sqrt{x} + b^3 \cdot c^2) \cdot ((1/2)^{1/3} \cdot (b^3 - (c^8 - 1) \cdot b^3/c^8 + b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)) + 10 \cdot ((4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot c + 3 \cdot b \cdot c) \cdot \log(-4 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot c + 3 \cdot b \cdot c) \cdot \log(-4 \cdot (4 \cdot (-1/1024 \cdot b^3 + 1/1024 \cdot (c^8 - 1) \cdot b^3/c^8 + 1/1024 \cdot b^3/c^8)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot c + 3 \cdot b \cdot c)$

$$) * (I * \sqrt{3} + 1) - b)^2 * b^2 * c^5 * \sqrt{x} - 4 * b^4 * c^5 * \sqrt{x} + 4 * b^4 * c^2 + 4 * b^4 * x - 4 * (2 * b^3 * c^5 * \sqrt{x} - b^3 * c^2) * (4 * (-1/1024 * b^3 + 1/1024 * (c^8 - 1) * b^3 / c^8 + 1/1024 * b^3 / c^8)^{1/3} * (I * \sqrt{3} + 1) - b)) + 20 * (b * c * x^4 - b * c) * \log(-(c^2 * x^3 + 2 * c * x^{3/2} + 1) / (c^2 * x^3 - 1)) / c$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*atanh(c\*x\*\*(3/2))),x)

[Out] Timed out

**Giac [C]** time = 1.49211, size = 296, normalized size = 1.56

$$\frac{1}{4} a x^4 + \frac{1}{320} \left( 40 x^4 \log \left( -\frac{c x^{\frac{3}{2}} + 1}{c x^{\frac{3}{2}} - 1} \right) + c \left( \frac{48 x^{\frac{5}{2}}}{c^2} - \frac{10 \sqrt{3} (-i \sqrt{3} - 1)^2 \arctan \left( \frac{\sqrt{3} \left( 2 \sqrt{x} + \left( -\frac{1}{c} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{1}{c} \right)^{\frac{1}{3}}} \right)}{c^{\frac{11}{3}}} + \frac{5 (-i \sqrt{3} - 1)^2 \log \left( x + \sqrt{x} \right)}{c^{\frac{11}{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arctanh(c\*x^(3/2))),x, algorithm="giac")

[Out] 1/4\*a\*x^4 + 1/320\*(40\*x^4\*log(-(c\*x^(3/2) + 1)/(c\*x^(3/2) - 1)) + c\*(48\*x^(5/2)/c^2 - 10\*sqrt(3)\*(-I\*sqrt(3) - 1)^2\*arctan(1/3\*sqrt(3)\*(2\*sqrt(x) + (-1/c)^(1/3))/(-1/c)^(1/3))/c^(11/3) + 5\*(-I\*sqrt(3) - 1)^2\*log(x + sqrt(x))\*(-1/c)^(1/3) + (-1/c)^(2/3))/c^(11/3) - 40\*(-1/c)^(2/3)\*log(abs(sqrt(x) - (-1/c)^(1/3)))/c^3 + 40\*sqrt(3)\*abs(c)^(4/3)\*arctan(1/3\*sqrt(3)\*c^(1/3)\*(2\*sqrt(x) + 1/c^(1/3))/c^5 - 20\*abs(c)^(4/3)\*log(x + sqrt(x)/c^(1/3) + 1/c^(2/3))/c^5 + 40\*log(abs(sqrt(x) - 1/c^(1/3))/c^(11/3))) \* b

### 3.214 $\int x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) dx$

**Optimal.** Leaf size=49

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) - \frac{b \tanh^{-1} \left( cx^{3/2} \right)}{3c^2} + \frac{bx^{3/2}}{3c}$$

[Out]  $(b*x^{(3/2)})/(3*c) - (b*ArcTanh[c*x^{(3/2)}])/(3*c^2) + (x^3*(a + b*ArcTanh[c*x^{(3/2)}]))/3$

**Rubi [A]** time = 0.0337865, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6097, 321, 329, 275, 206}

$$\frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) - \frac{b \tanh^{-1} \left( cx^{3/2} \right)}{3c^2} + \frac{bx^{3/2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^(3/2)]),x]

[Out]  $(b*x^{(3/2)})/(3*c) - (b*ArcTanh[c*x^{(3/2)}])/(3*c^2) + (x^3*(a + b*ArcTanh[c*x^{(3/2)}]))/3$

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 329

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int x^2 (a + b \tanh^{-1}(cx^{3/2})) dx &= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^{3/2})) - \frac{1}{2}(bc) \int \frac{x^{7/2}}{1 - c^2x^3} dx \\
&= \frac{bx^{3/2}}{3c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^{3/2})) - \frac{b \int \frac{\sqrt{x}}{1 - c^2x^3} dx}{2c} \\
&= \frac{bx^{3/2}}{3c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^{3/2})) - \frac{b \operatorname{Subst}\left(\int \frac{x^2}{1 - c^2x^6} dx, x, \sqrt{x}\right)}{c} \\
&= \frac{bx^{3/2}}{3c} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^{3/2})) - \frac{b \operatorname{Subst}\left(\int \frac{1}{1 - c^2x^2} dx, x, x^{3/2}\right)}{3c} \\
&= \frac{bx^{3/2}}{3c} - \frac{b \tanh^{-1}(cx^{3/2})}{3c^2} + \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^{3/2}))
\end{aligned}$$

**Mathematica [A]** time = 0.0263497, size = 75, normalized size = 1.53

$$\frac{ax^3}{3} + \frac{b \log(1 - cx^{3/2})}{6c^2} - \frac{b \log(cx^{3/2} + 1)}{6c^2} + \frac{bx^{3/2}}{3c} + \frac{1}{3}bx^3 \tanh^{-1}(cx^{3/2})$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)]), x]
```



[Out]  $(b*x^{(3/2)})/(3*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x^{(3/2)}])/3 + (b*Log[1 - c*x^{(3/2)}])/(6*c^2) - (b*Log[1 + c*x^{(3/2)}])/(6*c^2)$

**Maple [A]** time = 0.029, size = 57, normalized size = 1.2

$$\frac{x^3 a}{3} + \frac{b x^3}{3} \operatorname{Artanh}\left(cx^{\frac{3}{2}}\right) + \frac{b}{3c} x^{\frac{3}{2}} + \frac{b}{6c^2} \ln\left(cx^{\frac{3}{2}} - 1\right) - \frac{b}{6c^2} \ln\left(cx^{\frac{3}{2}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^(3/2))),x)`

[Out]  $1/3*x^3*a+1/3*b*x^3*arctanh(c*x^{(3/2)})+1/3*b*x^{(3/2)}/c+1/6/c^2*b*\ln(c*x^{(3/2)}-1)-1/6/c^2*b*\ln(c*x^{(3/2)}+1)$

**Maxima [A]** time = 0.995113, size = 78, normalized size = 1.59

$$\frac{1}{3} a x^3 + \frac{1}{6} \left( 2 x^3 \operatorname{artanh}\left(c x^{\frac{3}{2}}\right) + c \left( \frac{2 x^{\frac{3}{2}}}{c^2} - \frac{\log\left(c x^{\frac{3}{2}} + 1\right)}{c^3} + \frac{\log\left(c x^{\frac{3}{2}} - 1\right)}{c^3} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")`

[Out]  $1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^{(3/2)}) + c*(2*x^{(3/2)}/c^2 - \log(c*x^{(3/2)} + 1)/c^3 + \log(c*x^{(3/2)} - 1)/c^3))*b$

**Fricas [A]** time = 1.85607, size = 142, normalized size = 2.9

$$\frac{2 a c^2 x^3 + 2 b c x^{\frac{3}{2}} + (b c^2 x^3 - b) \log\left(-\frac{c^2 x^3 + 2 c x^{\frac{3}{2}} + 1}{c^2 x^3 - 1}\right)}{6 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")`

[Out]  $\frac{1}{6}(2ac^2x^3 + 2b*cx^{(3/2)} + (b*c^2*x^3 - b)*\log(-(c^2*x^3 + 2*c*x^{(3/2)} + 1)/(c^2*x^3 - 1)))/c^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c*x**(3/2))),x)`

[Out] Timed out

**Giac [A]** time = 1.22804, size = 97, normalized size = 1.98

$$\frac{1}{3}ax^3 + \frac{1}{6} \left( x^3 \log \left( -\frac{cx^{\frac{3}{2}} + 1}{cx^{\frac{3}{2}} - 1} \right) + c \left( \frac{2x^{\frac{3}{2}}}{c^2} - \frac{\log \left( \left| cx^{\frac{3}{2}} + 1 \right| \right)}{c^3} + \frac{\log \left( \left| cx^{\frac{3}{2}} - 1 \right| \right)}{c^3} \right) \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")`

[Out]  $\frac{1}{3}ax^3 + \frac{1}{6}(x^3*\log(-(c*x^{(3/2)} + 1)/(c*x^{(3/2)} - 1)) + c*(2*x^{(3/2)}/c^2 - \log(\text{abs}(c*x^{(3/2)} + 1))/c^3 + \log(\text{abs}(c*x^{(3/2)} - 1))/c^3))*b$

### 3.215 $\int x \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) dx$

**Optimal.** Leaf size=190

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) + \frac{b \log \left( c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1 \right)}{8c^{4/3}} - \frac{b \log \left( c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1 \right)}{8c^{4/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{4c^{4/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{4c^{4/3}}$$

[Out]  $(3*b*\text{Sqrt}[x])/(2*c) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2*c^{(1/3)}*\text{Sqrt}[x])/\text{Sqrt}[3]])/(4*c^{(4/3)}) - (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2*c^{(1/3)}*\text{Sqrt}[x])/\text{Sqrt}[3]])/(4*c^{(4/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*\text{Sqrt}[x]])/(2*c^{(4/3)}) + (x^2*(a + b*\text{ArcTanh}[c*x^{(3/2)}]))/2 + (b*\text{Log}[1 - c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/(8*c^{(4/3)}) - (b*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/(8*c^{(4/3)})$

**Rubi [A]** time = 0.238218, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {6097, 321, 329, 210, 634, 618, 204, 628, 206}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) + \frac{b \log \left( c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1 \right)}{8c^{4/3}} - \frac{b \log \left( c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1 \right)}{8c^{4/3}} + \frac{\sqrt{3}b \tan^{-1} \left( \frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{4c^{4/3}} - \frac{\sqrt{3}b \tan^{-1} \left( \frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{4c^{4/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*\text{ArcTanh}[c*x^{(3/2)}]), x]$

[Out]  $(3*b*\text{Sqrt}[x])/(2*c) + (\text{Sqrt}[3]*b*\text{ArcTan}[(1 - 2*c^{(1/3)}*\text{Sqrt}[x])/\text{Sqrt}[3]])/(4*c^{(4/3)}) - (\text{Sqrt}[3]*b*\text{ArcTan}[(1 + 2*c^{(1/3)}*\text{Sqrt}[x])/\text{Sqrt}[3]])/(4*c^{(4/3)}) - (b*\text{ArcTanh}[c^{(1/3)}*\text{Sqrt}[x]])/(2*c^{(4/3)}) + (x^2*(a + b*\text{ArcTanh}[c*x^{(3/2)}]))/2 + (b*\text{Log}[1 - c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/(8*c^{(4/3)}) - (b*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/(8*c^{(4/3)})$

#### Rule 6097

$\text{Int}[(a + \text{ArcTanh}[c*x^n])*(b*x^m), x\_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{(n-1)}*(d*x)^{(m+1)})/(1 - c^2*x^{(2*n)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 321

$\text{Int}[(c*x)^m*(a + b*x^n)^p, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^p, \text{Int}[(c*x)^m, x], x]$

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^
n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(m_), x_Symbol] := Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x (a + b \tanh^{-1}(cx^{3/2})) dx &= \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) - \frac{1}{4}(3bc) \int \frac{x^{5/2}}{1 - c^2x^3} dx \\
 &= \frac{3b\sqrt{x}}{2c} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) - \frac{(3b) \int \frac{1}{\sqrt{x}(1-c^2x^3)} dx}{4c} \\
 &= \frac{3b\sqrt{x}}{2c} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) - \frac{(3b) \text{Subst}\left(\int \frac{1}{1-c^2x^6} dx, x, \sqrt{x}\right)}{2c} \\
 &= \frac{3b\sqrt{x}}{2c} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) - \frac{b \text{Subst}\left(\int \frac{1}{1-c^{2/3}x^2} dx, x, \sqrt{x}\right)}{2c} - \frac{b \text{Subst}\left(\int \frac{1}{1-\sqrt[3]{c}x} dx, x, \sqrt{x}\right)}{2c} \\
 &= \frac{3b\sqrt{x}}{2c} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) + \frac{b \text{Subst}\left(\int \frac{-\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt[3]{c}x+c^{2/3}x^2} dx, x, \sqrt{x}\right)}{8c^{4/3}} \\
 &= \frac{3b\sqrt{x}}{2c} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2})) + \frac{b \log\left(1 - \sqrt[3]{c}\sqrt{x} + c^{2/3}x\right)}{8c^{4/3}} \\
 &= \frac{3b\sqrt{x}}{2c} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}} - \frac{b \tanh^{-1}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}} + \frac{1}{2}x^2 (a + b \tanh^{-1}(cx^{3/2}))
 \end{aligned}$$

**Mathematica [A]** time = 0.0337743, size = 222, normalized size = 1.17

$$\frac{ax^2}{2} + \frac{b \log\left(1 - \sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}} - \frac{b \log\left(\sqrt[3]{c}\sqrt{x} + 1\right)}{4c^{4/3}} + \frac{b \log\left(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1\right)}{8c^{4/3}} - \frac{b \log\left(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1\right)}{8c^{4/3}} - \frac{\sqrt{3}b \tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x}-1}{\sqrt{3}}\right)}{4c^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^(3/2)]), x]

[Out] (3\*b\*Sqrt[x])/(2\*c) + (a\*x^2)/2 - (Sqrt[3]\*b\*ArcTan[(-1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(4\*c^(4/3)) - (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(4\*c^(4/3)) + (b\*x^2\*ArcTanh[c\*x^(3/2)])/2 + (b\*Log[1 - c^(1/3)\*Sqrt[x]])/(4\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*Sqrt[x]])/(4\*c^(4/3)) + (b\*Log[1 - c^(1/3)\*Sqrt[x]])/(4\*c^(4/3)) - (b\*Log[1 + c^(1/3)\*Sqrt[x]])/(4\*c^(4/3))

$3) \cdot \sqrt{x} + c^{2/3} \cdot x] / (8 \cdot c^{4/3}) - (b \cdot \log[1 + c^{1/3} \cdot \sqrt{x} + c^{2/3} \cdot x]) / (8 \cdot c^{4/3})$

**Maple [A]** time = 0.036, size = 194, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^2}{2} \operatorname{Arctanh}\left(cx^{\frac{3}{2}}\right) + \frac{3b}{2c} \sqrt{x} + \frac{b}{4c^2} \ln\left(\sqrt{x} - \sqrt[3]{c^{-1}}\right) (c^{-1})^{-\frac{2}{3}} - \frac{b}{8c^2} \ln\left(x + \sqrt[3]{c^{-1}} \sqrt{x} + (c^{-1})^{\frac{2}{3}}\right) (c^{-1})^{-\frac{2}{3}} - \frac{b\sqrt{3}}{4c^2} \operatorname{arctan}\left(\frac{\sqrt{x} - \sqrt[3]{c^{-1}}}{\sqrt{x} + \sqrt[3]{c^{-1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^(3/2))),x)`

[Out]  $\frac{1}{2}ax^2 + \frac{1}{2}bx^2 \operatorname{arctanh}(cx^{3/2}) + \frac{3}{2}b \frac{x^{1/2}}{c} + \frac{1}{4}b \frac{c^2}{(1/c)^{2/3}} \ln(x^{1/2} - (1/c)^{1/3}) - \frac{1}{8}b \frac{c^2}{(1/c)^{2/3}} \ln(x + (1/c)^{1/3} x^{1/2} + (1/c)^{2/3}) - \frac{1}{4}b \frac{c^2}{(1/c)^{2/3}} 3^{1/2} \operatorname{arctan}\left(\frac{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} + 1}{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} - 1}\right) - \frac{1}{4}b \frac{c^2}{(1/c)^{2/3}} \ln(x^{1/2} + (1/c)^{1/3}) + \frac{1}{8}b \frac{c^2}{(1/c)^{2/3}} \ln(x - (1/c)^{1/3} x^{1/2} + (1/c)^{2/3}) - \frac{1}{4}b \frac{c^2}{(1/c)^{2/3}} 3^{1/2} \operatorname{arctan}\left(\frac{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} - 1}{1/3 \cdot 3^{1/2} \cdot (2/(1/c)^{1/3}) \cdot x^{1/2} + 1}\right)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 10.5287, size = 4730, normalized size = 24.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")`

```

[Out] 1/16*(8*a*c*x^2 + 4*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c*arctan(-1/24*(2*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 + 4*b^2*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 + b*c*sqrt(x)))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)))*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c^3 - 2*b*c^3)*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2) + sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^4 + 4*b^2*c^4 + 8*b^2*c^3*sqrt(x) - 4*(b*c^4 + b*c^3*sqrt(x))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b))*sqrt(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 - 4*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2))/b^3) - 2*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(1/2*(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - b*c + b*sqrt(x)) - 4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c*log((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c + b*c + b*sqrt(x)) - 8*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2)*c*arctan(-1/3*(2*sqrt((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + b^2*c^2 - b^2*c*sqrt(x) + b^2*x + (2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c^3 + b*c^3)*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2) + ((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^4 + b^2*c^4 - 2*b^2*c^3*sqrt(x) + 2*(b*c^4 - b*c^3*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b))*sqrt(3*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2 + 6*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*b + 3*b^2))/b^3) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c)*log(((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 + 4*b^2*c^2 + 4*b^2*c*sqrt(x) + 4*b^2*x - 2*(2*b*c^2 + b*c*sqrt(x))*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)) + 2*((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*c + 3*b*c)*log(4*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^2 + 4*b^2*c^2 - 4*b^2*c*sqrt(x) + 4*b^2*x + 4*(2*b*c^2 - b*c*sqrt(x))*(2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1

```

$$\frac{1}{128}b^3/c^4)^{1/3}*(I*\sqrt{3} + 1) - b)) + 4*(b*c*x^2 - b*c)*\log(-(c^2*x^3 + 2*c*x^{3/2} + 1)/(c^2*x^3 - 1)) + 24*b*\sqrt{x})/c$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*atanh(c\*x\*\*(3/2))),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh} \left( cx^{\frac{3}{2}} \right) + a \right) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^(3/2))),x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^(3/2)) + a)\*x, x)



### 3.216 $\int \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right) dx$

**Optimal.** Leaf size=170

$$ax + \frac{b \log \left( c^{2/3} x - \sqrt[3]{c} \sqrt{x} + 1 \right)}{4c^{2/3}} - \frac{b \log \left( c^{2/3} x + \sqrt[3]{c} \sqrt{x} + 1 \right)}{4c^{2/3}} - \frac{\sqrt{3} b \tan^{-1} \left( \frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{2c^{2/3}} + \frac{\sqrt{3} b \tan^{-1} \left( \frac{2\sqrt[3]{c}\sqrt{x} + 1}{\sqrt{3}} \right)}{2c^{2/3}} - \frac{b \tanh^{-1} \left( cx^{3/2} \right)}{c^2}$$

[Out] a\*x - (Sqrt[3]\*b\*ArcTan[(1 - 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(2\*c^(2/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(2\*c^(2/3)) - (b\*ArcTanh[c^(1/3)\*Sqrt[x]])/c^(2/3) + b\*x\*ArcTanh[c\*x^(3/2)] + (b\*Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/(4\*c^(2/3)) - (b\*Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/(4\*c^(2/3))

---

**Rubi [A]** time = 0.287018, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6091, 329, 296, 634, 618, 204, 628, 206}

$$ax + \frac{b \log \left( c^{2/3} x - \sqrt[3]{c} \sqrt{x} + 1 \right)}{4c^{2/3}} - \frac{b \log \left( c^{2/3} x + \sqrt[3]{c} \sqrt{x} + 1 \right)}{4c^{2/3}} - \frac{\sqrt{3} b \tan^{-1} \left( \frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}} \right)}{2c^{2/3}} + \frac{\sqrt{3} b \tan^{-1} \left( \frac{2\sqrt[3]{c}\sqrt{x} + 1}{\sqrt{3}} \right)}{2c^{2/3}} - \frac{b \tanh^{-1} \left( cx^{3/2} \right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*x^(3/2)], x]

[Out] a\*x - (Sqrt[3]\*b\*ArcTan[(1 - 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(2\*c^(2/3)) + (Sqrt[3]\*b\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/(2\*c^(2/3)) - (b\*ArcTanh[c^(1/3)\*Sqrt[x]])/c^(2/3) + b\*x\*ArcTanh[c\*x^(3/2)] + (b\*Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/(4\*c^(2/3)) - (b\*Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/(4\*c^(2/3))

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n))], x], x] /; FreeQ[{c, n}, x]

#### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(cx^{3/2})) dx &= ax + b \int \tanh^{-1}(cx^{3/2}) dx \\
&= ax + bx \tanh^{-1}(cx^{3/2}) - \frac{1}{2}(3bc) \int \frac{x^{3/2}}{1 - c^2x^3} dx \\
&= ax + bx \tanh^{-1}(cx^{3/2}) - (3bc) \text{Subst} \left( \int \frac{x^4}{1 - c^2x^6} dx, x, \sqrt{x} \right) \\
&= ax + bx \tanh^{-1}(cx^{3/2}) - \frac{b \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x} \right)}{\sqrt[3]{c}} - \frac{b \text{Subst} \left( \int \frac{-\frac{1}{2} - \frac{\sqrt[3]{c}x}{2}}{1 - \sqrt[3]{c}x + c^{2/3}x^2} dx, x, \sqrt{x} \right)}{\sqrt[3]{c}} \\
&= ax - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{c^{2/3}} + bx \tanh^{-1}(cx^{3/2}) + \frac{b \text{Subst} \left( \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{c}x + c^{2/3}x^2} dx, x, \sqrt{x} \right)}{4c^{2/3}} - \frac{b \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x} \right)}{4c^{2/3}} \\
&= ax - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{c^{2/3}} + bx \tanh^{-1}(cx^{3/2}) + \frac{b \log(1 - \sqrt[3]{c}\sqrt{x} + c^{2/3}x)}{4c^{2/3}} - \frac{b \log(1 + \sqrt[3]{c}\sqrt{x})}{4c^{2/3}} \\
&= ax - \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}} + \frac{\sqrt{3}b \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}} - \frac{b \tanh^{-1}(\sqrt[3]{c}\sqrt{x})}{c^{2/3}} + bx \tanh^{-1}(cx^{3/2})
\end{aligned}$$

**Mathematica [A]** time = 0.100893, size = 141, normalized size = 0.83

$$ax - \frac{b \left( -\log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) + 2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{c}\sqrt{x} + 1}{\sqrt{3}}\right) + 4 \tanh^{-1}(cx^{3/2}) \right)}{4c^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*x^(3/2)], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x^(3/2)] - (b\*(2\*Sqrt[3]\*ArcTan[(1 - 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]] - 2\*Sqrt[3]\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]] + 4\*ArcTanh[c^(1/3)\*Sqrt[x]] - Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x] + Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x]))/(4\*c^(2/3))

**Maple [A]** time = 0.032, size = 179, normalized size = 1.1

$$ax + bx \operatorname{Artanh}(cx^{\frac{3}{2}}) + \frac{b}{2c} \ln(\sqrt{x} - \sqrt[3]{c^{-1}}) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{b}{4c} \ln\left(x + \sqrt[3]{c^{-1}}\sqrt{x} + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{b\sqrt{3}}{2c} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \frac{\sqrt{x}}{\sqrt[3]{c^{-1}}} + \dots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctanh(c*x^(3/2)),x)
```

```
[Out] a*x+b*x*arctanh(c*x^(3/2))+1/2*b/c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/4*
b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b*3^(1/2)/c/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))-1/2*b/c/(1/c)^(1/3)*ln
(x^(1/2)+(1/c)^(1/3))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2
/3))+1/2*b*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-
1))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 10.7687, size = 4289, normalized size = 25.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="fricas")
```

```
[Out] a*x - 1/2*sqrt(3)*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)
*(I*sqrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3
/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*arctan(1/24*(4*sqrt(3)*sqrt((
(1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b
)^2*b^2*c*sqrt(x) + 4*b^4*c*sqrt(x) + 4*b^4*x + 4*b^4 - 2*(2*b^3*c*sqrt(x)
+ b^3)*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) +
1) + 2*b))*sqrt(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*s
qrt(3) + 1) + 2*b)^2 - 4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(
1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)*c - sqrt(3)*(((1/2)^(1/3)*(b^3 - (c^
2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^2 - 4*((1/2)^(1/3)
*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b + 4*b^2)

```

$$\begin{aligned}
& 3) \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b \cdot c^2 + \\
& 4 \cdot b^2 \cdot c^2 + 8 \cdot b^2 \cdot c \cdot \sqrt{x}) \cdot \sqrt{((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + \\
& b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 - 4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + \\
& b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b + 4 \cdot b^2)} / b^3) + 1/8 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + \\
& b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - 4 \cdot b) \cdot \log(((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + \\
& 2 \cdot b)^2 \cdot b^2 \cdot c \cdot \sqrt{x} + 4 \cdot b^4 \cdot c \cdot \sqrt{x} + 4 \cdot b^4 \cdot x + 4 \cdot b^4 - 2 \cdot (2 \cdot b^3 \cdot c \cdot \sqrt{x} \\
& (x) + b^3) \cdot ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)) + 1/4 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot \log(-4 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot b^2 \cdot c \cdot \sqrt{x} - 4 \cdot b^4 \cdot c \cdot \sqrt{x} + 4 \cdot b^4 \cdot x + 4 \cdot b^4 - 4 \cdot (2 \cdot b^3 \cdot c \cdot \sqrt{x} - b^3) \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)) - 1/4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot \log(-1/4 \cdot ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b)^2 \cdot c + ((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) + 2 \cdot b) \cdot b \cdot c - b^2 \cdot c + b^2 \cdot \sqrt{x}) - 1/2 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot \log(((1/2)^{1/3} \cdot (b^3 - (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot c + 2 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b \cdot c + b^2 \cdot c + b^2 \cdot \sqrt{x}) + 1/2 \cdot (b \cdot x - b) \cdot \log(-(c^2 \cdot x^3 + 2 \cdot c \cdot x^{3/2}) + 1) / (c^2 \cdot x^3 - 1)) - \sqrt{3} \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 + 6 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b + 3 \cdot b^2) \cdot \arctan(1/3 \cdot (((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot c^2 + 2 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b \cdot c^2 + b^2 \cdot c^2 - 2 \cdot b^2 \cdot c \cdot \sqrt{x} + \sqrt{-4 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 \cdot b^2 \cdot c \cdot \sqrt{x} - 4 \cdot b^4 \cdot c \cdot \sqrt{x} + 4 \cdot b^4 \cdot x + 4 \cdot b^4 - 4 \cdot (2 \cdot b^3 \cdot c \cdot \sqrt{x} - b^3) \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)}) \cdot c) \cdot \sqrt{3} \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b)^2 + 6 \cdot ((-1/16 \cdot b^3 + 1/16 \cdot (c^2 - 1) \cdot b^3/c^2 + 1/16 \cdot b^3/c^2)^{1/3} \cdot (I \cdot \sqrt{3} + 1) - b) \cdot b + 3 \cdot b^2) / b^3)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c\*x\*\*(3/2)),x)

[Out] Timed out

**Giac [A]** time = 1.19899, size = 251, normalized size = 1.48

$$\frac{1}{4} \left( \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + \frac{1}{|c|^{\frac{1}{3}}}\right)\right)|c|^{\frac{1}{3}}}{c^2} + \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} - \frac{1}{|c|^{\frac{1}{3}}}\right)\right)|c|^{\frac{1}{3}}}{c^2} - \frac{|c|^{\frac{1}{3}} \log\left(x + \frac{\sqrt{x}}{|c|^{\frac{1}{3}}} + \frac{1}{2}\right)}{c^2} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="giac")`

[Out] `1/4*(c*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3)))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/abs(c)^(1/3)))*abs(c)^(1/3)/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2 + 2*abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) + 2*x*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))*b + a*x`

$$3.217 \quad \int \frac{a+b \tanh^{-1}(cx^{3/2})}{x} dx$$

**Optimal.** Leaf size=34

$$-\frac{1}{3}b\text{PolyLog}(2, -cx^{3/2}) + \frac{1}{3}b\text{PolyLog}(2, cx^{3/2}) + a \log(x)$$

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^(3/2))])/3 + (b\*PolyLog[2, c\*x^(3/2)])/3

**Rubi [A]** time = 0.033444, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {6095, 5912}

$$-\frac{1}{3}b\text{PolyLog}(2, -cx^{3/2}) + \frac{1}{3}b\text{PolyLog}(2, cx^{3/2}) + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^(3/2)])/x, x]

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^(3/2))])/3 + (b\*PolyLog[2, c\*x^(3/2)])/3

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}(cx^{3/2})}{x} dx &= \frac{2}{3} \text{Subst} \left( \int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, x^{3/2} \right) \\ &= a \log(x) - \frac{1}{3}b\text{Li}_2(-cx^{3/2}) + \frac{1}{3}b\text{Li}_2(cx^{3/2}) \end{aligned}$$

**Mathematica [A]** time = 0.0196923, size = 32, normalized size = 0.94

$$\frac{1}{3}b \left( \text{PolyLog} \left( 2, cx^{3/2} \right) - \text{PolyLog} \left( 2, -cx^{3/2} \right) \right) + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^(3/2)])/x,x]

[Out] a\*Log[x] + (b\*(-PolyLog[2, -(c\*x^(3/2))] + PolyLog[2, c\*x^(3/2)]))/3

**Maple [B]** time = 0.036, size = 63, normalized size = 1.9

$$\frac{2a}{3} \ln \left( cx^{\frac{3}{2}} \right) + \frac{2b}{3} \ln \left( cx^{\frac{3}{2}} \right) \text{Arctanh} \left( cx^{\frac{3}{2}} \right) - \frac{b}{3} \text{dilog} \left( cx^{\frac{3}{2}} \right) - \frac{b}{3} \text{dilog} \left( cx^{\frac{3}{2}} + 1 \right) - \frac{b}{3} \ln \left( cx^{\frac{3}{2}} \right) \ln \left( cx^{\frac{3}{2}} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(3/2)))/x,x)

[Out] 2/3\*a\*ln(c\*x^(3/2))+2/3\*b\*ln(c\*x^(3/2))\*arctanh(c\*x^(3/2))-1/3\*b\*dilog(c\*x^(3/2))-1/3\*b\*dilog(c\*x^(3/2)+1)-1/3\*b\*ln(c\*x^(3/2))\*ln(c\*x^(3/2)+1)

**Maxima [B]** time = 1.59449, size = 84, normalized size = 2.47

$$-\frac{1}{3} \left( \log \left( cx^{\frac{3}{2}} \right) \log \left( -cx^{\frac{3}{2}} + 1 \right) + \text{Li}_2 \left( -cx^{\frac{3}{2}} + 1 \right) \right) b + \frac{1}{3} \left( \log \left( cx^{\frac{3}{2}} + 1 \right) \log \left( -cx^{\frac{3}{2}} \right) + \text{Li}_2 \left( cx^{\frac{3}{2}} + 1 \right) \right) b + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x,x, algorithm="maxima")

[Out] -1/3\*(log(c\*x^(3/2))\*log(-c\*x^(3/2) + 1) + dilog(-c\*x^(3/2) + 1))\*b + 1/3\*(log(c\*x^(3/2) + 1)\*log(-c\*x^(3/2)) + dilog(c\*x^(3/2) + 1))\*b + a\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b \text{artanh} \left( cx^{\frac{3}{2}} \right) + a}{x}, x \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^(3/2)) + a)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(3/2)))/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^(3/2)) + a)/x, x)

$$3.218 \quad \int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^2} dx$$

**Optimal.** Leaf size=172

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{x} - \frac{1}{4}bc^{2/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{4}bc^{2/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) - \frac{1}{2}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

[Out] -(Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 - 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 + b\*c^(2/3)\*ArcTanh[c^(1/3)\*Sqrt[x]] - (a + b\*ArcTanh[c\*x^(3/2)])/x - (b\*c^(2/3)\*Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4

**Rubi [A]** time = 0.220878, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {6097, 329, 210, 634, 618, 204, 628, 206}

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{x} - \frac{1}{4}bc^{2/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{4}bc^{2/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) - \frac{1}{2}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^(3/2)])/x^2,x]

[Out] -(Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 - 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 + b\*c^(2/3)\*ArcTanh[c^(1/3)\*Sqrt[x]] - (a + b\*ArcTanh[c\*x^(3/2)])/x - (b\*c^(2/3)\*Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_)^(m\_.)), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 329

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_.), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n)))/c^n]^p, x], x, (c\*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 210

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(n\_ - 1), x\_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s\*Cos[(2\*k\*Pi)/n]\*x)/(r^2 - 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x] + Int[(r + s\*Cos[(2\*k\*Pi)/n]\*x)/(r^2 + 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x]; (2\*r^2\*Int[1/(r^2 - s^2\*x^2), x])/(a\*n) + Dist[(2\*r)/(a\*n), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_ - 1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_ - 1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_.) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_ - 1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^{3/2})}{x^2} dx &= -\frac{a + b \tanh^{-1}(cx^{3/2})}{x} + \frac{1}{2}(3bc) \int \frac{1}{\sqrt{x}(1 - c^2x^3)} dx \\
&= -\frac{a + b \tanh^{-1}(cx^{3/2})}{x} + (3bc) \text{Subst} \left( \int \frac{1}{1 - c^2x^6} dx, x, \sqrt{x} \right) \\
&= -\frac{a + b \tanh^{-1}(cx^{3/2})}{x} + (bc) \text{Subst} \left( \int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x} \right) + (bc) \text{Subst} \left( \int \frac{1 - \frac{\sqrt[3]{c}x}{2}}{1 - \sqrt[3]{c}x + c^{2/3}} dx, x, \sqrt{x} \right) \\
&= bc^{2/3} \tanh^{-1}(\sqrt[3]{c}\sqrt{x}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{x} - \frac{1}{4}(bc^{2/3}) \text{Subst} \left( \int \frac{-\sqrt[3]{c} + 2c^{2/3}x}{1 - \sqrt[3]{c}x + c^{2/3}x^2} dx, x, \sqrt{x} \right) \\
&= bc^{2/3} \tanh^{-1}(\sqrt[3]{c}\sqrt{x}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{x} - \frac{1}{4}bc^{2/3} \log(1 - \sqrt[3]{c}\sqrt{x} + c^{2/3}x) + \frac{1}{4}bc^{2/3} \log(1 + \sqrt[3]{c}\sqrt{x} + c^{2/3}x) \\
&= -\frac{1}{2}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) + \frac{1}{2}\sqrt{3}bc^{2/3} \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) + bc^{2/3} \tanh^{-1}(\sqrt[3]{c}\sqrt{x}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0338964, size = 205, normalized size = 1.19

$$-\frac{a}{x} - \frac{1}{2}bc^{2/3} \log(1 - \sqrt[3]{c}\sqrt{x}) + \frac{1}{2}bc^{2/3} \log(\sqrt[3]{c}\sqrt{x} + 1) - \frac{1}{4}bc^{2/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{4}bc^{2/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) +$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^(3/2)])/x^2,x]

[Out] -(a/x) + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(-1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 + (Sqrt[3]\*b\*c^(2/3)\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/2 - (b\*ArcTanh[c\*x^(3/2)])/x - (b\*c^(2/3)\*Log[1 - c^(1/3)\*Sqrt[x]])/2 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*Sqrt[x]])/2 - (b\*c^(2/3)\*Log[1 - c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4 + (b\*c^(2/3)\*Log[1 + c^(1/3)\*Sqrt[x] + c^(2/3)\*x])/4

**Maple [A]** time = 0.034, size = 167, normalized size = 1.

$$-\frac{a}{x} - \frac{b}{x} \text{Artanh}\left(cx^{\frac{3}{2}}\right) - \frac{b}{2} \ln\left(\sqrt{x} - \sqrt[3]{c^{-1}}\right) (c^{-1})^{-\frac{2}{3}} + \frac{b}{4} \ln\left(x + \sqrt[3]{c^{-1}}\sqrt{x} + (c^{-1})^{\frac{2}{3}}\right) (c^{-1})^{-\frac{2}{3}} + \frac{b\sqrt{3}}{2} \arctan\left(\frac{\sqrt{3}}{3} \left(2 \frac{\sqrt{x}}{\sqrt[3]{c^{-1}}} + \sqrt[3]{c^{-1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(3/2)))/x^2,x)`

[Out] 
$$-a/x - b/x \operatorname{arctanh}(c x^{3/2}) - \frac{1}{2} b / (1/c)^{2/3} \ln(x^{1/2} - (1/c)^{1/3}) + \frac{1}{4} b / (1/c)^{2/3} \ln(x + (1/c)^{1/3} x^{1/2} + (1/c)^{2/3}) + \frac{1}{2} b / (1/c)^{2/3} 3^{1/2} \operatorname{arctan}\left(\frac{1}{3} 3^{1/2} \frac{2}{(1/c)^{1/3} x^{1/2} + 1}\right) + \frac{1}{2} b / (1/c)^{2/3} \ln(x^{1/2} + (1/c)^{1/3}) - \frac{1}{4} b / (1/c)^{2/3} \ln(x - (1/c)^{1/3} x^{1/2} + (1/c)^{2/3}) + \frac{1}{2} b / (1/c)^{2/3} 3^{1/2} \operatorname{arctan}\left(\frac{1}{3} 3^{1/2} \frac{2}{(1/c)^{1/3} x^{1/2} - 1}\right)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.93343, size = 647, normalized size = 3.76

$$2\sqrt{3}(-c^2)^{\frac{1}{3}} b x \operatorname{arctan}\left(\frac{2\sqrt{3}(-c^2)^{\frac{2}{3}}\sqrt{x}+\sqrt{3}c}{3c}\right) - 2\sqrt{3}b(c^2)^{\frac{1}{3}} x \operatorname{arctan}\left(\frac{2\sqrt{3}(c^2)^{\frac{2}{3}}\sqrt{x}-\sqrt{3}c}{3c}\right) + (-c^2)^{\frac{1}{3}} b x \log\left(c^2 x - (-c^2)^{\frac{1}{3}} c \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="fricas")`

[Out] 
$$-1/4*(2*\sqrt{3}*(-c^2)^{1/3}*b*x*\operatorname{arctan}(1/3*(2*\sqrt{3}*(-c^2)^{2/3}*\sqrt{x} + \sqrt{3}*c)/c) - 2*\sqrt{3}*(c^2)^{1/3}*x*\operatorname{arctan}(1/3*(2*\sqrt{3}*(c^2)^{2/3}*\sqrt{x} - \sqrt{3}*c)/c) + (-c^2)^{1/3}*b*x*\log(c^2*x - (-c^2)^{1/3}*c*\sqrt{x}) + (-c^2)^{2/3}) + b*(c^2)^{1/3}*x*\log(c^2*x - (c^2)^{1/3}*c*\sqrt{x} + (c^2)^{2/3}) - 2*(-c^2)^{1/3}*b*x*\log(c*\sqrt{x} + (-c^2)^{1/3}) - 2*b*(c^2)^{1/3}*x*\log(c*\sqrt{x} + (c^2)^{1/3}) + 2*b*\log(-(c^2*x^3 + 2*c*x^{3/2} + 1)/(c^2*x^3 - 1)) + 4*a)/x$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(3/2)))/x\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.46984, size = 232, normalized size = 1.35

$$\frac{1}{4} \left( \frac{2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + \frac{1}{|c|^{1/3}}\right)\right)}{|c|^{1/3}} + \frac{2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} - \frac{1}{|c|^{1/3}}\right)\right)}{|c|^{1/3}} + \frac{\log\left(x + \frac{\sqrt{x}}{|c|^{1/3}} + \frac{1}{2}\right)}{|c|^{1/3}} - \frac{\log\left(x - \frac{\sqrt{x}}{|c|^{1/3}} + \frac{1}{2}\right)}{|c|^{1/3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x^2,x, algorithm="giac")

[Out] 1/4\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*sqrt(x) + 1/abs(c)^(1/3))\*abs(c)^(1/3))/abs(c)^(1/3) + 2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*sqrt(x) - 1/abs(c)^(1/3))\*abs(c)^(1/3))/abs(c)^(1/3) + log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) - log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2\*log(sqrt(x) + 1/abs(c)^(1/3))/abs(c)^(1/3) - 2\*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/abs(c)^(1/3))\*b\*c - 1/2\*b\*log(-(c\*x^(3/2) + 1)/(c\*x^(3/2) - 1))/x - a/x

$$3.219 \quad \int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^3} dx$$

**Optimal.** Leaf size=188

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{2x^2} - \frac{1}{8}bc^{4/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{8}bc^{4/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{4}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

[Out]  $(-3*b*c)/(2*\text{Sqrt}[x]) + (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[(1 - 2*c^{(1/3)}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/4 - (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[(1 + 2*c^{(1/3)}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/4 + (b*c^{(4/3)}*\text{ArcTanh}[c^{(1/3)}*\text{Sqrt}[x]])/2 - (a + b*\text{ArcTanh}[c*x^{(3/2)}])/(2*x^2) - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/8 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/8$

**Rubi [A]** time = 0.286581, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6097, 325, 329, 296, 634, 618, 204, 628, 206}

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{2x^2} - \frac{1}{8}bc^{4/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{8}bc^{4/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{4}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*\text{ArcTanh}[c*x^{(3/2)}])/x^3, x]$

[Out]  $(-3*b*c)/(2*\text{Sqrt}[x]) + (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[(1 - 2*c^{(1/3)}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/4 - (\text{Sqrt}[3]*b*c^{(4/3)}*\text{ArcTan}[(1 + 2*c^{(1/3)}*\text{Sqrt}[x])/ \text{Sqrt}[3]])/4 + (b*c^{(4/3)}*\text{ArcTanh}[c^{(1/3)}*\text{Sqrt}[x]])/2 - (a + b*\text{ArcTanh}[c*x^{(3/2)}])/(2*x^2) - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/8 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)}*x])/8$

### Rule 6097

$\text{Int}[(a + b*\text{ArcTanh}[c*x^n])/x^{m+1}, x] := \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c^n)/(d*(m+1)), \text{Int}[(x^{n-1}*(d*x)^{m+1})/(1 - c^2*x^{2*n}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n, x\} \ \&\& \ \text{NeQ}[m, -1]$

### Rule 325

$\text{Int}[(a + b*x^n)^p/x^{m+1}, x] := \text{Simp}[(a + b*x^n)^p/(a*c*(m+1)), x] - \text{Dist}[(b*(m+n*(p+1)))]$

+ 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k\*(m + 1) - 1)\*(a + (b\*x^(k\*n))/c^n)^p, x], x, (c\*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 296

Int[(x\_)^(m\_)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r\*Cos[(2\*k\*m\*Pi)/n] - s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/(r^2 - 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x] + Int[(r\*Cos[(2\*k\*m\*Pi)/n] + s\*Cos[(2\*k\*(m + 1)\*Pi)/n]\*x)/(r^2 + 2\*r\*s\*Cos[(2\*k\*Pi)/n]\*x + s^2\*x^2), x]; (2\*r^(m + 2)\*Int[1/(r^2 - s^2\*x^2), x])/(a\*n\*s^m) + Dist[(2\*r^(m + 1))/(a\*n\*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

### Rule 634

Int[((d\_.) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d},



e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \tanh^{-1}(cx^{3/2})}{x^3} dx &= -\frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} + \frac{1}{4}(3bc) \int \frac{1}{x^{3/2}(1 - c^2x^3)} dx \\
 &= -\frac{3bc}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} + \frac{1}{4}(3bc^3) \int \frac{x^{3/2}}{1 - c^2x^3} dx \\
 &= -\frac{3bc}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} + \frac{1}{2}(3bc^3) \text{Subst}\left(\int \frac{x^4}{1 - c^2x^6} dx, x, \sqrt{x}\right) \\
 &= -\frac{3bc}{2\sqrt{x}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} + \frac{1}{2}(bc^{5/3}) \text{Subst}\left(\int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x}\right) + \frac{1}{2}(bc^{5/3}) \text{Subst}\left(\int \frac{1}{1 - c^{2/3}x^2} dx, x, \sqrt{x}\right) \\
 &= -\frac{3bc}{2\sqrt{x}} + \frac{1}{2}bc^{4/3} \tanh^{-1}(\sqrt[3]{c}\sqrt{x}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} - \frac{1}{8}(bc^{4/3}) \text{Subst}\left(\int \frac{-\sqrt[3]{c} + 2c^{2/3}}{1 - \sqrt[3]{c}x + c^{2/3}x^2} dx, x, \sqrt{x}\right) \\
 &= -\frac{3bc}{2\sqrt{x}} + \frac{1}{2}bc^{4/3} \tanh^{-1}(\sqrt[3]{c}\sqrt{x}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{2x^2} - \frac{1}{8}bc^{4/3} \log(1 - \sqrt[3]{c}\sqrt{x} + c^{2/3}x) + \frac{1}{8}bc^{4/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + c^{2/3}) \\
 &= -\frac{3bc}{2\sqrt{x}} + \frac{1}{4}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) - \frac{1}{4}\sqrt{3}bc^{4/3} \tan^{-1}\left(\frac{1 + 2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) + \frac{1}{2}bc^{4/3} \tanh^{-1}\left(\frac{1 - \sqrt[3]{c}\sqrt{x}}{1 + \sqrt[3]{c}\sqrt{x} + c^{2/3}x}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.058352, size = 220, normalized size = 1.17

$$-\frac{a}{2x^2} - \frac{1}{4}bc^{4/3} \log(1 - \sqrt[3]{c}\sqrt{x}) + \frac{1}{4}bc^{4/3} \log(\sqrt[3]{c}\sqrt{x} + 1) - \frac{1}{8}bc^{4/3} \log(c^{2/3}x - \sqrt[3]{c}\sqrt{x} + 1) + \frac{1}{8}bc^{4/3} \log(c^{2/3}x + \sqrt[3]{c}\sqrt{x} + c^{2/3})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^(3/2)])/x^3, x]

[Out] -a/(2\*x^2) - (3\*b\*c)/(2\*Sqrt[x]) - (Sqrt[3]\*b\*c^(4/3)\*ArcTan[(-1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/4 - (Sqrt[3]\*b\*c^(4/3)\*ArcTan[(1 + 2\*c^(1/3)\*Sqrt[x])/Sqrt[3]])/4

$$\begin{aligned} & \text{Sqrt}[3])/4 - (b*\text{ArcTanh}[c*x^{(3/2)}])/(2*x^2) - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)}*\text{S} \\ & \text{qrt}[x]])/4 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x]])/4 - (b*c^{(4/3)}*\text{Log}[1 - c^{(1/3)} \\ & *(1/3)*\text{Sqrt}[x] + c^{(2/3)*x}])/8 + (b*c^{(4/3)}*\text{Log}[1 + c^{(1/3)}*\text{Sqrt}[x] + c^{(2/3)} \\ & *x])/8 \end{aligned}$$

**Maple [A]** time = 0.037, size = 180, normalized size = 1.

$$-\frac{a}{2x^2} - \frac{b}{2x^2} \text{Arctanh}\left(cx^{\frac{3}{2}}\right) - \frac{3bc}{2} \frac{1}{\sqrt{x}} - \frac{bc}{4} \ln\left(\sqrt{x} - \sqrt[3]{c^{-1}}\right) \frac{1}{\sqrt[3]{c^{-1}}} + \frac{bc}{8} \ln\left(x + \sqrt[3]{c^{-1}}\sqrt{x} + (c^{-1})^{\frac{2}{3}}\right) \frac{1}{\sqrt[3]{c^{-1}}} - \frac{bc\sqrt{3}}{4} \arctan\left(\frac{\sqrt{3}x - \sqrt[3]{c^{-1}}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(3/2)))/x^3,x)

[Out] 
$$-1/2*a/x^2 - 1/2*b/x^2*\text{arctanh}(c*x^{(3/2)}) - 3/2*b*c/x^{(1/2)} - 1/4*b*c/(1/c)^{(1/3)} * \ln(x^{(1/2)} - (1/c)^{(1/3)}) + 1/8*b*c/(1/c)^{(1/3)} * \ln(x + (1/c)^{(1/3)}*x^{(1/2)} + (1/c)^{(2/3)}) - 1/4*b*c*3^{(1/2)}/(1/c)^{(1/3)} * \arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x^{(1/2)} + 1)) + 1/4*b*c/(1/c)^{(1/3)} * \ln(x^{(1/2)} + (1/c)^{(1/3)}) - 1/8*b*c/(1/c)^{(1/3)} * \ln(x - (1/c)^{(1/3)}*x^{(1/2)} + (1/c)^{(2/3)}) - 1/4*b*c*3^{(1/2)}/(1/c)^{(1/3)} * \arctan(1/3*3^{(1/2)}*(2/(1/c)^{(1/3)}*x^{(1/2)} - 1))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.99585, size = 618, normalized size = 3.29

$$2\sqrt{3}b(-c)^{\frac{1}{3}}cx^2\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{1}{3}}\sqrt{x} - \frac{1}{3}\sqrt{3}\right) + 2\sqrt{3}bc^{\frac{4}{3}}x^2\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}\sqrt{x} - \frac{1}{3}\sqrt{3}\right) + b(-c)^{\frac{1}{3}}cx^2\log\left(cx + (-c)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x^3,x, algorithm="fricas")

[Out] 
$$-1/8*(2*\sqrt{3}*b*(-c)^{(1/3)}*c*x^2*\arctan(2/3*\sqrt{3}*(-c)^{(1/3)}*\sqrt{x} - 1/3*\sqrt{3}) + 2*\sqrt{3}*b*c^{(4/3)}*x^2*\arctan(2/3*\sqrt{3}*c^{(1/3)}*\sqrt{x} - 1/3*\sqrt{3}) + b*(-c)^{(1/3)}*c*x^2*\log(c*x + (-c)^{(2/3)}*\sqrt{x} - (-c)^{(1/3)}) + b*c^{(4/3)}*x^2*\log(c*x - c^{(2/3)}*\sqrt{x} + c^{(1/3)}) - 2*b*(-c)^{(1/3)}*c*x^2*\log(c*\sqrt{x} - (-c)^{(2/3)}) - 2*b*c^{(4/3)}*x^2*\log(c*\sqrt{x} + c^{(2/3)}) + 12*b*c*x^{(3/2)} + 2*b*\log(-(c^2*x^3 + 2*c*x^{(3/2)} + 1)/(c^2*x^3 - 1)) + 4*a)/x^2$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(3/2)))/x\*\*3,x)

[Out] Timed out

**Giac [A]** time = 1.51374, size = 270, normalized size = 1.44

$$-\frac{1}{8}bc^3 \left( \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + \frac{1}{|c|^{\frac{1}{3}}}\right)\right)|c|^{\frac{1}{3}}}{c^2} + \frac{2\sqrt{3}|c|^{\frac{1}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} - \frac{1}{|c|^{\frac{1}{3}}}\right)\right)|c|^{\frac{1}{3}}}{c^2} - \frac{|c|^{\frac{1}{3}} \log\left(x + \frac{\sqrt{x}}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))/x^3,x, algorithm="giac")

[Out] 
$$-1/8*b*c^3*(2*\sqrt{3}*abs(c)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*\sqrt{x} + 1/abs(c))^{(1/3)}*abs(c)^{(1/3)})/c^2 + 2*\sqrt{3}*abs(c)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*\sqrt{x} - 1/abs(c))^{(1/3)}*abs(c)^{(1/3)})/c^2 - abs(c)^{(1/3)}*\log(x + \sqrt{x}/abs(c)^{(1/3)} + 1/abs(c)^{(2/3)})/c^2 + abs(c)^{(1/3)}*\log(x - \sqrt{x}/abs(c)^{(1/3)} + 1/abs(c)^{(2/3)})/c^2 - 2*abs(c)^{(1/3)}*\log(\sqrt{x} + 1/abs(c)^{(1/3)})/c^2 + 2*abs(c)^{(1/3)}*\log(\sqrt{x} - 1/abs(c)^{(1/3)})/c^2 - 1/4*b*\log(-(c*x^{(3/2)} + 1)/(c*x^{(3/2)} - 1))/x^2 - 1/2*(3*b*c*x^{(3/2)} + a)/x^2$$

$$3.220 \quad \int \frac{a+b \tanh^{-1}(cx^{3/2})}{x^4} dx$$

**Optimal.** Leaf size=47

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{3x^3} + \frac{1}{3}bc^2 \tanh^{-1}(cx^{3/2}) - \frac{bc}{3x^{3/2}}$$

[Out]  $-(b*c)/(3*x^{(3/2)}) + (b*c^2*ArcTanh[c*x^{(3/2)}])/3 - (a + b*ArcTanh[c*x^{(3/2)}])/(3*x^3)$

**Rubi [A]** time = 0.0322121, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6097, 325, 329, 275, 206}

$$-\frac{a+b \tanh^{-1}(cx^{3/2})}{3x^3} + \frac{1}{3}bc^2 \tanh^{-1}(cx^{3/2}) - \frac{bc}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^(3/2)])/x^4,x]

[Out]  $-(b*c)/(3*x^{(3/2)}) + (b*c^2*ArcTanh[c*x^{(3/2)}])/3 - (a + b*ArcTanh[c*x^{(3/2)}])/(3*x^3)$

### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n])/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

### Rule 325

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*c^n\*(m + 1)), Int[(c\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 329

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k =
  Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \tanh^{-1}(cx^{3/2})}{x^4} dx &= -\frac{a + b \tanh^{-1}(cx^{3/2})}{3x^3} + \frac{1}{2}(bc) \int \frac{1}{x^{5/2}(1 - c^2x^3)} dx \\
&= -\frac{bc}{3x^{3/2}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{3x^3} + \frac{1}{2}(bc^3) \int \frac{\sqrt{x}}{1 - c^2x^3} dx \\
&= -\frac{bc}{3x^{3/2}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{3x^3} + (bc^3) \text{Subst} \left( \int \frac{x^2}{1 - c^2x^6} dx, x, \sqrt{x} \right) \\
&= -\frac{bc}{3x^{3/2}} - \frac{a + b \tanh^{-1}(cx^{3/2})}{3x^3} + \frac{1}{3}(bc^3) \text{Subst} \left( \int \frac{1}{1 - c^2x^2} dx, x, x^{3/2} \right) \\
&= -\frac{bc}{3x^{3/2}} + \frac{1}{3}bc^2 \tanh^{-1}(cx^{3/2}) - \frac{a + b \tanh^{-1}(cx^{3/2})}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0289697, size = 73, normalized size = 1.55

$$-\frac{a}{3x^3} - \frac{1}{6}bc^2 \log(1 - cx^{3/2}) + \frac{1}{6}bc^2 \log(cx^{3/2} + 1) - \frac{bc}{3x^{3/2}} - \frac{b \tanh^{-1}(cx^{3/2})}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^4, x]
```

[Out]  $-a/(3*x^3) - (b*c)/(3*x^{(3/2)}) - (b*ArcTanh[c*x^{(3/2)}])/(3*x^3) - (b*c^2*Log[1 - c*x^{(3/2)}])/6 + (b*c^2*Log[1 + c*x^{(3/2)}])/6$

**Maple [A]** time = 0.036, size = 55, normalized size = 1.2

$$-\frac{a}{3x^3} - \frac{b}{3x^3} \operatorname{Arctanh}\left(cx^{\frac{3}{2}}\right) - \frac{bc^2}{6} \ln\left(cx^{\frac{3}{2}} - 1\right) + \frac{bc^2}{6} \ln\left(cx^{\frac{3}{2}} + 1\right) - \frac{bc}{3} x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^(3/2)))/x^4,x)`

[Out]  $-1/3*a/x^3 - 1/3*b/x^3*arctanh(c*x^{(3/2)}) - 1/6*b*c^2*\ln(c*x^{(3/2)}-1) + 1/6*b*c^2*\ln(c*x^{(3/2)}+1) - 1/3*b*c/x^{(3/2)}$

**Maxima [A]** time = 0.984514, size = 69, normalized size = 1.47

$$\frac{1}{6} \left( \left( c \log\left(cx^{\frac{3}{2}} + 1\right) - c \log\left(cx^{\frac{3}{2}} - 1\right) - \frac{2}{x^{\frac{3}{2}}} \right) c - \frac{2 \operatorname{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="maxima")`

[Out]  $1/6*((c*\log(c*x^{(3/2)} + 1) - c*\log(c*x^{(3/2)} - 1) - 2/x^{(3/2)})*c - 2*arctanh(c*x^{(3/2)})/x^3)*b - 1/3*a/x^3$

**Fricas [A]** time = 1.87849, size = 132, normalized size = 2.81

$$\frac{2bcx^{\frac{3}{2}} - (bc^2x^3 - b) \log\left(-\frac{c^2x^3 + 2cx^{\frac{3}{2}} + 1}{c^2x^3 - 1}\right) + 2a}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="fricas")`

[Out]  $-1/6*(2*b*c*x^{(3/2)} - (b*c^2*x^3 - b)*\log(-(c^2*x^3 + 2*c*x^{(3/2)} + 1)/(c^2*x^3 - 1)) + 2*a)/x^3$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**(3/2)))/x**4,x)`

[Out] Timed out

**Giac [A]** time = 1.2122, size = 90, normalized size = 1.91

$$\frac{1}{6}bc^2 \log\left(cx^{\frac{3}{2}} + 1\right) - \frac{1}{6}bc^2 \log\left(cx^{\frac{3}{2}} - 1\right) - \frac{b \log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{6x^3} - \frac{bcx^{\frac{3}{2}} + a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="giac")`

[Out]  $1/6*b*c^2*\log(c*x^{(3/2)} + 1) - 1/6*b*c^2*\log(c*x^{(3/2)} - 1) - 1/6*b*\log(-(c*x^{(3/2)} + 1)/(c*x^{(3/2)} - 1))/x^3 - 1/3*(b*c*x^{(3/2)} + a)/x^3$

$$3.221 \quad \int x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2 dx$$

**Optimal.** Leaf size=101

$$-\frac{\left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2}{3c^2} + \frac{2abx^{3/2}}{3c} + \frac{1}{3}x^3 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2 + \frac{b^2 \log \left( 1 - c^2x^3 \right)}{3c^2} + \frac{2b^2x^{3/2} \tanh^{-1} \left( cx^{3/2} \right)}{3c}$$

[Out] (2\*a\*b\*x^(3/2))/(3\*c) + (2\*b^2\*x^(3/2)\*ArcTanh[c\*x^(3/2)])/(3\*c) - (a + b\*ArcTanh[c\*x^(3/2)])^2/(3\*c^2) + (x^3\*(a + b\*ArcTanh[c\*x^(3/2)])^2)/3 + (b^2\*Log[1 - c^2\*x^3])/(3\*c^2)

**Rubi [F]** time = 0.0244631, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(a + b\*ArcTanh[c\*x^(3/2)])^2,x]

[Out] Defer[Int][x^2\*(a + b\*ArcTanh[c\*x^(3/2)])^2, x]

Rubi steps

$$\int x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2 dx = \int x^2 \left( a + b \tanh^{-1} \left( cx^{3/2} \right) \right)^2 dx$$

**Mathematica [A]** time = 0.0928518, size = 122, normalized size = 1.21

$$\frac{a^2c^2x^3 + 2abcx^{3/2} + b(a + b) \log \left( 1 - cx^{3/2} \right) - ab \log \left( cx^{3/2} + 1 \right) + 2bcx^{3/2} \tanh^{-1} \left( cx^{3/2} \right) \left( acx^{3/2} + b \right) + b^2 \left( c^2x^3 - 1 \right) \tanh^{-1} \left( cx^{3/2} \right)}{3c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^(3/2)])^2,x]



[Out]  $(2abcx^{3/2} + a^2c^2x^3 + 2b^2cx^{3/2})(b + acx^{3/2})\text{ArcTanh}[cx^{3/2}] + b^2(-1 + c^2x^3)\text{ArcTanh}[cx^{3/2}]^2 + b(a + b)\text{Log}[1 - cx^{3/2}] - ab\text{Log}[1 + cx^{3/2}] + b^2\text{Log}[1 + cx^{3/2}]/(3c^2)$

**Maple [B]** time = 0.049, size = 284, normalized size = 2.8

$$\frac{x^3a^2}{3} + \frac{x^3b^2}{3} \left( \text{Arctanh}\left(cx^{\frac{3}{2}}\right) \right)^2 + \frac{2b^2}{3c} x^{\frac{3}{2}} \text{Arctanh}\left(cx^{\frac{3}{2}}\right) + \frac{b^2}{3c^2} \text{Arctanh}\left(cx^{\frac{3}{2}}\right) \ln\left(cx^{\frac{3}{2}} - 1\right) - \frac{b^2}{3c^2} \text{Arctanh}\left(cx^{\frac{3}{2}}\right) \ln\left(cx^{\frac{3}{2}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arctanh(c*x^(3/2)))^2,x)`

[Out]  $1/3x^3a^2 + 1/3b^2x^3\text{arctanh}(cx^{3/2})^2 + 2/3b^2x^{3/2}\text{arctanh}(cx^{3/2})/c + 1/3/c^2b^2\text{arctanh}(cx^{3/2})\ln(cx^{3/2}-1) - 1/3/c^2b^2\text{arctanh}(cx^{3/2})\ln(cx^{3/2}+1) - 1/6/c^2b^2\ln(cx^{3/2}-1)\ln(1/2+1/2cx^{3/2}) + 1/12/c^2b^2\ln(cx^{3/2}-1)^2 + 1/3/c^2b^2\ln(cx^{3/2}-1) + 1/3/c^2b^2\ln(cx^{3/2}+1) - 1/6/c^2b^2\ln(-1/2cx^{3/2}+1/2)\ln(cx^{3/2}+1) + 1/6/c^2b^2\ln(-1/2cx^{3/2}+1/2)\ln(1/2+1/2cx^{3/2}) + 1/12/c^2b^2\ln(cx^{3/2}+1)^2 + 2/3abx^3\text{arctanh}(cx^{3/2}) + 2/3abx^{3/2}/c + 1/3/c^2ab\ln(cx^{3/2}-1) - 1/3/c^2ab\ln(cx^{3/2}+1)$

**Maxima [B]** time = 1.00881, size = 251, normalized size = 2.49

$$\frac{1}{3}b^2x^3 \text{artanh}\left(cx^{\frac{3}{2}}\right)^2 + \frac{1}{3}a^2x^3 + \frac{1}{3}\left(2x^3 \text{artanh}\left(cx^{\frac{3}{2}}\right) + c\left(\frac{2x^{\frac{3}{2}}}{c^2} - \frac{\log\left(cx^{\frac{3}{2}} + 1\right)}{c^3} + \frac{\log\left(cx^{\frac{3}{2}} - 1\right)}{c^3}\right)\right)ab + \frac{1}{12}\left(4c\left(\frac{2x^{\frac{3}{2}}}{c^2} - \frac{\log\left(cx^{\frac{3}{2}} + 1\right)}{c^3} + \frac{\log\left(cx^{\frac{3}{2}} - 1\right)}{c^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="maxima")`

[Out]  $1/3b^2x^3\text{arctanh}(cx^{3/2})^2 + 1/3a^2x^3 + 1/3(2x^3\text{arctanh}(cx^{3/2}) + c(2x^{3/2}/c^2 - \log(cx^{3/2} + 1)/c^3 + \log(cx^{3/2} - 1)/c^3))ab + 1/12(4c(2x^{3/2}/c^2 - \log(cx^{3/2} + 1)/c^3 + \log(cx^{3/2} - 1)/c^3)\text{arctanh}(cx^{3/2}) - (2(\log(cx^{3/2} - 1) - 2)\log(cx^{3/2} + 1) - \log(cx^{3/2} + 1)^2 - \log(cx^{3/2} - 1)^2 - 4\log(cx^{3/2} - 1))/c^2) * b^2$

---

**Fricas [B]** time = 1.99773, size = 402, normalized size = 3.98

$$\frac{4a^2c^2x^3 + 8abcx^{\frac{3}{2}} + (b^2c^2x^3 - b^2) \log\left(-\frac{c^2x^3 + 2cx^{\frac{3}{2}} + 1}{c^2x^3 - 1}\right)^2 + 4(abc^2 - ab + b^2) \log\left(cx^{\frac{3}{2}} + 1\right) - 4(abc^2 - ab - b^2) \log\left(cx^{\frac{3}{2}} - 1\right)}{12c^2}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(3/2)))^2,x, algorithm="fricas")

[Out] 1/12\*(4\*a^2\*c^2\*x^3 + 8\*a\*b\*c\*x^(3/2) + (b^2\*c^2\*x^3 - b^2)\*log(-(c^2\*x^3 + 2\*c\*x^(3/2) + 1)/(c^2\*x^3 - 1))^2 + 4\*(a\*b\*c^2 - a\*b + b^2)\*log(c\*x^(3/2) + 1) - 4\*(a\*b\*c^2 - a\*b - b^2)\*log(c\*x^(3/2) - 1) + 4\*(a\*b\*c^2\*x^3 + b^2\*c\*x^(3/2) - a\*b\*c^2)\*log(-(c^2\*x^3 + 2\*c\*x^(3/2) + 1)/(c^2\*x^3 - 1)))/c^2

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*atanh(c\*x\*\*(3/2)))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( b \operatorname{artanh}\left( cx^{\frac{3}{2}} \right) + a \right)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^(3/2)))^2,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^(3/2)) + a)^2\*x^2, x)

$$3.222 \quad \int \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{x} dx$$

**Optimal.** Leaf size=156

$$-\frac{2}{3}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^{3/2}}\right)(a + b \tanh^{-1}(cx^{3/2})) + \frac{2}{3}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^{3/2}} - 1\right)(a + b \tanh^{-1}(cx^{3/2})) + \frac{1}{3}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - cx^{3/2}}\right)(a + b \tanh^{-1}(cx^{3/2})) - \frac{1}{3}b^2 \operatorname{PolyLog}\left(3, \frac{2}{1 - cx^{3/2}} - 1\right)(a + b \tanh^{-1}(cx^{3/2}))$$

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

**Rubi [A]** time = 0.319783, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{2}{3}b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1 - cx^{3/2}}\right)(a + b \tanh^{-1}(cx^{3/2})) + \frac{2}{3}b \operatorname{PolyLog}\left(2, \frac{2}{1 - cx^{3/2}} - 1\right)(a + b \tanh^{-1}(cx^{3/2})) + \frac{1}{3}b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1 - cx^{3/2}}\right)(a + b \tanh^{-1}(cx^{3/2})) - \frac{1}{3}b^2 \operatorname{PolyLog}\left(3, \frac{2}{1 - cx^{3/2}} - 1\right)(a + b \tanh^{-1}(cx^{3/2}))$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTanh[c*x^(3/2)])^2/x, x]
```

```
[Out] (4*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))])/3 - (2*b*(a + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/3 + (2*b*(a + b*ArcTanh[c*x^(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/3 + (b^2*PolyLog[3, 1 - 2/(1 - c*x^(3/2))])/3 - (b^2*PolyLog[3, -1 + 2/(1 - c*x^(3/2))])/3
```

### Rule 6095

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)^(p_.)/(x_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
```

### Rule 5914

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.)^(p_.)/(x_), x_Symbol] := Simp[2*(a + b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Dist[2*b*c*p, Int[(a + b*ArcTanh[c*x])^(p - 1)*ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 6052

```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := Dist[1/2, Int[(Log[1 + u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]*(a + b*ArcTanh[c*x])^p)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 5948

```
Int[((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbol] := Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

Rule 6058

```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)])*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2), x_Symbol] := -Simp[((a + b*ArcTanh[c*x])^p*PolyLog[2, 1 - u])/(2*c*d), x] + Dist[(b*p)/2, Int[((a + b*ArcTanh[c*x])^(p - 1)*PolyLog[2, 1 - u])/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^{3/2}))^2}{x} dx &= \frac{2}{3} \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))^2}{x} dx, x, x^{3/2} \right) \\
&= \frac{4}{3} (a + b \tanh^{-1}(cx^{3/2}))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^{3/2}} \right) - \frac{1}{3} (8bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))}{1 - cx} dx, x, x^{3/2} \right) \\
&= \frac{4}{3} (a + b \tanh^{-1}(cx^{3/2}))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^{3/2}} \right) + \frac{1}{3} (4bc) \text{Subst} \left( \int \frac{(a + b \tanh^{-1}(cx))}{1 - c^2x} dx, x, x^{3/2} \right) \\
&= \frac{4}{3} (a + b \tanh^{-1}(cx^{3/2}))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^{3/2}} \right) - \frac{2}{3} b (a + b \tanh^{-1}(cx^{3/2})) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^{3/2}} \right) \\
&= \frac{4}{3} (a + b \tanh^{-1}(cx^{3/2}))^2 \tanh^{-1} \left( 1 - \frac{2}{1 - cx^{3/2}} \right) - \frac{2}{3} b (a + b \tanh^{-1}(cx^{3/2})) \text{Li}_2 \left( 1 - \frac{2}{1 - cx^{3/2}} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.132115, size = 167, normalized size = 1.07

$$\frac{1}{3} \left( b \left( 2 \text{PolyLog} \left( 2, \frac{cx^{3/2} + 1}{1 - cx^{3/2}} \right) (a + b \tanh^{-1}(cx^{3/2})) - 2 \text{PolyLog} \left( 2, \frac{cx^{3/2} + 1}{cx^{3/2} - 1} \right) (a + b \tanh^{-1}(cx^{3/2})) + b \left( \text{PolyLog} \left( 3, \frac{1 + cx^{3/2}}{1 - cx^{3/2}} \right) - \text{PolyLog} \left( 3, \frac{1 + cx^{3/2}}{-1 + cx^{3/2}} \right) \right) \right) \right) / 3$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^(3/2)])^2/x,x]

[Out] (4\*(a + b\*ArcTanh[c\*x^(3/2)])^2\*ArcTanh[1 + 2/(-1 + c\*x^(3/2))] + b\*(2\*(a + b\*ArcTanh[c\*x^(3/2)])\*PolyLog[2, (1 + c\*x^(3/2))/(1 - c\*x^(3/2))] - 2\*(a + b\*ArcTanh[c\*x^(3/2)])\*PolyLog[2, (1 + c\*x^(3/2))/(-1 + c\*x^(3/2))] + b\*(-PolyLog[3, (1 + c\*x^(3/2))/(1 - c\*x^(3/2))] + PolyLog[3, (1 + c\*x^(3/2))/(-1 + c\*x^(3/2))]))/3

---

**Maple [C]** time = 0.334, size = 785, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(3/2)))^2/x,x)

```
[Out] 2/3*a^2*ln(c*x^(3/2))+2/3*b^2*ln(c*x^(3/2))*arctanh(c*x^(3/2))^2-2/3*b^2*ar
ctanh(c*x^(3/2))*polylog(2,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))+1/3*b^2*polylog(3
,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))-2/3*b^2*arctanh(c*x^(3/2))^2*ln((c*x^(3/2)+
1)^2/(-c^2*x^3+1)-1)+2/3*b^2*arctanh(c*x^(3/2))^2*ln(1-(c*x^(3/2)+1)/(-c^2*x
^3+1)^(1/2))+4/3*b^2*arctanh(c*x^(3/2))*polylog(2,(c*x^(3/2)+1)/(-c^2*x^3+
1)^(1/2))-4/3*b^2*polylog(3,(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+2/3*b^2*arcta
nh(c*x^(3/2))^2*ln(1+(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+4/3*b^2*arctanh(c*x^
(3/2))*polylog(2,-(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))-4/3*b^2*polylog(3,-(c*x
^(3/2)+1)/(-c^2*x^3+1)^(1/2))-1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^
3+1)-1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3
+1)+1))^2*arctanh(c*x^(3/2))^2+1/3*I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x
^3+1)-1))*csgn(I/((c*x^(3/2)+1)^2/(-c^2*x^3+1)+1))*csgn(I*((c*x^(3/2)+1)^2/
(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3+1)+1))*arctanh(c*x^(3/2))^2+1/3*
I*b^2*Pi*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^3
+1)+1))^3*arctanh(c*x^(3/2))^2-1/3*I*b^2*Pi*csgn(I/((c*x^(3/2)+1)^2/(-c^2*x
^3+1)+1))*csgn(I*((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)/((c*x^(3/2)+1)^2/(-c^2*x^
3+1)+1))^2*arctanh(c*x^(3/2))^2-2/3*a*b*ln(c*x^(3/2))*ln(c*x^(3/2)+1)+4/3*a
*b*ln(c*x^(3/2))*arctanh(c*x^(3/2))-2/3*a*b*dilog(c*x^(3/2))-2/3*a*b*dilog(
c*x^(3/2)+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}b^2 \int \frac{\log\left(cx^{\frac{3}{2}}+1\right)^2}{x} dx - \frac{1}{2}b^2 \int \frac{\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}+1\right)}{x} dx + \frac{1}{4}b^2 \int \frac{\log\left(-cx^{\frac{3}{2}}+1\right)^2}{x} dx + ab \int \frac{\log\left(cx^{\frac{3}{2}}+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*integrate(log(c*x^(3/2) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*x^(3
/2) + 1)*log(-c*x^(3/2) + 1)/x, x) + 1/4*b^2*integrate(log(-c*x^(3/2) + 1)^
2/x, x) + a*b*integrate(log(c*x^(3/2) + 1)/x, x) - a*b*integrate(log(-c*x^(
3/2) + 1)/x, x) + a^2*log(x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 \operatorname{artanh}\left(cx^{\frac{3}{2}}\right)^2 + 2ab \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a^2}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))^2/x,x, algorithm="fricas")

[Out] integral((b^2\*arctanh(c\*x^(3/2))^2 + 2\*a\*b\*arctanh(c\*x^(3/2)) + a^2)/x, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*(3/2)))\*\*2/x,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))^2/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^(3/2)) + a)^2/x, x)

$$3.223 \quad \int \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{x^4} dx$$

**Optimal.** Leaf size=96

$$\frac{1}{3}c^2 \left(a + b \tanh^{-1}(cx^{3/2})\right)^2 - \frac{2bc \left(a + b \tanh^{-1}(cx^{3/2})\right)}{3x^{3/2}} - \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{3x^3} - \frac{1}{3}b^2c^2 \log(1 - c^2x^3) + b^2c^2 \log(x)$$

[Out]  $(-2*b*c*(a + b*ArcTanh[c*x^(3/2)]))/(3*x^(3/2)) + (c^2*(a + b*ArcTanh[c*x^(3/2)])^2)/3 - (a + b*ArcTanh[c*x^(3/2)])^2/(3*x^3) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 - c^2*x^3])/3$

**Rubi [F]** time = 0.0237288, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^(3/2)])^2/x^4, x]

[Out] Defer[Int][(a + b\*ArcTanh[c\*x^(3/2)])^2/x^4, x]

Rubi steps

$$\int \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{x^4} dx = \int \frac{\left(a + b \tanh^{-1}(cx^{3/2})\right)^2}{x^4} dx$$

**Mathematica [A]** time = 0.139447, size = 123, normalized size = 1.28

$$\frac{1}{3} \left( -\frac{a^2}{x^3} - bc^2(a + b) \log(1 - cx^{3/2}) + bc^2(a - b) \log(cx^{3/2} + 1) - \frac{2abc}{x^{3/2}} - \frac{2b \tanh^{-1}(cx^{3/2})(a + bcx^{3/2})}{x^3} + \frac{b^2(c^2x^3 - 1) \tanh^{-1}(cx^{3/2})}{x^3} \right)$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*ArcTanh[c\*x^(3/2)])^2/x^4,x]

[Out]  $(-a^2/x^3) - (2*a*b*c)/x^{(3/2)} - (2*b*(a + b*c*x^{(3/2)})*ArcTanh[c*x^{(3/2)}])/x^3 + (b^2*(-1 + c^2*x^3)*ArcTanh[c*x^{(3/2)}]^2)/x^3 + 3*b^2*c^2*Log[x] - b*(a + b)*c^2*Log[1 - c*x^{(3/2)}] + (a - b)*b*c^2*Log[1 + c*x^{(3/2)}])/3$

**Maple [F]** time = 0.28, size = 0, normalized size = 0.

$$\int \frac{1}{x^4} \left( a + b \operatorname{Arctanh} \left( cx^{\frac{3}{2}} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^(3/2)))^2/x^4,x)

[Out] int((a+b\*arctanh(c\*x^(3/2)))^2/x^4,x)

**Maxima [B]** time = 0.998003, size = 236, normalized size = 2.46

$$\frac{1}{3} \left( \left( c \log \left( cx^{\frac{3}{2}} + 1 \right) - c \log \left( cx^{\frac{3}{2}} - 1 \right) - \frac{2}{x^{\frac{3}{2}}} \right) c - \frac{2 \operatorname{artanh} \left( cx^{\frac{3}{2}} \right)}{x^3} \right) ab + \frac{1}{12} \left( \left( 2 \left( \log \left( cx^{\frac{3}{2}} - 1 \right) - 2 \right) \log \left( cx^{\frac{3}{2}} + 1 \right) - \log \left( cx^{\frac{3}{2}} + 1 \right)^2 - \log \left( cx^{\frac{3}{2}} - 1 \right)^2 - 4 \log \left( cx^{\frac{3}{2}} - 1 \right) + 12 \log(x) \right) c^2 + 4 \left( c \log \left( cx^{\frac{3}{2}} + 1 \right) - c \log \left( cx^{\frac{3}{2}} - 1 \right) - \frac{2}{x^{\frac{3}{2}}} \right) c * \operatorname{arctanh} \left( cx^{\frac{3}{2}} \right) \right) b^2 - \frac{1}{3} b^2 * \operatorname{arctanh} \left( cx^{\frac{3}{2}} \right)^2 / x^3 - \frac{1}{3} a^2 / x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^(3/2)))^2/x^4,x, algorithm="maxima")

[Out]  $1/3*((c*\log(c*x^{(3/2)} + 1) - c*\log(c*x^{(3/2)} - 1) - 2/x^{(3/2)})*c - 2*\operatorname{arctanh}(c*x^{(3/2)})/x^3)*a*b + 1/12*((2*(\log(c*x^{(3/2)} - 1) - 2)*\log(c*x^{(3/2)} + 1) - \log(c*x^{(3/2)} + 1)^2 - \log(c*x^{(3/2)} - 1)^2 - 4*\log(c*x^{(3/2)} - 1) + 12*\log(x))*c^2 + 4*(c*\log(c*x^{(3/2)} + 1) - c*\log(c*x^{(3/2)} - 1) - 2/x^{(3/2)})*c*\operatorname{arctanh}(c*x^{(3/2)}))*b^2 - 1/3*b^2*\operatorname{arctanh}(c*x^{(3/2)})^2/x^3 - 1/3*a^2/x^3$

**Fricas [B]** time = 1.89078, size = 402, normalized size = 4.19

$$24 b^2 c^2 x^3 \log(\sqrt{x}) + 4 (ab - b^2) c^2 x^3 \log \left( cx^{\frac{3}{2}} + 1 \right) - 4 (ab + b^2) c^2 x^3 \log \left( cx^{\frac{3}{2}} - 1 \right) - 8 abc x^{\frac{3}{2}} + (b^2 c^2 x^3 - b^2) \log \left( -\frac{c^2 x^3}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(24*b^2*c^2*x^3*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x^3*log(c*x^(3/2) + 1) - 4*(a*b + b^2)*c^2*x^3*log(c*x^(3/2) - 1) - 8*a*b*c*x^(3/2) + (b^2*c^2*x^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^(3/2) + a*b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/x^3
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**(3/2)))**2/x**4,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b \operatorname{artanh}\left(cx^{\frac{3}{2}}\right) + a\right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^(3/2)) + a)^2/x^4, x)
```

### 3.224 $\int x^2 (a + b \tanh^{-1}(cx^n)) dx$

**Optimal.** Leaf size=66

$$\frac{1}{3}x^3(a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{n+3} \text{Hypergeometric2F1}\left(1, \frac{n+3}{2n}, \frac{3(n+1)}{2n}, c^2x^{2n}\right)}{3(n+3)}$$

[Out]  $(x^3(a + b \text{ArcTanh}[c x^n]))/3 - (b*c*n*x^{(3+n)}*\text{Hypergeometric2F1}[1, (3+n)/(2*n), (3*(1+n))/(2*n), c^2*x^{(2*n)}])/(3*(3+n))$

**Rubi [A]** time = 0.0305945, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 364}

$$\frac{1}{3}x^3(a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{n+3} {}_2F_1\left(1, \frac{n+3}{2n}; \frac{3(n+1)}{2n}; c^2x^{2n}\right)}{3(n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + b*\text{ArcTanh}[c*x^n]), x]$

[Out]  $(x^3*(a + b*\text{ArcTanh}[c*x^n]))/3 - (b*c*n*x^{(3+n)}*\text{Hypergeometric2F1}[1, (3+n)/(2*n), (3*(1+n))/(2*n), c^2*x^{(2*n)}])/(3*(3+n))$

#### Rule 6097

$\text{Int}[(a + \text{ArcTanh}[c*(x)^n]*(b))*(d*(x))^m, x\_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcTanh}[c*x^n])/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(x^{n-1}*(d*x)^{m+1})/(1 - c^2*x^{2n}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, m, n\}, x\} \ \&\& \ \text{NeQ}[m, -1]$

#### Rule 364

$\text{Int}[(c*(x))^m*(a + (b)*(x)^n)^p, x\_Symbol] \rightarrow \text{Simp}[(a^p*(c*x)^{m+1}*\text{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])/(c*(m+1)), x] /;$   
 $\text{FreeQ}\{a, b, c, m, n, p\}, x\} \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ (\text{ILtQ}[p, 0] \ || \ \text{GtQ}[a, 0])$

#### Rubi steps

$$\int x^2 (a + b \tanh^{-1}(cx^n)) dx = \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{1}{3}(bcn) \int \frac{x^{2+n}}{1 - c^2x^{2n}} dx$$

$$= \frac{1}{3}x^3 (a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{3+n} {}_2F_1\left(1, \frac{3+n}{2n}; \frac{3(1+n)}{2n}; c^2x^{2n}\right)}{3(3+n)}$$

**Mathematica [A]** time = 0.0519168, size = 73, normalized size = 1.11

$$\frac{bcnx^{n+3} \text{Hypergeometric2F1}\left(1, \frac{n+3}{2n}, \frac{n+3}{2n} + 1, c^2x^{2n}\right)}{3(n+3)} + \frac{ax^3}{3} + \frac{1}{3}bx^3 \tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcTanh[c\*x^n]), x]

[Out] (a\*x^3)/3 + (b\*x^3\*ArcTanh[c\*x^n])/3 - (b\*c\*n\*x^(3 + n)\*Hypergeometric2F1[1, (3 + n)/(2\*n), 1 + (3 + n)/(2\*n), c^2\*x^(2\*n)])/(3\*(3 + n))

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int x^2 (a + b \text{Artanh}(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arctanh(c\*x^n)), x)

[Out] int(x^2\*(a+b\*arctanh(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}ax^3 + \frac{1}{6}\left(x^3 \log(cx^n + 1) - x^3 \log(-cx^n + 1) + 3n \int \frac{x^2}{3(cx^n + 1)} dx + 3n \int \frac{x^2}{3(cx^n - 1)} dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arctanh(c\*x^n)), x, algorithm="maxima")

[Out]  $\frac{1}{3}ax^3 + \frac{1}{6}(x^3 \log(cx^n + 1) - x^3 \log(-cx^n + 1) + 3n \int \frac{1}{3x^2/(cx^n + 1)}, x) + 3n \int \frac{1}{3x^2/(cx^n - 1)}, x) * b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(bx^2 \operatorname{artanh}(cx^n) + ax^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="fricas")`

[Out] `integral(b*x^2*arctanh(c*x^n) + a*x^2, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*atanh(c*x**n)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^n) + a)*x^2, x)`

### 3.225 $\int x \left( a + b \tanh^{-1}(cx^n) \right) dx$

**Optimal.** Leaf size=67

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1}(cx^n) \right) - \frac{bcnx^{n+2} \operatorname{Hypergeometric2F1} \left( 1, \frac{n+2}{2n}, \frac{1}{2} \left( \frac{2}{n} + 3 \right), c^2 x^{2n} \right)}{2(n+2)}$$

[Out]  $(x^2*(a + b*\operatorname{ArcTanh}[c*x^n]))/2 - (b*c*n*x^{(2 + n)}*\operatorname{Hypergeometric2F1}[1, (2 + n)/(2*n), (3 + 2/n)/2, c^2*x^{(2*n)}])/ (2*(2 + n))$

**Rubi [A]** time = 0.0243278, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6097, 364}

$$\frac{1}{2}x^2 \left( a + b \tanh^{-1}(cx^n) \right) - \frac{bcnx^{n+2} {}_2F_1 \left( 1, \frac{n+2}{2n}; \frac{1}{2} \left( 3 + \frac{2}{n} \right); c^2 x^{2n} \right)}{2(n+2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*(a + b*\operatorname{ArcTanh}[c*x^n]), x]$

[Out]  $(x^2*(a + b*\operatorname{ArcTanh}[c*x^n]))/2 - (b*c*n*x^{(2 + n)}*\operatorname{Hypergeometric2F1}[1, (2 + n)/(2*n), (3 + 2/n)/2, c^2*x^{(2*n)}])/ (2*(2 + n))$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*
n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

#### Rubi steps

$$\int x(a + b \tanh^{-1}(cx^n)) dx = \frac{1}{2}x^2(a + b \tanh^{-1}(cx^n)) - \frac{1}{2}(bcn) \int \frac{x^{1+n}}{1 - c^2x^{2n}} dx$$

$$= \frac{1}{2}x^2(a + b \tanh^{-1}(cx^n)) - \frac{bcnx^{2+n} {}_2F_1\left(1, \frac{2+n}{2n}; \frac{1}{2}\left(3 + \frac{2}{n}\right); c^2x^{2n}\right)}{2(2+n)}$$

**Mathematica [A]** time = 0.0445054, size = 73, normalized size = 1.09

$$-\frac{bcnx^{n+2} \text{Hypergeometric2F1}\left(1, \frac{n+2}{2n}, \frac{n+2}{2n} + 1, c^2x^{2n}\right)}{2(n+2)} + \frac{ax^2}{2} + \frac{1}{2}bx^2 \tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^n]), x]

[Out] (a\*x^2)/2 + (b\*x^2\*ArcTanh[c\*x^n])/2 - (b\*c\*n\*x^(2 + n)\*Hypergeometric2F1[1, (2 + n)/(2\*n), 1 + (2 + n)/(2\*n), c^2\*x^(2\*n)])/(2\*(2 + n))

**Maple [F]** time = 0.21, size = 0, normalized size = 0.

$$\int x(a + b \text{Artanh}(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arctanh(c\*x^n)), x)

[Out] int(x\*(a+b\*arctanh(c\*x^n)), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ax^2 + \frac{1}{4}\left(x^2 \log(cx^n + 1) - x^2 \log(-cx^n + 1) + 2n \int \frac{x}{2(cx^n + 1)} dx + 2n \int \frac{x}{2(cx^n - 1)} dx\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arctanh(c\*x^n)), x, algorithm="maxima")

[Out]  $\frac{1}{2}ax^2 + \frac{1}{4}(x^2 \log(cx^n + 1) - x^2 \log(-cx^n + 1) + 2n \int \frac{1}{2x/(cx^n + 1)}, x) + 2n \int \frac{1}{2x/(cx^n - 1)}, x) * b$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(bx \operatorname{artanh}(cx^n) + ax, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="fricas")`

[Out] `integral(b*x*arctanh(c*x^n) + a*x, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(a + b \operatorname{atanh}(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*atanh(c*x**n)),x)`

[Out] `Integral(x*(a + b*atanh(c*x**n)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^n) + a)*x, x)`



### 3.226 $\int (a + b \tanh^{-1}(cx^n)) dx$

**Optimal.** Leaf size=58

$$-\frac{bcnx^{n+1}\text{Hypergeometric2F1}\left(1, \frac{n+1}{2n}, \frac{1}{2}\left(\frac{1}{n} + 3\right), c^2x^{2n}\right)}{n+1} + ax + bx \tanh^{-1}(cx^n)$$

[Out] a\*x + b\*x\*ArcTanh[c\*x^n] - (b\*c\*n\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, c^2\*x^(2\*n)])/(1 + n)

**Rubi [A]** time = 0.0279346, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6091, 364}

$$ax - \frac{bcnx^{n+1} {}_2F_1\left(1, \frac{n+1}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2x^{2n}\right)}{n+1} + bx \tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcTanh[c\*x^n], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x^n] - (b\*c\*n\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, c^2\*x^(2\*n)])/(1 + n)

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a]]/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int (a + b \tanh^{-1}(cx^n)) dx &= ax + b \int \tanh^{-1}(cx^n) dx \\
&= ax + bx \tanh^{-1}(cx^n) - (bcn) \int \frac{x^n}{1 - c^2 x^{2n}} dx \\
&= ax + bx \tanh^{-1}(cx^n) - \frac{bcn x^{1+n} {}_2F_1\left(1, \frac{1+n}{2n}; \frac{1}{2}\left(3 + \frac{1}{n}\right); c^2 x^{2n}\right)}{1+n}
\end{aligned}$$

**Mathematica [A]** time = 0.0316411, size = 58, normalized size = 1.

$$-\frac{bcn x^{n+1} \text{Hypergeometric2F1}\left(1, \frac{n+1}{2n}, \frac{1}{2}\left(\frac{1}{n} + 3\right), c^2 x^{2n}\right)}{n+1} + ax + bx \tanh^{-1}(cx^n)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcTanh[c\*x^n], x]

[Out] a\*x + b\*x\*ArcTanh[c\*x^n] - (b\*c\*n\*x^(1 + n)\*Hypergeometric2F1[1, (1 + n)/(2\*n), (3 + n^(-1))/2, c^2\*x^(2\*n)])/(1 + n)

**Maple [F]** time = 0.191, size = 0, normalized size = 0.

$$\int a + b \text{Artanh}(cx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b\*arctanh(c\*x^n), x)

[Out] int(a+b\*arctanh(c\*x^n), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \left( n \int \frac{1}{cx^n + 1} dx + n \int \frac{1}{cx^n - 1} dx + x \log(cx^n + 1) - x \log(-cx^n + 1) \right) b + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^n),x, algorithm="maxima")

[Out] 1/2\*(n\*integrate(1/(c\*x^n + 1), x) + n\*integrate(1/(c\*x^n - 1), x) + x\*log(c\*x^n + 1) - x\*log(-c\*x^n + 1))\*b + a\*x

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}(b \operatorname{artanh}(cx^n) + a, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^n),x, algorithm="fricas")

[Out] integral(b\*arctanh(c\*x^n) + a, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*atanh(c\*x\*\*n),x)

[Out] Integral(a + b\*atanh(c\*x\*\*n), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{artanh}(cx^n) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arctanh(c\*x^n),x, algorithm="giac")

[Out] integrate(b\*arctanh(c\*x^n) + a, x)

$$3.227 \quad \int \frac{a+b \tanh^{-1}(cx^n)}{x} dx$$

**Optimal.** Leaf size=36

$$-\frac{b \operatorname{PolyLog}(2, -cx^n)}{2n} + \frac{b \operatorname{PolyLog}(2, cx^n)}{2n} + a \log(x)$$

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^n)])/(2\*n) + (b\*PolyLog[2, c\*x^n])/(2\*n)

**Rubi [A]** time = 0.0350596, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6095, 5912}

$$-\frac{b \operatorname{PolyLog}(2, -cx^n)}{2n} + \frac{b \operatorname{PolyLog}(2, cx^n)}{2n} + a \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^n])/x, x]

[Out] a\*Log[x] - (b\*PolyLog[2, -(c\*x^n)])/(2\*n) + (b\*PolyLog[2, c\*x^n])/(2\*n)

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] :> Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])^p/2, x] + Simp[(b\*PolyLog[2, c\*x])^p/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{a+b \tanh^{-1}(cx^n)}{x} dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b \tanh^{-1}(cx)}{x} dx, x, x^n\right)}{n} \\ &= a \log(x) - \frac{b \operatorname{Li}_2(-cx^n)}{2n} + \frac{b \operatorname{Li}_2(cx^n)}{2n} \end{aligned}$$

**Mathematica [C]** time = 0.0750161, size = 39, normalized size = 1.08

$$\frac{bcx^n \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2n}\right)}{n} + a \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])/x, x]

[Out] (b\*c\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2\*x^(2\*n)])/n + a\*Log[x]

**Maple [B]** time = 0.036, size = 76, normalized size = 2.1

$$\frac{a \ln(cx^n)}{n} + \frac{b \ln(cx^n) \text{Artanh}(cx^n)}{n} - \frac{b \text{dilog}(cx^n)}{2n} - \frac{b \text{dilog}(cx^n + 1)}{2n} - \frac{b \ln(cx^n) \ln(cx^n + 1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))/x, x)

[Out] 1/n\*a\*ln(c\*x^n)+1/n\*b\*ln(c\*x^n)\*arctanh(c\*x^n)-1/2/n\*b\*dilog(c\*x^n)-1/2/n\*b\*dilog(c\*x^n+1)-1/2/n\*b\*ln(c\*x^n)\*ln(c\*x^n+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \left( n \int \frac{\log(x)}{c x x^n + x} dx + n \int \frac{\log(x)}{c x x^n - x} dx + \log(cx^n + 1) \log(x) - \log(-cx^n + 1) \log(x) \right) b + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x, x, algorithm="maxima")

[Out] 1/2\*(n\*integrate(log(x)/(c\*x\*x^n + x), x) + n\*integrate(log(x)/(c\*x\*x^n - x), x) + log(c\*x^n + 1)\*log(x) - log(-c\*x^n + 1)\*log(x))\*b + a\*log(x)

**Fricas [B]** time = 1.81852, size = 456, normalized size = 12.67

$$bn \log(c \cosh(n \log(x)) + c \sinh(n \log(x)) + 1) \log(x) - bn \log(-c \cosh(n \log(x)) - c \sinh(n \log(x)) + 1) \log(x) - b$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x,x, algorithm="fricas")

[Out] 
$$-1/2*(b*n*\log(c*\cosh(n*\log(x)) + c*\sinh(n*\log(x)) + 1)*\log(x) - b*n*\log(-c*\cosh(n*\log(x)) - c*\sinh(n*\log(x)) + 1)*\log(x) - b*n*\log(x)*\log(-(c*\cosh(n*\log(x)) + c*\sinh(n*\log(x)) + 1)/(c*\cosh(n*\log(x)) + c*\sinh(n*\log(x)) - 1)) - 2*a*n*\log(x) - b*\operatorname{dilog}(c*\cosh(n*\log(x)) + c*\sinh(n*\log(x))) + b*\operatorname{dilog}(-c*\cosh(n*\log(x)) - c*\sinh(n*\log(x))))/n$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*n))/x,x)

[Out] Integral((a + b\*atanh(c\*x\*\*n))/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^n) + a)/x, x)

$$3.228 \quad \int \frac{a+b \tanh^{-1}(cx^n)}{x^2} dx$$

**Optimal.** Leaf size=67

$$\frac{bcnx^{n-1} \text{Hypergeometric2F1}\left(1, -\frac{1-n}{2n}, \frac{1}{2}\left(3 - \frac{1}{n}\right), c^2x^{2n}\right)}{1-n} - \frac{a+b \tanh^{-1}(cx^n)}{x}$$

[Out] -((a + b\*ArcTanh[c\*x^n])/x) - (b\*c\*n\*x^(-1 + n)\*Hypergeometric2F1[1, -(1 - n)/(2\*n), (3 - n^(-1))/2, c^2\*x^(2\*n)])/(1 - n)

**Rubi [A]** time = 0.032358, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 364}

$$\frac{a+b \tanh^{-1}(cx^n)}{x} - \frac{bcnx^{n-1} {}_2F_1\left(1, -\frac{1-n}{2n}; \frac{1}{2}\left(3 - \frac{1}{n}\right); c^2x^{2n}\right)}{1-n}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^n])/x^2, x]

[Out] -((a + b\*ArcTanh[c\*x^n])/x) - (b\*c\*n\*x^(-1 + n)\*Hypergeometric2F1[1, -(1 - n)/(2\*n), (3 - n^(-1))/2, c^2\*x^(2\*n)])/(1 - n)

#### Rule 6097

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[(x^(n - 1)\*(d\*x)^(m + 1))/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

#### Rule 364

Int[((c\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(a^p\*(c\*x)^(m + 1)\*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b\*x^n)/a])]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^n)}{x^2} dx = -\frac{a + b \tanh^{-1}(cx^n)}{x} + (bcn) \int \frac{x^{-2+n}}{1 - c^2 x^{2n}} dx$$

$$= -\frac{a + b \tanh^{-1}(cx^n)}{x} - \frac{bcn x^{-1+n} {}_2F_1\left(1, -\frac{1-n}{2n}; \frac{1}{2}\left(3 - \frac{1}{n}\right); c^2 x^{2n}\right)}{1-n}$$

**Mathematica [A]** time = 0.0609311, size = 66, normalized size = 0.99

$$\frac{bcn x^{n-1} \text{Hypergeometric2F1}\left(1, \frac{n-1}{2n}, \frac{n-1}{2n} + 1, c^2 x^{2n}\right)}{n-1} - \frac{a}{x} - \frac{b \tanh^{-1}(cx^n)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])/x^2,x]

[Out] -(a/x) - (b\*ArcTanh[c\*x^n])/x + (b\*c\*n\*x^(-1 + n)\*Hypergeometric2F1[1, (-1 + n)/(2\*n), 1 + (-1 + n)/(2\*n), c^2\*x^(2\*n)])/(-1 + n)

**Maple [F]** time = 0.102, size = 0, normalized size = 0.

$$\int \frac{a + b \text{Artanh}(cx^n)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))/x^2,x)

[Out] int((a+b\*arctanh(c\*x^n))/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( n \int \frac{1}{cx^2 x^n + x^2} dx + n \int \frac{1}{cx^2 x^n - x^2} dx + \frac{\log(cx^n + 1) - \log(-cx^n + 1)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^2,x, algorithm="maxima")



[Out]  $-1/2*(n*\int(1/(c*x^2*x^n + x^2), x) + n*\int(1/(c*x^2*x^n - x^2), x) + (\log(c*x^n + 1) - \log(-c*x^n + 1))/x)*b - a/x$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( \frac{b \operatorname{artanh}(cx^n) + a}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="fricas")`

[Out] `integral((b*arctanh(c*x^n) + a)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{atanh}(cx^n)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**n))/x**2,x)`

[Out] `Integral((a + b*atanh(c*x**n))/x**2, x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^n) + a)/x^2, x)`

$$3.229 \quad \int \frac{a+b \tanh^{-1}(cx^n)}{x^3} dx$$

**Optimal.** Leaf size=70

$$\frac{bcnx^{n-2} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{2}{n}\right), \frac{1}{2}\left(3 - \frac{2}{n}\right), c^2x^{2n}\right)}{2(2-n)} - \frac{a + b \tanh^{-1}(cx^n)}{2x^2}$$

[Out]  $-(a + b \text{ArcTanh}[c*x^n])/(2*x^2) - (b*c*n*x^{(-2 + n)} \text{Hypergeometric2F1}[1, (1 - 2/n)/2, (3 - 2/n)/2, c^2*x^{(2*n)}])/(2*(2 - n))$

**Rubi [A]** time = 0.0311192, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 364}

$$-\frac{a + b \tanh^{-1}(cx^n)}{2x^2} - \frac{bcnx^{n-2} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2}{n}\right); \frac{1}{2}\left(3 - \frac{2}{n}\right); c^2x^{2n}\right)}{2(2-n)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b \text{ArcTanh}[c*x^n])/x^3, x]$

[Out]  $-(a + b \text{ArcTanh}[c*x^n])/(2*x^2) - (b*c*n*x^{(-2 + n)} \text{Hypergeometric2F1}[1, (1 - 2/n)/2, (3 - 2/n)/2, c^2*x^{(2*n)}])/(2*(2 - n))$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

#### Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^n)}{x^3} dx = -\frac{a + b \tanh^{-1}(cx^n)}{2x^2} + \frac{1}{2}(bcn) \int \frac{x^{-3+n}}{1 - c^2x^{2n}} dx$$

$$= -\frac{a + b \tanh^{-1}(cx^n)}{2x^2} - \frac{bcnx^{-2+n} {}_2F_1\left(1, \frac{1}{2}\left(1 - \frac{2}{n}\right); \frac{1}{2}\left(3 - \frac{2}{n}\right); c^2x^{2n}\right)}{2(2-n)}$$

**Mathematica [A]** time = 0.0411021, size = 73, normalized size = 1.04

$$\frac{bcnx^{n-2} \text{Hypergeometric2F1}\left(1, \frac{n-2}{2n}, \frac{n-2}{2n} + 1, c^2x^{2n}\right)}{2(n-2)} - \frac{a}{2x^2} - \frac{b \tanh^{-1}(cx^n)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])/x^3, x]

[Out] -a/(2\*x^2) - (b\*ArcTanh[c\*x^n])/(2\*x^2) + (b\*c\*n\*x^(-2 + n)\*Hypergeometric2F1[1, (-2 + n)/(2\*n), 1 + (-2 + n)/(2\*n), c^2\*x^(2\*n)])/(2\*(-2 + n))

**Maple [F]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{a + b \text{Artanh}(cx^n)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))/x^3, x)

[Out] int((a+b\*arctanh(c\*x^n))/x^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4} \left( 2n \int \frac{1}{2(cx^3x^n + x^3)} dx + 2n \int \frac{1}{2(cx^3x^n - x^3)} dx + \frac{\log(cx^n + 1) - \log(-cx^n + 1)}{x^2} \right) b - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^3,x, algorithm="maxima")

[Out] -1/4\*(2\*n\*integrate(1/2/(c\*x^3\*x^n + x^3), x) + 2\*n\*integrate(1/2/(c\*x^3\*x^n - x^3), x) + (log(c\*x^n + 1) - log(-c\*x^n + 1))/x^2)\*b - 1/2\*a/x^2

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx^n) + a}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^3,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^n) + a)/x^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*n))/x\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^3,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^n) + a)/x^3, x)

$$3.230 \quad \int \frac{a+b \tanh^{-1}(cx^n)}{x^4} dx$$

**Optimal.** Leaf size=72

$$\frac{bcnx^{n-3} \text{Hypergeometric2F1}\left(1, -\frac{3-n}{2n}, -\frac{3(1-n)}{2n}, c^2x^{2n}\right)}{3(3-n)} - \frac{a+b \tanh^{-1}(cx^n)}{3x^3}$$

[Out]  $-(a + b \text{ArcTanh}[c*x^n])/(3*x^3) - (b*c*n*x^{(-3+n)}*\text{Hypergeometric2F1}[1, -(3-n)/(2*n), (-3*(1-n))/(2*n), c^2*x^{(2*n)}])/(3*(3-n))$

**Rubi [A]** time = 0.0342286, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {6097, 364}

$$-\frac{a+b \tanh^{-1}(cx^n)}{3x^3} - \frac{bcnx^{n-3} {}_2F_1\left(1, -\frac{3-n}{2n}; -\frac{3(1-n)}{2n}; c^2x^{2n}\right)}{3(3-n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^n])/x^4, x]

[Out]  $-(a + b \text{ArcTanh}[c*x^n])/(3*x^3) - (b*c*n*x^{(-3+n)}*\text{Hypergeometric2F1}[1, -(3-n)/(2*n), (-3*(1-n))/(2*n), c^2*x^{(2*n)}])/(3*(3-n))$

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol]
:> Simp[((d*x)^(m+1)*(a + b*ArcTanh[c*x^n]))/(d*(m+1)), x] - Dist[(b*c*n)/(d*(m+1)), Int[(x^(n-1)*(d*x)^(m+1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 364

```
Int[((c_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_)^(p_.)), x_Symbol] :> Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b*x^n)/a)])/((c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

#### Rubi steps

$$\int \frac{a + b \tanh^{-1}(cx^n)}{x^4} dx = -\frac{a + b \tanh^{-1}(cx^n)}{3x^3} + \frac{1}{3}(bcn) \int \frac{x^{-4+n}}{1 - c^2 x^{2n}} dx$$

$$= -\frac{a + b \tanh^{-1}(cx^n)}{3x^3} - \frac{bcnx^{-3+n} {}_2F_1\left(1, -\frac{3-n}{2n}; -\frac{3(1-n)}{2n}; c^2 x^{2n}\right)}{3(3-n)}$$

**Mathematica [A]** time = 0.0416435, size = 73, normalized size = 1.01

$$\frac{bcnx^{n-3} \text{Hypergeometric2F1}\left(1, \frac{n-3}{2n}, \frac{n-3}{2n} + 1, c^2 x^{2n}\right)}{3(n-3)} - \frac{a}{3x^3} - \frac{b \tanh^{-1}(cx^n)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])/x^4, x]

[Out] -a/(3\*x^3) - (b\*ArcTanh[c\*x^n])/(3\*x^3) + (b\*c\*n\*x^(-3 + n)\*Hypergeometric2F1[1, (-3 + n)/(2\*n), 1 + (-3 + n)/(2\*n), c^2\*x^(2\*n)])/(3\*(-3 + n))

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{a + b \text{Artanh}(cx^n)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))/x^4, x)

[Out] int((a+b\*arctanh(c\*x^n))/x^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6} \left( 3n \int \frac{1}{3(cx^4 x^n + x^4)} dx + 3n \int \frac{1}{3(cx^4 x^n - x^4)} dx + \frac{\log(cx^n + 1) - \log(-cx^n + 1)}{x^3} \right) b - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^4,x, algorithm="maxima")

[Out] -1/6\*(3\*n\*integrate(1/3/(c\*x^4\*x^n + x^4), x) + 3\*n\*integrate(1/3/(c\*x^4\*x^n - x^4), x) + (log(c\*x^n + 1) - log(-c\*x^n + 1))/x^3)\*b - 1/3\*a/x^3

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{artanh}(cx^n) + a}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^4,x, algorithm="fricas")

[Out] integral((b\*arctanh(c\*x^n) + a)/x^4, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*atanh(c\*x\*\*n))/x\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{artanh}(cx^n) + a}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))/x^4,x, algorithm="giac")

[Out] integrate((b\*arctanh(c\*x^n) + a)/x^4, x)

$$\mathbf{3.231} \quad \int x \left( a + b \tanh^{-1} (cx^n) \right)^2 dx$$

**Optimal.** Leaf size=16

$$\text{Unintegrable} \left( x \left( a + b \tanh^{-1} (cx^n) \right)^2, x \right)$$

[Out] Unintegrable[x\*(a + b\*ArcTanh[c\*x^n])^2, x]

**Rubi [A]** time = 0.0149523, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int x \left( a + b \tanh^{-1} (cx^n) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[x\*(a + b\*ArcTanh[c\*x^n])^2,x]

[Out] Defer[Int][x\*(a + b\*ArcTanh[c\*x^n])^2, x]

Rubi steps

$$\int x \left( a + b \tanh^{-1} (cx^n) \right)^2 dx = \int x \left( a + b \tanh^{-1} (cx^n) \right)^2 dx$$

**Mathematica [A]** time = 8.61828, size = 0, normalized size = 0.

$$\int x \left( a + b \tanh^{-1} (cx^n) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[x\*(a + b\*ArcTanh[c\*x^n])^2,x]

[Out] Integrate[x\*(a + b\*ArcTanh[c\*x^n])^2, x]



**Maple [A]** time = 0.162, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{Arctanh}(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arctanh(c*x^n))^2,x)`

[Out] `int(x*(a+b*arctanh(c*x^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} b^2 x^2 \log(-cx^n + 1)^2 + \frac{1}{2} a^2 x^2 - \int -\frac{(b^2 c x x^n - b^2 x) \log(cx^n + 1)^2 + 4(abc x x^n - abx) \log(cx^n + 1) + (4abx - (b^2 c n + \dots))}{4(cx^n - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")`

[Out] `1/8*b^2*x^2*log(-c*x^n + 1)^2 + 1/2*a^2*x^2 - integrate(-1/4*((b^2*c*x*x^n - b^2*x)*log(c*x^n + 1)^2 + 4*(a*b*c*x*x^n - a*b*x)*log(c*x^n + 1) + (4*a*b*x - (b^2*c*n + 4*a*b*c))*x*x^n - 2*(b^2*c*x*x^n - b^2*x)*log(c*x^n + 1))*log(-c*x^n + 1)/(c*x^n - 1), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^2 x \operatorname{artanh}(cx^n)^2 + 2 abx \operatorname{artanh}(cx^n) + a^2 x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x*arctanh(c*x^n)^2 + 2*a*b*x*arctanh(c*x^n) + a^2*x, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*x, x)
```

$$3.232 \quad \int (a + b \tanh^{-1}(cx^n))^2 dx$$

**Optimal.** Leaf size=14

$$\text{Unintegrable}\left(\left(a + b \tanh^{-1}(cx^n)\right)^2, x\right)$$

[Out] Unintegrable[(a + b\*ArcTanh[c\*x^n])^2, x]

**Rubi [A]** time = 0.0063323, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (a + b \tanh^{-1}(cx^n))^2 dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^n])^2, x]

Rubi steps

$$\int (a + b \tanh^{-1}(cx^n))^2 dx = \int (a + b \tanh^{-1}(cx^n))^2 dx$$

**Mathematica [A]** time = 1.89704, size = 0, normalized size = 0.

$$\int (a + b \tanh^{-1}(cx^n))^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^n])^2, x]

**Maple [A]** time = 0.187, size = 0, normalized size = 0.

$$\int (a + b \operatorname{Artanh}(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arctanh(c*x^n))^2,x)`

[Out] `int((a+b*arctanh(c*x^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} b^2 x \log(-cx^n + 1)^2 + a^2 x - \int -\frac{(b^2 cx^n - b^2) \log(cx^n + 1)^2 + 4(abcx^n - ab) \log(cx^n + 1) + 2(2ab - (b^2 cn + 2abc)x^n)}{4(cx^n - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))^2,x, algorithm="maxima")`

[Out] `1/4*b^2*x*log(-c*x^n + 1)^2 + a^2*x - integrate(-1/4*((b^2*c*x^n - b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (b^2*c*n + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^n - 1), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^2 \operatorname{artanh}(cx^n)^2 + 2ab \operatorname{artanh}(cx^n) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))^2,x, algorithm="fricas")`

[Out] `integral(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2, x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{atanh}(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2, x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2, x)
```

$$3.233 \quad \int \frac{(a + b \tanh^{-1}(cx^n))^2}{x} dx$$

**Optimal.** Leaf size=148

$$-\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx^n}\right)(a + b \tanh^{-1}(cx^n))}{n} + \frac{b \operatorname{PolyLog}\left(2, \frac{2}{1-cx^n} - 1\right)(a + b \tanh^{-1}(cx^n))}{n} + \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-cx^n}\right)}{2n}$$

[Out] (2\*(a + b\*ArcTanh[c\*x^n])^2\*ArcTanh[1 - 2/(1 - c\*x^n)]/n - (b\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, 1 - 2/(1 - c\*x^n)]/n + (b\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, -1 + 2/(1 - c\*x^n)]/n + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^n)]/(2\*n) - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^n)]/(2\*n)

**Rubi [A]** time = 0.313463, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {6095, 5914, 6052, 5948, 6058, 6610}

$$-\frac{b \operatorname{PolyLog}\left(2, 1 - \frac{2}{1-cx^n}\right)(a + b \tanh^{-1}(cx^n))}{n} + \frac{b \operatorname{PolyLog}\left(2, \frac{2}{1-cx^n} - 1\right)(a + b \tanh^{-1}(cx^n))}{n} + \frac{b^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1-cx^n}\right)}{2n}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcTanh[c\*x^n])^2/x, x]

[Out] (2\*(a + b\*ArcTanh[c\*x^n])^2\*ArcTanh[1 - 2/(1 - c\*x^n)]/n - (b\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, 1 - 2/(1 - c\*x^n)]/n + (b\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, -1 + 2/(1 - c\*x^n)]/n + (b^2\*PolyLog[3, 1 - 2/(1 - c\*x^n)]/(2\*n) - (b^2\*PolyLog[3, -1 + 2/(1 - c\*x^n)]/(2\*n)

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5914

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Simp[2\*(a + b\*ArcTanh[c\*x])^p\*ArcTanh[1 - 2/(1 - c\*x)], x] - Dist[2\*b\*c\*p, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*ArcTanh[1 - 2/(1 - c\*x)]/(1 - c^2\*x^2), x], x] /; F

reeQ[{a, b, c}, x] && IGtQ[p, 1]

### Rule 6052

Int[(ArcTanh[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Dist[1/2, Int[(Log[1 + u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] - Dist[1/2, Int[(Log[1 - u]\*(a + b\*ArcTanh[c\*x])^p)/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 5948

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.)/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := Simp[(a + b\*ArcTanh[c\*x])^(p + 1)/(b\*c\*d\*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2\*d + e, 0] && NeQ[p, -1]

### Rule 6058

Int[(Log[u\_]\*((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.))^(p\_.))/((d\_) + (e\_.)\*(x\_)^2), x\_Symbol] := -Simp[((a + b\*ArcTanh[c\*x])^p\*PolyLog[2, 1 - u])/(2\*c\*d), x] + Dist[(b\*p)/2, Int[((a + b\*ArcTanh[c\*x])^(p - 1)\*PolyLog[2, 1 - u])/(d + e\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2\*d + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c\*x))^2, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x} dx &= \frac{\text{Subst}\left(\int \frac{(a+b \tanh^{-1}(cx))^2}{x} dx, x, x^n\right)}{n} \\
&= \frac{2(a + b \tanh^{-1}(cx^n))^2 \tanh^{-1}\left(1 - \frac{2}{1-cx^n}\right)}{n} - \frac{(4bc) \text{Subst}\left(\int \frac{(a+b \tanh^{-1}(cx)) \tanh^{-1}\left(1 - \frac{2}{1-cx}\right)}{1-c^2x^2} dx, x, x^n\right)}{n} \\
&= \frac{2(a + b \tanh^{-1}(cx^n))^2 \tanh^{-1}\left(1 - \frac{2}{1-cx^n}\right)}{n} + \frac{(2bc) \text{Subst}\left(\int \frac{(a+b \tanh^{-1}(cx)) \log\left(\frac{2}{1-cx}\right)}{1-c^2x^2} dx, x, x^n\right)}{n} \\
&= \frac{2(a + b \tanh^{-1}(cx^n))^2 \tanh^{-1}\left(1 - \frac{2}{1-cx^n}\right)}{n} - \frac{b(a + b \tanh^{-1}(cx^n)) \text{Li}_2\left(1 - \frac{2}{1-cx^n}\right)}{n} + \frac{b(a + b \tanh^{-1}(cx^n)) \text{Li}_2\left(\frac{2}{1-cx^n}\right)}{n} \\
&= \frac{2(a + b \tanh^{-1}(cx^n))^2 \tanh^{-1}\left(1 - \frac{2}{1-cx^n}\right)}{n} - \frac{b(a + b \tanh^{-1}(cx^n)) \text{Li}_2\left(1 - \frac{2}{1-cx^n}\right)}{n} + \frac{b(a + b \tanh^{-1}(cx^n)) \text{Li}_2\left(\frac{2}{1-cx^n}\right)}{n}
\end{aligned}$$

**Mathematica [A]** time = 0.128001, size = 146, normalized size = 0.99

$$\frac{\frac{1}{2}b \left( 2\text{PolyLog}\left(2, \frac{cx^n+1}{1-cx^n}\right) (a + b \tanh^{-1}(cx^n)) - 2\text{PolyLog}\left(2, \frac{cx^n+1}{cx^n-1}\right) (a + b \tanh^{-1}(cx^n)) + b \left( \text{PolyLog}\left(3, \frac{cx^n+1}{cx^n-1}\right) - \text{PolyLog}\left(3, \frac{cx^n+1}{1-cx^n}\right) \right) \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])^2/x,x]

[Out] (2\*(a + b\*ArcTanh[c\*x^n])^2\*ArcTanh[1 + 2/(-1 + c\*x^n)] + (b\*(2\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, (1 + c\*x^n)/(1 - c\*x^n)] - 2\*(a + b\*ArcTanh[c\*x^n])\*PolyLog[2, (1 + c\*x^n)/(-1 + c\*x^n)] + b\*(-PolyLog[3, (1 + c\*x^n)/(1 - c\*x^n)] + PolyLog[3, (1 + c\*x^n)/(-1 + c\*x^n)])))/2)/n

**Maple [C]** time = 0.33, size = 880, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))^2/x,x)



```
[Out] 1/n*a^2*ln(c*x^n)+1/n*b^2*ln(c*x^n)*arctanh(c*x^n)^2-1/n*b^2*arctanh(c*x^n)
*polylog(2,-(c*x^n+1)^2/(-c^2*(x^n)^2+1))+1/2/n*b^2*polylog(3,-(c*x^n+1)^2/
(-c^2*(x^n)^2+1))-1/n*b^2*arctanh(c*x^n)^2*ln((c*x^n+1)^2/(-c^2*(x^n)^2+1)-
1)+1/n*b^2*arctanh(c*x^n)^2*ln(1-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*
arctanh(c*x^n)*polylog(2,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2/n*b^2*polylog(
3,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/n*b^2*arctanh(c*x^n)^2*ln(1+(c*x^n+1)
/(-c^2*(x^n)^2+1)^(1/2))+2/n*b^2*arctanh(c*x^n)*polylog(2,-(c*x^n+1)/(-c^2*
(x^n)^2+1)^(1/2))-2/n*b^2*polylog(3,-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+1/2*
I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)
^2+1)+1))^3*arctanh(c*x^n)^2+1/2*I/n*b^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)
^2+1)-1))*csgn(I/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*csgn(I*((c*x^n+1)^2/(-c^
2*(x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*arctanh(c*x^n)^2-1/2*I/n*
b^2*Pi*csgn(I/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))*csgn(I*((c*x^n+1)^2/(-c^2*(
x^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))^2*arctanh(c*x^n)^2-1/2*I/n*b
^2*Pi*csgn(I*((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1))*csgn(I*((c*x^n+1)^2/(-c^2*(x
^n)^2+1)-1)/((c*x^n+1)^2/(-c^2*(x^n)^2+1)+1))^2*arctanh(c*x^n)^2-1/n*a*b*ln
(c*x^n)*ln(c*x^n+1)+2/n*a*b*ln(c*x^n)*arctanh(c*x^n)-1/n*a*b*dilog(c*x^n)-1
/n*a*b*dilog(c*x^n+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} b^2 \log(-cx^n + 1)^2 \log(x) + a^2 \log(x) - \int -\frac{(b^2 cx^n - b^2) \log(cx^n + 1)^2 + 4(abcx^n - ab) \log(cx^n + 1) + 2(2ab - (b^2 c x^n - b^2) \log(cx^n + 1))}{4(cxx^n - x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(-c*x^n + 1)^2*log(x) + a^2*log(x) - integrate(-1/4*((b^2*c*x^n
- b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (
b^2*c*n*log(x) + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*x*x^n - x), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2 \operatorname{artanh}(cx^n)^2 + 2ab \operatorname{artanh}(cx^n) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^n))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atanh(c*x**n))**2/x,x)
```

```
[Out] Integral((a + b*atanh(c*x**n))**2/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2/x, x)
```

$$3.234 \quad \int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable} \left( \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2}, x \right)$$

[Out] Unintegrable[(a + b\*ArcTanh[c\*x^n])^2/x^2, x]

**Rubi [A]** time = 0.02298, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^n])^2/x^2, x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^n])^2/x^2, x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2} dx = \int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2} dx$$

**Mathematica [A]** time = 12.3515, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])^2/x^2, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^n])^2/x^2, x]

**Maple [A]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Arctanh}(cx^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))^2/x^2,x)

[Out] int((a+b\*arctanh(c\*x^n))^2/x^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{b^2 \log(-cx^n + 1)^2}{4x} - \frac{a^2}{x} - \int -\frac{(b^2 cx^n - b^2) \log(cx^n + 1)^2 + 4(abcx^n - ab) \log(cx^n + 1) + 2(2ab + (b^2 cn - 2abc)x^n - (b^2 cx^n - b^2) \log(-cx^n + 1)) \log(cx^n + 1)}{4(cx^2 x^n - x^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))^2/x^2,x, algorithm="maxima")

[Out] -1/4\*b^2\*log(-c\*x^n + 1)^2/x - a^2/x - integrate(-1/4\*((b^2\*c\*x^n - b^2)\*log(c\*x^n + 1)^2 + 4\*(a\*b\*c\*x^n - a\*b)\*log(c\*x^n + 1) + 2\*(2\*a\*b + (b^2\*c\*n - 2\*a\*b\*c)\*x^n - (b^2\*c\*x^n - b^2)\*log(c\*x^n + 1))\*log(-c\*x^n + 1))/(c\*x^2\*x^n - x^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(cx^n)^2 + 2ab \operatorname{artanh}(cx^n) + a^2}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))^2/x^2,x, algorithm="fricas")

[Out] `integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^2, x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{atanh}(cx^n))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**n))**2/x**2,x)`

[Out] `Integral((a + b*atanh(c*x**n))**2/x**2, x)`

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^n) + a)^2/x^2, x)`

$$3.235 \quad \int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^3} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(a + b \tanh^{-1}(cx^n))^2}{x^3}, x\right)$$

[Out] Unintegrable[(a + b\*ArcTanh[c\*x^n])^2/x^3, x]

**Rubi [A]** time = 0.0233473, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcTanh[c\*x^n])^2/x^3, x]

[Out] Defer[Int] [(a + b\*ArcTanh[c\*x^n])^2/x^3, x]

Rubi steps

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^3} dx = \int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^3} dx$$

**Mathematica [A]** time = 12.3165, size = 0, normalized size = 0.

$$\int \frac{(a + b \tanh^{-1}(cx^n))^2}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcTanh[c\*x^n])^2/x^3, x]

[Out] Integrate[(a + b\*ArcTanh[c\*x^n])^2/x^3, x]

**Maple [A]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{Artanh}(cx^n))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arctanh(c\*x^n))^2/x^3,x)

[Out] int((a+b\*arctanh(c\*x^n))^2/x^3,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{b^2 \log(-cx^n + 1)^2}{8x^2} - \frac{a^2}{2x^2} - \int -\frac{(b^2 cx^n - b^2) \log(cx^n + 1)^2 + 4(abcx^n - ab) \log(cx^n + 1) + (4ab + (b^2 cn - 4abc)x^n - 4(cx^3 x^n - x^3))}{4(cx^3 x^n - x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))^2/x^3,x, algorithm="maxima")

[Out] -1/8\*b^2\*log(-c\*x^n + 1)^2/x^2 - 1/2\*a^2/x^2 - integrate(-1/4\*((b^2\*c\*x^n - b^2)\*log(c\*x^n + 1)^2 + 4\*(a\*b\*c\*x^n - a\*b)\*log(c\*x^n + 1) + (4\*a\*b + (b^2\*c\*n - 4\*a\*b\*c)\*x^n - 2\*(b^2\*c\*x^n - b^2)\*log(c\*x^n + 1))\*log(-c\*x^n + 1))/(c\*x^3\*x^n - x^3), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{artanh}(cx^n)^2 + 2ab \operatorname{artanh}(cx^n) + a^2}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arctanh(c\*x^n))^2/x^3,x, algorithm="fricas")

[Out] `integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^3, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*atanh(c*x**n))**2/x**3,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="giac")`

[Out] `integrate((b*arctanh(c*x^n) + a)^2/x^3, x)`



$$3.236 \quad \int \frac{\tanh^{-1}(ax^n)}{x} dx$$

**Optimal.** Leaf size=30

$$\frac{\text{PolyLog}(2, ax^n)}{2n} - \frac{\text{PolyLog}(2, -ax^n)}{2n}$$

[Out] -PolyLog[2, -(a\*x^n)]/(2\*n) + PolyLog[2, a\*x^n]/(2\*n)

**Rubi [A]** time = 0.0213019, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6095, 5912}

$$\frac{\text{PolyLog}(2, ax^n)}{2n} - \frac{\text{PolyLog}(2, -ax^n)}{2n}$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a\*x^n]/x, x]

[Out] -PolyLog[2, -(a\*x^n)]/(2\*n) + PolyLog[2, a\*x^n]/(2\*n)

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x]) /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(ax^n)}{x} dx &= \frac{\text{Subst}\left(\int \frac{\tanh^{-1}(ax)}{x} dx, x, x^n\right)}{n} \\ &= -\frac{\text{Li}_2(-ax^n)}{2n} + \frac{\text{Li}_2(ax^n)}{2n} \end{aligned}$$

**Mathematica [C]** time = 0.0386013, size = 33, normalized size = 1.1

$$\frac{ax^n \text{HypergeometricPFQ}\left(\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, a^2 x^{2n}\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a\*x^n]/x, x]

[Out] (a\*x^n\*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2\*x^(2\*n)]) / n

**Maple [B]** time = 0.034, size = 61, normalized size = 2.

$$\frac{\ln(ax^n) \text{Artanh}(ax^n)}{n} - \frac{\text{dilog}(ax^n)}{2n} - \frac{\text{dilog}(ax^n + 1)}{2n} - \frac{\ln(ax^n) \ln(ax^n + 1)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a\*x^n)/x, x)

[Out] 1/n\*ln(a\*x^n)\*arctanh(a\*x^n)-1/2/n\*dilog(a\*x^n)-1/2/n\*dilog(a\*x^n+1)-1/2/n\*ln(a\*x^n)\*ln(a\*x^n+1)

**Maxima [B]** time = 1.26638, size = 198, normalized size = 6.6

$$-\frac{1}{2} an \left( \frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an} \right) \log(x) + \frac{1}{2} an \left( \frac{\log(ax^n+1)\log(x) - \log(ax^n-1)\log(x)}{an} - \frac{n \log(ax^n+1)\log(x) + 1}{an^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^n)/x, x, algorithm="maxima")

[Out] -1/2\*a\*n\*(log((a\*x^n + 1)/a)/(a\*n) - log((a\*x^n - 1)/a)/(a\*n))\*log(x) + 1/2\*a\*n\*((log(a\*x^n + 1)\*log(x) - log(a\*x^n - 1)\*log(x))/(a\*n) - (n\*log(a\*x^n + 1)\*log(x) + dilog(-a\*x^n))/(a\*n^2) + (n\*log(-a\*x^n + 1)\*log(x) + dilog(a\*x^n))/(a\*n^2)) + arctanh(a\*x^n)\*log(x)

---

**Fricas [B]** time = 1.73074, size = 423, normalized size = 14.1

$$n \log(a \cosh(n \log(x)) + a \sinh(n \log(x)) + 1) \log(x) - n \log(-a \cosh(n \log(x)) - a \sinh(n \log(x)) + 1) \log(x) - n$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^n)/x,x, algorithm="fricas")

[Out] 
$$\frac{-1/2*(n*\log(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) + 1)*\log(x) - n*\log(-a*\cosh(n*\log(x)) - a*\sinh(n*\log(x)) + 1)*\log(x) - n*\log(x)*\log(-(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) + 1)/(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x)) - 1)) - \operatorname{dilog}(a*\cosh(n*\log(x)) + a*\sinh(n*\log(x))) + \operatorname{dilog}(-a*\cosh(n*\log(x)) - a*\sinh(n*\log(x))))}{n}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{atanh}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a\*x\*\*n)/x,x)

[Out] Integral(atanh(a\*x\*\*n)/x, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{artanh}(ax^n)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^n)/x,x, algorithm="giac")

[Out] integrate(arctanh(a\*x^n)/x, x)

$$3.237 \quad \int \frac{\tanh^{-1}(ax^5)}{x} dx$$

**Optimal.** Leaf size=24

$$\frac{1}{10} \text{PolyLog}(2, ax^5) - \frac{1}{10} \text{PolyLog}(2, -ax^5)$$

[Out] -PolyLog[2, -(a\*x^5)]/10 + PolyLog[2, a\*x^5]/10

**Rubi [A]** time = 0.0212865, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {6095, 5912}

$$\frac{1}{10} \text{PolyLog}(2, ax^5) - \frac{1}{10} \text{PolyLog}(2, -ax^5)$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[a\*x^5]/x, x]

[Out] -PolyLog[2, -(a\*x^5)]/10 + PolyLog[2, a\*x^5]/10

#### Rule 6095

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)^(n\_)])\*(b\_.)^(p\_.)/(x\_), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*ArcTanh[c\*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]

#### Rule 5912

Int[((a\_.) + ArcTanh[(c\_.)\*(x\_)])\*(b\_.)/(x\_), x\_Symbol] := Simp[a\*Log[x], x] + (-Simp[(b\*PolyLog[2, -(c\*x)])]/2, x] + Simp[(b\*PolyLog[2, c\*x])/2, x] /; FreeQ[{a, b, c}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\tanh^{-1}(ax^5)}{x} dx &= \frac{1}{5} \text{Subst} \left( \int \frac{\tanh^{-1}(ax)}{x} dx, x, x^5 \right) \\ &= -\frac{1}{10} \text{Li}_2(-ax^5) + \frac{\text{Li}_2(ax^5)}{10} \end{aligned}$$

**Mathematica [A]** time = 0.0112493, size = 22, normalized size = 0.92

$$\frac{1}{10} \left( \text{PolyLog} \left( 2, ax^5 \right) - \text{PolyLog} \left( 2, -ax^5 \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[a\*x^5]/x,x]

[Out] (-PolyLog[2, -(a\*x^5)] + PolyLog[2, a\*x^5])/10

**Maple [C]** time = 0.095, size = 85, normalized size = 3.5

$$\ln(x) \operatorname{Artanh}(ax^5) + \frac{1}{2} \sum_{_R1=\text{RootOf}(a_Z^5-1)} \ln(x) \ln\left(\frac{-_R1-x}{_R1}\right) + \operatorname{dilog}\left(\frac{-_R1-x}{_R1}\right) - \frac{1}{2} \sum_{_R1=\text{RootOf}(a_Z^5+1)} \ln(x) \ln\left(\frac{-_R1-x}{_R1}\right) + \operatorname{dilog}\left(\frac{-_R1-x}{_R1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(a\*x^5)/x,x)

[Out] ln(x)\*arctanh(a\*x^5)+1/2\*sum(ln(x)\*ln((\_R1-x)/\_R1)+dilog((\_R1-x)/\_R1),\_R1=RootOf(\_Z^5\*a-1))-1/2\*sum(ln(x)\*ln((\_R1-x)/\_R1)+dilog((\_R1-x)/\_R1),\_R1=RootOf(\_Z^5\*a+1))

**Maxima [B]** time = 0.989419, size = 140, normalized size = 5.83

$$-\frac{1}{2} a \left( \frac{\log(ax^5 + 1)}{a} - \frac{\log(ax^5 - 1)}{a} \right) \log(x) - \frac{1}{10} a \left( \frac{\log(ax^5 - 1) \log(ax^5) + \operatorname{Li}_2(-ax^5 + 1)}{a} - \frac{\log(ax^5 + 1) \log(-ax^5)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^5)/x,x, algorithm="maxima")

[Out] -1/2\*a\*(log(a\*x^5 + 1)/a - log(a\*x^5 - 1)/a)\*log(x) - 1/10\*a\*((log(a\*x^5 - 1)\*log(a\*x^5) + dilog(-a\*x^5 + 1))/a - (log(a\*x^5 + 1)\*log(-a\*x^5) + dilog(a\*x^5 + 1))/a) + arctanh(a\*x^5)\*log(x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{artanh}(ax^5)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^5)/x,x, algorithm="fricas")

[Out] integral(arctanh(a\*x^5)/x, x)

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(a\*x\*\*5)/x,x)

[Out] Exception raised: KeyError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{artanh}(ax^5)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(a\*x^5)/x,x, algorithm="giac")

[Out] integrate(arctanh(a\*x^5)/x, x)

$$3.238 \quad \int \tanh^{-1} \left( \frac{1}{x} \right) dx$$

Optimal. Leaf size=19

$$\frac{1}{2} \log(1 - x^2) + x \tanh^{-1} \left( \frac{1}{x} \right)$$

[Out] x\*ArcTanh[x^(-1)] + Log[1 - x^2]/2

**Rubi [A]** time = 0.006243, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {6091, 263, 260}

$$\frac{1}{2} \log(1 - x^2) + x \tanh^{-1} \left( \frac{1}{x} \right)$$

Antiderivative was successfully verified.

[In] Int[ArcTanh[x^(-1)], x]

[Out] x\*ArcTanh[x^(-1)] + Log[1 - x^2]/2

#### Rule 6091

Int[ArcTanh[(c\_.)\*(x\_)^(n\_)], x\_Symbol] := Simp[x\*ArcTanh[c\*x^n], x] - Dist[c\*n, Int[x^n/(1 - c^2\*x^(2\*n)), x], x] /; FreeQ[{c, n}, x]

#### Rule 263

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[x^(m + n\*p)\*(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]

#### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

#### Rubi steps

$$\begin{aligned}
 \int \tanh^{-1}\left(\frac{1}{x}\right) dx &= x \tanh^{-1}\left(\frac{1}{x}\right) + \int \frac{1}{\left(1 - \frac{1}{x^2}\right)x} dx \\
 &= x \tanh^{-1}\left(\frac{1}{x}\right) + \int \frac{x}{-1 + x^2} dx \\
 &= x \tanh^{-1}\left(\frac{1}{x}\right) + \frac{1}{2} \log(1 - x^2)
 \end{aligned}$$

**Mathematica [A]** time = 0.0020759, size = 17, normalized size = 0.89

$$\frac{1}{2} \log(x^2 - 1) + x \tanh^{-1}\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[ArcTanh[x^(-1)], x]

[Out] x\*ArcTanh[x^(-1)] + Log[-1 + x^2]/2

**Maple [A]** time = 0.043, size = 30, normalized size = 1.6

$$x \operatorname{Artanh}(x^{-1}) + \frac{\ln(x^{-1} - 1)}{2} - \ln(x^{-1}) + \frac{\ln(x^{-1} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arctanh(1/x), x)

[Out] x\*arctanh(1/x)+1/2\*ln(1/x-1)-ln(1/x)+1/2\*ln(1/x+1)

**Maxima [A]** time = 0.954859, size = 20, normalized size = 1.05

$$x \operatorname{artanh}\left(\frac{1}{x}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(arctanh(1/x),x, algorithm="maxima")

[Out] x\*arctanh(1/x) + 1/2\*log(x^2 - 1)

---

**Fricas [A]** time = 1.60687, size = 63, normalized size = 3.32

$$\frac{1}{2}x \log\left(\frac{x+1}{x-1}\right) + \frac{1}{2} \log(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(1/x),x, algorithm="fricas")

[Out] 1/2\*x\*log((x + 1)/(x - 1)) + 1/2\*log(x^2 - 1)

---

**Sympy [A]** time = 0.258037, size = 15, normalized size = 0.79

$$x \operatorname{atanh}\left(\frac{1}{x}\right) + \log(x + 1) - \operatorname{atanh}\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(atanh(1/x),x)

[Out] x\*atanh(1/x) + log(x + 1) - atanh(1/x)

---

**Giac [A]** time = 1.17986, size = 38, normalized size = 2.

$$\frac{1}{2}x \log\left(-\frac{\frac{1}{x} + 1}{\frac{1}{x} - 1}\right) + \frac{1}{2} \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arctanh(1/x),x, algorithm="giac")

[Out] 1/2\*x\*log(-(1/x + 1)/(1/x - 1)) + 1/2\*log(abs(x^2 - 1))

$$3.239 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable} \left( (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3, x \right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^3, x]

**Rubi [A]** time = 0.0235968, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^3,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3 dx$$

**Mathematica [A]** time = 7.97124, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^3,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^3, x]

**Maple [A]** time = 0.232, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^n))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^n))^3,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^n))^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^n))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx)^m b^3 \operatorname{artanh}(cx^n)^3 + 3 (dx)^m ab^2 \operatorname{artanh}(cx^n)^2 + 3 (dx)^m a^2 b \operatorname{artanh}(cx^n) + (dx)^m a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^n))^3,x, algorithm="fricas")

[Out] integral((d\*x)^m\*b^3\*arctanh(c\*x^n)^3 + 3\*(d\*x)^m\*a\*b^2\*arctanh(c\*x^n)^2 + 3\*(d\*x)^m\*a^2\*b\*arctanh(c\*x^n) + (d\*x)^m\*a^3, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**3,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^3*(d*x)^m, x)
```

$$3.240 \quad \int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2 dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left((dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2, x\right)$$

[Out] Unintegrable[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^2, x]

**Rubi [A]** time = 0.0241501, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2 dx = \int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2 dx$$

**Mathematica [A]** time = 5.79071, size = 0, normalized size = 0.

$$\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n])^2, x]

**Maple [A]** time = 0.214, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Arctanh}(cx^n))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^n))^2,x)

[Out] int((d\*x)^m\*(a+b\*arctanh(c\*x^n))^2,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^n))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((dx)^m b^2 \operatorname{artanh}(cx^n)^2 + 2 (dx)^m ab \operatorname{artanh}(cx^n) + (dx)^m a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arctanh(c\*x^n))^2,x, algorithm="fricas")

[Out] integral((d\*x)^m\*b^2\*arctanh(c\*x^n)^2 + 2\*(d\*x)^m\*a\*b\*arctanh(c\*x^n) + (d\*x)^m\*a^2, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atanh(c*x**n))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)^2*(d*x)^m, x)
```

### 3.241 $\int (dx)^m \left( a + b \tanh^{-1}(cx^n) \right) dx$

**Optimal.** Leaf size=84

$$\frac{x(dx)^m \left( a + b \tanh^{-1}(cx^n) \right)}{m+1} - \frac{bcnx^{n+1}(dx)^m \operatorname{Hypergeometric2F1} \left( 1, \frac{m+n+1}{2n}, \frac{m+3n+1}{2n}, c^2x^{2n} \right)}{(m+1)(m+n+1)}$$

[Out] (x\*(d\*x)^m\*(a + b\*ArcTanh[c\*x^n]))/(1 + m) - (b\*c\*n\*x^(1 + n)\*(d\*x)^m\*Hypergeometric2F1[1, (1 + m + n)/(2\*n), (1 + m + 3\*n)/(2\*n), c^2\*x^(2\*n)])/((1 + m)\*(1 + m + n))

**Rubi [A]** time = 0.0441134, antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {6097, 20, 364}

$$\frac{(dx)^{m+1} \left( a + b \tanh^{-1}(cx^n) \right)}{d(m+1)} - \frac{bcnx^{n+1}(dx)^m {}_2F_1 \left( 1, \frac{m+n+1}{2n}; \frac{m+3n+1}{2n}; c^2x^{2n} \right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n]),x]

[Out] ((d\*x)^(1 + m)\*(a + b\*ArcTanh[c\*x^n]))/(d\*(1 + m)) - (b\*c\*n\*x^(1 + n)\*(d\*x)^m\*Hypergeometric2F1[1, (1 + m + n)/(2\*n), (1 + m + 3\*n)/(2\*n), c^2\*x^(2\*n)])/((1 + m)\*(1 + m + n))

#### Rule 6097

```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTanh[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 - c^2*x^(2*n)), x], x] /;
FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

#### Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol]
:> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]
```



Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
)]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \tanh^{-1}(cx^n)) dx &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^n))}{d(1+m)} - \frac{(bcn) \int \frac{x^{-1+n}(dx)^{1+m}}{1-c^2x^{2n}} dx}{d(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^n))}{d(1+m)} - \frac{(bcnx^{-m}(dx)^m) \int \frac{x^{m+n}}{1-c^2x^{2n}} dx}{1+m} \\ &= \frac{(dx)^{1+m} (a + b \tanh^{-1}(cx^n))}{d(1+m)} - \frac{bcnx^{1+n}(dx)^m {}_2F_1\left(1, \frac{1+m+n}{2n}; \frac{1+m+3n}{2n}; c^2x^{2n}\right)}{(1+m)(1+m+n)} \end{aligned}$$

**Mathematica [A]** time = 0.0870418, size = 77, normalized size = 0.92

$$\frac{x(dx)^m \left( (m+n+1)(a + b \tanh^{-1}(cx^n)) - bcnx^n \operatorname{Hypergeometric2F1}\left(1, \frac{m+n+1}{2n}, \frac{m+3n+1}{2n}, c^2x^{2n}\right) \right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcTanh[c\*x^n]), x]

[Out] (x\*(d\*x)^m\*((1 + m + n)\*(a + b\*ArcTanh[c\*x^n]) - b\*c\*n\*x^n\*Hypergeometric2F1[1, (1 + m + n)/(2\*n), (1 + m + 3\*n)/(2\*n), c^2\*x^(2\*n)]))/(1 + m)\*(1 + m + n))

**Maple [F]** time = 0.279, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{Artanh}(cx^n)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arctanh(c\*x^n)), x)

[Out] `int((d*x)^m*(a+b*arctanh(c*x^n)),x)`

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((dx)^m b \operatorname{artanh}(cx^n) + (dx)^m a, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="fricas")`

[Out] `integral((d*x)^m*b*arctanh(c*x^n) + (d*x)^m*a, x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a+b*atanh(c*x**n)),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{artanh}(cx^n) + a) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*arctanh(c*x^n) + a)*(d*x)^m, x)
```

$$3.242 \quad \int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b \tanh^{-1}(cx^n)}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^n]), x]

**Rubi [A]** time = 0.0269594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^n]), x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^n]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx = \int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx$$

**Mathematica [A]** time = 0.372964, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b \tanh^{-1}(cx^n)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^n]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^n]), x]

---

**Maple [A]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{Arctanh}(cx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^n)),x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^n)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^n)),x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arctanh(c\*x^n) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{artanh}(cx^n) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^n)),x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arctanh(c\*x^n) + a), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atanh(c*x**n)),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{artanh}(cx^n) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

$$3.243 \quad \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2} dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}\left(\frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2}, x\right)$$

[Out] Unintegrable[(d\*x)^m/(a + b\*ArcTanh[c\*x^n])^2, x]

**Rubi [A]** time = 0.0264206, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcTanh[c\*x^n])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2} dx$$

**Mathematica [A]** time = 1.7533, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \tanh^{-1}(cx^n)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^n])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcTanh[c\*x^n])^2, x]

**Maple [A]** time = 0.275, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{Arctanh}(cx^n))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arctanh(c\*x^n))^2,x)

[Out] int((d\*x)^m/(a+b\*arctanh(c\*x^n))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2(c^2 d^m x e^{(m \log(x) + 2n \log(x))} - d^m x x^m)}{b^2 c n x^n \log(cx^n + 1) - b^2 c n x^n \log(-cx^n + 1) + 2 a b c n x^n} + \int -\frac{2(c^2 d^m (m + n + 1) e^{(m \log(x) + 2n \log(x))} - d^m (m - n + 1) x^m)}{b^2 c n x^n \log(cx^n + 1) - b^2 c n x^n \log(-cx^n + 1) + 2 a b c n x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^n))^2,x, algorithm="maxima")

[Out] 2\*(c^2\*d^m\*x\*e^(m\*log(x) + 2\*n\*log(x)) - d^m\*x\*x^m)/(b^2\*c\*n\*x^n\*log(c\*x^n + 1) - b^2\*c\*n\*x^n\*log(-c\*x^n + 1) + 2\*a\*b\*c\*n\*x^n) + integrate(-2\*(c^2\*d^m\*(m + n + 1)\*e^(m\*log(x) + 2\*n\*log(x)) - d^m\*(m - n + 1)\*x^m)/(b^2\*c\*n\*x^n\*log(c\*x^n + 1) - b^2\*c\*n\*x^n\*log(-c\*x^n + 1) + 2\*a\*b\*c\*n\*x^n), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{artanh}(cx^n)^2 + 2ab \operatorname{artanh}(cx^n) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arctanh(c\*x^n))^2,x, algorithm="fricas")



[Out] `integral((d*x)^m/(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m/(a+b*atanh(c*x**n))**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{artanh}(cx^n) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="giac")`

[Out] `integrate((d*x)^m/(b*arctanh(c*x^n) + a)^2, x)`



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```

```
56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+^') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```



```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157 ElementaryFunctionQ := proc(func)
158     member(func,[
159         exp,log,ln,
160         sin,cos,tan,cot,sec,csc,
161         arcsin,arccos,arctan,arccot,arcsec,arccsc,
162         sinh,cosh,tanh,coth,sech,csch,
163         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
164 end proc:
165
166 SpecialFunctionQ := proc(func)
167     member(func,[
168         erf,erfc,erfi,
169         FresnelS,FresnelC,
170         Ei,Ei,Li,Si,Ci,Shi,Chi,
171         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
172         EllipticF,EllipticE,EllipticPi])
173 end proc:
174
175 HypergeometricFunctionQ := proc(func)
176     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
177 end proc:
178
179 AppellFunctionQ := proc(func)
180     member(func,[AppellF1])
181 end proc:
182
183 # u is a sum or product. rest(u) returns all but the
184 # first term or factor of u.
185 rest := proc(u) local v;
186     if nops(u)=2 then
187         op(2,u)
188     else
189         apply(op(0,u),op(2..nops(u),u))
190     end if
191 end proc:
192
193 #leafcount(u) returns the number of nodes in u.
194 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```

```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
          sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```



```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```